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CTRL+ALT+DEFEAT

Team Reference Document

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<https://github.com/ctrl-alt-Defeat-icpc>

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1. STL

1.1. bitscroll

```
__builtin_ctz(x); // first 1 from left (index)
__builtin_popcount(x); // count of 1 in numbers bit
__builtin_ctzll(x); // for long long
__builtin_popcountll(x); // ...
```

1.2. 128 bit

```
__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}

void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}
```

```
bool cmp(__int128 x, __int128 y) { return x > y; }

int main() {
    __int128 x = read();
    print(x);
    cout << endl;
    return 0;
}
```

2. Segment Tree

2.1. easy implementation

```
const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];

void build() { // build the tree
    for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] +
t[i<<1|1];
}

void modify(int p, int value) { // set value at
position p
    for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] =
t[p] + t[p^1];
}

int query(int l, int r) { // sum on interval [l, r]
    int res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) res += t[l++];
        if (r&1) res += t[--r];
    }
    return res;
}

int main() {
```

```
scanf("%d", &n);
for (int i = 0; i < n; ++i) scanf("%d", t + n +
i);
build();
modify(0, 2);
printf("%d\n", query(3, 11));
return 0;
}
```

2.2. with lazy propagation

```
const int N = 1e5 + 5;
int n;
int seg[2 * N], lazy[2 * N], a[N];
int segSize;

void build(int u = 1, int ul = 0, int ur = n) {
    if (ur - ul < 2) {
        seg[u] = a[ul];
        return;
    }
    int mid = (ul + ur) / 2;
    build(u * 2, ul, mid);
    build(u * 2 + 1, mid, ur);
    seg[u] = seg[u * 2] + seg[u * 2 + 1];
}

void upd(int u, int ul, int ur, int x) {
    lazy[u] += x;
    seg[u] += (ur - ul) * x;
}

void shift(int u, int ul, int ur) {
    int mid = (ul + ur) / 2;
    upd(u * 2, ul, mid, lazy[u]);
    upd(u * 2 + 1, mid, ur, lazy[u]);
    lazy[u] = 0;
}
```

```
void increase(int l, int r, int x, int u = 1, int ul
= 0, int ur = n) {
    if (l >= ur || ul >= r) return;
    if (l <= ul && ur <= r) {
        upd(u, ul, ur, x);
        return;
    }
    shift(u, ul, ur);
    int mid = (ul + ur) / 2;
    increase(l, r, x, u * 2, ul, mid);
    increase(l, r, x, u * 2 + 1, mid, ur);
    seg[u] = seg[u * 2] + seg[u * 2 + 1];
}

int sum(int l, int r, int u = 1, int ul = 0, int ur
= n) {
    if (l >= ur || ul >= r) return 0;
    if (l <= ul && ur <= r) return seg[u];
    shift(u, ul, ur);
    int mid = (ul + ur) / 2;
    return sum(l, r, u * 2, ul, mid) + sum(l, r, u *
2 + 1, mid, ur);
}

void showSegments() {
    for (int i = 0; i < segSize; i++)
        cout << seg[i] << ' ';
    cout << endl;
}

void Main() {
    cin >> n;
    segSize = 2;
    while (segSize / 2 <= n) segSize *= 2;
    for (int i = 0; i < n; i++)
        cin >> a[i];
    build();
}
```

```
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0); cout.tie(0);
    Main();
    return 0;
}
```

3. Math

3.1. choose

```
#define ll long long
const int N = 2e3 + 5;
const ll M = 1e9 + 7;
ll fact[N], inv[N];
int r, n, q;

ll exp(ll b, ll p, ll m) {
    b %= m;
    ll result = 1;
    while(p) {
        if(p % 2)
            result = result * b % m;
        b = b * b % m;
        p /= 2;
    }
    return result;
}
```

```
void preProcess() {
    fact[0] = 1;
    for(int i = 1; i < N; i++)
        fact[i] = fact[i - 1] * i % M;
    inv[N - 1] = exp(fact[N - 1], M - 2, M);
    for(int i = N - 1; i > 0; i--)
        inv[i - 1] = inv[i] * i % M;
}
```

```
ll choose(int n, int r) {
    if(r > n) return 0;
    return fact[n] * inv[r] % M * inv[n - r] % M;
}

void Main() {
    cin >> q;
    while(q--) {
        cin >> n >> r;
        cout << choose(n, r) << '\n';
    }
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0); cout.tie(0);
    preProcess();
    Main();
    return 0;
}
```

3.2. gcd

```
int gcd(int a, int b) {
    return b ? gcd(b, a % b) : a;
}

// fast version...
int gcd(int a, int b) {
    if (!a || !b)
        return a | b;
    unsigned shift = __builtin_ctz(a | b);
    a >>= __builtin_ctz(a);
    do {
        b >>= __builtin_ctz(b);
        if (a > b)
            swap(a, b);
    }
```

```
        b -= a;
    } while (b);
    return a << shift;
}
```

3.3. compressing

```
// compressing
sort(temp_values, temp_values + n);
int numOfUnique = unique(temp_values, temp_values + n) - temp_values;
for(int i = 0; i < n; i++)
    h[i] = lower_bound(temp_values, temp_values + numOfUnique, h[i]) - temp_values;
```

3.4. lower bound and upper bound

```
int main() {
    vector<int> v = {11, 34, 56, 67, 89};

    // Finding lower bound of 56
    cout << *lower_bound(v.begin(), v.end(), 56)
        << endl;

    // Finding upper bound of 56
    cout << *upper_bound(v.begin(), v.end(), 56);
    return 0;
}
Output:
56
67
```

4. Graph

4.1. BFS

```
#define distance d
const int maxN = 1e5 + 10, oo = 1e9;
vector<int> adj[maxN];
int distance[maxN];
queue<int> q;
```

```

void BFS(int n, int r) {
    for (int i=1; i<=n; i++) distance[i] = oo;
    distance[r] = 0;
    q.push(r);
    while(q.size()) {
        int v = q.front();
        q.pop();

        for (auto u : adj[v])
            if(distance[u] > distance[v] + 1) {
                distance[u] = distance[v] + 1;
                q.push(u);
            }
    }
}

```

```

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0);
    int n, m; cin >> n >> m;
    for (int i=0; i<m; i++) {
        int u, v; cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    BFS(n, 1);
    for (int i=1; i<=n; i++)
        cout << i << ':' << distance[i] << '\n';
}

```

4.2. bipartite

```

const int maxN = 1e5 + 10;
vector<int> adj[maxN];
bool mark[maxN];
int color[maxN];
bool bipartite = true;

```

```

void DFS(int v, int parent) {
    mark[v] = true;

    if(parent != -1) color[v] = 1 - color[parent];
    else color[v] = 1;

    for (auto u : adj[v]) {
        if(!mark[u])
            DFS(u, v);
        else if(color[u] == color[v])
            bipartite = false;
    }
}

```

```

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0);
    int n, m; cin >> n >> m;
    for (int i=0; i<m; i++) {
        int u, v; cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    for (int i=1; i<=n; i++) {
        if(mark[i]) continue ;

        DFS(i, -1); //root does not have parent.

    }
    if(bipartite) cout << "Graph Is Bipartite\n";
    else cout << "Graph Is Not Bipartite\n";
}

```

4.3. cycle finding

```

const int maxN = 1e5 + 10;
vector<int> adj[maxN];
bool mark[maxN];

```

```

bool cycle_found = false;

void DFS(int v, int parent) {
    mark[v] = true;

    for (auto u : adj[v]) {
        if(!mark[u]) DFS(u, v); //u's parent is v.
        else if(u != parent) cycle_found = true;
    }
}

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0);
    int n, m; cin >> n >> m;
    for (int i=0; i<m; i++) {
        int u, v; cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    for (int i=1; i<=n; i++) {
        if(mark[i]) continue ;
        DFS(i, -1); //root does not have parent.
    }
    if(cycle_found) cout << "Graph has Cycle\n";
    else cout << "Graph does not have Cycle\n";
}

```

4.4. DFS

```

const int maxN = 1e5 + 10;
vector<int> adj[maxN];
bool mark[maxN];
vector<int> component;

```

```

void DFS(int v) {
    mark[v] = true;
    component.push_back(v);
}

```

```

    for (auto u : adj[v])
        if(!mark[u]) DFS(u);
}

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0);
    int n, m; cin >> n >> m;
    for (int i=0; i<m; i++) {
        int u, v; cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    for (int i=1; i<=n; i++) {
        if(mark[i]) continue;
        component.clear();
        DFS(i);
        for (auto v : component)
            cout << v << ' ';
        cout << '\n';
    }
}

```

4.5. floyd-warshall

// Implementing floyd warshall algorithm

```

void floydWarshall(int graph[][nV]) {
    int matrix[nV][nV], i, j, k;
    for (i = 0; i < nV; i++)
        for (j = 0; j < nV; j++)
            matrix[i][j] = graph[i][j];
    // Adding vertices individually
    for (k = 0; k < nV; k++) {
        for (i = 0; i < nV; i++) {
            for (j = 0; j < nV; j++) {
                if (matrix[i][k] + matrix[k][j] <
matrix[i][j])
                    matrix[i][j] = matrix[i][k] +
matrix[k][j];
            }
        }
    }
}

```

```

    }
}

// printMatrix(matrix);
}

4.6. prim
// Function to find sum of weights of edges of the
Minimum Spanning Tree.
int spanningTree(int V, int E, vector<vector<int>>
&edges) {
    // Create an adjacency list representation of
the graph
    vector<vector<int>> adj[V];
    // Fill the adjacency list with edges and their
weights
    for (int i = 0; i < E; i++) {
        int u = edges[i][0];
        int v = edges[i][1];
        int wt = edges[i][2];
        adj[u].push_back({v, wt});
        adj[v].push_back({u, wt});
    }
    // Create a priority queue to store edges with
their weights
    priority_queue<pair<int,int>,
vector<pair<int,int>>, greater<pair<int,int>>> pq;
    // Create a visited array to keep track of
visited vertices
    vector<bool> visited(V, false);
    // Variable to store the result (sum of edge
weights)
    int res = 0;

    // Start with vertex 0
    pq.push({0, 0});
}

```

```

    // Perform Prim's algorithm to find the Minimum
Spanning Tree
    while(!pq.empty()){
        auto p = pq.top();
        pq.pop();

        int wt = p.first; // Weight of the edge
        int u = p.second; // Vertex connected to
the edge

        if(visited[u] == true){
            continue; // Skip if the vertex is
already visited
        }

        res += wt; // Add the edge weight to the
result
        visited[u] = true; // Mark the vertex as
visited
        // Explore the adjacent vertices
        for(auto v : adj[u]){
            // v[0] represents the vertex and v[1]
represents the edge weight
            if(visited[v[0]] == false){
                pq.push({v[1], v[0]}); // Add the
adjacent edge to the priority queue
            }
        }
    }

    return res; // Return the sum of edge weights
of the Minimum Spanning Tree
}

int main() {
    vector<vector<int>> graph = {{0, 1, 5},
                                {1, 2, 3},
                                {0, 2, 1}};

    cout << spanningTree(3, 3, graph) << endl;
}

```

```
return 0;
```

4.7. shortest cycle

```
//this code works for simple graphs.
```

```
const int maxN = 1010, oo = 1e9;
```

```
vector<int> adj[maxN];
```

```
int deleted, distances[maxN];
```

```
queue<int> q;
```

```
void BFS(int n, int r) {
```

```
    for (int i=1; i<=n; i++) distances[i] = oo;
```

```
    distances[r] = 0;
```

```
    q.push(r);
```

```
    while(q.size()) {
```

```
        int v = q.front();
```

```
        q.pop();
```

```
        for (auto u : adj[v]) {
```

```
            if(v == r && u == deleted) continue;
```

```
//ignore deleted edge.
```

```
            if(distances[u] > distances[v] + 1) {
```

```
                distances[u] = distances[v] + 1;
```

```
                q.push(u);
```

```
            }
```

```
        }
```

```
    }
```

```
}
```

```
int main() {
```

```
    ios_base::sync_with_stdio(0); cin.tie(0);
```

```
    int n, m; cin >> n >> m;
```

```
    for (int i=0; i<m; i++) {
```

```
        int u, v; cin >> u >> v;
```

```
        adj[u].push_back(v); adj[v].push_back(u);
```

```
    }
```

```
    int length = oo;
```

```
    for (int i=1; i<=n; i++) {
        for (auto u : adj[i]) {
            deleted = u;
            BFS(n, i);
            length = min(length, distances[u] + 1);
        }
    }
    if(length == oo) cout << "Graph Does Not Have
Cycle\n";
    else cout << "Minimum Cycle Length is : " <<
length << '\n';
}
```

4.8. topological sort

```
int n; // number of vertices
```

```
vector<vector<int>> adj; // adjacency list of graph
```

```
vector<bool> visited;
```

```
vector<int> ans;
```

```
void dfs(int v) {
```

```
    visited[v] = true;
```

```
    for (int u : adj[v]) {
```

```
        if (!visited[u])
```

```
            dfs(u);
```

```
    }
```

```
    ans.push_back(v);
```

```
}
```

```
void topological_sort() {
```

```
    visited.assign(n, false);
```

```
    ans.clear();
```

```
    for (int i = 0; i < n; ++i) {
```

```
        if (!visited[i]) {
```

```
            dfs(i);
```

```
        }
```

```
    }
```

```
    reverse(ans.begin(), ans.end());
```

```
}
```

4.9. lowest common Ancestor

```
struct LCA {
```

```
    vector<int> height, euler, first, segtree;
```

```
    vector<bool> visited;
```

```
    int n;
```

```
LCA(vector<vector<int>> &adj, int root = 0) {
```

```
    n = adj.size();
```

```
    height.resize(n);
```

```
    first.resize(n);
```

```
    euler.reserve(n * 2);
```

```
    visited.assign(n, false);
```

```
    dfs(adj, root);
```

```
    int m = euler.size();
```

```
    segtree.resize(m * 4);
```

```
    build(1, 0, m - 1);
```

```
}
```

```
void dfs(vector<vector<int>> &adj, int node, int
h = 0) {
```

```
    visited[node] = true;
```

```
    height[node] = h;
```

```
    first[node] = euler.size();
```

```
    euler.push_back(node);
```

```
    for (auto to : adj[node]) {
```

```
        if (!visited[to]) {
```

```
            dfs(adj, to, h + 1);
```

```
            euler.push_back(node);
```

```
        }
```

```
    }
```

```
}
```

```
void build(int node, int b, int e) {
```

```
    if (b == e) {
```

```
        segtree[node] = euler[b];
```

```

    } else {
        int mid = (b + e) / 2;
        build(node << 1, b, mid);
        build(node << 1 | 1, mid + 1, e);
        int l = segtree[node << 1], r =
segtree[node << 1 | 1];
        segtree[node] = (height[l] < height[r])
? l : r;
    }
}

int query(int node, int b, int e, int L, int R)
{
    if (b > R || e < L)
        return -1;
    if (b >= L && e <= R)
        return segtree[node];
    int mid = (b + e) >> 1;

    int left = query(node << 1, b, mid, L, R);
    int right = query(node << 1 | 1, mid + 1, e,
L, R);
    if (left == -1) return right;
    if (right == -1) return left;
    return height[left] < height[right] ? left :
right;
}

int lca(int u, int v) {
    int left = first[u], right = first[v];
    if (left > right)
        swap(left, right);
    return query(1, 0, euler.size() - 1, left,
right);
}
};

```

4.10. lowest common Ancestor (binary lifting)

```

int n, l;
vector<vector<int>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>> up;

void dfs(int v, int p)
{
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool is_ancestor(int u, int v)
{
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v)
{
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))

```

```

        u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}

```

4.11. hungarian algorithm (assignment problem)

```

vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
for (int i=1; i<=n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv (m+1, INF);
    vector<bool> used (m+1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j=1; j<=m; ++j)
            if (!used[j]) {
                int cur = A[i0][j]-u[i0]-v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j=0; j<=m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
        else

```



```

        minv[j] -= delta;
    }
    j0 = j1;
} while (p[j0] != 0);
do {
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
} while (j0);
}

```

4.12. 2SAT

```

struct TwoSatSolver {
    int n_vars;
    int n_vertices;
    vector<vector<int>> adj, adj_t;
    vector<bool> used;
    vector<int> order, comp;
    vector<bool> assignment;

    TwoSatSolver(int _n_vars) : n_vars(_n_vars),
        n_vertices(2 * n_vars), adj(n_vertices),
        adj_t(n_vertices), used(n_vertices), order(),
        comp(n_vertices, -1), assignment(n_vars) {
        order.reserve(n_vertices);
    }

    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u])
                dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {

```

```

            if (comp[u] == -1)
                dfs2(u, cl);
        }
    }

    bool solve_2SAT() {
        order.clear();
        used.assign(n_vertices, false);
        for (int i = 0; i < n_vertices; ++i) {
            if (!used[i])
                dfs1(i);
        }

        comp.assign(n_vertices, -1);
        for (int i = 0, j = 0; i < n_vertices; ++i) {
            int v = order[n_vertices - i - 1];
            if (comp[v] == -1)
                dfs2(v, j++);
        }

        assignment.assign(n_vars, false);
        for (int i = 0; i < n_vertices; i += 2) {
            if (comp[i] == comp[i + 1])
                return false;
            assignment[i / 2] = comp[i] > comp[i +
1];
        }
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool
nb) {
        // na and nb signify whether a and b are to
        be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;

```

```

        int neg_b = b ^ 1;
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        adj_t[b].push_back(neg_a);
        adj_t[a].push_back(neg_b);
    }

    static void example_usage() {
        TwoSatSolver solver(3); // a, b, c
        solver.add_disjunction(0, false, 1,
true); // a v not b
        solver.add_disjunction(0, true, 1,
true); // not a v not b
        solver.add_disjunction(1, false, 2, false);
// b v c
        solver.add_disjunction(0, false, 0, false);
// a v a
        assert(solver.solve_2SAT() == true);
        auto expected = vector<bool>(True, False,
True);
        assert(solver.assignment == expected);
    }
};

```

4.13. Heavy-light decomposition

```

vector<int> parent, depth, heavy, head, pos;
int cur_pos;

int dfs(int v, vector<vector<int>> const& adj) {
    int size = 1;
    int max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;
            int c_size = dfs(c, adj);
            size += c_size;
            if (c_size > max_c_size)

```

```

        max_c_size = c_size, heavy[v] = c;
    }
}

return size;
}

void decompose(int v, int h, vector<vector<int>>
const& adj) {
    head[v] = h, pos[v] = cur_pos++;
    if (heavy[v] != -1)
        decompose(heavy[v], h, adj);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c, adj);
    }
}

void init(vector<vector<int>> const& adj) {
    int n = adj.size();
    parent = vector<int>(n);
    depth = vector<int>(n);
    heavy = vector<int>(n, -1);
    head = vector<int>(n);
    pos = vector<int>(n);
    cur_pos = 0;

    dfs(0, adj);
    decompose(0, 0, adj);
}

int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]])
    {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max =
segment_tree_query(pos[head[b]], pos[b]);

```

```

        res = max(res, cur_heavy_path_max);
    }
    if (depth[a] > depth[b])
        swap(a, b);
    int last_heavy_path_max =
segment_tree_query(pos[a], pos[b]);
    res = max(res, last_heavy_path_max);
    return res;
}

```

5. Data Structures

5.1. Array

```

int main() {
    array<int, 5> arr = {1, 2, 3, 4, 5};

    // Accessing elements
    cout << "Element at index 2: " << arr[2] <<
endl;

    // Size of the array
    cout << "Size of array: " << arr.size() << endl;

    // Fill array with a value
    arr.fill(10);
    cout << "Array after fill: ";
    for (int num : arr)
        cout << num << " ";
    cout << endl;

    return 0;
}

```

5.2. bitset

```

int main() {
    bitset<8> b1; // All bits initialized to 0
    bitset<8> b2("11001010");

```

```

// Set bit at index 3 to 1
b1.set(3);
cout << "b1 after set(3): " << b1 << endl;

// Reset all bits of b2
b2.reset();
cout << "b2 after reset: " << b2 << endl;

// Flip all bits of b1
b1.flip();
cout << "b1 after flip: " << b1 << endl;

// Access the bit at index 2
cout << "b1[2]: " << b1[2] << endl;

// Test if bit at index 2 is set to 1
if (b1.test(2))
{
    cout << "Bit 2 is set to 1." << endl;
}

// Count the number of 1's in b1
cout << "Number of 1's in b1: " << b1.count() <<
endl;

return 0;
}

```

5.3. deque

```

int main() {
    deque<int> deq = {1, 2, 3, 4, 5};

    // Adding elements
    deq.push_front(0); // Add at front
    deq.push_back(6);  // Add at back

```

```
// Removing elements
deq.pop_front(); // Remove from front
deq.pop_back(); // Remove from back

// Accessing elements
cout << "First element: " << deq.front() << endl;
cout << "Last element: " << deq.back() << endl;

// Iterating through deque
cout << "Deque elements: ";
for (int x : deq)
    cout << x << " ";
cout << endl;

// Size of deque
cout << "Deque size: " << deq.size() << endl;

return 0;
}
```

5.4. link list

```
int main() {
    list<int> lst = {1, 2, 3, 4, 5};

    // Adding elements
    lst.push_back(6); // Add to the end
    lst.push_front(0); // Add to the front

    // Removing elements
    lst.pop_back(); // Remove from the end
    lst.pop_front(); // Remove from the front

    // Iterating through list
    cout << "List elements: ";
    for (int x : lst)
        cout << x << " ";
}
```

```
cout << endl;

// Size of list
cout << "List size: " << lst.size() << endl;

return 0;
}
```

5.5. Map

```
int main() {
    map<string, int> m;

    // Insert key-value pairs
    m["apple"] = 3;
    m["banana"] = 2;
    m["orange"] = 5;

    // Access value by key
    cout << "Value for apple: " << m["apple"] << endl;

    // Iterate through map
    cout << "Map elements: ";
    for (auto &pair : m)
    {
        cout << pair.first << ": " << pair.second << " ";
    }
    cout << endl;

    // Remove an element
    m.erase("banana");
    cout << "Map after erase: ";
    for (auto &pair : m)
    {
        cout << pair.first << ": " << pair.second << " ";
    }
}
```

```
}
cout << endl;

// Check if key exists
if (m.find("orange") != m.end())
{
    cout << "Orange is in the map." << endl;
}

return 0;
}
```

5.6. priority queue

```
int main() {
    priority_queue<int> pq;

    // Insert elements into the priority queue
    pq.push(10);
    pq.push(30);
    pq.push(20);

    cout << "Top element (max priority): " << pq.top() << endl;

    // Pop the top element
    pq.pop();
    cout << "Top element after pop: " << pq.top() << endl;

    // Size of the priority queue
    cout << "Size of priority queue: " << pq.size() << endl;

    return 0;
}
```

5.7. queue

```
int main() {
    queue<int> q;

    // Push elements onto the queue
    q.push(10);
    q.push(20);
    q.push(30);
    cout << "Front element: " << q.front() << endl;

    // Pop an element from the queue
    q.pop();
    cout << "Front element after pop: " << q.front()
<< endl;

    // Size of the queue
    cout << "Size of queue: " << q.size() << endl;

    // Check if the queue is empty
    if (q.empty()) {
        cout << "Queue is empty." << endl;
    }
    else {
        cout << "Queue is not empty." << endl;
    }

    return 0;
}
```

5.8. set

```
int main() {
    set<int> s;

    // Insert elements
    s.insert(10);
    s.insert(20);
    s.insert(15);
```

```
// Print elements in sorted order
cout << "Set elements: ";
for (int num : s)
    cout << num << " ";
cout << endl;

// Check if an element exists
if (s.find(15) != s.end())
{
    cout << "15 is in the set." << endl;
}

// Remove an element
s.erase(10);
cout << "Set after erase: ";
for (int num : s)
    cout << num << " ";
cout << endl;

// Size of the set
cout << "Size of set: " << s.size() << endl;

return 0;
}
```

5.9. stack

```
int main() {
    stack<int> s;

    // Push elements onto the stack
    s.push(10);
    s.push(20);
    s.push(30);
    cout << "Top element after push: " << s.top() <<
endl;
```

```
// Pop an element from the stack
s.pop();
cout << "Top element after pop: " << s.top() <<
endl;

// Check if stack is empty
if (s.empty()) {
    cout << "Stack is empty." << endl;
}
else {
    cout << "Stack is not empty." << endl;
}

// Size of the stack
cout << "Size of stack: " << s.size() << endl;

return 0;
}
```

5.10. unordered map

```
int main() {
    unordered_map<int, string> um;

    // Inserting elements
    um[1] = "apple";
    um[2] = "banana";
    um[3] = "cherry";

    // Accessing elements
    cout << "Key 2 maps to: " << um[2] << endl;

    // Checking if key exists
    if (um.find(4) == um.end())
        cout << "Key 4 not found!" << endl;

    // Iterating through unordered map
    cout << "Unordered map elements: ";
```

```

    for (auto &pair : um)
    {
        cout << pair.first << " -> " << pair.second
<< " | ";
    }
    cout << endl;

    return 0;
}

```

5.11. unordered set

```

int main() {
    unordered_set<int> us;

    // Insert elements
    us.insert(10);
    us.insert(20);
    us.insert(15);

    // Print elements
    cout << "Unordered set elements: ";
    for (int num : us)
        cout << num << " ";
    cout << endl;

    // Check if an element exists
    if (us.find(15) != us.end())
    {
        cout << "15 is in the unordered set." <<
endl;
    }

    // Remove an element
    us.erase(10);
    cout << "Unordered set after erase: ";
    for (int num : us)
        cout << num << " ";
}

```

```

    cout << endl;

    return 0;
}

```

5.12. vector

```

int main() {
    // Create a vector of integers
    vector<int> v;

    // Add elements using push_back
    v.push_back(10);
    v.push_back(20);
    v.push_back(30);
    cout << "Vector after push_back: ";
    for (int num : v)
        cout << num << " ";
    cout << endl;

    // Accessing elements using at() and indexing
    cout << "Element at index 1: " << v.at(1) <<
endl;
    cout << "Element at index 0: " << v[0] << endl;

    // Pop an element from the back
    v.pop_back();
    cout << "Vector after pop_back: ";
    for (int num : v)
        cout << num << " ";
    cout << endl;

    // Insert an element at a specific position
    v.insert(v.begin() + 1, 25); // Insert 25 at
index 1
    cout << "Vector after insert: ";
    for (int num : v)
        cout << num << " ";
}

```

```

    cout << endl;

    // Remove an element from the vector
    v.erase(v.begin() + 1); // Remove the element at
index 1
    cout << "Vector after erase: ";
    for (int num : v)
        cout << num << " ";
    cout << endl;

    // Resize the vector
    v.resize(5, 50); // Resize to size 5, fill new
elements with 50
    cout << "Vector after resize: ";
    for (int num : v)
        cout << num << " ";
    cout << endl;

    // Get the size of the vector
    cout << "Size of vector: " << v.size() << endl;

    // Clear the vector
    v.clear();
    cout << "Vector after clear: " << v.size() << "
(size is now zero)" << endl;

    return 0;
}

```

5.13. DS cheatsheet

1. Vector

- Description: Dynamic array that allows fast random access.
- Methods:
 - push_back(x): Add element x to the end.
 - pop_back(): Remove the last element.
 - at(i): Access the element at index i (bounds checked).
 - operator[]: Access the element at index i (no bounds check).
 - size(): Return the number of elements.

- `empty()`: Check if the vector is empty.
- `resize(n, x)`: Resize the vector to size n and fill new elements with x.
- `clear()`: Remove all elements.

2. Stack

- Description: Last-In-First-Out (LIFO) structure, used for backtracking problems.
- Methods:
 - `push(x)`: Add element x to the top.
 - `pop()`: Remove the top element.
 - `top()`: Get the top element.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the stack is empty.

3. Queue

- Description: First-In-First-Out (FIFO) structure, ideal for problems involving processing in order.
- Methods:
 - `push(x)`: Add element x to the back.
 - `pop()`: Remove the front element.
 - `front()`: Get the front element.
 - `back()`: Get the back element.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the queue is empty.

4. Priority Queue (Max-Heap by default)

- Description: A heap-based structure that always gives the maximum element.
- Methods:
 - `push(x)`: Add element x to the queue.
 - `pop()`: Remove the largest element.
 - `top()`: Get the largest element.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the queue is empty.

5. Set

- Description: Collection of unique elements in sorted order.
- Methods:
 - `insert(x)`: Add element x.
 - `erase(x)`: Remove element x.
 - `find(x)`: Check if element x exists.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the set is empty.
 - `clear()`: Remove all elements.

6. Map

- Description: Stores key-value pairs in sorted order based on keys.
- Methods:
 - `insert({key, value})`: Add key-value pair.
 - `erase(key)`: Remove element by key.
 - `find(key)`: Check if a key exists.
 - `operator[]`: Access the value associated with a key.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the map is empty.
 - `clear()`: Remove all elements.

7. Unordered Set

- Description: Collection of unique elements with no specific order.
- Methods:
 - `insert(x)`: Add element x.
 - `erase(x)`: Remove element x.
 - `find(x)`: Check if element x exists.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the unordered set is empty.

8. Unordered Map

- Description: Stores key-value pairs with no specific order.
- Methods:
 - `insert({key, value})`: Add key-value pair.
 - `erase(key)`: Remove element by key.
 - `find(key)`: Check if a key exists.
 - `operator[]`: Access the value associated with a key.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the unordered map is empty.

9. Bitset

- Description: A space-efficient container for a fixed-size sequence of bits (0 or 1).
- Methods:
 - `set(i)`: Set bit at index i to 1.
 - `reset(i)`: Set bit at index i to 0.
 - `flip(i)`: Toggle the bit at index i.
 - `test(i)`: Check if the bit at index i is 1.
 - `count()`: Count the number of bits set to 1.
 - `size()`: Return the number of bits.
 - `operator[]`: Access the bit at index i.
 - `to_string()`: Convert bitset to string.

10. Array

- Description: Fixed-size array used for fast access, but size cannot be changed after initialization.
- Methods:
 - `fill(x)`: Fill all elements with the value x.
 - `size()`: Return the number of elements.
 - `operator[]`: Access element at index i.
 - `at(i)`: Access element at index i with bounds checking.
 - `front()`: Get the first element.
 - `back()`: Get the last element.

11. Deque

- Description: Double-ended queue that allows fast insertion and removal at both ends.
- Methods:
 - `push_front(x)`: Add element x to the front.
 - `push_back(x)`: Add element x to the back.
 - `pop_front()`: Remove the front element.
 - `pop_back()`: Remove the back element.
 - `front()`: Get the front element.
 - `back()`: Get the back element.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the deque is empty.

12. Linked List (Using STL List)

- Description: Doubly linked list that allows fast insertion and deletion at both ends.
- Methods:
 - `push_back(x)`: Add element x to the back.
 - `push_front(x)`: Add element x to the front.
 - `pop_back()`: Remove the last element.
 - `pop_front()`: Remove the first element.
 - `size()`: Return the number of elements.
 - `empty()`: Check if the list is empty.
 - `front()`: Get the first element.
 - `back()`: Get the last element.
 - `clear()`: Remove all elements.

6. Dynamic Programming

6.1. counting paths matrix

```
// Function to count the number of unique paths in a matrix
int countPaths(int n, int m) {
```

```

vector<vector<int>> dp(n, vector<int>(m, 0));

// Starting point: only one way to be at the
start
dp[0][0] = 1;

// Fill the DP table for first row and first
column
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++) {
        if (i > 0)
            dp[i][j] += dp[i - 1][j]; // From
top
        if (j > 0)
            dp[i][j] += dp[i][j - 1]; // From
left
    }
}

return dp[n - 1][m - 1]; // Return the number of
paths to bottom-right corner
}

int main() {
    int n = 3, m = 3; // Example
matrix dimensions
    cout << countPaths(n, m) << endl; // Output the
result
    return 0;
}

```

6.2. edit distance

```

// Function to compute the Edit Distance
int editDistance(string str1, string str2, int m,
int n) {
    vector<vector<int>> dp(m + 1, vector<int>(n +
1));

```

```

    for (int i = 0; i <= m; i++) {
        for (int j = 0; j <= n; j++) {
            if (i == 0)
                dp[i][j] = j;
            else if (j == 0)
                dp[i][j] = i;
            else if (str1[i - 1] == str2[j - 1])
                dp[i][j] = dp[i - 1][j - 1];
            else
                dp[i][j] = 1 + min({dp[i - 1][j -
1], dp[i][j - 1], dp[i - 1][j]});
        }
    }
    return dp[m][n];
}

int main() {
    string str1 = "sitting", str2 = "kitten";
    int m = str1.length(), n = str2.length();

    cout << "Edit Distance: " << editDistance(str1,
str2, m, n) << endl;
    return 0;
}

```

6.3. Egg Dropping

```

// Function to find the minimum number of attempts
needed
int eggDrop(int eggs, int floors)
{
    vector<vector<int>> dp(eggs + 1,
vector<int>(floors + 1, 0));

    for (int i = 1; i <= eggs; i++)
        dp[i][0] = 0;
    for (int j = 0; j <= floors; j++)

```

```

        dp[1][j] = j;

    for (int i = 2; i <= eggs; i++)
    {
        for (int j = 2; j <= floors; j++)
        {
            dp[i][j] = INT_MAX;
            for (int x = 1; x <= j; x++)
            {
                dp[i][j] = min(dp[i][j], 1 +
max(dp[i - 1][x - 1], dp[i][j - x]));
            }
        }
    }
    return dp[eggs][floors];
}

int main()
{
    int eggs = 2, floors = 10;
    cout << "Minimum attempts: " << eggDrop(eggs,
floors) << endl;
    return 0;
}

```

6.4. fibonacci

// Fibonacci sequence using dynamic programming
(Memoization)

```

// Function to compute Fibonacci number
int fib(int n, vector<int> &dp)
{
    // Base cases
    if (n <= 1)
        return n;

    // If the value is already computed, return it

```

```

    if (dp[n] != -1)
        return dp[n];

    // Store the computed value in dp array
    dp[n] = fib(n - 1, dp) + fib(n - 2, dp);
    return dp[n];
}

int main()
{
    int n = 10;
    vector<int> dp(n + 1, -1); // Initialize dp
    array with -1

    cout << "Fibonacci of " << n << " is " << fib(n,
dp) << endl;
    return 0;
}

```

6.5. knapsack 01

```

// Function to solve the 0/1 Knapsack problem
int knapsack(int W, vector<int> &wt, vector<int>
&val, int n)
{
    vector<vector<int>> dp(n + 1, vector<int>(W + 1,
0)); // DP table

    for (int i = 1; i <= n; i++)
    {
        for (int w = 0; w <= W; w++)
        {
            if (wt[i - 1] <= w)
                dp[i][w] = max(dp[i - 1][w], val[i -
1] + dp[i - 1][w - wt[i - 1]]);
            else
                dp[i][w] = dp[i - 1][w];
        }
    }
}

```

```

    }
    return dp[n][W];
}

int main()
{
    int W = 100; // Capacity of the knapsack
    vector<int> val = {80, 24, 23, 22, 21}; // Values of the items
    vector<int> wt = {80, 25, 25, 25, 25}; // Weights of the items
    int n = val.size(); // Number of items

    cout << "Maximum value in knapsack: " <<
knapsack(W, wt, val, n) << endl;
    return 0;
}

```

6.6. LCS

```

// Function to compute the length of the Longest
Common Subsequence
int lcs(string X, string Y, int m, int n)
{
    vector<vector<int>> dp(m + 1, vector<int>(n + 1,
0)); // DP table

    for (int i = 1; i <= m; i++)
    {
        for (int j = 1; j <= n; j++)
        {
            if (X[i - 1] == Y[j - 1])
                dp[i][j] = 1 + dp[i - 1][j - 1];
            else
                dp[i][j] = max(dp[i - 1][j], dp[i][j
- 1]);
        }
    }
}

```

```

    }
    return dp[m][n];
}

int main()
{
    string X = "AGGTAB", Y = "GTXAYB";
    int m = X.length(), n = Y.length();

    cout << "Length of Longest Common Subsequence: "
<< lcs(X, Y, m, n) << endl;
    return 0;
}

```

6.7. LIS

```

// Function to find the length of the Longest
Increasing Subsequence
int lis(vector<int> &arr, int n)
{
    vector<int> dp(n, 1); // DP array, initialized
to 1

    for (int i = 1; i < n; i++)
    {
        for (int j = 0; j < i; j++)
        {
            if (arr[i] > arr[j])
                dp[i] = max(dp[i], dp[j] + 1);
        }
    }
    return *max_element(dp.begin(), dp.end());
}

int main()
{
}

```



```
vector<int> arr = {10, 22, 9, 33, 21, 50, 41, 60};
int n = arr.size();

cout << "Length of Longest Increasing Subsequence: " << lis(arr, n) << endl;
return 0;
}
```

6.8. LPS

```
// Function to compute the length of the Longest Palindromic Subsequence
int lps(string s)
{
    int n = s.length();
    vector<vector<int>> dp(n, vector<int>(n, 0));

    for (int i = 0; i < n; i++)
        dp[i][i] = 1; // Single character is a palindrome

    for (int len = 2; len <= n; len++)
    {
        for (int i = 0; i < n - len + 1; i++)
        {
            int j = i + len - 1;
            if (s[i] == s[j])
                dp[i][j] = 2 + dp[i + 1][j - 1];
            else
                dp[i][j] = max(dp[i + 1][j], dp[i][j - 1]);
        }
    }
    return dp[0][n - 1];
}

int main()
```

```
{
    string s = "bbabcbcab";
    cout << "Length of Longest Palindromic Subsequence: " << lps(s) << endl;
    return 0;
}
```

6.9. MCM

```
// Function to compute the minimum number of scalar multiplications
int matrixChainMultiplication(vector<int> &dims, int n) {
    vector<vector<int>> dp(n, vector<int>(n, 0)); // DP table

    for (int len = 2; len < n; len++)
    {
        for (int i = 1; i < n - len + 1; i++)
        {
            int j = i + len - 1;
            dp[i][j] = INT_MAX;
            for (int k = i; k < j; k++)
            {
                int q = dp[i][k] + dp[k + 1][j] +
                    dims[i - 1] * dims[k] * dims[j];
                dp[i][j] = min(dp[i][j], q);
            }
        }
    }
    return dp[1][n - 1];
}

int main()
{
    vector<int> dims = {10, 20, 30, 40, 30};
    int n = dims.size();
```

```
    cout << "Minimum number of scalar multiplications: " <<
        matrixChainMultiplication(dims, n) << endl;
    return 0;
}
```

6.10. Minimum Coin Change

```
// Function to find the minimum number of coins required to make a total
int minCoins(const vector<int> &coins, int total)
{
    int n = coins.size();
    vector<int> dp(total + 1, INT_MAX); // Initialize DP array with infinity
    dp[0] = 0; // Base case: 0 coins needed to make total 0

    // Fill DP array
    for (int i = 0; i < n; i++)
    {
        for (int j = coins[i]; j <= total; j++)
        {
            if (dp[j - coins[i]] != INT_MAX)
            {
                dp[j] = min(dp[j], dp[j - coins[i]] + 1); // Minimize coin count
            }
        }
    }

    return dp[total] == INT_MAX ? -1 : dp[total]; // Return -1 if not possible
}

int main()
{
```

```

vector<int> coins = {1, 2, 5};           //
Example denominations
int total = 11;                          //
Target total
cout << minCoins(coins, total) << endl; //
Output the result
return 0;
}

```

6.11. optimal BST

// Function to calculate the minimum search cost for an optimal BST

```

int optimalBST(const vector<int> &freq)
{
    int n = freq.size();
    vector<vector<int>> dp(n, vector<int>(n, 0));

    // Fill the DP table for subarrays of increasing
length
    for (int len = 1; len <= n; len++)
    { // len is the range length
        for (int i = 0; i <= n - len; i++)
        {
            int j = i + len - 1;
            dp[i][j] = INT_MAX;
            int sum = 0;
            // Calculate sum of frequencies from i
to j
            for (int k = i; k <= j; k++)
            {
                sum += freq[k];
            }
            // Try each k as the root and calculate
the minimum cost
            for (int k = i; k <= j; k++)
            {

```

```

                int cost = (k == i ? 0 : dp[i][k -
1]) + (k == j ? 0 : dp[k + 1][j]);
                dp[i][j] = min(dp[i][j], cost +
sum);
            }
        }
    }
    return dp[0][n - 1]; // The minimum cost for the
entire range
}

int main()
{
    vector<int> freq = {34, 8, 50, 13}; // Example
frequencies of keys
    cout << optimalBST(freq) << endl; // Output
the minimum cost
    return 0;
}

```

6.12. partition problem

// Function to determine if a given set can be partitioned into two subsets

```

bool canPartition(vector<int> &nums)
{
    int sum = 0;
    for (int num : nums)
        sum += num;

    if (sum % 2 != 0)
        return false;

    int target = sum / 2;
    vector<bool> dp(target + 1, false);
    dp[0] = true;

    for (int num : nums)

```

```

{
    for (int j = target; j >= num; j--)
    {
        dp[j] = dp[j] || dp[j - num];
    }
}
return dp[target];
}

int main()
{
    vector<int> nums = {1, 5, 11, 5};
    cout << "Can partition: " << (canPartition(nums)
? "Yes" : "No") << endl;
    return 0;
}

```

6.13. Regular Expression Matching

```

bool isMatch(const string &s, const string &p) {
    int m = s.size(), n = p.size();

    // DP table dp[i][j] will be true if s[0...i-1]
matches p[0...j-1]
    vector<vector<bool>> dp(m + 1, vector<bool>(n +
1, false));

    // Base case: empty string matches empty pattern
    dp[0][0] = true;

    // Handle patterns like "a*" or "."* where "*"
can match 0 occurrence
    for (int j = 1; j <= n; j++)
    {
        if (p[j - 1] == '*')
        {
            dp[0][j] = dp[0][j - 2];
        }
    }
}

```

```

    }

    // Fill the dp table
    for (int i = 1; i <= m; i++)
    {
        for (int j = 1; j <= n; j++)
        {
            if (p[j - 1] == s[i - 1] || p[j - 1] ==
'.')
            {
                dp[i][j] = dp[i - 1][j - 1]; //
Character matches
            }
            else if (p[j - 1] == '*')
            {
                // '*' matches zero occurrence or
one/more occurrences of the preceding character
                dp[i][j] = dp[i][j - 2] || (dp[i -
1][j] && (s[i - 1] == p[j - 2] || p[j - 2] == '.'));
            }
        }
    }

    return dp[m][n]; // Final answer whether the
whole string matches the pattern
}

int main()
{
    string s = "aab";
    string p = "c*a*b";

    if (isMatch(s, p))
    {
        cout << "The string matches the pattern." <<
endl;
    }
    else

```

```

    {
        cout << "The string does not match the
pattern." << endl;
    }

    return 0;
}

6.14. ROD cutting
// Function to compute the maximum profit from
cutting a rod
int rodCutting(vector<int> &prices, int n)
{
    vector<int> dp(n + 1, 0); // DP array

    for (int i = 1; i <= n; i++)
    {
        for (int j = 1; j <= i; j++)
        {
            dp[i] = max(dp[i], prices[j - 1] + dp[i
- j]);
        }
    }

    return dp[n];
}

int main()
{
    vector<int> prices = {1, 5, 8, 9, 10, 17, 17,
20};
    int n = prices.size();

    cout << "Maximum profit from rod cutting: " <<
rodCutting(prices, n) << endl;
    return 0;
}

```

6.15. subset sum

```

// Function to determine if there's a subset with
sum equal to the target
bool subsetSum(vector<int> &nums, int sum)
{
    int n = nums.size();
    vector<vector<bool>> dp(n + 1, vector<bool>(sum
+ 1, false));

    for (int i = 0; i <= n; i++)
        dp[i][0] = true;

    for (int i = 1; i <= n; i++)
    {
        for (int j = 1; j <= sum; j++)
        {
            if (nums[i - 1] <= j)
                dp[i][j] = dp[i - 1][j] || dp[i -
1][j - nums[i - 1]];
            else
                dp[i][j] = dp[i - 1][j];
        }
    }

    return dp[n][sum];
}

int main()
{
    vector<int> nums = {3, 34, 4, 12, 5, 2};
    int sum = 9;

    cout << "Subset sum possible: " <<
(subsetSum(nums, sum) ? "Yes" : "No") << endl;
    return 0;
}

```

6.16. two player game

```
// Function to find the maximum sum player A can get
int maxCoins(const vector<int> &coins)
{
    int n = coins.size();
    vector<vector<int>> dp(n, vector<int>(n, 0));

    // Base case: when there's only one coin, player
    // A takes it
    for (int i = 0; i < n; i++)
    {
        dp[i][i] = coins[i];
    }

    // Fill DP table for subarrays of length 2 to n
    for (int len = 2; len <= n; len++)
    {
        for (int i = 0; i < n - len + 1; i++)
        {
            int j = i + len - 1;
            dp[i][j] = max(coins[i] + min(dp[i +
2][j], dp[i + 1][j - 1]),
                           coins[j] + min(dp[i +
1][j - 1], dp[i][j - 2]));
        }
    }

    return dp[0][n - 1]; // Maximum sum player A can
    get
}

int main()
{
    vector<int> coins = {8, 15, 3, 7}; // Example
    coins array
    cout << maxCoins(coins) << endl;    // Output the
    result
    return 0;
}
```

6.17. word break

```
// Function to check if a word can be segmented into
// words from a dictionary
bool wordBreak(string s, unordered_set<string>
&wordDict)
{
    int n = s.length();
    vector<bool> dp(n + 1, false);
    dp[0] = true;

    for (int i = 1; i <= n; i++)
    {
        for (int j = 0; j < i; j++)
        {
            if (dp[j] && wordDict.find(s.substr(j, i
- j)) != wordDict.end())
            {
                dp[i] = true;
                break;
            }
        }
    }
    return dp[n];
}

int main()
{
    string s = "leetcode";
    unordered_set<string> wordDict = {"leet",
"code"};
    cout << "Can break: " << (wordBreak(s, wordDict)
? "Yes" : "No") << endl;
    return 0;
}
```

6.18. dp cheatsheet

1. Fibonacci Sequence (Memoization)

- Explanation: This algorithm calculates the nth Fibonacci number using memoization to store previously computed results, avoiding redundant calculations.
- When to Use: When you need to compute Fibonacci numbers for large values of `n` efficiently (i.e., for recursive problems that involve overlapping subproblems).

2. 0/1 Knapsack Problem

- Explanation: Given a set of items with weights and values, the goal is to determine the maximum value that can be obtained by putting items in a knapsack without exceeding the weight capacity.
- When to Use: In optimization problems where you need to maximize profit or value while respecting constraints (e.g., weight, space).

3. Minimum Coin Change

- Explanation: This algorithm calculates the minimum number of coins needed to make a given total using a set of coin denominations. It uses dynamic programming to store the results for all possible totals.
- When to Use: When you need to find the fewest coins needed to form a specific amount (e.g., for making change, budget optimization).

4. Longest Common Subsequence (LCS)

- Explanation: This algorithm finds the longest subsequence common to two sequences (strings, arrays, etc.). It uses a dynamic programming table to store intermediate results.
- When to Use: When comparing two sequences (e.g., DNA sequences, text comparison, diff tools) and need the longest subsequence they share.

5. Edit Distance (Levenshtein Distance)

CTRL+ALT+DEFEAT – Hakim Sabzevari University

- Explanation: This algorithm calculates the minimum number of operations (insertions, deletions, substitutions) needed to convert one string into another.

- When to Use: When you need to compare two strings and find the minimum edit operations required (e.g., spell checkers, natural language processing).

6. Longest Increasing Subsequence (LIS)

- Explanation: This algorithm finds the length of the longest increasing subsequence in a sequence of numbers. The subsequence need not be contiguous.

- When to Use: When you need to find the longest increasing subsequence in a sequence of numbers (e.g., stock price prediction, finding trends).

7. Matrix Chain Multiplication

- Explanation: This algorithm calculates the most efficient way to multiply a chain of matrices by minimizing the number of scalar multiplications.

- When to Use: When multiplying multiple matrices and you need to minimize the cost of multiplication (e.g., in computer graphics, optimization problems).

8. Rod Cutting Problem

- Explanation: This algorithm finds the maximum profit you can obtain by cutting a rod of length `n` into smaller pieces and selling them, based on the prices for each length.

- When to Use: When you need to solve problems related to cutting materials into pieces to maximize profit (e.g., resource allocation, profit optimization).

9. Subset Sum Problem

- Explanation: This algorithm checks if there is a subset of a given set of numbers that adds up to a target sum. It uses dynamic programming to keep track of achievable sums.

- When to Use: When you need to check whether a subset exists with a given sum (e.g., partitioning problems, subset analysis).

10. Egg Dropping Problem

- Explanation: This algorithm determines the minimum number of attempts required to find the highest floor from which an egg can be dropped without breaking, given `k` eggs and `n` floors.

- When to Use: In optimization problems where you need to minimize the number of trials in a worst-case scenario (e.g., testing, fault tolerance, hardware).

11. Partition Problem

- Explanation: This algorithm checks whether a given set can be partitioned into two subsets such that their sums are equal. It uses dynamic programming to check for subset sums.

- When to Use: When dividing a set of numbers into two equal subsets (e.g., load balancing, resource allocation).

12. Longest Palindromic Subsequence (LPS)

- Explanation: This algorithm finds the longest subsequence within a string that is a palindrome. It uses dynamic programming to build a table based on matching characters.

- When to Use: When you need to find the longest palindromic subsequence in a string (e.g., text processing, bioinformatics).

13. Word Break Problem

- Explanation: This algorithm checks if a string can be segmented into a space-separated sequence of words from a dictionary. It uses dynamic programming to store results for substrings.

- When to Use: When you need to determine whether a string can be split into valid words (e.g., for tokenizing sentences, text segmentation).

14. Regular Expression Matching

- Explanation: This algorithm checks if a string matches a pattern with . (any character) and * (zero or more of the previous character). It uses dynamic programming to track matching results for substrings.

- When to Use: When matching strings to patterns with wildcards like . and * (e.g., text matching, search engines, file pattern matching).

15. Optimal Binary Search Tree

- Explanation: This algorithm finds the minimum cost to construct a binary search tree based on the frequencies of elements. It uses dynamic programming to calculate the optimal cost for different subarrays.

- When to Use: When you need to minimize the cost of searching elements with different access frequencies (e.g., database indexing, search optimization).

16. 2 Player Game

- Explanation: This algorithm calculates the maximum sum player A can collect in a turn-based game where players alternate picking coins from either end of the array. It uses dynamic programming to determine the optimal strategy for player A.

- When to Use: When you need to calculate the optimal strategy in a turn-based game with alternating choices (e.g., maximizing outcomes in competitive games).

17. Counting Paths in Matrix

- Explanation: This algorithm calculates the number of unique paths from the top-left corner to the bottom-right corner of an $n \times m$ matrix. It uses dynamic programming to count paths by combining the results from adjacent cells.

- When to Use: When you need to find the number of distinct paths in a grid, where you can only move right or down (e.g., grid-based traversal problems).

7. More

7.1. longest polindrom substring (Mancher)

// O(n)

```
class Solution {
public:
    string longestPalindrome(std::string s) {
        if (s.length() <= 1) return s;

        // Preprocess the string with '#' characters
        // to handle even-length palindromes
        string modified_s = "#";
        for (char c : s) {
            modified_s += c;
            modified_s += '#';
        }

        int n = modified_s.size();
        vector<int> dp(n, 0); // dp array to store
        // the radius of the palindrome centered at each
        // character
        int center = 0, right = 0; // Initialize
        // center and right boundary
        int max_len = 1; // Maximum length of
        // palindrome found
        string max_str = s.substr(0, 1); //
        // Initialize the max palindrome substring with the
        // first character

        for (int i = 0; i < n; i++) {
```

```
            // If i is within the current right
            // boundary, use previously calculated values to
            // minimize comparisons
            if (i < right) {
                dp[i] = min(right - i, dp[2 * center
                - i]);
            }

            // Expand around center i
            while (i - dp[i] - 1 >= 0 && i + dp[i] +
            1 < n && modified_s[i - dp[i] - 1] == modified_s[i +
            dp[i] + 1]) {
                dp[i]++;
            }

            // Update center and right boundary if
            // we've expanded beyond the current right
            if (i + dp[i] > right) {
                center = i;
                right = i + dp[i];
            }

            // Update max_len and max_str if a
            // longer palindrome is found
            if (dp[i] > max_len) {
                max_len = dp[i];
                max_str = modified_s.substr(i -
                dp[i], 2 * dp[i] + 1);
                max_str.erase(remove(max_str.begin()
                , max_str.end(), '#'), max_str.end());
            }

            return max_str;
        }
    };
```

7.2. median of 2 soted array

```
class Solution {
public:
    double findMedianSortedArrays(vector<int>
    &nums1, vector<int> &nums2) {
        int n = nums1.size();
        int m = nums2.size();
        if(n > m) return
        findMedianSortedArrays(nums2, nums1);
        int size = n + m;
        int left = (size + 1) / 2;
        int low = 0, high = n;
        int l1, l2, r1, r2, mid1, mid2;
        while(low <= high) {
            mid1 = (low + high) >> 1;
            mid2 = left - mid1;
            r1 = mid1 < n ? nums1[mid1] : 1e9;
            r2 = mid2 < m ? nums2[mid2] : 1e9;
            l1 = mid1 - 1 >= 0 ? nums1[mid1 - 1] : -
            1e9;
            l2 = mid2 - 1 >= 0 ? nums2[mid2 - 1] : -
            1e9;

            if(l1 <= r2 && l2 <= r1)
                if(size % 2) return max(l1, l2);
                else return
                (static_cast<double>(max(l1, l2) + min(r1, r2))) /
                2.0;
            else if(l1 > r2) high = mid1 - 1;
            else low = mid1 + 1;
        }

        return 0;
    }
};
```

10. other

10.1. useful geo

Area of triangle with sides a, b, c: $\sqrt{S \cdot (S-a) \cdot (S-b) \cdot (S-c)}$ where $S = (a+b+c)/2$

Area of equilateral triangle: $s^2 \cdot \sqrt{3} / 4$ where is side lenght

Pyramid and cones volume: $1/3 \cdot \text{area}(\text{base}) \cdot \text{height}$

if $p_1=(x_1, y_1)$, $p_2=(x_2, y_2)$, $p_3=(x_3, y_3)$ are points on circle, the center is

$$x = -((x_2^2 - x_1^2 + y_2^2 - y_1^2) \cdot (y_3 - y_2) - (x_2^2 - x_3^2 + y_2^2 - y_3^2) \cdot (y_1 - y_2)) / (2 \cdot (x_1 - x_2) \cdot (y_3 - y_2) - 2 \cdot (x_3 - x_2) \cdot (y_1 - y_2))$$

$$y = -((y_2^2 - y_1^2 + x_2^2 - x_1^2) \cdot (x_3 - x_2) - (y_2^2 - y_3^2 + x_2^2 - x_3^2) \cdot (x_1 - x_2)) / (2 \cdot (y_1 - y_2) \cdot (x_3 - x_2) - 2 \cdot (y_3 - y_2) \cdot (x_1 - x_2))$$

10.2. number of primes

- 30: 10
- 60: 17
- 100: 25
- 1000: 168
- 10000: 1229
- 100000: 9592
- 1000000: 78498
- 10000000: 664579

10.3. Factorials

- 1: 1
- 2: 2
- 3: 6
- 4: 24
- 5: 120
- 6: 720
- 7: 5040
- 8: 40320
- 9: 362880
- 10: 3628800
- 11: 39916800
- 12: 479001600
- 13: 6227020800
- 14: 87178291200
- 15: 1307674368000

10.4. power of 3

- 1: 3
- 2: 9
- 3: 27
- 4: 81
- 5: 243
- 6: 729
- 7: 2187
- 8: 6561
- 9: 19683
- 10: 59049
- 11: 177147
- 12: 531441
- 13: 1594323
- 14: 4782969
- 15: 14348907
- 16: 43046721
- 17: 129140163
- 18: 387420489
- 19: 1162261467
- 20: 3486784401

10.5. C(2n, n)

- 1: 2
- 2: 6
- 3: 20
- 4: 70
- 5: 252
- 6: 924
- 7: 3432
- 8: 12870
- 9: 48620
- 10: 184756
- 11: 705432
- 12: 2704156
- 13: 10400600
- 14: 40116600
- 15: 155117520

10.6. Most Divisor

- $\leq 1e2$: 60 with 12 divisors
- $\leq 1e3$: 840 with 32 divisors
- $\leq 1e4$: 7560 with 64 divisors
- $\leq 1e5$: 83160 with 128 divisors

- $\leq 1e6$: 720720 with 240 divisors
- $\leq 1e7$: 8648640 with 448 divisors
- $\leq 1e8$: 73513440 with 768 divisors
- $\leq 1e9$: 735134400 with 1344 divisors
- $\leq 1e10$: 6983776800 with 2304 divisors
- $\leq 1e11$: 97772875200 with 4032 divisors
- $\leq 1e12$: 963761198400 with 6720 divisors
- $\leq 1e13$: 9316358251200 with 10752 divisors
- $\leq 1e14$: 97821761637600 with 17280 divisors
- $\leq 1e15$: 866421317361600 with 26880 divisors
- $\leq 1e16$: 8086598962041600 with 41472 divisors
- $\leq 1e17$: 74801040398884800 with 64512 divisors
- $\leq 1e18$: 897612484786617600 with 103680 divisors

Useful formulas

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ — number of ways to choose k objects out of n
 $\binom{n+k-1}{k-1}$ — number of ways to choose k objects out of n with repetitions

$\frac{n!}{m}$ — Stirling numbers of the first kind; number of permutations of n elements with k cycles

$$\binom{n+1}{m} = \binom{n}{m} + \binom{n}{m-1}$$

$(x)_n = x(x-1)(x-2)\dots(x-n+1) = \sum_{k=0}^n (-1)^k \binom{n}{k} x^{n-k}$
 $\frac{n!}{m!}$ — Stirling numbers of the second kind; number of partitions of set $1, 2, \dots, n$ into k disjoint subsets.

$$\binom{n+m}{n} = \binom{n}{k} + \binom{n}{k-1}$$

$$\binom{n}{k} (x)_k = x^n$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \text{Catalan numbers}$$

$$C(x) = \frac{1}{1-4x}$$

Binomial transform

If $a_n = \sum_{k=0}^n \binom{n}{k} b_k$, then $b_n = \sum_{k=0}^n (-1)^k \binom{n}{k} a_k$

$$a = (1 \ x \ x^2 \ \dots) \quad b = (1 \ (x+1) \ (x+1)^2 \ \dots)$$

$$a_i = \frac{1}{i!} b_i = \frac{1}{i!} i!$$

Burnside's lemma

Let G be a group of *action* on set X (Ex.: cyclic shifts of array, rotations and symmetries of n matrix, ...)

Call two objects x and y *equivalent* if there is an action f that transforms x to y : $f(x) = y$.

The number of equivalence classes then can be calculated as follows: $C = \frac{1}{G} \sum_{f \in G} X^f$, where X^f

is the set of *fixed points* of f : $X^f = \{x \mid f(x) = x\}$

Generating functions

Ordinary generating function (o.g.f.) for sequence

$$a_0 \ a_1 \ a_2 \ \dots \ a_n \ \dots \text{ is } A(x) = \sum_{n=0}^{\infty} a_n x^n$$

Exponential generating function (e.g.f.) for

$$\text{sequence } a_0 \ a_1 \ a_2 \ \dots \ a_n \ \dots \text{ is } A(x) = \sum_{n=0}^{\infty} \frac{a_n x^n}{n!}$$

$$B(x) = A(x) \quad b_n = n \cdot a_n$$

$c_n = \sum_{k=0}^n a_k b_{n-k}$ (o.g.f. convolution)
 $c_n = \sum_{k=0}^n a_k b_{n-k}$ (e.g.f. convolution, compute with FFT using $a_n = \frac{a_n}{n!}$)

General linear recurrences

If $a_n = \sum_{k=1}^n b_k a_{n-k}$, then $A(x) = \frac{a_0}{1 - B(x)}$. We also can compute all a_n with Divide-and-Conquer algorithm in $O(n \log^2 n)$.

Inverse polynomial modulo x^l

Given $A(x)$, find $B(x)$ such that $A(x)B(x) \equiv 1 \pmod{x^l}$ for some $Q(x)$

1. Start with $B_0(x) = \frac{1}{a_0}$
2. Double the length of $B(x)$:
 $B_{k+1}(x) = (B_k(x)^2 A(x) + 2B_k(x)) \pmod{x^{2^{k+1}}}$

Fast subset convolution

Given array a_i of size 2^k , calculate $b_i = \sum_{j \& i = i} b_j$

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      a[i + (1 << b)] += a[i]
```

Hadamard transform

Treat array a of size 2^k as k -dimensional array of size $2 \times 2 \times \dots \times 2$, calculate FFT of that array:

```
for b = 0..k-1
  for i = 0..2^k-1
    if (i & (1 << b)) != 0:
      u = a[i], v = a[i + (1 << b)]
      a[i] = u + v
      a[i + (1 << b)] = u - v
```