Booklet 1

PHYSICAL QUANTITIES, SCALARS AND VECTORS

Specification:

1.1 Physical quantities

1.1.1 **Describe all physical quantities as consisting of a numerical magnitude and unit;**

1.1.2 **State the base units of mass, length, time, current, temperature, and amount of substance and be able to express other quantities in terms of these units;**

1.1.3 **Recall and use the prefixes T, G, M, k, c, m, μ, n, p and f, and present these in standard form;**

1.2 Scalars and vectors

1.2.1 **Distinguish between and give examples of scalar and vector quantity;**

1.2.2 **Resolve a vector into two perpendicular components;**

1.2.3 **Calculate the resultant of two coplanar vectors by calculation or scale drawing, with calculations limited to two perpendicular vectors;**

1.2.4 **Solve problems that include two or three coplanar forces acting at a point, in the context of equilibrium;**

Methodist College Belfast, 2022

Physical quantity

A Physical quantity is a *physical property* that can be *measured* or *calculated* from other physical property or properties. A particular value of a physical quantity is expressed as the product of a **numerical magnitude** and a **unit** and it is written as the product of a number and a unit abbreviation.

Physical quantity	
Mass	3 kg
Distance	50 km
Speed	20 km

Table 1: Examples of physical quantities and particular values of the physical quantity.

Base of physical quantities

All physical quantities can be expressed **only** in terms of 6[1] fundamental physical quantities. These 6 physical quantities are known as **base physical quantities**. In spite of the fact that we may select between different bases, a *general convention* is to use the following base physical quantities: **mass**, **length**, **time**, **electric current**, **temperature**, and **amount of substance**.

Example 1: Acceleration in terms of base physical quantities.

$$\textbf{acceleration} = \frac{\Delta\text{velocity}}{\Delta\text{time}} = \frac{\Delta\text{displacement}}{\Delta\text{time}^2} = \frac{\textbf{length}}{\textbf{time}^2}$$

Example 2: Energy in terms of base physical quantities.

$$\textbf{energy} = \text{energy} =$$

Base units

Once the base of physical quantities has been chosen, we must **scale** the base by assigning specific units to each quantity. These units are the **base units**. We could arbitrarily assign units to our base quantities but instead, we are going to embrace the general convention of using the **International System of Units** (SI units). You must memorise the following table (SPEC. 1.1.2):

[1] There are in fact 7 physical quantities and therefore 7 base units. Candela is **not** considered by CCEA.

Base physical quantities & SI base units			
Physical quantity	**Symbol**	**SI units**	**SI unit name**
Mass	m	kg	Kilogram
Length	ℓ	m	Metre
Time	t	s	Second
Electric current	I	A	Ampere
Temperature	T	K	Kelvin
Amount of substance	n	mol	Mole

Table 2: Physical quantities, and their SI base units.

In some special occasions during A-level you will be using units out of the SI, for example in quantum physics you will consider energy in electron-volts instead of Joules, distance in Light years instead of metres in astrophysics and a few others.

Units of derived physical quantities

As stated before, we can express the units of **any physical quantity** in terms of the **base units**. To do so it is necessary to have an equation relating the derived physical quantity in terms of the base physical quantities.

Example 1: Derived quantity: **Area**.
Any equation for the calculation of an area can be used to derive the units. The simplest equation with dimensions of area is the area of a square,

$$[\text{Area}] = \underbrace{[\text{side}]}_{\text{m}} \times \underbrace{[\text{side}]}_{\text{m}} = \text{m} \times \text{m} = \text{m}^2$$

- Notice that brackets here should be read as units of the bracketed quantity. This is a standard notation. i.e. [Area] means units of area.

Example 2: Derived quantity: **Velocity**.
The simplest equation with dimensions of area is the area of a square,

$$[\text{velocity}] = \frac{[\text{displacement}]}{[\text{time}]} = \frac{\text{m}}{\text{s}} = \text{m} \cdot \text{s}^{-1}$$

- Notice that instead of using the GCSE notation m/s we now also use m$\cdot$s^{-1} you must know this new form of writing the units. Remember:

$$x = x^1 \qquad 1/x = \frac{1}{x} = x^{-1} \qquad 1/x^2 = \frac{1}{x^2} = x^{-2}$$

Example 3: Derived quantity: **Force**.

$$[\text{Force}] = [\text{mass}]\times[\text{acceleration}] = [\text{mass}]\times\frac{[\text{velocity}]}{[\text{time}]} = [\text{mass}]\times\frac{[\text{displacement}]}{[\text{time}]^2} = \text{kg}\cdot\text{m}\cdot\text{s}^{-2}$$

In this case we have a name for this derived unit, **Newton**.

$$[F] = \text{N} = \text{kg}\cdot\text{m}\cdot\text{s}^{-2}$$

Example 4: Derived quantity: **Energy**.
Any energy equation can be used to derive the units of energy.

$$[\text{Work}] = [\text{force}] \times [\text{displacement}] = \underbrace{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}}_{\text{force}}\cdot\text{m} =$$

We could have used the equation for the kinetic energy and obtain the same

$$[\text{Kinetic energy}] = \left[\frac{1}{2}\right] \times [\text{mass}] \times [\text{velocity}^2] = \text{kg} \times (\underbrace{\text{m}\cdot\text{s}^{-1}}_{\text{velocity}})^2 = \text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$$

- Notice that we ignore the term [1/2] because it is dimensionless i.e. does not contribute to the units.
- Notice that we obtain the energy same units following both methods.
- In this case we also have a name for this derived unit, **Joule**.

$$[E] = \text{J} = \text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$$

Units of derived physical quantities questions/homework

(1)

(2) Express the following derived units in terms of the SI base units. The first one has been done for you: [1] p. 3.

Derived Unit	in Base Units	Power of each base unit			
		m	s	kg	A
m s^{-2}	m s^{-2}	1	-2	0	0
J	kg m^2 s^{-2}	2	-2	1	0
N	kg m s^{-2}	1	-2	1	0
C	A s	0	1	0	1
Ω	m^2 kg s^{-3} A^{-2}	2	-3	1	-2
Pa	kg m^{-1} s^{-2}	-1	-2	1	0
N C^{-1}	kg m s^{-3} A^{-1}	1	-3	1	-1
V m^{-1}	kg m s^{-3} A^{-1}	1	-3	1	-1

(3) *Find the units:* Determine the **SI unit** of each of the following quantities. Let $[a]$= m s^{-2}, $[t]$=s, $[v]$= m s^{-1}, and $[x]$= m. [2]

(a) $\dfrac{v^2}{ax}$ **(b)** $\dfrac{at^2}{2}$ **(c)** $2\pi\sqrt{\dfrac{x}{a}}$

Solution:

(a)

$$\boxed{\frac{[v]^2}{[a][x]} = \frac{\mathrm{m}^2 \cdot \mathrm{s}^{-2}}{\mathrm{m} \cdot \mathrm{s}^{-2} \cdot \mathrm{m}} = 1 \qquad \text{(dimensionless)}}$$

(b) We can neglect the 2 dividing as it is dimensionless.

$$\boxed{[a] \cdot [t]^2 = \mathrm{m} \cdot \mathrm{s}^{-2} \cdot \mathrm{s}^2 = \mathrm{m}}$$

(c) We can neglect the factor 2π as it is dimensionless.

$$\boxed{\sqrt{\frac{[x]}{[a]}} = \sqrt{\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{m}} \cdot \mathrm{s}^{-2}}} = \mathrm{s}}$$

Definition of the SI base units

In 2019 the most fundamental of all the units, time, was redefined.

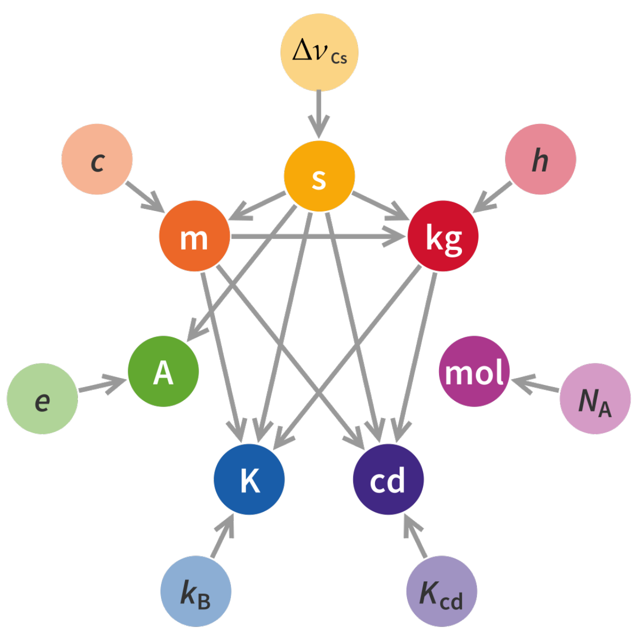

Figure 1: 2019 redefinition of the SI base units.

Importance of units

A wrong use of units can lead to disaster.

"Schoolkid blunder brought down Mars probe"

NASA lost his $125 million Mars Climate Orbiter spacecraft as a result of a mistake that would shame a first-year physics student, failing to convert Imperial units to metric.

"Gimli slider"

good practice can help you to,
detect if you are using a correct equation.
detect if all the units used in the equation are compatible.
This are very common mistakes that an A-Level physics student does.

Metric prefixes

These prefixes are used **in front of the units** to change the magnitude. It is a quick way to re-scale the unit. You must memorise this metric prefixes (SPEC. 1.1.3):

YouTube video on Power of Ten (https://www.youtube.com/watch?v=bhofN1xX6u0)

Factor	Prefix	Symbol
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T

Factor	Prefix	Symbol
10^{-3}	mili	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

- **Example 1: Newton to micro Newton.**
  0.00037 N $\longrightarrow$ x $\boldsymbol{\mu}$N

$$0.00037\,\cancel{N} \times \underbrace{\frac{1\ \mu N}{10^{-6}\,\cancel{N}}}_{N \to\ \mu N} = 370\ \mu N$$

- **Example 2: giga to mega square metre.**
  0.05 $\mathbf{G}\mathrm{m}^2 \longrightarrow$ x $\mathbf{M}\mathrm{m}^2$

$$0.05\,\cancel{Gm^2} \times \underbrace{\frac{10^9\,\cancel{m^2}}{1\,\cancel{Gm^2}}}_{Gm^2 \to m^2} \times \underbrace{\frac{1\ Mm^2}{10^6\,\cancel{m^2}}}_{m^2 \to Mm^2} = 50\ Mm^2$$

- **Example 3: micro to giga Pascal.**
  5×10^{10} $\boldsymbol{\mu}$Pa $\longrightarrow$ x **G**Pa

$$5 \times 10^{10}\ \mu\text{Pa} \times \underbrace{\frac{10^{-6}\ \text{Pa}}{1\ \cancel{\mu\text{Pa}}}}_{\mu\text{Pa}\to\ \text{Pa}} \times \underbrace{\frac{1\ \text{GPa}}{10^{9}\ \cancel{\text{Pa}}}}_{\text{Pa}\to\ \text{GPa}} = 5 \times 10^{10} \times 10^{-15}\ \text{GPa} = 5.0 \times 10^{-5}\ \text{GPa}$$

Metric prefixes questions/homework:

(1) *Prefix:* Complete the following puns based on the **SI** metric prefixes. [2]

- **(a)** 1 **millionth** of a fish equals. **Solution:** A micro fish.
- **(b)** 1 **trillion** pins equal. **Solution:** A tera pin.
- **(c)** 1 **thousan** legs equal. **Solution:** A kilo leg.
- **(d)** 1 **billionth** of a chocolate bar equal. **Solution:** A nano chocolate bar.
- **(e)** 1 **quadrillionth** of a boy equals. **Solution:** A femto boy.
- **(f)** 1 **trillionth** of a door equals. **Solution:** A pico door.
- **(g)** 1 **billion** microphones equal. **Solution:** A giga microphone.
- **(h)** 1 **million** pains equal. **Solution:** A mega pain.
- **(i)** 2000 mockingbirds equal. **Solution:** 2 kilo mockingbirds.
- **(j)** 1021 piccolos equal. **Solution:** 1021 kilo picolos.

(1) Convert the following quantities to the given units or prefixes and unit.

Before	**After**
3 km (into m)	$3\ \text{km} \times \frac{1000\ \text{m}}{1\ \text{km}} = 3000\ \text{m}$
76 g (into mg)	$76\ \text{g} \times \frac{1000\ \text{mg}}{1\ \text{g}} = 76000\ \text{mg}$
4 m (into mm)	$4\ \text{m} \times \frac{1000\ \text{mm}}{1\ \text{m}} = 4000\ \text{mm}$
5.5 kg (into μg)	$5.5\ \text{kg} \times \frac{1000\ \text{g}}{1\ \text{kg}} \times \frac{1\ \mu\text{g}}{10^{-6}\ \text{g}} = 5500000000\ \mu\text{g}$
0.46 m (into nm)	$0.46\ \text{m} \times \frac{10^{-9}\ \text{nm}}{1\ \text{m}} = 0.00000000046\ \text{nm}$
6.008 g (into kg)	$6.008\ \text{g} \times \frac{1\ \text{kg}}{1000\ \text{g}} = 0.006008\ \text{kg}$
2 km^2 (into m^2)	$2\ \text{km}^2 \times \frac{1000\ \text{m}^2}{1\ \text{km}^2} = 2000\ \text{m}^2$
100 ml (into l)	$100\ \text{ml} \times \frac{1\ \text{l}}{1000\ \text{ml}} = 0.1\ \text{l}$
101325 Pa (into MPa)	
0.050508 J (into mJ)	

(2) *Tom Duff*, at Bell Labs is reported to have said that "π seconds is a nano-century". Show that he was more or less correct. [2]

Solution:

Let us convert the nano-century into seconds.

$$1 \text{ nano-century} = 1 \times 10^{-9} \times 100\ \text{yr} = 10^{-9} \times 10^{2}\ \text{yr} = 10^{-7}\ \text{yr}$$

converting years to seconds,

$$10^{-7}\ \text{yr} \cdot \frac{365\ \text{day}}{1\ \text{yr}} \cdot \frac{24\ \text{h}}{1\ \text{day}} \cdot \frac{60\ \text{min}}{1\ \text{h}} \cdot \frac{60\ \text{s}}{1\ \text{min}} = 3.15\ \text{s} \sim \pi\ \text{s}$$

(3) *J. V. Neumann sentence:* The mathematician John von Neumann is reputed to have said that a lecture should never last longer than a "microcentury". How long is this in more familiar time units? [2]

Solution:

Let us convert the micro-century into minutes.

$$1 \text{ micro-century} = 1 \times 10^{-6} \times 100\ \text{yr} = 10^{-6} \times 10^{2}\ \text{yr} = 10^{-4}\ \text{yr}$$

converting years to minutes,

$$10^{-4}\ \mathrm{yr}\cdot\frac{365\ \mathrm{day}}{1\ \mathrm{yr}}\cdot\frac{24\ \mathrm{h}}{1\ \mathrm{day}}\cdot\frac{60\ \mathrm{min}}{1\ \mathrm{h}}\sim 53\ \mathrm{min}$$

(4) *Nanoacres:* The nanoacre is a unit invented by computer engineers as a joke. It is said that one nanoacre of an integrated circuit (a computer chip) costs about the same to develop as an acre of real estate. Determine the length of the side of a square with an area of one nanoacre in. [2]

(a) inches

(b) millimeters

Change of units

In some occasions the units in a problem are not given in the SI system or you are requested to give an answer in units out of the SI system. In this cases you will need to change units. To change units you need to know how many times you can fit one unit in the other. This is known as **conversion factor**. Some examples of conversion factors are,

From	To	Conversion factor
meters	millimetres	$\frac{1000\,\mathrm{mm}}{1\,\mathrm{m}}$ (In one meter you can fit 1000 millimetres)
miles	kilometres	$\frac{1.61\,\mathrm{km}}{1\,\mathrm{mi}}$ (In one mile you can fit 1.61 kilometres)
minutes	seconds	$\frac{60\,\mathrm{s}}{1\,\mathrm{min}}$ (In one minute you can fit 60 seconds)

Table 3: Examples of conversion factors between different units.

The use of conversion factors is the most **systematic** and **safest** method to change units. Find some examples of this method below.

Example 1: Time: Base unit. 3 hours $\longrightarrow$ x seconds

$$3\ \mathrm{h} = 3\cancel{\mathrm{h}}\times\underbrace{\frac{60\ \cancel{\mathrm{min}}}{1\ \cancel{\mathrm{h}}}}_{\mathrm{h}\to\ \mathrm{min}}\times\underbrace{\frac{60\ \mathrm{s}}{1\ \cancel{\mathrm{min}}}}_{\mathrm{min}\to\ \mathrm{sec}} = 10800\ \mathrm{s}$$

Example 2: Time: Base unit, reverse. 10800 s $\longrightarrow$ x h

$$10800\ \mathrm{s} = 10800\cancel{\mathrm{s}}\times\underbrace{\frac{1\ \cancel{\mathrm{min}}}{60\ \cancel{\mathrm{s}}}}_{\mathrm{s}\to\ \mathrm{min}}\times\underbrace{\frac{1\ \mathrm{h}}{60\ \cancel{\mathrm{min}}}}_{\mathrm{min}\to\ \mathrm{h}} = 3\ \mathrm{h}$$

Example 3: Speed: 450 kn (knots=nautical mile/hour) $\longrightarrow$ x m/s

$$450\ \text{kn} = 450\,\frac{\cancel{\text{NM}}}{\cancel{\text{h}}} \times \overbrace{\underbrace{\frac{1.852\ \cancel{\text{km}}}{1\ \cancel{\text{NM}}}}_{\text{NM}\to\text{km}} \times \underbrace{\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}}_{\text{km}\to\text{m}}}^{\text{length conversion}} \times \overbrace{\underbrace{\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}}_{\text{h}^{-1}\to\text{min}^{-1}} \times \underbrace{\frac{1\ \cancel{\text{min}}}{60\ \text{s}}}_{\text{min}\to\text{sec}}}^{\text{time conversion}} = 232\ \text{m}\cdot\text{s}^{-1}$$

Example 4: Density. 1 g/cm^3 $\longrightarrow$ kg/m^3

$$1\,\frac{\text{g}}{\text{cm}^3} = 1\,\frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \underbrace{\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}}_{\text{g}\to\text{kg}} \times \underbrace{\frac{(100\ \cancel{\text{cm}})^3}{(1\ \text{m})^3}}_{\text{cm}^{-3}\to\text{m}^{-3}} = 1000 \times \frac{\text{kg}}{\text{m}^3} = 1\,\frac{\text{Mg}}{\text{m}^3}$$

Dimensional analysis

Dimensional analysis is a powerful technique that can be applied to different situations We are going to cover the most likely situations where you need to apply dimensional analysis. 3 typical applications of dimensional analysis in CCEA past paper problems:

1. Homogeneity of physics equations (validating equations)

Units must be **consistent** on both sides of the equal sign.

Example: Is this equation dimensionally correct?

$$\frac{Ev}{a^3t^3} = 5m \qquad \text{where:} \qquad \begin{array}{ll} E = & \text{Energy} \\ v = & \text{velocity} \\ a = & \text{acceleration} \\ t = & \text{time} \\ m = & \text{mass} \end{array}$$

Substituting the physical quantities by its **SI base units** in the previous equation (on page 3 we wrote the energy (E) in terms of its SI base units):

$$\frac{\overbrace{(\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2})}^{E}\cdot\overbrace{(\text{m}\cdot\text{s}^{-1})}^{v}}{\underbrace{(\text{m}\cdot\text{s}^{-2})^3}_{a^3}\ \underbrace{\text{s}^3}_{t^3}} = 5\ \overbrace{\text{kg}}^{m}$$

The numbers dividing or multiplying the **units equation** do not affect the units and can be neglected.

$$\frac{(\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2})\cdot(\text{m}\cdot\text{s}^{-1})}{(\text{m}\cdot\text{s}^{-2})^3\ \text{s}^3} = \cancel{5}\ \text{kg}$$

Simplifying the expression,

$$\frac{\mathrm{kg} \cdot \cancel{\mathrm{m}^3} \cdot \cancel{\mathrm{s}^{-3}}}{\cancel{\mathrm{m}^3} \cdot \cancel{\mathrm{s}^{-3}}} = \mathrm{kg} \Longrightarrow \boxed{\text{The equation is homogeneous}}$$

(but not necessarily meaningful)

2. Apply homogeneity to derive units of constants:

Example: What are the units of the unknown constant G?

$$F = G\,\frac{m_1 \cdot m_2}{d^2} \qquad \text{where:} \qquad \begin{aligned} F &= \text{force} \\ m_1 &= \text{mass} \\ m_2 &= \text{mass} \\ d &= \text{distance} \end{aligned}$$

Solution:
Replace the known physical quantities by its **SI base units** in the previous equation (on page 3 we wrote the force (F) in terms of its SI base units,

$$\overbrace{(\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2})}^{F} = [G]\,\frac{\overbrace{\mathrm{kg}}^{m_1} \cdot \overbrace{\mathrm{kg}}^{m_2}}{\underbrace{\mathrm{m}^2}_{d^2}}$$

Solve the equation for the units of G represented as $[G]$ (do it!),

$$\cancel{\mathrm{kg}} \cdot \mathrm{m} \cdot \mathrm{s}^{-2} = [G]\,\frac{\cancel{\mathrm{kg}} \cdot \mathrm{kg}}{\mathrm{m}^2}$$

$$\mathrm{m}^3 \cdot \mathrm{s}^{-2} = [G]\,\mathrm{kg}$$

$$\boxed{[\boldsymbol{G}] = \mathrm{kg}^{-1} \cdot \mathrm{m}^3 \cdot \mathrm{s}^{-2}}$$

This is the equation used to calculate gravitational force and G is known as universal gravitational constant. This will be studied in A2.

3. The argument of the following functions (log, exp, sin, cos, tan, . . .) must be always adimensional.

4. Guessing equations from parameters of a system.

Example: Derive the equation for the period T of a pendulum just considering its main parameters: Length ℓ, mass m and gravity g. You can consider the following functional dependence.

$$T = \ell^x\, g^y\, m^z$$

Solution:

In this case let us replace the quantities by the base dimensions,

$$\underbrace{T^1 \cdot L^0 \cdot M^0}_{\text{period}} = (\underbrace{L}_{\text{length}})^x (\underbrace{L \cdot T^{-2}}_{\text{gravity}})^y (\underbrace{M}_{\text{mass}})^z$$

this equation must be true for every dimension,

$$\text{for time dimension: } T^1 = T^{-2y} \implies 1 = -2y \implies y = -1/2$$
$$\text{for length dimension: } L^0 = L^x \cdot L^{-\frac{1}{2}} \implies 0 = x - 1/2 \implies x = 1/2$$
$$\text{for mass dimension: } M^0 = M^z \implies z = 0 \ \ \text{(does not depend on mass!)}$$

Therefore, the period of a pendulum has the following dependence on the length and gravity:

$$\boxed{T \sim \sqrt{\frac{\ell}{g}}}$$

(there is a prefactor 2π that can't be obtained with this analysis)

Dimensional analysis questions/homework:

(1) A simple pendulum consists of a mass on the end of a length of string. If the length of the string is $\boldsymbol{\ell}$ and $\boldsymbol{g}$ is the acceleration of free fall, then the time to complete one oscillation, called the period, is $\boldsymbol{T}$, where: [3] p. 7 pb. 3.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Show that the **base units** of both sides of the equation are identical.

Solution:

Replacing the quantities symbols in the equations by its units,

$$[T] = \left[\sqrt{\frac{\ell}{g}}\right]$$

$$\mathrm{s} = \sqrt{\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{m}} \cdot \mathrm{s}^{-2}}} = \mathrm{s}$$

(2) A mass attached to a spring will oscillate up and down when disturbed. The period $\boldsymbol{T}$ of such oscillations is given by: [3] p. 7 pb. 4.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The mass is m and k is the spring constant, ie the force needed to stretch the spring by 1 m. The units of k are $\mathrm{Nm^{-1}}$. Show that the equation is **homogeneous** in terms of the **base units** on each side.

Solution:

If we replace the quantity symbols by its units in the equation,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

we obtain the following 'unit' equation. The dimensionless numbers can be neglected as they do not affect the units.

$$\mathrm{s} = \sqrt{\frac{\mathrm{kg}}{\mathrm{N}\cdot\mathrm{m}^{-1}}}$$

squaring the equation both sides, and replacing the derived unit Newton by its base units ($\mathrm{N}=\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2}$),

$$\mathrm{s}^2 = \frac{\mathrm{kg}}{\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2}\cdot\mathrm{m}^{-1}}$$

Simplifying the expression,

$$\mathrm{s}^2 = \mathrm{s}^2$$

therefore, the equation is homogeneous.

(3) *Dimensional consistency:* Determine which of the following expressions are dimensionally consistent. Let [a]= m/s^2, [t]=s, [v]= m/s, and [x]= m. [2]

(a) $v^2 = 2ax$ **(b)** $x = v\,t + \frac{1}{2}a\,t^3$ **(c)** $\tan\theta = \frac{v}{x}$

(d) $v = x\,t$ **(e)** $x = a\,t^{\frac{2}{3}}$ **(f)** $t = \sqrt{\frac{v^2}{ax}}$

(4) *Determine* which of the following expressions are dimensionally consistent. Let [a]= m/s^2, [t]=s, [v]= m/s, and [x]= m. [2]

(a) $v = xt$

(b) $x = vt$

(c) $t = vx$

(d) $x = at + \frac{1}{2}at^2$

(e) $x = vt + \frac{1}{2}at^3$

(f) $v^2 = 2ax^2$

(g) $x = \sqrt{\frac{2a}{t}}$

(h) $t = \sqrt{\frac{2x}{a}}$

(i) $t = \sqrt{\frac{v^2}{ax}}$

(j) $\tan(\theta) = \frac{x}{y}$

(k) $\log(\theta) = \frac{y}{x}$

(l) $\cos(\theta) = \frac{v}{at}$

Solution:

(a)

$$\boxed{\cancel{m} \cdot s^{-1} = \cancel{m} \cdot s \implies s^{-1} \neq s \implies \text{Incompatible}}$$

(b)

$$\boxed{\cancel{m} = \cancel{m} \cdot \cancel{s}^{-1} \cdot \cancel{s} \implies 1 = 1 \implies \text{Compatible}}$$

(c)

$$\boxed{s = m \cdot s^{-1} \cdot m \implies s^2 \neq m^2 \implies \text{Incompatible}}$$

(d) We can neglect the factor 1/2 as it is dimensionless.

$$m = m \cdot s^{\cancel{1/2}-1} \cdot \cancel{s} + \frac{1}{\cancel{2}} m \cdot \cancel{s}^{-2} \cdot \cancel{s}^{2}$$

$$m \neq m \cdot s^{-1} + m \implies \text{First term incompatible, second term compatible}$$

$$\boxed{\text{In general, incompatible}}$$

(e) We can neglect the factor 1/2 as it is dimensionless.

$$m = m \cdot \cancel{s}^{-1} \cdot \cancel{s} + \frac{1}{\cancel{2}} m \cdot \cancel{s}^{\cancel{1/2}} \cdot s^{\cancel{3}\,1}$$

$$m \neq m + m \cdot s^{-1} \implies \text{First term compatible, second term incompatible}$$

$$\boxed{\text{In general, incompatible}}$$

(f) We can neglect the factor 2 as it is dimensionless.

$$\boxed{m^2 \cdot \cancel{s}^{-2} = m \cdot \cancel{s}^{-2} \cdot m^2 \implies m^2 \neq m^3 \implies \text{Incompatible}}$$

(g) We can neglect the factor 2 as it is dimensionless.

$$\boxed{m = \sqrt{\frac{m \cdot s^{-2}}{s}} \implies m^2 \neq m \cdot s^{-3} \implies \text{Incompatible}}$$

(h) We can neglect the factor 2 as it is dimensionless.

$$\boxed{s = \sqrt{\frac{\cancel{m}}{\cancel{m} \cdot s^{-2}}} \implies s = s \implies \text{Compatible}}$$

(i)

$$\boxed{s = \sqrt{\frac{m^2 \cdot s^{-2}}{m \cdot s^{-2} \cdot m}} \implies s \neq 1 \implies \text{Incompatible}}$$

(j) tan is dimensionless (the argument θ is also dimensionless).

$$\boxed{[\tan(\theta)] = \frac{\mathrm{m}}{\mathrm{m}} \Longrightarrow 1 = 1 \Longrightarrow \text{Compatible}}$$

(k) log is dimensionless (the argument θ is also dimensionless).

$$\boxed{[\log(\theta)] = \frac{\mathrm{m}}{\mathrm{m}} \Longrightarrow 1 = 1 \Longrightarrow \text{Compatible}}$$

(l) cos is dimensionless (the argument θ is also dimensionless).

$$\boxed{[\cos(\theta)] = \frac{\mathrm{m}\cdot\mathrm{s}^{-1}}{\mathrm{m}\cdot\mathrm{s}^{-2}\cdot\mathrm{s}} \Longrightarrow 1 = 1 \Longrightarrow \text{Compatible}}$$

(5) *Constants dimensions:* Determine the SI units of the constants C_1 and C_2 so that the following expressions are dimensionally correct. [2]

(a) $x = C_1 + C_2\,t$

(b) $x = C_1\,t + \frac{1}{2}C_2\,t^2$

(c) $v = C_1\,t + C_2\,t^2$

(d) $x = C_1\,\sin(2\pi C_2\,t)$

(d) $v = \frac{1}{2}C_1\,e^{-C_2\,t}$

(6) *Coupled derivation:* Derive the unit for the quantity P given the following equations. [2]

$$v = \frac{x}{t}$$
$$a = \frac{v}{t}$$
$$F = ma$$
$$W = Fx$$
$$P = \frac{W}{t}$$

(7) In the formula below the symbol U is measured in kg $\mathrm{m}^2/\mathrm{s}^2$ (i.e. energy units, Joules) and x stands for length. What are the units of k?

$$U = \frac{1}{2}k\,x^2$$

Solution:

We can neglect the factor 1/2 in the equation as it is dimensionless.

$$[U] = [k]\,[x]^2$$

$$\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2} = [k]\,\mathrm{m}^2$$

Isolating k,

$$\boxed{[k] = \mathrm{kg} \cdot \mathrm{s}^{-2} \qquad \text{Spring constant N/m}}$$

(8) *Superman's land:* On the planet Krypton the same laws of Physics apply as on the Earth. However, the inhabitants of Krypton have decided to use **force** (F), **acceleration** (a) and **time** (t) as their **base units** (instead of Mass (m), Length (l), and time (t)). What are the base units of **energy** on the planet Krypton? [3] p. 7 pb. 2.

Solution:

From Mass Length Time to Force Acceleration Time.
In the old base: $[E]=\mathrm{ML^2T^{-2}}$
In the new base: $[E]=\mathrm{F}^x\mathrm{A}^y\mathrm{T}^z$
Expressing new base in terms of the old:
$\mathrm{F=MLT^{-2}}$
$\mathrm{A=LT^{-2}}$
$\mathrm{T=T}$
Solving the equation:

$$\mathrm{ML^2T^{-2}} = \mathrm{F}...$$

$\mathrm{ML^2T^{-2}}=\mathrm{F}^x\mathrm{A}^y\mathrm{T}^z=(\mathrm{MLT^{-2}})^x\ (\mathrm{LT^{-2}})^y\mathrm{T}^z$ We can solve the problem:

$$\begin{aligned} \text{For M: } 1 &= x \\ \text{For L: } 2 &= x + y \\ \text{For T: } -2 &= -2x - 2y + z \end{aligned}$$

$$x = 1$$

$$y = 1$$

$$z = 2$$

The dimensions of energy in the new base are:

$$[\mathrm{E}]=\mathrm{FAT^2}$$

There is a unit name for force (Newton) and time (second) there is no name of unit acceleration so we will define the unit of aceleration as (positive feeling) and represent by letter a.

$$\boxed{[\mathrm{E}]=\mathrm{N\,a\,s^2}}$$

Scalars and vectors

Graphical addition/subtraction of vectors

References

[1] A. Machacek, J. Crowter, and L. Jardine−Wright, Mastering Essential pre-university PHYSICS. 2018.

[2] G. Elert, "The physics hyper textbook."

[3] P. Carson and R. White, Physics for CCEA AS Level. 2016.