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1 Radius Calculation

To calculate the radius r out of the radian measure l and the angle φ (the difference between the endpoint angle and the starting angle) the basic equation 1 for calculating the perimeter of a circle was settled as starting point. Equation 4 shows the final result of the formula transformations. Figure 1 shows the parameters of the calculation. For more information see [1].

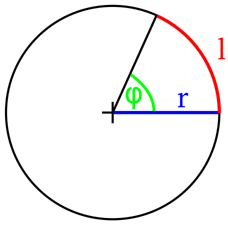

Figure 1: Arc showing the parameters of the calculation.

$$U = 2 \cdot r \cdot \pi \qquad U \ldots \text{perimeter} \tag{1}$$

$$\frac{U}{l} = \frac{2 \cdot r \cdot \pi}{\varphi \cdot r} \qquad \Rightarrow \qquad U = 2 \cdot l \cdot \frac{\pi}{\varphi} \qquad r \ldots \text{radius} \tag{2}$$

$$2 \cdot l \cdot \frac{\pi}{\varphi} = 2 \cdot r \cdot \pi \qquad l \ldots \text{radian measure} \tag{3}$$

$$r = \frac{l}{\varphi} \qquad \varphi \ldots \text{angle} \tag{4}$$

To calculate the angle, it is essential to differentiate between left and right curve:

left: $\varphi = \varphi_{out} - \varphi_{in}$

right: $\varphi = \varphi_{in} - \varphi_{out}$

where φ_{out} is the output angle and φ_{in} the input angle.

2 End Point Calculation

The next formula transformations will show how the endpoint coordinates are calculated in all three cases, namely left curve, right curve, and straight track.

2.1 Straight Track

The situation for a straight track is shown in figure 2. For any straight trails equation 23 and equation 6 can be used. Since it is a straight trail, the input angle and the output angle are the same.

$$x_{end} = x_{begin} + l \cdot \cos(\varphi_{in}) \tag{5}$$

$$y_{end} = y_{begin} + l \cdot \sin(\varphi_{in}) \tag{6}$$

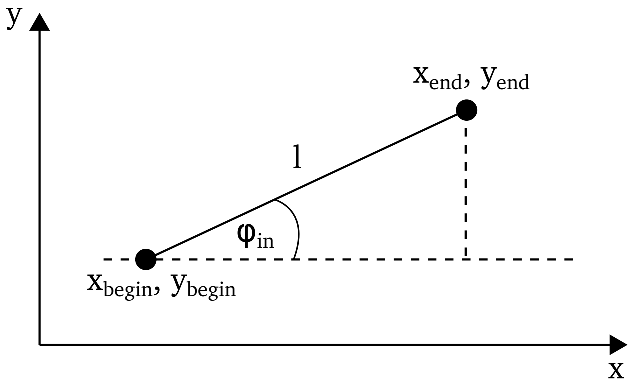

Figure 2: The parameters of a straight track.

2.2 Curves

Since every track piece exactly has one radius, it can be seen as an isosceles triangle. The isosceles sides have the length of radius r, whose calculation is shown in equation 4.To calculate the difference between starting point and endpoint the sine and cosine functions will be used. So in addition to the length of the track between starting point and endpoint there is a need for the angle of that track.

For the slightly more complicated calculation of the end point of a curve, first of all the coordinates of the center of the circle was calculated (as sort of reference point). To get a better idea from where this formulas originate, have a look at figure 3 and figure 4. Sine and cosine roles are switched due to the fact that the input and output angles are relative to the negative y-axis. So there is a rotation of $\frac{\pi}{2}$

$$x_{origin} = x_{begin} - r \cdot \sin(\varphi_{in}) \tag{7}$$
$$y_{origin} = y_{begin} + r \cdot \cos(\varphi_{in}) \tag{8}$$

Starting from the origin, the relative distance between center and the endpoint can be calculated. The resulting equations of the endpoint calculation for the left curve are shown in equation 11 and equation 12.

$$x_{end} = x_{origin} + r \cdot \sin(\varphi_{out}) \tag{9}$$
$$y_{end} = y_{origin} - r \cdot \cos(\varphi_{out}) \tag{10}$$
$$x_{end} = x_{begin} + r \cdot (\sin(\varphi_{out}) - \sin(\varphi_{in})) \tag{11}$$
$$y_{end} = y_{begin} + r \cdot (\cos(\varphi_{in}) - \cos(\varphi_{out})) \tag{12}$$

If we want to calculate the right curve just invert the + sign in front of r. This is shown in equation 13 and equation 14. To get a better idea of why this difference appears have a look at figure 5 and figure 6.

$$x_{end} = x_{begin} - r \cdot (\sin(\varphi_{out}) - \sin(\varphi_{in})) \tag{13}$$
$$y_{end} = y_{begin} - r \cdot (\cos(\varphi_{in}) - \cos(\varphi_{out})) \tag{14}$$

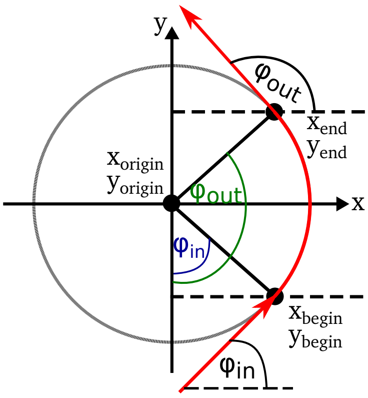

Figure 3: Left curve as a track piece.

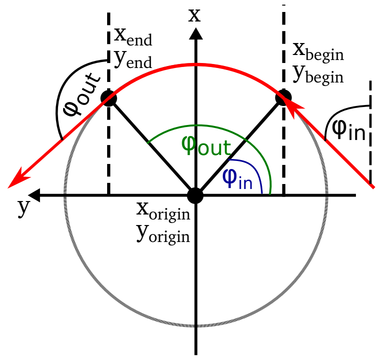

Figure 4: Left curve turned around 90°.

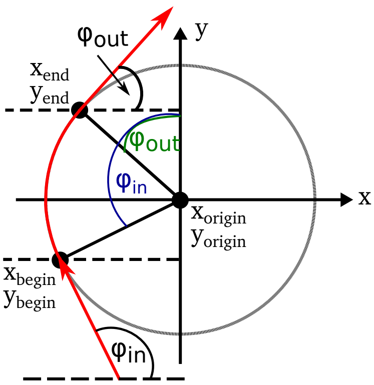

Figure 5: Right curve as a track piece.

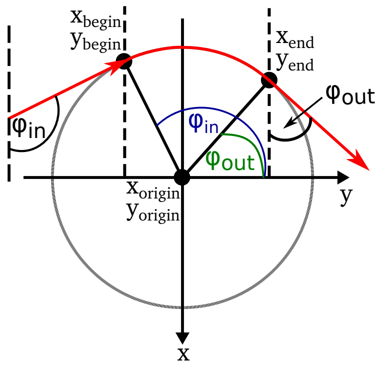

Figure 6: Right curve turned around -90°.

3 Car Flip Simulation

The car flip simulation requires to calculate the centripetal force [2] and the friction force [3]. If the centripetal force gets greater than the friction force, the car will get thrown out of the track. For more information, take a look on the theory about (un)banked turns, e.g. [4] and [5]. Since the AI is going to receive values from the gyro sensor, we need to transfer the calculated force to the corresponding sensor values. This will be one of the inputs for the AI.

$$F_R = m \cdot g \cdot \mu \qquad F_R \ldots \text{friction force} \tag{15}$$

For calculating the friction force, we have some constant values. The mass m of the car, the gravitational acceleration g, and μ which describes the friction coefficient. The friction coefficient μ could take on different values on different places of the track, but this consideration is a possible extension for future use. For easier calculation, it will be determined as constant.

An additional problem appears if the wheels are spinning through. In this case, the friction coefficient will change (it gets more tiny). There are cases in which just one wheel spins through, or two wheels. In these cases the friction force has to be calculated a little bit more complicated. Since only 2 of the 4 wheels of the Carrera car really touch the ground, the total friction force can be calculated as shown in equation 16. The masses m_i can simplified be assumed as $\frac{m}{2}$.

$$F_{Rs} = g \cdot \sum_{i=1}^{2} (m_i \cdot \mu_i) \qquad F_{Rs} \ldots \text{friction force spin through} \tag{16}$$

The coefficient can take on 2 different values:

1. spin through value
2. normal spin value

The maximum velocity v_{max} with which the car can drive an unbanked curve is shown in equation 18. The parameter r describes the radius of the curve which was calculated in equation 4. For more information about this equation, take a look at [6].

$$v \leq \sqrt{\mu_{Haft} \cdot r \cdot g} \tag{17}$$

$$v_{max} = \sqrt{\mu_{Haft} \cdot r \cdot g} \tag{18}$$

$$\omega_{max} = \frac{v_{max}}{r} = \frac{\sqrt{\mu_{Haft} \cdot r \cdot g}}{r} = \sqrt{\frac{\mu_{Haft} \cdot g}{r}} \tag{19}$$

$$\tag{20}$$

The actual velocity v of the car is determined by the rotational speed n of the wheels with diameter d_w, as shown in equation 21.

$$v = \omega \cdot r_w = 2\pi n \cdot r_w = d_w \cdot \pi \cdot n \tag{21}$$

4 Gyro Sensor

The gyro sensor provides 3 values, which represent a rotation in each of the 3 dimensions we know. The mathematical dependencies between ω and the gyro rotation around the z − axis is defined as follows.

$$Z_{gyro} = a \cdot \omega \qquad Z_{gyro} \ldots \text{gyro rotation around z-axis} \tag{22}$$

The parameter a represents a constant factor which is the difference between ω and Z_{gyro}. In fig. 7 you can see which force shows in which direction. In addition there is an axis of abscissas so you know where the z-axis shows to (for Z_{gyro}).

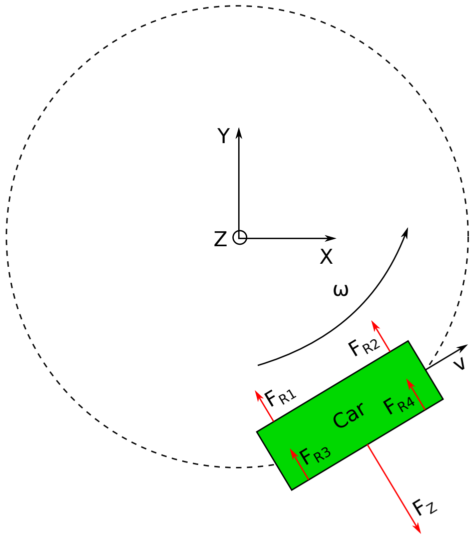

Figure 7: Forces in a curve.

5 Driving Algorithm Concept

5.1 Overview

The approach of the new driving algorithm is as follows. At first, the ghostcar drives a non-competitive initial round where it takes the measurement of the track with an optical sensor and then calculates the maximum velocity for each measured position of the track. This mapping of the track consists of pairs $(pos_n, v_{max}[pos_n]))$ which map a track position pos_n to an associated maximum allowed velocity at this position $v_{max}[pos_n]$ as demonstrated in figure 8.

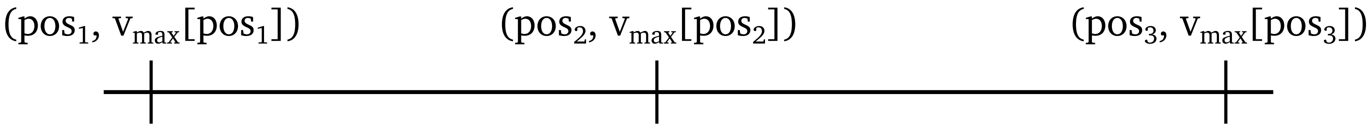

Figure 8: Mapping of the particular maximum allowed velocities to the corresponding recorded positions of the track.

The track mapping is then used to control the actual velocity of the ghostcar. For a rather optimal driving behavior and speed, it is essential to brake early enough and not only when the car starts to slip because then it is already too late. To brake early enough, one has to look ahead in the track mapping and adapt the current velocity appropriate to the following track sections. Now the question is how far has to be look ahead. To stay in either case on the track, it must be possible to brake timely from the current velocity to the smallest maximum velocity. The smallest maximum velocity is the lowest velocity of all maximum velocities and thus the lowest velocity that is needed on a certain point of the track to stay on the track.

5.2 Calculation of the Maximum Velocity

As already mentioned, the track mapping consists of pairs $(pos_n, v_{max}[pos_n]))$ which map a track position pos_n to an associated maximum allowed velocity at this position $v_{max}[pos_n]$. The maximum allowed velocity at a certain position is the maximum velocity at which the car stays barely on the track. To get thrown out of the track, the car has to tilt so far that the bolt of the car is out of the track. Therefore, the maximum allowed velocity is exactly the velocity at which the car just doesn't start to tilt. This whole situation is shown in figure 9.

The maximum allowed velocity $v_{max}[pos_n]$ for a certian position pos_n can be calculated by the following equations:

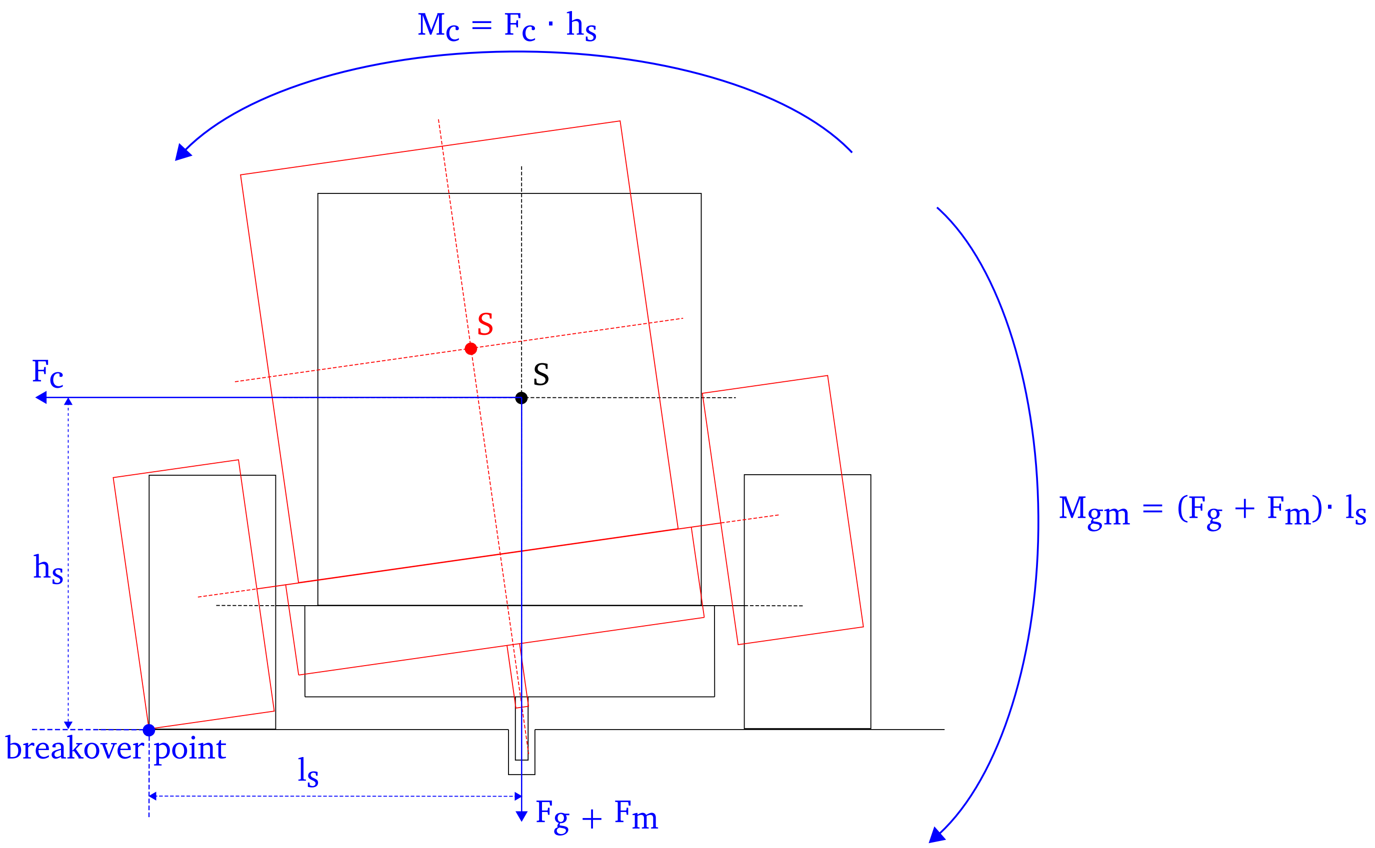

Figure 9: Forces and torques that act on the ghostcar while driving a right hand bend.

$$\sum M = M_{gm} - M_c = 0 \tag{23}$$

$$= (F_g + F_m) \cdot l_s - F_c \cdot h_s \tag{24}$$

$$F_c \cdot h_s = (F_g + F_m) \cdot l_s \tag{25}$$

$$\frac{m}{r} \cdot v_{max}^2 \cdot h_s = (m \cdot g + F_m) \cdot l_s \tag{26}$$

$$v_{max}^2 = \frac{(m \cdot g + F_m) \cdot l_s}{\frac{m}{r} \cdot h_s} \tag{27}$$

$$v_{max} = \sqrt{\frac{(m \cdot g + F_m) \cdot l_s}{m \cdot h_s} \cdot r} \tag{28}$$

$$v_{max} = K_{car} \cdot \sqrt{r}, \qquad K_{car} = \sqrt{\frac{(m \cdot g + F_m) \cdot l_s}{m \cdot h_s}} \tag{29}$$

In order to stay on the track and not to tilt, the torque against ground M_{gm} has to be greater as or at least equal to the overturning torque M_c. The force that leads to the torque against ground is the sum of the weight force $F_g = m \cdot g$ and the magnetic force F_m. On the other side, the force which is responsible for the overturning torque is the centrifugal force $F_c = \frac{m}{r} \cdot v^2$ [7]. After inserting and transposing the equations appropriately, one gets a function for the maximum allowed velocity v_{max}. This function can now be used to compute the maximum allowed velocity for each recorded position of the track. As one can see in the equations above, the maximum allowed velocity depends on a constant factor K_{car} and the square root of the radius r of the particular curve. The constant factor K_{car} can be determined by trial and error. The radius of the particular curves can be calculated by $r = \frac{v}{\omega}$, where v is the actual velocity of the ghostcar and ω is the angular velocity of the ghostcar measured by the gyro sensor of the car.

Now that we have the track mapping with the recorded positions and the maximum allowed velocity to each position, we can calculate the needed look-ahead-distance.

5.3 Calculation of the look-ahead-distance

To calculate the needed look-ahead-distance, one has to calculate the distance that is needed by the car to be able to timely brake from the current velocity to the smallest maximum velocity. So, the ghostcar has to be able to timely brake from the current position to the position with the smallest maximum velocity. This means, the distance between the current position and the position with the smallest maximum velocity has to be big enough to be able to brake in this distance from the current velocity down to the smallest maximum velocity. This situation is visualized in figure 10. Therefore, one has to know the acceleration with which the ghostcar brakes. Since the ghostcar uses an electric motor, the acceleration is actually not constant and has to be evaluated by trial and error.

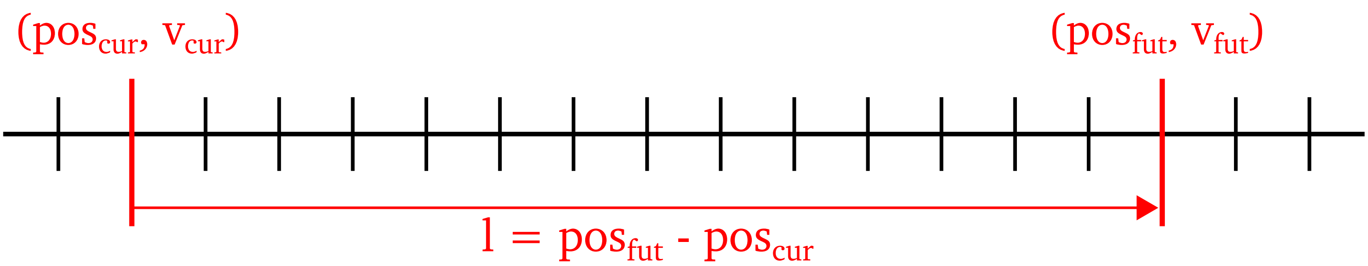

Figure 10: Visualization of the look-ahead-distance l.

Since the measured positions during driving aren't every time exacty the positions which were recorded in the initial round, this actual measured positions have to be mapped to a recorded $(pos_n, v_{max}[pos_n]))$ pair. This is demonstrated in figure 11 and 12.

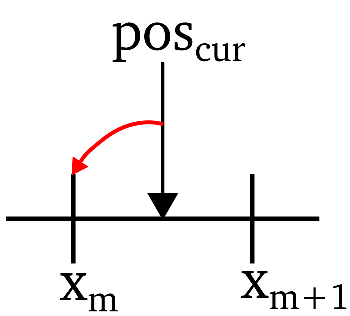

Figure 11: Mapping of the current position to a recorded $(pos_n, v_{max}[pos_n]))$ pair.

Now we have to determine the equations for calculation of the needed look-ahead-distance. Therefore, we have to evaluate the distance that is needed to brake from the current velocity to the target velocity [8]. To do so, at first the time t needed to brake from the initial velocity v_i to the final velocity v_f with a certain acceleration a has to be calculated:

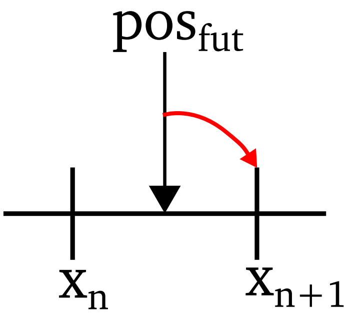

Figure 12: Mapping of the future position to a recorded $(pos_n, v_{max}[pos_n]))$ pair.

$$v_f - v_i = a \cdot t \tag{30}$$
$$t = \frac{v_f - v_i}{a} \tag{31}$$
$$t = \frac{v_{min} - v_{cur}}{a} \tag{32}$$
$$\tag{33}$$

Knowing the time t needed to brake from the current velocity v_{cur} down to the smallest maximum velocity v_{min}, one can compute the distance l which the car drives in this time while braking with a certain constant acceleration a. This distance l is the needed look-ahead-distance.

$$l = v_i \cdot t + \frac{a \cdot t^2}{2} \tag{34}$$
$$l = v_{cur} \cdot t + \frac{a \cdot t^2}{2} \tag{35}$$
$$l = v_{cur} \cdot \frac{v_{min} - v_{cur}}{a} + \frac{a \cdot \left(\frac{v_{min} - v_{cur}}{a}\right)^2}{2} \tag{36}$$
$$l = v_{cur} \cdot \frac{v_{min} - v_{cur}}{a} + \frac{(v_{min} - v_{cur})^2}{2a} \tag{37}$$
$$\tag{38}$$

As one can see, the needed look-ahead-distance l depends on the current velocity v_{cur} (determined by the position changes and the time needed for that changes), the smallest maximum velocity v_{min} (smallest velocity of all calculated maximum velocities) and the braking acceleration a (evaluated by trial and error).

References

Online sources

[1] URL: https://de.wikipedia.org/wiki/Kreisbogen (visited on 03/15/2017) (cit. on p. 1).

[2] URL: https://en.wikipedia.org/wiki/Centripetal_force (visited on 03/15/2017) (cit. on p. 4).

[3] URL: https://en.wikipedia.org/wiki/Friction (visited on 03/15/2017) (cit. on p. 4).

[4] URL: http://www.batesville.k12.in.us/physics/phynet/mechanics/circular%20motion/an_unbanked_turn.htm (visited on 03/15/2017) (cit. on p. 4).

[5] URL: https://en.wikipedia.org/wiki/Banked_turn (visited on 03/15/2017) (cit. on p. 4).

[6] URL: http://physikunterricht-online.de/jahrgang-10/kurvenfahrten-zentripetalkraft/ (visited on 03/22/2017) (cit. on p. 4).

[7] URL: https://en.wikipedia.org/wiki/Centrifugal_force (visited on 03/15/2017) (cit. on p. 7).

[8] URL: http://www.dummies.com/education/science/physics/how-to-calculate-time-and-distance-from-acceleration-and-velocity/ (visited on 06/09/2017) (cit. on p. 8).