GARCH, GAS, SV, and MSGARCH models:

Do we really need all of them for forecasting daily volatility?

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- Given its importance, numerous approaches have been developed for forecasting daily volatility.
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Goal

Offer insights to assist researchers and practitioners in selecting the most appropriate model given the specific characteristics of their data.

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Hereafter, let $r_t = (P_t - P_{t-1})/P_{t-1}$ denote the return at time t, where P_t represents the closing price at time t.

Models

GARCH model

Assumes that the conditional variance at time t is fully determined by past squared returns and its own past values. In its simplest form, the model is specified as:

$$r_t = \sigma_t \epsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where $\omega > 0$ and $\alpha, \beta \geq 0$ are model parameters, σ_t^2 represents the conditional variance (or squared volatility) at time t, and the innovation term ϵ_t has zero-mean and unit-variance.

In this study we considered the standard Normal and Student-t innovation distributions.

Models

SV model

Assumes that the log-conditional variance evolves stochastically following an AR(1) process. Its dynamics can be described as follows:

$$r_t = \exp(h_t/2)\epsilon_t, \tag{3}$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma\eta_t, \tag{4}$$

where h_t is the log conditional variance at time t, μ, ϕ and σ are parameters to be estimated, $\eta_t \sim N(0,1)$. In this study, ϵ_t follows either a standardized Normal or Student-t distribution.

Models

MSGARCH model

This specification allows for multiple volatility regimes. In its simplest form, the dynamics can be described as follows:

$$r_t = \sigma_t^{(k)} \epsilon_t, \tag{5}$$

$$\sigma_t^{2(k)} = \omega^{(k)} + \alpha^{(k)} r_{t-1}^2 + \beta^{(k)} \sigma_{t-1}^{2(k)}, \tag{6}$$

where $\omega^{(k)} > 0$ and $\alpha^{(k)}, \beta^{(k)} \geq 0$ are the model parameters in regime k, $\sigma_t^{2(k)}$ denotes the conditional variance in regime k at time t, and ϵ_t follows either a standardized Normal or a standardized Student-t distribution.

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The regime-switching mechanism is governed by the latent process $\{S_t\}$, assumed to be a first-order Markov chain with transition probability matrix Π. Its elements are given by

$$\pi_{ij} = \mathbb{P}(S_t = j \mid S_{t-1} = i), \tag{7}$$

representing the probability of moving from state i at time $t-1$ to state j at time t.

Models

GAS

Its central idea is that the dynamic behaviour of time-varying parameters depends on their own past values and the score of the conditional density function (hence the name *score model*).

Let $r_t|\mathcal{F}_{t-1} \sim p(r_t;\theta_t)$ with $\theta_t \in \mathbb{R}^p$ being a vector of time-varying parameters fully characterising $p(\cdot)$. Then, in the general, unrestricted, GAS specification, the dynamics of θ_t is given by

$$\theta_{t+1} = \kappa + As_t + B\theta_t, \tag{8}$$

where $s_t = S_t(\theta_t)\nabla_t(r_t,\theta_t)$, with $\nabla_t(r_t,\theta_t)$ being the score of the conditional density function and $S_t(\theta_t) = I_t(\theta_t)^{-\gamma}$ with typical values of $\gamma \in \{0, 1/2, 1\}$, and $\kappa_{p\times 1}$, $A_{p\times p}$ and $B_{p\times p}$.

Models

- Parameters are estimated by Maximum Likelohood
- For SV, the procedure of Wahl (2018) is used.
- In all cases, we are interested in $\mathbb{V}(r_{T+1}|\mathcal{F}_t)$, where $\mathcal{F}_t$ is the information available up to time t

Monte Carlo Simulations

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- Innovation distributions: standardized Normal and Student-t_7

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- The four models previously described are used both as true DGPs and for generating one-step-ahead volatility forecasts
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- Parameter values were selected to reflect characteristics of both emerging and developed markets.

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- Series are contaminated with a single additive outlier in the final month

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Parameters

More than 20 parameter vector configurations were designed to closely replicate patterns observed in real data.

Monte Carlo Simulations

Model	Parameter values 1	Parameter values 2
GARCH	$\omega = 0.18, \alpha = 0.09, \beta = 0.89$	$\omega = 0.37, \alpha = 0.14, \beta = 0.77$
GAS	$\kappa = 0.03, A = 0.22, B = 0.98$	$\kappa = 0.06, A = 0.34, B = 0.92$
SV	$\mu = 1.74, \phi = 0.97, \sigma_\eta = 0.17$	$\mu = 1.15, \phi = 0.90, \sigma_\eta = 0.36$
MSGARCH	$\omega_1 = 0.005, \alpha_1 = 0.025, \beta_1 = 0.95$ $\omega_2 = 0.1, \alpha_2 = 0.25, \beta_2 = 0.70$ $P = \begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix}$	$\omega_1 = 0.01, \alpha_1 = 0.16, \beta_1 = 0.30$ $\omega_2 = 0.18, \alpha_2 = 0.46, \beta_2 = 0.20$ $P = \begin{bmatrix} 0.98 & 0.05 \\ 0.02 & 0.95 \end{bmatrix}$

Table 1: Two parameter configurations (over 20) used in the Monte Carlo experiment

Monte Carlo Simulations

Loss Function	Formula	Loss Function	Formula
MSE	$R^{-1}\sum_{i=1}^{R}\left(\hat{\sigma}_i^2-\sigma_i^2\right)^2$	MAE	$R^{-1}\sum_{i=1}^{R}\left\lvert\hat{\sigma}_i^2-\sigma_i^2\right\rvert$
QLIKE	$R^{-1}\sum_{i=1}^{R}\left(\frac{\sigma_i^2}{\hat{\sigma}_i^2}-\log\frac{\sigma_i^2}{\hat{\sigma}_i^2}-1\right)$	$\mathrm{MAE}_{\mathrm{L}}$	$R^{-1}\sum_{i=1}^{R}\left\lvert\log\hat{\sigma}_i^2-\log\sigma_i^2\right\rvert$
$\mathrm{MSE}_{\mathrm{L}}$	$R^{-1}\sum_{i=1}^{R}\left(\log\hat{\sigma}_i^2-\log\sigma_i^2\right)^2$	$\mathrm{MAE}_{\mathrm{Sd}}$	$R^{-1}\sum_{i=1}^{R}\left\lvert\hat{\sigma}_i-\sigma_i\right\rvert$
$\mathrm{MSE}_{\mathrm{Sd}}$	$R^{-1}\sum_{i=1}^{R}\left(\hat{\sigma}_i-\sigma_i\right)^2$	$\mathrm{MAE}_{\mathrm{P}}$	$R^{-1}\sum_{i=1}^{R}\left\lvert\frac{\hat{\sigma}_i}{\sigma_i}-1\right\rvert$
$\mathrm{MSE}_{\mathrm{P}}$	$R^{-1}\sum_{i=1}^{R}\left(\frac{\hat{\sigma}_i}{\sigma_i}-1\right)^2$		

Table 2: Loss functions employed in the evaluation of volatility forecasts.

Monte Carlo Simulations

Loss Function	Formula	Loss Function	Formula
MSE	$R^{-1}\sum_{i=1}^{R}\left(\hat{\sigma}_i^2-\sigma_i^2\right)^2$	MAE	$R^{-1}\sum_{i=1}^{R}\left\lvert\hat{\sigma}_i^2-\sigma_i^2\right\rvert$
QLIKE	$R^{-1}\sum_{i=1}^{R}\left(\frac{\sigma_i^2}{\hat{\sigma}_i^2}-\log\frac{\sigma_i^2}{\hat{\sigma}_i^2}-1\right)$	$\mathrm{MAE_L}$	$R^{-1}\sum_{i=1}^{R}\left\lvert\log\hat{\sigma}_i^2-\log\sigma_i^2\right\rvert$
$\mathrm{MSE_L}$	$R^{-1}\sum_{i=1}^{R}\left(\log\hat{\sigma}_i^2-\log\sigma_i^2\right)^2$	$\mathrm{MAE_{Sd}}$	$R^{-1}\sum_{i=1}^{R}\lvert\hat{\sigma}_i-\sigma_i\rvert$
$\mathrm{MSE_{Sd}}$	$R^{-1}\sum_{i=1}^{R}(\hat{\sigma}_i-\sigma_i)^2$	$\mathrm{MAE_P}$	$R^{-1}\sum_{i=1}^{R}\left\lvert\frac{\hat{\sigma}_i}{\sigma_i}-1\right\rvert$
$\mathrm{MSE_P}$	$R^{-1}\sum_{i=1}^{R}\left(\frac{\hat{\sigma}_i}{\sigma_i}-1\right)^2$		

Table 2: Loss functions employed in the evaluation of volatility forecasts.

To select the best model (or set of best models) the model confidence set of Hansen et al. (2011) was used

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GARCH	N = 500	GARCH-N	0.0608	0.0034	0.0068	0.0048	0.0070	0.1670	0.0608	0.0497	0.0607
		GARCH-T	0.0538	0.0027	0.0055	0.0039	0.0054	0.1569	0.0565	0.0463	0.0563
		GAS-N	2.4896	0.0064	0.0107	0.0247	0.0236	0.2422	0.0691	0.0606	0.0716
		GAS-T	0.0730	0.0035	0.0073	0.0053	0.0068	0.1851	0.0651	0.0541	0.0644
		MS-N	0.1189	0.0062	0.0124	0.0089	0.0128	0.2402	0.0863	0.0709	0.0859
		MS-T	0.0992	0.0049	0.0100	0.0073	0.0096	0.2187	0.0773	0.0640	0.0764
		SV-N	0.1443	0.0086	0.0186	0.0122	0.0149	0.2838	0.1100	0.0871	0.1015
		SV-T	0.1114	0.0062	0.0133	0.0090	0.0109	0.2398	0.0904	0.0726	0.0845
	N = 1000	GARCH-N	0.0282	0.0014	0.0028	0.0020	0.0028	0.1136	0.0406	0.0335	0.0405
		GARCH-T	0.0245	0.0012	0.0023	0.0017	0.0023	0.1048	0.0374	0.0308	0.0374
		GAS-N	1.9691	0.0045	0.0071	0.0197	0.0171	0.1910	0.0513	0.0461	0.0536
		GAS-T	0.0617	0.0023	0.0046	0.0038	0.0044	0.1498	0.0507	0.0429	0.0502
		MS-N	0.0817	0.0039	0.0077	0.0058	0.0081	0.1896	0.0666	0.0554	0.0669
		MS-T	0.0566	0.0028	0.0056	0.0041	0.0055	0.1611	0.0568	0.0471	0.0564
		SV-N	0.1209	0.0072	0.0153	0.0099	0.0128	0.2703	0.1046	0.0828	0.0974
		SV-T	0.0999	0.0050	0.0106	0.0074	0.0091	0.2254	0.0839	0.0676	0.0789
	N = 2500	GARCH-N	0.0100	0.0006	0.0012	0.0008	0.0012	0.0720	0.0262	0.0214	0.0262
		GARCH-T	0.0080	0.0005	0.0010	0.0007	0.0010	0.0660	0.0243	0.0197	0.0243
		GAS-N	0.0841	0.0018	0.0035	0.0036	0.0040	0.1243	0.0397	0.0343	0.0402
		GAS-T	0.0590	0.0016	0.0033	0.0030	0.0031	0.1246	0.0418	0.0354	0.0414
		MS-N	0.0355	0.0019	0.0038	0.0027	0.0039	0.1339	0.0479	0.0395	0.0482
		MS-T	0.0303	0.0011	0.0023	0.0019	0.0023	0.1034	0.0352	0.0297	0.0352
		SV-N	0.1098	0.0065	0.0137	0.0088	0.0116	0.2644	0.1025	0.0810	0.0960
		SV-T	0.1034	0.0044	0.0093	0.0068	0.0080	0.2176	0.0803	0.0649	0.0759

Table 3: Forecast evaluation under **uncontaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GAS	N = 500	GARCH-N	0.0758	0.0045	0.0089	0.0059	0.0094	0.1868	0.0714	0.0567	0.0719
		GARCH-T	0.0744	0.0042	0.0082	0.0056	0.0089	0.1803	0.0684	0.0546	0.0692
		GAS-N	0.1552	0.0075	0.0144	0.0106	0.0168	0.2396	0.0889	0.0716	0.0907
		GAS-T	0.0419	0.0029	0.0058	0.0036	0.0059	0.1488	0.0581	0.0457	0.0580
		MS-N	0.1412	0.0088	0.0173	0.0114	0.0184	0.2678	0.1033	0.0819	0.1039
		MS-T	0.0959	0.0062	0.0125	0.0080	0.0122	0.2214	0.0861	0.0679	0.0852
		SV-N	0.1325	0.0098	0.0210	0.0125	0.0172	0.2933	0.1212	0.0929	0.1121
		SV-T	0.0797	0.0058	0.0121	0.0073	0.0105	0.2215	0.0896	0.0694	0.0845
	N = 1000	GARCH-N	0.0589	0.0030	0.0059	0.0042	0.0063	0.1590	0.0593	0.0476	0.0597
		GARCH-T	0.0530	0.0028	0.0054	0.0038	0.0058	0.1514	0.0565	0.0454	0.0570
		GAS-N	0.1198	0.0057	0.0107	0.0082	0.0130	0.2040	0.0743	0.0605	0.0761
		GAS-T	0.0221	0.0015	0.0030	0.0019	0.0029	0.1052	0.0408	0.0322	0.0405
		MS-N	0.1003	0.0069	0.0134	0.0086	0.0152	0.2258	0.0870	0.0691	0.0887
		MS-T	0.0696	0.0044	0.0087	0.0056	0.0089	0.1827	0.0703	0.0557	0.0701
		SV-N	0.1119	0.0082	0.0173	0.0103	0.0146	0.2808	0.1155	0.0887	0.1075
		SV-T	0.0651	0.0046	0.0096	0.0059	0.0084	0.2099	0.0844	0.0656	0.0799
	N = 2500	GARCH-N	0.0465	0.0022	0.0043	0.0032	0.0047	0.1337	0.0487	0.0396	0.0495
		GARCH-T	0.0425	0.0020	0.0039	0.0030	0.0044	0.1277	0.0465	0.0379	0.0472
		GAS-N	0.1219	0.0049	0.0089	0.0076	0.0120	0.1851	0.0650	0.0539	0.0674
		GAS-T	0.0079	0.0005	0.0010	0.0007	0.0010	0.0639	0.0245	0.0195	0.0244
		MS-N	0.0745	0.0049	0.0092	0.0060	0.0116	0.1828	0.0706	0.0559	0.0727
		MS-T	0.0377	0.0023	0.0045	0.0030	0.0046	0.1354	0.0509	0.0408	0.0510
		SV-N	0.0982	0.0072	0.0152	0.0090	0.0131	0.2743	0.1127	0.0866	0.1057
		SV-T	0.0533	0.0036	0.0076	0.0047	0.0068	0.2000	0.0796	0.0621	0.0760

Table 4: Forecast evaluation under **uncontaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

		Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: SV	N = 500	GARCH-N	0.5510	0.0434	0.0784	0.0487	0.1153	0.5507	0.2178	0.1708	0.2396
		GARCH-T	0.5214	0.0420	0.0759	0.0468	0.1098	0.5436	0.2150	0.1687	0.2381
		GAS-N	0.7431	0.0475	0.0842	0.0561	0.1320	0.5795	0.2235	0.1770	0.2495
		GAS-T	0.4303	0.0362	0.0671	0.0403	0.0887	0.5136	0.2061	0.1606	0.2242
		MS-N	0.5349	0.0444	0.0813	0.0494	0.1153	0.5611	0.2242	0.1751	0.2443
		MS-T	0.5231	0.0413	0.0765	0.0472	0.1026	0.5594	0.2219	0.1739	0.2406
		SV-N	0.5815	0.0391	0.0843	0.0527	0.0726	0.5548	0.2278	0.1755	0.2128
		SV-T	0.4341	0.0322	0.0646	0.0397	0.0685	0.4977	0.2016	0.1564	0.2028
	N = 1000	GARCH-N	0.4757	0.0380	0.0693	0.0427	0.0975	0.5223	0.2067	0.1621	0.2269
		GARCH-T	0.4715	0.0383	0.0694	0.0426	0.0985	0.5239	0.2076	0.1627	0.2295
		GAS-N	0.6814	0.0444	0.0769	0.0511	0.1383	0.5464	0.2129	0.1679	0.2389
		GAS-T	0.4005	0.0339	0.0628	0.0376	0.0824	0.4996	0.2004	0.1562	0.2181
		MS-N	0.4892	0.0413	0.0763	0.0456	0.1038	0.5415	0.2181	0.1697	0.2375
		MS-T	0.4728	0.0387	0.0713	0.0433	0.0971	0.5340	0.2128	0.1664	0.2314
		SV-N	0.5298	0.0353	0.0761	0.0479	0.0650	0.5376	0.2190	0.1695	0.2048
		SV-T	0.3978	0.0291	0.0581	0.0360	0.0617	0.4767	0.1920	0.1494	0.1937
	N = 2500	GARCH-N	0.4618	0.0369	0.0671	0.0414	0.0942	0.5115	0.2025	0.1587	0.2227
		GARCH-T	0.4654	0.0377	0.0682	0.0419	0.0976	0.5170	0.2048	0.1605	0.2270
		GAS-N	0.7886	0.0451	0.0772	0.0534	0.1437	0.5494	0.2118	0.1675	0.2395
		GAS-T	0.3921	0.0334	0.0617	0.0369	0.0815	0.4920	0.1975	0.1539	0.2155
		MS-N	0.4681	0.0410	0.0742	0.0438	0.1069	0.5297	0.2144	0.1664	0.2370
		MS-T	0.4307	0.0361	0.0662	0.0398	0.0909	0.5072	0.2028	0.1583	0.2221
		SV-N	0.5254	0.0347	0.0743	0.0471	0.0646	0.5330	0.2160	0.1675	0.2031
		SV-T	0.3888	0.0284	0.0565	0.0351	0.0608	0.4688	0.1886	0.1468	0.1910

Table 5: Forecast evaluation under **uncontaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

		Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: MSGARCH	N = 500	GARCH-N	0.0095	0.0056	0.0116	0.0025	0.0105	0.0610	0.0786	0.0343	0.0775
		GARCH-T	0.0076	0.0050	0.0101	0.0021	0.0099	0.0584	0.0754	0.0329	0.0755
		GAS-N	0.0115	0.0060	0.0121	0.0027	0.0121	0.0642	0.0809	0.0356	0.0807
		GAS-T	0.0090	0.0055	0.0112	0.0024	0.0104	0.0620	0.0794	0.0347	0.0785
		MS-N	0.0076	0.0052	0.0106	0.0021	0.0100	0.0591	0.0773	0.0335	0.0762
		MS-T	0.0095	0.0054	0.0112	0.0024	0.0102	0.0611	0.0780	0.0341	0.0764
		SV-N	0.0201	0.0150	0.0332	0.0061	0.0250	0.1051	0.1490	0.0620	0.1341
		SV-T	0.0161	0.0112	0.0245	0.0047	0.0189	0.0880	0.1220	0.0513	0.1111
	N = 1000	GARCH-N	0.0049	0.0031	0.0063	0.0013	0.0060	0.0444	0.0580	0.0251	0.0577
		GARCH-T	0.0044	0.0030	0.0060	0.0012	0.0058	0.0437	0.0569	0.0247	0.0569
		GAS-N	0.0066	0.0038	0.0075	0.0016	0.0078	0.0505	0.0638	0.0281	0.0644
		GAS-T	0.0067	0.0039	0.0080	0.0017	0.0075	0.0522	0.0665	0.0292	0.0660
		MS-N	0.0053	0.0033	0.0068	0.0014	0.0066	0.0479	0.0613	0.0268	0.0610
		MS-T	0.0057	0.0030	0.0060	0.0013	0.0060	0.0454	0.0570	0.0251	0.0568
		SV-N	0.0166	0.0133	0.0288	0.0052	0.0226	0.1022	0.1450	0.0603	0.1317
		SV-T	0.0133	0.0092	0.0199	0.0038	0.0159	0.0813	0.1118	0.0472	0.1027
	N = 2500	GARCH-N	0.0027	0.0018	0.0037	0.0007	0.0035	0.0348	0.0457	0.0198	0.0455
		GARCH-T	0.0026	0.0018	0.0037	0.0007	0.0036	0.0360	0.0467	0.0203	0.0468
		GAS-N	0.0294	0.0039	0.0071	0.0026	0.0105	0.0505	0.0580	0.0264	0.0593
		GAS-T	0.0058	0.0031	0.0063	0.0014	0.0058	0.0471	0.0592	0.0262	0.0587
		MS-N	0.0021	0.0015	0.0030	0.0006	0.0030	0.0321	0.0415	0.0181	0.0415
		MS-T	0.0021	0.0014	0.0028	0.0006	0.0028	0.0306	0.0390	0.0171	0.0390
		SV-N	0.0161	0.0127	0.0273	0.0050	0.0218	0.1026	0.1455	0.0605	0.1329
		SV-T	0.0124	0.0084	0.0180	0.0035	0.0146	0.0792	0.1084	0.0459	0.1000

Table 6: Forecast evaluation under **uncontaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GARCH	N = 500	GARCH-N	3.0883	0.1346	0.2001	0.1837	0.5500	1.2254	0.3537	0.3240	0.4873
		GARCH-T	3.1632	0.1421	0.2104	0.1914	0.5799	1.2572	0.3629	0.3326	0.5047
		GAS-N	> 10	0.7279	0.5480	1.1870	> 10	2.9836	0.5240	0.5635	1.2506
		GAS-T	0.3745	0.0195	0.0355	0.0274	0.0477	0.4594	0.1529	0.1310	0.1710
		MS-N	> 10	0.4670	0.1858	0.8437	> 10	1.7158	0.2822	0.2763	0.7470
		MS-T	2.0181	0.0850	0.1237	0.1144	0.3986	0.8429	0.2521	0.2266	0.3355
		SV-N	1.2415	0.0617	0.0946	0.0808	0.2407	0.6494	0.2054	0.1800	0.2581
		SV-T	0.2877	0.0164	0.0300	0.0223	0.0398	0.3886	0.1333	0.1126	0.1464
	N = 1000	GARCH-N	2.8981	0.1419	0.2165	0.1879	0.5286	1.2992	0.3838	0.3484	0.5255
		GARCH-T	2.9288	0.1456	0.2216	0.1914	0.5453	1.3150	0.3895	0.3532	0.5351
		GAS-N	¿ 10	2.4645	0.7976	4.3400	¿ 10	6.7300	0.6696	0.7664	3.1338
		GAS-T	0.3323	0.0182	0.0333	0.0250	0.0441	0.4436	0.1500	0.1276	0.1675
		MS-N	2.2681	0.1139	0.1699	0.1471	0.4476	1.0159	0.3090	0.2764	0.4223
		MS-T	1.9027	0.0972	0.1494	0.1272	0.3523	0.9658	0.2956	0.2639	0.3918
		SV-N	1.1118	0.0622	0.0982	0.0793	0.2137	0.6791	0.2167	0.1896	0.2749
		SV-T	0.3140	0.0184	0.0330	0.0245	0.0465	0.4010	0.1365	0.1157	0.1529
	N = 2500	GARCH-N	2.6449	0.1410	0.2203	0.1836	0.4778	1.3199	0.3980	0.3583	0.5390
		GARCH-T	2.6555	0.1429	0.2228	0.1853	0.4874	1.3283	0.4009	0.3608	0.5438
		GAS-N	¿ 10	1.2223	0.9560	1.9592	¿ 10	4.6304	0.7861	0.8607	2.0082
		GAS-T	0.3220	0.0181	0.0330	0.0244	0.0434	0.4429	0.1515	0.1282	0.1690
		MS-N	2.0581	0.1143	0.1783	0.1453	0.3938	1.0940	0.3419	0.3024	0.4560
		MS-T	2.0453	0.1096	0.1714	0.1419	0.3795	1.0995	0.3378	0.3013	0.4470
		SV-N	1.0545	0.0628	0.1016	0.0793	0.1981	0.7094	0.2280	0.1991	0.2887
		SV-T	0.3410	0.0204	0.0366	0.0267	0.0515	0.4218	0.1442	0.1220	0.1632

Table 7: Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GAS	N = 500	GARCH-N	2.3374	0.1265	0.1979	0.1592	0.4280	1.1736	0.3661	0.3223	0.4919
		GARCH-T	2.2496	0.1293	0.2022	0.1591	0.4346	1.1698	0.3681	0.3230	0.4971
		GAS-N	> 10	0.5658	0.4565	0.8755	> 10	2.4650	0.5154	0.5090	1.0801
		GAS-T	0.3279	0.0183	0.0331	0.0242	0.0449	0.4211	0.1467	0.1224	0.1646
		MS-N	1.4286	0.0842	0.1324	0.1015	0.2885	0.8304	0.2749	0.2350	0.3573
		MS-T	1.3299	0.0694	0.1088	0.0871	0.2532	0.7555	0.2462	0.2120	0.3146
		SV-N	0.7552	0.0474	0.0793	0.0581	0.1415	0.5895	0.2047	0.1711	0.2423
		SV-T	0.2182	0.0142	0.0259	0.0177	0.0342	0.3384	0.1237	0.1009	0.1360
	N = 1000	GARCH-N	2.4847	0.1440	0.2245	0.1760	0.4877	1.2687	0.3990	0.3503	0.5430
		GARCH-T	2.4037	0.1442	0.2251	0.1740	0.4859	1.2633	0.4004	0.3504	0.5446
		GAS-N	> 10	0.6541	0.6814	0.9031	> 10	2.9270	0.6593	0.6602	1.3132
		GAS-T	0.2777	0.0167	0.0305	0.0216	0.0404	0.4003	0.1423	0.1176	0.1588
		MS-N	1.4652	0.0981	0.1544	0.1127	0.3298	0.9060	0.3073	0.2602	0.4046
		MS-T	1.1638	0.0768	0.1237	0.0904	0.2454	0.8100	0.2739	0.2324	0.3499
		SV-N	0.8344	0.0564	0.0924	0.0663	0.1723	0.6341	0.2187	0.1838	0.2705
		SV-T	0.2195	0.0150	0.0271	0.0184	0.0371	0.3332	0.1222	0.0996	0.1360
	N = 2500	GARCH-N	2.4426	0.1515	0.2367	0.1810	0.5081	1.3132	0.4175	0.3652	0.5690
		GARCH-T	2.4032	0.1516	0.2369	0.1798	0.5081	1.3093	0.4180	0.3649	0.5695
		GAS-N	> 10	0.8662	0.9124	1.1191	8.7939	3.5148	0.7818	0.7963	1.6479
		GAS-T	0.2436	0.0160	0.0292	0.0200	0.0383	0.3867	0.1401	0.1148	0.1561
		MS-N	1.5815	0.1109	0.1753	0.1257	0.3644	0.9950	0.3378	0.2863	0.4486
		MS-T	1.4069	0.0912	0.1471	0.1086	0.2873	0.9248	0.3086	0.2637	0.3996
		SV-N	0.8587	0.0620	0.1008	0.0708	0.1911	0.6581	0.2282	0.1915	0.2883
		SV-T	0.2146	0.0162	0.0291	0.0190	0.0403	0.3349	0.1247	0.1010	0.1404

Table 8: Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: SV	N = 500	GARCH-N	2.3405	0.1669	0.2518	0.1810	0.6163	1.1962	0.4074	0.3438	0.5685
		GARCH-T	2.4600	0.1786	0.2682	0.1924	0.6637	1.2478	0.4235	0.3583	0.5971
		GAS-N	9.1394	0.3148	0.3923	0.3864	2.1378	1.6785	0.4840	0.4360	0.7924
		GAS-T	0.6880	0.0609	0.1036	0.0635	0.1748	0.6623	0.2568	0.2037	0.3083
		MS-N	1.3548	0.1124	0.1775	0.1166	0.3800	0.9082	0.3345	0.2721	0.4364
		MS-T	1.3345	0.1042	0.1659	0.1111	0.3481	0.8894	0.3246	0.2650	0.4190
		SV-N	0.9202	0.0695	0.1257	0.0812	0.1940	0.7007	0.2725	0.2157	0.2999
		SV-T	0.5707	0.0504	0.0886	0.0535	0.1379	0.5877	0.2329	0.1828	0.2683
	N = 1000	GARCH-N	2.3598	0.1754	0.2641	0.1874	0.6468	1.2409	0.4237	0.3575	0.5955
		GARCH-T	2.4155	0.1828	0.2745	0.1940	0.6760	1.2714	0.4337	0.3663	0.6133
		GAS-N	¿ 10	0.4004	0.4537	0.5307	6.7466	1.9746	0.5362	0.4885	0.9334
		GAS-T	0.6289	0.0565	0.0965	0.0587	0.1597	0.6332	0.2473	0.1955	0.2956
		MS-N	1.3620	0.1172	0.1839	0.1195	0.3989	0.9275	0.3420	0.2782	0.4506
		MS-T	1.2798	0.1058	0.1677	0.1103	0.3529	0.8911	0.3264	0.2663	0.4247
		SV-N	1.0067	0.0784	0.1337	0.0878	0.2468	0.7289	0.2771	0.2219	0.3210
		SV-T	0.5215	0.0463	0.0816	0.0491	0.1245	0.5656	0.2244	0.1760	0.2573
	N = 2500	GARCH-N	2.3625	0.1828	0.2751	0.1924	0.6745	1.2698	0.4357	0.3671	0.6160
		GARCH-T	2.4527	0.1899	0.2848	0.1998	0.7046	1.3023	0.4450	0.3758	0.6327
		GAS-N	8.1638	0.4008	0.5167	0.4575	2.2197	2.0234	0.5905	0.5340	0.9894
		GAS-T	0.5982	0.0553	0.0944	0.0567	0.1563	0.6225	0.2445	0.1928	0.2920
		MS-N	1.5132	0.1330	0.2056	0.1336	0.4676	0.9959	0.3649	0.2980	0.4922
		MS-T	1.3460	0.1144	0.1796	0.1174	0.3907	0.9322	0.3416	0.2788	0.4504
		SV-N	1.0485	0.0838	0.1396	0.0917	0.2693	0.7564	0.2848	0.2292	0.3400
		SV-T	0.5036	0.0456	0.0801	0.0478	0.1235	0.5575	0.2216	0.1736	0.2547

Table 9: Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: MSGARCH	N = 500	GARCH-N	0.1912	0.0959	0.1429	0.0386	0.3809	0.2621	0.2713	0.1313	0.3647
		GARCH-T	0.1789	0.0997	0.1517	0.0390	0.3680	0.2726	0.2862	0.1379	0.3840
		GAS-N	¿10	1.7069	0.4714	0.9703	¿10	1.4028	0.4055	0.2584	2.1100
		GAS-T	0.0316	0.0212	0.0378	0.0083	0.0551	0.1224	0.1472	0.0665	0.1652
		MS-N	0.1143	0.0625	0.0947	0.0243	0.2428	0.1921	0.2115	0.0994	0.2718
		MS-T	0.1218	0.0612	0.0921	0.0244	0.2422	0.1918	0.2081	0.0984	0.2670
		SV-N	0.0457	0.0316	0.0572	0.0123	0.0851	0.1357	0.1725	0.0757	0.1814
		SV-T	0.0296	0.0204	0.0369	0.0079	0.0531	0.1092	0.1368	0.0606	0.1463
	N = 1000	GARCH-N	0.1700	0.0980	0.1489	0.0381	0.3568	0.2653	0.2774	0.1340	0.3742
		GARCH-T	0.1550	0.0962	0.1500	0.0368	0.3275	0.2704	0.2890	0.1383	0.3841
		GAS-N	3.7793	0.5344	0.5033	0.2460	8.2986	0.6975	0.4706	0.2680	1.0039
		GAS-T	0.0246	0.0182	0.0326	0.0069	0.0459	0.1121	0.1371	0.0615	0.1531
		MS-N	0.1083	0.0640	0.0972	0.0243	0.2418	0.1932	0.2148	0.1006	0.2780
		MS-T	0.1204	0.0686	0.1032	0.0264	0.2631	0.2046	0.2232	0.1055	0.2908
		SV-N	0.0420	0.0312	0.0550	0.0116	0.0863	0.1345	0.1699	0.0749	0.1842
		SV-T	0.0241	0.0179	0.0325	0.0068	0.0451	0.1022	0.1286	0.0569	0.1380
	N = 2500	GARCH-N	0.1627	0.0995	0.1525	0.0381	0.3495	0.2630	0.2763	0.1333	0.3753
		GARCH-T	0.1506	0.0972	0.1526	0.0368	0.3238	0.2721	0.2926	0.1398	0.3896
		GAS-N	2.6077	0.5596	0.5983	0.2486	4.9391	0.7648	0.5448	0.3080	1.1039
		GAS-T	0.0223	0.0168	0.0302	0.0063	0.0418	0.1056	0.1300	0.0582	0.1451
		MS-N	0.0979	0.0639	0.0997	0.0237	0.2231	0.2011	0.2296	0.1064	0.2932
		MS-T	0.1238	0.0745	0.1140	0.0284	0.2703	0.2221	0.2438	0.1150	0.3178
		SV-N	0.0420	0.0314	0.0549	0.0117	0.0864	0.1372	0.1714	0.0760	0.1887
		SV-T	0.0237	0.0178	0.0322	0.0067	0.0450	0.1028	0.1284	0.0570	0.1394

Table 10: Forecast evaluation under **contaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GARCH	N = 500	GARCH-N	1.0683	0.0919	0.1420	0.0934	0.3209	0.6657	0.2707	0.2097	0.3605
		GARCH-T	1.0718	0.0948	0.1445	0.0947	0.3387	0.6541	0.2645	0.2056	0.3581
		GAS-N	> 10	0.5513	0.4396	0.7427	> 10	1.7941	0.4144	0.3859	0.9636
		GAS-T	0.1926	0.0197	0.0350	0.0198	0.0512	0.2964	0.1377	0.1002	0.1553
		MS-N	0.7403	0.0672	0.1042	0.0667	0.2345	0.5161	0.2166	0.1653	0.2816
		MS-T	0.7028	0.0612	0.0958	0.0620	0.2098	0.5031	0.2103	0.1608	0.2693
		SV-N	0.2893	0.0311	0.0538	0.0302	0.0873	0.3450	0.1627	0.1174	0.1815
		SV-T	0.1848	0.0198	0.0356	0.0196	0.0506	0.2849	0.1357	0.0975	0.1476
	N = 1000	GARCH-N	0.9879	0.0905	0.1403	0.0900	0.3093	0.6383	0.2608	0.2019	0.3508
		GARCH-T	0.9925	0.0920	0.1416	0.0908	0.3190	0.6327	0.2575	0.1998	0.3492
		GAS-N	> 10	0.5204	0.5124	0.6299	7.1201	1.8003	0.4716	0.4321	0.9912
		GAS-T	0.1772	0.0189	0.0335	0.0186	0.0488	0.2809	0.1306	0.0950	0.1479
		MS-N	0.6571	0.0638	0.1002	0.0621	0.2140	0.4990	0.2128	0.1614	0.2757
		MS-T	0.7404	0.0685	0.1069	0.0677	0.2348	0.5282	0.2217	0.1693	0.2887
		SV-N	0.2807	0.0306	0.0527	0.0295	0.0854	0.3433	0.1615	0.1167	0.1826
		SV-T	0.2111	0.0228	0.0402	0.0223	0.0603	0.2974	0.1405	0.1013	0.1561
	N = 2500	GARCH-N	0.9891	0.0925	0.1431	0.0913	0.3159	0.6334	0.2579	0.2001	0.3503
		GARCH-T	0.9898	0.0936	0.1444	0.0919	0.3220	0.6328	0.2578	0.2000	0.3513
		GAS-N	> 10	0.5666	0.5944	0.6420	5.2625	1.9374	0.5267	0.4822	1.0928
		GAS-T	0.1752	0.0189	0.0334	0.0185	0.0488	0.2723	0.1268	0.0922	0.1445
		MS-N	0.7056	0.0693	0.1091	0.0673	0.2294	0.5264	0.2235	0.1699	0.2924
		MS-T	0.8006	0.0765	0.1196	0.0749	0.2577	0.5605	0.2336	0.1792	0.3093
		SV-N	0.2854	0.0314	0.0538	0.0301	0.0884	0.3482	0.1629	0.1180	0.1868
		SV-T	0.2437	0.0268	0.0466	0.0258	0.0729	0.3215	0.1508	0.1092	0.1712

Table 11: Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in developed markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: GAS	N = 500	GARCH-N	1.0568	0.0988	0.1511	0.0936	0.3623	0.6491	0.2827	0.2107	0.3801
		GARCH-T	1.0268	0.1024	0.1564	0.0951	0.3655	0.6464	0.2808	0.2099	0.3824
		GAS-N	> 10	0.9756	0.4697	1.1284	> 10	2.1705	0.4428	0.4002	1.4168
		GAS-T	0.1729	0.0210	0.0365	0.0189	0.0564	0.2679	0.1355	0.0943	0.1557
		MS-N	0.5787	0.0613	0.0956	0.0553	0.2185	0.4579	0.2144	0.1545	0.2746
		MS-T	0.6544	0.0586	0.0912	0.0558	0.2070	0.4538	0.2080	0.1511	0.2655
		SV-N	0.2208	0.0295	0.0516	0.0258	0.0812	0.3183	0.1684	0.1146	0.1855
		SV-T	0.1554	0.0210	0.0370	0.0183	0.0563	0.2607	0.1378	0.0939	0.1521
	N = 1000	GARCH-N	1.0954	0.1080	0.1641	0.1004	0.3882	0.6671	0.2876	0.2157	0.3951
		GARCH-T	1.0536	0.1073	0.1632	0.0986	0.3819	0.6564	0.2838	0.2127	0.3908
		GAS-N	> 10	0.8242	0.6147	0.9185	> 10	2.1632	0.5295	0.4797	1.3530
		GAS-T	0.1561	0.0197	0.0343	0.0176	0.0525	0.2472	0.1259	0.0874	0.1453
		MS-N	0.5677	0.0640	0.1011	0.0568	0.2165	0.4699	0.2210	0.1590	0.2844
		MS-T	0.5654	0.0604	0.0963	0.0551	0.1962	0.4637	0.2159	0.1561	0.2754
		SV-N	0.2338	0.0321	0.0548	0.0274	0.0912	0.3244	0.1693	0.1160	0.1922
		SV-T	0.1723	0.0240	0.0416	0.0205	0.0658	0.2731	0.1431	0.0980	0.1615
	N = 2500	GARCH-N	1.0651	0.1107	0.1680	0.1011	0.3961	0.6648	0.2868	0.2153	0.3974
		GARCH-T	1.0506	0.1108	0.1680	0.1007	0.3963	0.6616	0.2859	0.2145	0.3966
		GAS-N	> 10	0.8064	0.7435	0.8791	> 10	2.2819	0.5954	0.5434	1.4013
		GAS-T	0.1492	0.0198	0.0344	0.0173	0.0526	0.2378	0.1215	0.0842	0.1412
		MS-N	0.6244	0.0732	0.1151	0.0641	0.2443	0.5057	0.2365	0.1708	0.3094
		MS-T	0.6361	0.0697	0.1101	0.0629	0.2299	0.4970	0.2285	0.1664	0.2979
		SV-N	0.2392	0.0340	0.0574	0.0285	0.0980	0.3277	0.1703	0.1170	0.1970
		SV-T	0.1910	0.0275	0.0470	0.0231	0.0767	0.2904	0.1519	0.1041	0.1740

Table 12: Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in developed markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: SV	N = 500	GARCH-N	1.7035	0.2143	0.3061	0.1660	0.9247	0.9706	0.4411	0.3206	0.6421
		GARCH-T	2.1332	0.2660	0.3613	0.2026	1.2964	1.0746	0.4723	0.3489	0.7283
		GAS-N	> 10	0.9192	0.5236	1.5360	> 10	2.6725	0.5231	0.4422	1.4309
		GAS-T	0.5958	0.1019	0.1640	0.0736	0.3384	0.6164	0.3212	0.2192	0.4058
		MS-N	1.0378	0.1570	0.2325	0.1140	0.6370	0.7631	0.3749	0.2629	0.5169
		MS-T	1.0500	0.1541	0.2307	0.1142	0.6040	0.7766	0.3782	0.2663	0.5175
		SV-N	0.6464	0.0878	0.1598	0.0758	0.2644	0.5870	0.3105	0.2102	0.3329
		SV-T	0.5342	0.0863	0.1468	0.0658	0.2703	0.5650	0.3007	0.2031	0.3531
	N = 1000	GARCH-N	1.8119	0.2300	0.3218	0.1767	1.0447	0.9958	0.4460	0.3266	0.6635
		GARCH-T	2.1239	0.2630	0.3589	0.2023	1.2628	1.0774	0.4701	0.3485	0.7227
		GAS-N	> 10	0.6912	0.5914	0.7223	> 10	1.8911	0.5569	0.4632	1.2376
		GAS-T	0.5854	0.0999	0.1613	0.0723	0.3299	0.6105	0.3185	0.2172	0.4013
		MS-N	1.0032	0.1575	0.2342	0.1135	0.6250	0.7655	0.3774	0.2644	0.5198
		MS-T	1.0130	0.1561	0.2305	0.1130	0.6436	0.7684	0.3756	0.2642	0.5178
		SV-N	0.6907	0.0945	0.1666	0.0801	0.3113	0.6013	0.3139	0.2139	0.3440
		SV-T	0.5127	0.0815	0.1398	0.0629	0.2508	0.5473	0.2912	0.1966	0.3394
	N = 2500	GARCH-N	1.8262	0.2388	0.3302	0.1807	1.1230	0.9991	0.4466	0.3275	0.6738
		GARCH-T	2.0609	0.2644	0.3597	0.2005	1.2920	1.0676	0.4688	0.3467	0.7230
		GAS-N	> 10	0.6324	0.6026	0.5729	> 10	1.7504	0.5737	0.4688	1.1966
		GAS-T	0.5652	0.0992	0.1602	0.0710	0.3280	0.6071	0.3180	0.2166	0.4011
		MS-N	1.1017	0.1746	0.2535	0.1242	0.7540	0.8041	0.3921	0.2763	0.5524
		MS-T	0.9973	0.1607	0.2354	0.1140	0.6929	0.7713	0.3791	0.2661	0.5258
		SV-N	0.6786	0.0961	0.1667	0.0795	0.3315	0.5991	0.3129	0.2132	0.3473
		SV-T	0.4942	0.0801	0.1368	0.0611	0.2481	0.5396	0.2873	0.1940	0.3357

Table 13: Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in developed markets

Monte Carlo Simulations

DGP	N	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
DGP: MSGARCH	N = 500	GARCH-N	0.1129	0.3905	0.4421	0.0478	2.8826	0.1783	0.4410	0.1333	0.8227
		GARCH-T	0.1088	0.3466	0.3775	0.0437	2.6597	0.1566	0.3599	0.1126	0.6975
		GAS-N	5.1360	> 10	> 10	5.0656	> 10	> 10	> 10	1.2573	1.9577
		GAS-T	0.0263	0.0768	0.1173	0.0114	0.2938	0.0809	0.2330	0.0650	0.2941
		MS-N	0.0731	0.2190	0.2491	0.0282	1.6653	0.1164	0.2916	0.0871	0.5026
		MS-T	0.0723	0.2254	0.2508	0.0285	1.7606	0.1145	0.2816	0.0850	0.5005
		SV-N	0.0322	0.1360	0.2243	0.0178	0.5179	0.1155	0.3813	0.0994	0.4257
		SV-T	0.0316	0.1216	0.2010	0.0167	0.4601	0.1094	0.3506	0.0928	0.3903
	N = 1000	GARCH-N	0.1053	0.3888	0.4332	0.0466	2.8596	0.1712	0.4169	0.1272	0.7988
		GARCH-T	0.1041	0.3541	0.3845	0.0436	2.7213	0.1551	0.3570	0.1119	0.7026
		GAS-N	0.8070	> 10	> 10	7.8892	> 10	> 10	> 10	1.6321	3.1176
		GAS-T	0.0198	0.0708	0.1098	0.0097	0.2566	0.0748	0.2274	0.0620	0.2844
		MS-N	0.0626	0.2200	0.2433	0.0266	1.7278	0.1058	0.2662	0.0797	0.4815
		MS-T	0.0667	0.2379	0.2616	0.0285	1.8585	0.1078	0.2673	0.0806	0.5032
		SV-N	0.0301	0.1376	0.2258	0.0174	0.5189	0.1146	0.3823	0.0993	0.4315
		SV-T	0.0296	0.1333	0.2177	0.0169	0.5047	0.1123	0.3710	0.0969	0.4210
	N = 2500	GARCH-N	0.1063	0.3931	0.4326	0.0471	2.9347	0.1697	0.4039	0.1247	0.7905
		GARCH-T	0.1058	0.3649	0.3934	0.0446	2.8559	0.1571	0.3591	0.1130	0.7154
		GAS-N	1.5851	> 10	> 10	> 10	> 10	> 10	> 10	2.0218	4.2721
		GAS-T	0.0186	0.0683	0.1061	0.0093	0.2455	0.0732	0.2254	0.0612	0.2811
		MS-N	0.0656	0.2273	0.2473	0.0274	1.8238	0.1012	0.2531	0.0760	0.4766
		MS-T	0.0681	0.2443	0.2663	0.0290	1.9376	0.1032	0.2566	0.0774	0.5000
		SV-N	0.0300	0.1400	0.2283	0.0175	0.5321	0.1148	0.3840	0.0997	0.4371
		SV-T	0.0296	0.1384	0.2258	0.0173	0.5237	0.1140	0.3812	0.0990	0.4340

Table 14: Forecast evaluation under **contaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in developed markets

Empirical Application

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- The Model Confidence Set of Hansen et al. (2011) was used to select the best set of models

Empirical Application

	Min	Q_1	Med	Mean	Q_3	Max	Skew	Kurt	Sd	ACF_1
MMM	-12.9450	-0.5931	0.0540	0.0394	0.7338	22.9906	0.6217	26.4331	1.4631	-0.0438
AMZN	-14.0494	-0.9025	0.0938	0.1136	1.1906	15.7457	0.2606	9.2383	2.0610	-0.0190
AXP	-14.8187	-0.7041	0.0777	0.0751	0.9279	21.8823	0.8175	22.6249	1.8245	-0.0533
AMGN	-9.5846	-0.7233	0.0317	0.0614	0.8461	11.8180	0.3778	8.8610	1.5232	-0.0755
AAPL	-12.8647	-0.7397	0.1004	0.1129	1.0357	11.9808	-0.0434	8.1919	1.7551	-0.0400
BA	-23.8484	-0.9198	0.0679	0.0635	1.0459	24.3186	0.1755	21.1000	2.2543	0.0540
CAT	-14.2822	-0.8497	0.0590	0.0760	1.0278	10.3321	-0.1333	6.7862	1.8298	0.0033
CVX	-22.1248	-0.7318	0.0703	0.0468	0.8295	22.7407	-0.2220	26.8621	1.6872	-0.0671
CSCO	-16.2107	-0.6509	0.0520	0.0481	0.7960	15.9505	-0.4466	18.6962	1.6295	-0.0638
KO	-9.6725	-0.4607	0.0566	0.0388	0.5729	6.4796	-0.6067	11.8785	1.0721	-0.0340
HD	-19.7938	-0.5921	0.0940	0.0891	0.8301	13.7508	-0.6515	18.0384	1.4627	-0.0424
HON	-12.0868	-0.6024	0.0713	0.0655	0.7542	15.0684	0.0125	12.3544	1.4405	-0.0309
INTC	-26.0585	-0.9213	0.0566	0.0320	1.0257	19.5213	-0.7099	18.7537	2.0229	-0.0703
IBM	-12.8507	-0.6128	0.0556	0.0376	0.7120	11.3010	-0.4608	12.7177	1.4006	-0.0331
JNJ	-10.0379	-0.4471	0.0312	0.0386	0.5695	7.9977	-0.1134	12.0442	1.0521	-0.0681
JPM	-14.9649	-0.7608	0.0586	0.0719	0.9074	18.0125	0.2216	12.8850	1.7483	-0.0978
MCD	-15.8753	-0.4841	0.0750	0.0585	0.6014	18.1255	0.3910	33.9165	1.1706	-0.1008
MRK	-9.8630	-0.6197	0.0310	0.0484	0.7436	10.4080	-0.0100	9.6857	1.3077	-0.0638
MSFT	-14.7390	-0.7078	0.0698	0.0900	0.9262	14.2169	0.0265	10.6409	1.6110	-0.1046
NKE	-19.9809	-0.7633	0.0508	0.0607	0.9280	15.5314	0.0250	17.1611	1.7571	-0.0378
PG	-8.7373	-0.4713	0.0568	0.0444	0.5846	12.0090	0.1688	14.9902	1.0764	-0.0766
GS	-12.7910	-0.8520	0.0549	0.0555	0.9845	17.5803	0.0073	11.5460	1.8007	-0.0649
TRV	-20.8004	-0.5772	0.1044	0.0612	0.7408	13.2902	-1.1312	25.3817	1.4064	-0.1548
UNH	-17.2769	-0.6889	0.0957	0.0932	0.8602	12.7989	-0.0642	12.5191	1.6006	-0.0654
VZ	-7.4978	-0.5835	0.0495	0.0317	0.6449	9.2705	0.0711	8.3111	1.1627	-0.0359
V	-13.5472	-0.6974	0.1312	0.0861	0.8665	14.9973	0.1765	13.3194	1.5741	-0.0965
WMT	-11.3758	-0.5144	0.0682	0.0589	0.6375	11.7085	0.1154	18.9946	1.2087	-0.0578
DIS	-13.1632	-0.6788	0.0445	0.0503	0.8230	14.4123	0.2035	13.5132	1.6423	-0.0512
CRM	-19.7371	-0.9626	0.0860	0.1029	1.1906	26.0449	0.4990	14.0377	2.2822	-0.0275

Empirical Application

	MSE								QLIKE							
	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T
MMM	3.811	4.367	4.460	2.095	2.036	2.916	0.705	0.656	0.340	0.347	0.374	0.285	0.278	0.298	0.377	0.304
AMZN	6.598	6.922	4.303	7.349	5.648	7.229	2.896	2.386	0.274	0.272	0.284	0.272	0.252	0.268	0.385	0.300
AXP	7.114	6.763	4.254	5.992	4.613	7.500	1.747	1.446	0.319	0.308	0.336	0.308	0.280	0.307	0.369	0.310
AMGN	1.466	1.710	1.533	1.240	1.392	1.433	0.893	0.796	0.251	0.266	0.277	0.248	0.243	0.261	0.440	0.379
AAPL	3.255	3.915	2.605	4.532	2.773	3.606	1.437	1.258	0.285	0.296	0.290	0.308	0.274	0.292	0.354	0.309
BA	12.445	17.245	7.899	15.445	10.850	19.435	9.753	8.852	0.235	0.257	0.221	0.257	0.220	0.261	0.676	0.595
CAT	3.485	4.365	3.281	4.187	2.955	3.982	1.539	1.349	0.233	0.254	0.240	0.252	0.221	0.250	0.325	0.283
CVX	3.395	3.965	2.909	3.306	2.950	3.594	1.439	1.345	0.237	0.246	0.237	0.239	0.225	0.223	0.348	0.344
CSCO	2.043	2.289	1.848	1.807	1.973	2.108	0.578	0.488	0.339	0.315	0.385	0.289	0.316	0.315	0.361	0.284
KO	0.297	0.350	0.315	0.333	0.325	0.399	0.217	0.210	0.189	0.183	0.214	0.195	0.194	0.184	0.313	0.273
HD	1.209	1.343	1.290	1.246	1.080	1.266	0.889	0.828	0.194	0.200	0.203	0.203	0.189	0.197	0.335	0.308
HON	1.316	1.597	1.069	1.473	1.128	1.509	0.619	0.554	0.222	0.233	0.227	0.229	0.212	0.223	0.297	0.276
INTC	12.367	15.986	13.572	12.255	9.763	10.668	4.192	3.204	0.339	0.353	0.359	0.308	0.294	0.306	0.600	0.413
IBM	1.421	1.958	1.241	1.424	1.349	1.621	0.430	0.409	0.282	0.297	0.309	0.269	0.281	0.288	0.328	0.260
JNJ	0.288	0.313	0.248	0.286	0.276	0.289	0.211	0.206	0.193	0.194	0.202	0.194	0.191	0.191	0.270	0.248
JPM	2.824	3.619	3.770	3.072	2.607	4.017	0.925	0.809	0.255	0.272	0.303	0.260	0.246	0.258	0.296	0.274
MCD	0.419	0.443	0.353	0.486	0.458	0.607	0.301	0.281	0.247	0.250	0.271	0.258	0.259	0.258	0.441	0.367
MRK	0.745	0.918	0.685	0.739	0.819	0.763	0.476	0.425	0.222	0.226	0.260	0.219	0.225	0.220	0.352	0.289
MSFT	2.444	3.168	1.941	3.368	2.308	2.724	0.934	0.836	0.264	0.277	0.279	0.285	0.266	0.263	0.324	0.277
NKE	3.548	4.266	2.955	3.881	4.349	4.682	1.299	0.997	0.326	0.332	0.382	0.305	0.328	0.335	0.475	0.332
PG	0.332	0.337	0.298	0.324	0.270	0.314	0.226	0.215	0.181	0.180	0.195	0.188	0.176	0.180	0.316	0.276
GS	3.024	3.197	3.362	2.719	2.400	3.504	1.350	1.256	0.227	0.230	0.251	0.219	0.208	0.228	0.294	0.281
TRV	1.376	1.961	1.193	1.488	1.243	2.065	0.823	0.703	0.209	0.230	0.230	0.211	0.204	0.221	0.342	0.276
UNH	1.807	2.239	1.939	1.613	1.651	1.892	1.153	0.996	0.234	0.251	0.243	0.230	0.228	0.238	0.408	0.318
VZ	0.574	0.672	0.617	0.480	0.518	0.638	0.282	0.273	0.320	0.315	0.354	0.302	0.319	0.320	0.475	0.424
V	1.980	2.007	1.432	1.852	1.541	1.848	0.731	0.649	0.304	0.308	0.307	0.305	0.303	0.296	0.392	0.351
WMT	0.726	0.628	0.721	0.580	0.608	0.628	0.338	0.311	0.312	0.307	0.356	0.262	0.292	0.277	0.403	0.324
DIS	4.821	5.702	3.879	3.880	3.505	4.395	1.772	1.534	0.307	0.312	0.318	0.275	0.275	0.289	0.421	0.342
CRM	12.925	15.807	17.502	10.292	8.529	9.766	3.461	2.858	0.311	0.329	0.401	0.288	0.290	0.289	0.444	0.337

Figure 1: Out-of-sample average MSE (left panel) and QLIKE (right panel) forecasting performance of assets in the Dow Jones Average Index

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