Physics Challenge for Teachers and Students

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Solution to September 2017 *Challenge*

A fox trot

A rabbit is chasing a fox (hey, why not?). The fox is trotting in a straight line at a constant speed v while the rabbit is running at a speed $u > v$ in such a way that its velocity is always directed toward the fox. Initially, the fox and the rabbit are separated by a distance L, and their velocities are perpendicular to each other. What is the distance covered by the rabbit by the time it catches up with the fox?

Solution:

The time t required for the rabbit to catch up to the fox is given by:

$$t = \frac{uL}{u^2 - v^2}. \quad \text{(A)}$$

This can be shown in several ways. One can use Eq. (1) of Ref. 1 with $R = L$ and the initial angle θ_0 between the velocity vectors of the fox and rabbit equal to 90°. Alternatively, one can use Eq. (4) of Ref. 2 with $R = L$ based on an elegant non-calculus method of solution. A third way to derive this result is to adapt the *Physics Challenge* solution of Ref. 3 as follows. Set $d = L$ and $\Delta = 0$ so that Eq. (1) of that solution becomes

$$L = \int_0^t (u - v\cos\alpha)dt \quad \Rightarrow \quad uL = \int_0^t (u^2 - uv\cos\alpha)dt, \quad \text{(B)}$$

after rewriting the follower's speed V as u. Likewise Eq. (2) of that solution becomes

$$0 = \int_0^t (v - u\cos\alpha)dt \quad \Rightarrow \quad 0 = \int_0^t (v^2 - uv\cos\alpha)dt. \quad \text{(C)}$$

after again rewriting the follower's speed V as u. Subtracting Eq. (C) from (B) gives

$$uL = \int_0^t (u^2 - v^2)dt = (u^2 - v^2)t, \quad \text{(D)}$$

which rearranges into Eq. (A).

Given Eq. (A), the distance s covered by the rabbit during the chase is

$$s = ut = \frac{u^2 L}{u^2 - v^2}. \quad \text{(E)}$$

As a check on this result, note that it correctly predicts $s = L$ when $v = 0$ (i.e., the fox stands still) and $s \to \infty$ when $v \to u$ (i.e., the fox approaches the rabbit's speed and so can always just stay out of reach).

1. C.E. Mungan, "The pursuit of a plane by a homing missile," *Phys. Teach.* **53**, 68–69 (Feb. 2015).
2. M. Kagan, "Thinking outside of the rectangular box," *Phys. Teach.* **51**, 215–217 (Apr. 2013).
3. N.C. Hernández, "A futile chase," *Phys. Teach.* **50**, 374 (Sep. 2012) with solution at http://aapt.scitation.org/doi/suppl/10.1119/1.4745698/suppl_file/phys_challenge_answers_sep_2012.pdf

(Submitted by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

We also recognize the following successful contributors:

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Guidelines for contributors

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address ***challenges@aapt.org***. Each message will receive an automatic acknowledgment.
- If your name is—for instance—Sean Spicer, please name the file "**Spicer17Sept**" (do not include your first initial) when submitting the September 2017 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at ***korsunbo@post.harvard.edu***; however, please do not send your solutions to this address.

Many thanks to all contributors; we hope to hear from many more of you in the futre.
We also hope to see more submissions of the original problems – thank you in advance!

– Boris Korsunsky, Column Editor