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## IBS-ISI Young Ambassador Winners' Presentation Session

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#### GARCH, GAS, SV, and MSGARCH models:

Do we really need all of them for forecasting daily volatility?

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### **Motivation**

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- 1. **Benchmarks:** If we truly want practitioners to adopt the new procedure we are proposing, it must outperform strong benchmarks. It is very difficult to convince people from other fields to use our methods if they believe they offer no real advantage.
- 2. **High-dimensional time series:** Even in high-dimensional settings, we still need appropriate univariate volatility models in the first stage of the procedure.
- Machine learning in social media communications: Many of these works focus mainly on "horse races" comparisons between models, sometimes with little or no understanding of how the procedures actually work.

### Introduction

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- Given its importance, numerous approaches have been developed for forecasting daily volatility.
- While several options benefit researchers and experienced practitioners, they pose significant challenges for (untrained) practitioners, who must choose among these models for their daily tasks, often with limited or no information to guide their decisions.

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#### Main goals and contribution

• Offer insights to help researchers and practitioners in selecting the most appropriate volatility model for their data (based on user-friendly implementations).

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#### Main goals and contribution

- Offer insights to help researchers and practitioners in selecting the most appropriate volatility model for their data (based on user-friendly implementations).
- Using user-friendly implementations: provide a strong benchmark, a solid first step (univariate) for High-Dimensional volatility modelling, and emphasizes the importance of understanding how the procedures actually work.

We consider four widely used families of models for forecasting volatility:

• **GARCH**: Generalized Autoregressive Conditional Heteroskedasticity, introduced by Bollerslev (1986).

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- **GAS**: Generalized Autoregressive Score model, introduced independently by Creal et al. (2013) and Harvey (2013).

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Hereafter, let  $r_t = (P_t - P_{t-1})/P_{t-1} \approx \log(P_t/P_{t-1})$  denote the return at time t, where  $P_t$  represents the closing price at time t. We assume  $\mathbb{E}(r_t|\mathcal{F}_{t-1}) = 0$ 

#### **GARCH** model

Assumes that the conditional variance at time t is fully determined by past squared returns and its own past values. In its simplest form, the model is specified as:

$$r_t = \sigma_t \epsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where  $\omega > 0$  and  $\alpha, \beta \geq 0$  are model parameters,  $\sigma_t^2$  represents the conditional variance (or squared volatility) at time t, and the innovation term  $\epsilon_t$  has zero-mean and unit-variance.

In this study we considered the standard Normal and Student-t innovation distributions.

#### **SV** model

Assumes that the log-conditional variance evolves stochastically following an AR(1) process. Its dynamics can be described as follows:

$$r_t = \exp(h_t/2)\epsilon_t,\tag{3}$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma \eta_t, \tag{4}$$

where  $h_t$  is the log conditional variance at time t,  $\mu$ ,  $\phi$  and  $\sigma$  are parameters to be estimated,  $\eta_t \sim N(0,1)$ . In this study,  $\epsilon_t$  follows either a standardized Normal or Student-t distribution.

#### **MSGARCH** model

This specification allows for multiple volatility regimes. In its simplest form, the dynamics can be described as follows:

$$r_t = \sigma_t^{(k)} \epsilon_t, \tag{5}$$

$$\sigma_t^{2(k)} = \omega^{(k)} + \alpha^{(k)} r_{t-1}^2 + \beta^{(k)} \sigma_{t-1}^{2(k)}, \tag{6}$$

where  $\omega^{(k)} > 0$  and  $\alpha^{(k)}, \beta^{(k)} \ge 0$  are the model parameters in regime k,  $\sigma_t^{2(k)}$  denotes the conditional variance in regime k at time t, and  $\epsilon_t$  follows either a standardized Normal or a standardized Student-t distribution.

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The regime-switching mechanism is governed by the latent process  $\{S_t\}$ , assumed to be a first-order Markov chain with transition probability matrix  $\Pi$ . Its elements are given by

$$\pi_{ij} = \mathbb{P}(S_t = j \mid S_{t-1} = i), \tag{7}$$

representing the probability of moving from state i at time t-1 to state j at time t.

#### **GAS**

Its central idea is that the dynamic behaviour of time-varying parameters depends on their own past values and the score of the conditional density function (hence the name *score model*).

Let  $r_t|\mathcal{F}_{t-1} \sim p(r_t; \theta_t)$  with  $\theta_t \in \mathbb{R}^p$  being a vector of time-varying parameters fully characterising  $p(\cdot)$ . Then, in the general, unrestricted, GAS specification, the dynamics of  $\theta_t$  is given by

$$\theta_{t+1} = \kappa + As_t + B\theta_t, \tag{8}$$

where  $s_t = S_t(\theta_t) \nabla_t(r_t, \theta_t)$ , with  $\nabla_t(r_t, \theta_t)$  being the score of the conditional density function and  $S_t(\theta_t) = I_t(\theta_t)^{-\gamma}$  with typical values of  $\gamma \in \{0, 1/2, 1\}$ , and  $\kappa_{p \times 1}$ ,  $A_{p \times p}$  and  $B_{p \times p}$ .

#### **GAS**

When the parameter space is restricted, it is common to use a mapping function  $\Lambda(\cdot)$  such that

$$\theta_{t+1} = \Lambda(\tilde{\theta}_{t+1}), \tag{9}$$

$$\tilde{\theta}_{t+1} = \tilde{\kappa} + \tilde{A}s_t + \tilde{B}\tilde{\theta}_t. \tag{10}$$

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In particular, setting  $\gamma = 0$  and using an exponential function for the time-varying scale parameter under a Student-t distribution assumption, we obtain the Beta-t-EGARCH model of Harvey and Sucarrat (2014).

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In particular, setting  $\gamma=0$  and using an exponential function for the time-varying scale parameter under a Student-t distribution assumption, we obtain the Beta-t-EGARCH model of Harvey and Sucarrat (2014).

$$r_t = \sigma_t \epsilon_t, \tag{11}$$

$$\log(\sigma_t) = \delta + \phi \log(\sigma_{t-1}) + \kappa \left( \frac{(\nu+1)r_{t-1}^2}{\nu \sigma_{t-1}^2 + r_{t-1}^2} - 1 \right)$$
 (12)

- Parameters are estimated by Maximum Likelohood
- For SV, the procedure of Wahl (2018) is used.
- In all cases, we are interested in  $\mathbb{V}(r_{T+1}|\mathcal{F}_t)$ , where  $\mathcal{F}_t$  is the information available up to time t

#### Simulation setup

• The four models previously described are used both as true DGPs and for generating one-step-ahead volatility forecasts

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#### **Parameters**

More than 20 parameter vector configurations were designed to closely replicate patterns observed in real data.

Model	Parameter values 1	Parameter values 2
GARCH	$\omega=0.18, \alpha=0.09, \beta=0.89$	$\omega = 0.37, \alpha = 0.14, \beta = 0.77$
GAS	$\kappa = 0.03, A = 0.22, B = 0.98$	$\kappa = 0.06, A = 0.34, B = 0.92$
SV	$\mu = 1.74, \phi = 0.97, \sigma_{\eta} = 0.17$	$\mu = 1.15, \phi = 0.90, \sigma_{\eta} = 0.36$
MSGARCH	$\omega_1 = 0.005, \alpha_1 = 0.025, \beta_1 = 0.95$ $\omega_2 = 0.1, \alpha_2 = 0.25, \beta_2 = 0.70$ $P = \begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix}$	$\omega_1 = 0.01, \alpha_1 = 0.16, \beta_1 = 0.30$ $\omega_2 = 0.18, \alpha_2 = 0.46, \beta_2 = 0.20$ $P = \begin{bmatrix} 0.98 & 0.05 \\ 0.02 & 0.95 \end{bmatrix}$

 $\textbf{Table 1:} \ \, \textbf{Two parameter configurations (over 20) used in the Monte Carlo experiment}$ 

Loss Function	Formula	Loss Function	Formula
MSE	$R^{-1} \sum_{i=1}^{R} (\hat{\sigma}_{i}^{2} - \sigma_{i}^{2})^{2}$	MAE	$R^{-1}\sum_{i=1}^{R}\left \hat{\sigma}_{i}^{2}-\sigma_{i}^{2}\right $
QLIKE	$R^{-1}\sum_{i=1}^{R}\left(rac{\sigma_i^2}{\hat{\sigma}_i^2}-\lograc{\sigma_i^2}{\hat{\sigma}_i^2}-1 ight)$	$\mathrm{MAE_{L}}$	$R^{-1}\sum_{i=1}^{R}\left \log\hat{\sigma}_{i}^{2}-\log\sigma_{i}^{2}\right $
$\mathrm{MSE}_{\mathrm{L}}$	$R^{-1}\sum_{i=1}^{R} \left(\log \hat{\sigma}_i^2 - \log \sigma_i^2\right)^2$	$MAE_{Sd}$	$R^{-1} \sum_{i=1}^{R}  \hat{\sigma}_i - \sigma_i $ $R^{-1} \sum_{i=1}^{R} \left  \frac{\hat{\sigma}_i}{\sigma_i} - 1 \right $
$\mathrm{MSE}_{\mathrm{Sd}}$	$R^{-1}\sum_{i=1}^{R}(\hat{\sigma}_i-\sigma_i)^2$	$MAE_{P}$	$R^{-1}\sum_{i=1}^{R} \left  \frac{\sigma_i}{\sigma_i} - 1 \right $
MSE <sub>P</sub>	$R^{-1}\sum_{i=1}^{R}\left(rac{\hat{\sigma}_{i}}{\sigma_{i}}-1 ight)^{2}$		101

**Table 2:** Loss functions employed in the evaluation of volatility forecasts.

Loss Function	Formula	Loss Function	Formula
MSE	$R^{-1} \sum_{i=1}^{R} (\hat{\sigma}_{i}^{2} - \sigma_{i}^{2})^{2}$	MAE	$R^{-1}\sum_{i=1}^{R}\left \hat{\sigma}_{i}^{2}-\sigma_{i}^{2}\right $
QLIKE	$R^{-1}\sum_{i=1}^{R}\left(rac{\sigma_i^2}{\hat{\sigma}_i^2}-\lograc{\sigma_i^2}{\hat{\sigma}_i^2}-1 ight)$	$\mathrm{MAE_{L}}$	$R^{-1}\sum_{i=1}^{R}\left \log\hat{\sigma}_{i}^{2}-\log\sigma_{i}^{2}\right $
$\mathrm{MSE}_{\mathrm{L}}$	$R^{-1}\sum_{i=1}^{R} \left(\log \hat{\sigma}_i^2 - \log \sigma_i^2\right)^{2'}$	$MAE_{Sd}$	$R^{-1}\sum_{i=1}^{R}\left \hat{\sigma}_{i}-\sigma_{i}\right $
$\mathrm{MSE}_{\mathrm{Sd}}$	$R^{-1}\sum_{i=1}^{R}(\hat{\sigma}_i-\sigma_i)^2$	$MAE_{P}$	$R^{-1}\sum_{i=1}^{R} \left  \hat{\sigma}_i - \sigma_i  ight $ $R^{-1}\sum_{i=1}^{R} \left  \frac{\hat{\sigma}_i}{\sigma_i} - 1  ight $
$MSE_{P}$	$R^{-1}\sum_{i=1}^{R}\left(rac{\hat{\sigma}_{i}}{\sigma_{i}}-1 ight)^{2}$		101

**Table 2:** Loss functions employed in the evaluation of volatility forecasts.

To select the best model (or set of best models) the model confidence set of Hansen et al. (2011) was used

	Мо	del	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_{L}$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	0.0608	0.0034	0.0068	0.0048	0.0070	0.1670	0.0608	0.0497	0.0607
		GARCH-T	0.0538	0.0027	0.0055	0.0039	0.0054	0.1569	0.0565	0.0463	0.0563
	0	GAS-N	2.4896	0.0064	0.0107	0.0247	0.0236	0.2422	0.0691	0.0606	0.0716
	200	GAS-T	0.0730	0.0035	0.0073	0.0053	0.0068	0.1851	0.0651	0.0541	0.0644
	∥ Z	MS-N	0.1189	0.0062	0.0124	0.0089	0.0128	0.2402	0.0863	0.0709	0.0859
	2	MS-T	0.0992	0.0049	0.0100	0.0073	0.0096	0.2187	0.0773	0.0640	0.0764
		SV-N	0.1443	0.0086	0.0186	0.0122	0.0149	0.2838	0.1100	0.0871	0.1015
		SV-T	0.1114	0.0062	0.0133	0.0090	0.0109	0.2398	0.0904	0.0726	0.0845
		GARCH-N	0.0282	0.0014	0.0028	0.0020	0.0028	0.1136	0.0406	0.0335	0.0405
_		GARCH-T	0.0245	0.0012	0.0023	0.0017	0.0023	0.1048	0.0374	0.0308	0.0374
Ş	0	GAS-N	1.9691	0.0045	0.0071	0.0197	0.0171	0.1910	0.0513	0.0461	0.0536
GARCH	1000	GAS-T	0.0617	0.0023	0.0046	0.0038	0.0044	0.1498	0.0507	0.0429	0.0502
	II	MS-N	0.0817	0.0039	0.0077	0.0058	0.0081	0.1896	0.0666	0.0554	0.0669
DGP:	Z	MS-T	0.0566	0.0028	0.0056	0.0041	0.0055	0.1611	0.0568	0.0471	0.0564
		SV-N	0.1209	0.0072	0.0153	0.0099	0.0128	0.2703	0.1046	0.0828	0.0974
		SV-T	0.0999	0.0050	0.0106	0.0074	0.0091	0.2254	0.0839	0.0676	0.0789
		GARCH-N	0.0100	0.0006	0.0012	0.0008	0.0012	0.0720	0.0262	0.0214	0.0262
		GARCH-T	0.0080	0.0005	0.0010	0.0007	0.0010	0.0660	0.0243	0.0197	0.0243
	0	GAS-N	0.0841	0.0018	0.0035	0.0036	0.0040	0.1243	0.0397	0.0343	0.0402
	2500	GAS-T	0.0590	0.0016	0.0033	0.0030	0.0031	0.1246	0.0418	0.0354	0.0414
	II	MS-N	0.0355	0.0019	0.0038	0.0027	0.0039	0.1339	0.0479	0.0395	0.0482
	Z	MS-T	0.0303	0.0011	0.0023	0.0019	0.0023	0.1034	0.0352	0.0297	0.0352
		SV-N	0.1098	0.0065	0.0137	0.0088	0.0116	0.2644	0.1025	0.0810	0.0960
		SV-T	0.1034	0.0044	0.0093	0.0068	0.0080	0.2176	0.0803	0.0649	0.0759

**Table 3:** Forecast evaluation under **uncontaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	М	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	0.0758	0.0045	0.0089	0.0059	0.0094	0.1868	0.0714	0.0567	0.0719
		GARCH-T	0.0744	0.0042	0.0082	0.0056	0.0089	0.1803	0.0684	0.0546	0.0692
	200	GAS-N	0.1552	0.0075	0.0144	0.0106	0.0168	0.2396	0.0889	0.0716	0.0907
		GAS-T	0.0419	0.0029	0.0058	0.0036	0.0059	0.1488	0.0581	0.0457	0.0580
	 Z	MS-N	0.1412	0.0088	0.0173	0.0114	0.0184	0.2678	0.1033	0.0819	0.1039
	~	MS-T	0.0959	0.0062	0.0125	0.0080	0.0122	0.2214	0.0861	0.0679	0.0852
		SV-N	0.1325	0.0098	0.0210	0.0125	0.0172	0.2933	0.1212	0.0929	0.1121
		SV-T	0.0797	0.0058	0.0121	0.0073	0.0105	0.2215	0.0896	0.0694	0.0845
		GARCH-N	0.0589	0.0030	0.0059	0.0042	0.0063	0.1590	0.0593	0.0476	0.0597
		GARCH-T	0.0530	0.0028	0.0054	0.0038	0.0058	0.1514	0.0565	0.0454	0.0570
GAS	8	GAS-N	0.1198	0.0057	0.0107	0.0082	0.0130	0.2040	0.0743	0.0605	0.0761
	1000	GAS-T	0.0221	0.0015	0.0030	0.0019	0.0029	0.1052	0.0408	0.0322	0.0405
DGP:	II	MS-N	0.1003	0.0069	0.0134	0.0086	0.0152	0.2258	0.0870	0.0691	0.0887
ă	Z	MS-T	0.0696	0.0044	0.0087	0.0056	0.0089	0.1827	0.0703	0.0557	0.0701
		SV-N	0.1119	0.0082	0.0173	0.0103	0.0146	0.2808	0.1155	0.0887	0.1075
		SV-T	0.0651	0.0046	0.0096	0.0059	0.0084	0.2099	0.0844	0.0656	0.0799
		GARCH-N	0.0465	0.0022	0.0043	0.0032	0.0047	0.1337	0.0487	0.0396	0.0495
		GARCH-T	0.0425	0.0020	0.0039	0.0030	0.0044	0.1277	0.0465	0.0379	0.0472
	0	GAS-N	0.1219	0.0049	0.0089	0.0076	0.0120	0.1851	0.0650	0.0539	0.0674
	2500	GAS-T	0.0079	0.0005	0.0010	0.0007	0.0010	0.0639	0.0245	0.0195	0.0244
	II	MS-N	0.0745	0.0049	0.0092	0.0060	0.0116	0.1828	0.0706	0.0559	0.0727
	Z	MS-T	0.0377	0.0023	0.0045	0.0030	0.0046	0.1354	0.0509	0.0408	0.0510
		SV-N	0.0982	0.0072	0.0152	0.0090	0.0131	0.2743	0.1127	0.0866	0.1057
		SV-T	0.0533	0.0036	0.0076	0.0047	0.0068	0.2000	0.0796	0.0621	0.0760

**Table 4:** Forecast evaluation under **uncontaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

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		GARCH-N	0.5510	0.0434	0.0784	0.0487	0.1153	0.5507	0.2178	0.1708	0.2396
		GARCH-T	0.5214	0.0420	0.0759	0.0468	0.1098	0.5436	0.2150	0.1687	0.2381
	9	GAS-N	0.7431	0.0475	0.0842	0.0561	0.1320	0.5795	0.2235	0.1770	0.2495
	200	GAS-T	0.4303	0.0362	0.0671	0.0403	0.0887	0.5136	0.2061	0.1606	0.2242
		MS-N	0.5349	0.0444	0.0813	0.0494	0.1153	0.5611	0.2242	0.1751	0.2443
	Z	MS-T	0.5231	0.0413	0.0765	0.0472	0.1026	0.5594	0.2219	0.1739	0.2406
		SV-N	0.5815	0.0391	0.0843	0.0527	0.0726	0.5548	0.2278	0.1755	0.2128
		SV-T	0.4341	0.0322	0.0646	0.0397	0.0685	0.4977	0.2016	0.1564	0.2028
		GARCH-N	0.4757	0.0380	0.0693	0.0427	0.0975	0.5223	0.2067	0.1621	0.2269
		GARCH-T	0.4715	0.0383	0.0694	0.0426	0.0985	0.5239	0.2076	0.1627	0.2295
>	0	GAS-N	0.6814	0.0444	0.0769	0.0511	0.1383	0.5464	0.2129	0.1679	0.2389
SV:	1000	GAS-T	0.4005	0.0339	0.0628	0.0376	0.0824	0.4996	0.2004	0.1562	0.2181
DGP:	II	MS-N	0.4892	0.0413	0.0763	0.0456	0.1038	0.5415	0.2181	0.1697	0.2375
	Z	MS-T	0.4728	0.0387	0.0713	0.0433	0.0971	0.5340	0.2128	0.1664	0.2314
		SV-N	0.5298	0.0353	0.0761	0.0479	0.0650	0.5376	0.2190	0.1695	0.2048
		SV-T	0.3978	0.0291	0.0581	0.0360	0.0617	0.4767	0.1920	0.1494	0.1937
		GARCH-N	0.4618	0.0369	0.0671	0.0414	0.0942	0.5115	0.2025	0.1587	0.2227
		GARCH-T	0.4654	0.0377	0.0682	0.0419	0.0976	0.5170	0.2048	0.1605	0.2270
	0	GAS-N	0.7886	0.0451	0.0772	0.0534	0.1437	0.5494	0.2118	0.1675	0.2395
	2500	GAS-T	0.3921	0.0334	0.0617	0.0369	0.0815	0.4920	0.1975	0.1539	0.2155
	II	MS-N	0.4681	0.0410	0.0742	0.0438	0.1069	0.5297	0.2144	0.1664	0.2370
	Z	MS-T	0.4307	0.0361	0.0662	0.0398	0.0909	0.5072	0.2028	0.1583	0.2221
		SV-N	0.5254	0.0347	0.0743	0.0471	0.0646	0.5330	0.2160	0.1675	0.2031
		SV-T	0.3888	0.0284	0.0565	0.0351	0.0608	0.4688	0.1886	0.1468	0.1910

**Table 5:** Forecast evaluation under **uncontaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	Мо	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	MAE <sub>L</sub>	$MAE_{Sd}$	$MAE_P$
		GARCH-N	0.0095	0.0056	0.0116	0.0025	0.0105	0.0610	0.0786	0.0343	0.0775
		GARCH-T	0.0076	0.0050	0.0101	0.0021	0.0099	0.0584	0.0754	0.0329	0.0755
	0	GAS-N	0.0115	0.0060	0.0121	0.0027	0.0121	0.0642	0.0809	0.0356	0.0807
	200	GAS-T	0.0090	0.0055	0.0112	0.0024	0.0104	0.0620	0.0794	0.0347	0.0785
	II	MS-N	0.0076	0.0052	0.0106	0.0021	0.0100	0.0591	0.0773	0.0335	0.0762
	Z	MS-T	0.0095	0.0054	0.0112	0.0024	0.0102	0.0611	0.0780	0.0341	0.0764
		SV-N	0.0201	0.0150	0.0332	0.0061	0.0250	0.1051	0.1490	0.0620	0.1341
		SV-T	0.0161	0.0112	0.0245	0.0047	0.0189	0.0880	0.1220	0.0513	0.1111
		GARCH-N	0.0049	0.0031	0.0063	0.0013	0.0060	0.0444	0.0580	0.0251	0.0577
$\Xi$		GARCH-T	0.0044	0.0030	0.0060	0.0012	0.0058	0.0437	0.0569	0.0247	0.0569
MSGARCH	00	GAS-N	0.0066	0.0038	0.0075	0.0016	0.0078	0.0505	0.0638	0.0281	0.0644
99	1000	GAS-T	0.0067	0.0039	0.0080	0.0017	0.0075	0.0522	0.0665	0.0292	0.0660
ž	II	MS-N	0.0053	0.0033	0.0068	0.0014	0.0066	0.0479	0.0613	0.0268	0.0610
DGP:	Z	MS-T	0.0057	0.0030	0.0060	0.0013	0.0060	0.0454	0.0570	0.0251	0.0568
ă		SV-N	0.0166	0.0133	0.0288	0.0052	0.0226	0.1022	0.1450	0.0603	0.1317
		SV-T	0.0133	0.0092	0.0199	0.0038	0.0159	0.0813	0.1118	0.0472	0.1027
		GARCH-N	0.0027	0.0018	0.0037	0.0007	0.0035	0.0348	0.0457	0.0198	0.0455
		GARCH-T	0.0026	0.0018	0.0037	0.0007	0.0036	0.0360	0.0467	0.0203	0.0468
	0	GAS-N	0.0294	0.0039	0.0071	0.0026	0.0105	0.0505	0.0580	0.0264	0.0593
	2500	GAS-T	0.0058	0.0031	0.0063	0.0014	0.0058	0.0471	0.0592	0.0262	0.0587
	II	MS-N	0.0021	0.0015	0.0030	0.0006	0.0030	0.0321	0.0415	0.0181	0.0415
	Z	MS-T	0.0021	0.0014	0.0028	0.0006	0.0028	0.0306	0.0390	0.0171	0.0390
		SV-N	0.0161	0.0127	0.0273	0.0050	0.0218	0.1026	0.1455	0.0605	0.1329
		SV-T	0.0124	0.0084	0.0180	0.0035	0.0146	0.0792	0.1084	0.0459	0.1000

**Table 6:** Forecast evaluation under **uncontaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	Me	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	3.0883	0.1346	0.2001	0.1837	0.5500	1.2254	0.3537	0.3240	0.4873
		GARCH-T	3.1632	0.1421	0.2104	0.1914	0.5799	1.2572	0.3629	0.3326	0.5047
	200	GAS-N	> 10	0.7279	0.5480	1.1870	> 10	2.9836	0.5240	0.5635	1.2506
		GAS-T	0.3745	0.0195	0.0355	0.0274	0.0477	0.4594	0.1529	0.1310	0.1710
	∥ Z	MS-N	> 10	0.4670	0.1858	0.8437	> 10	1.7158	0.2822	0.2763	0.7470
	~	MS-T	2.0181	0.0850	0.1237	0.1144	0.3986	0.8429	0.2521	0.2266	0.3355
		SV-N	1.2415	0.0617	0.0946	0.0808	0.2407	0.6494	0.2054	0.1800	0.2581
		SV-T	0.2877	0.0164	0.0300	0.0223	0.0398	0.3886	0.1333	0.1126	0.1464
		GARCH-N	2.8981	0.1419	0.2165	0.1879	0.5286	1.2992	0.3838	0.3484	0.5255
_		GARCH-T	2.9288	0.1456	0.2216	0.1914	0.5453	1.3150	0.3895	0.3532	0.5351
$\tilde{z}$	9	GAS-N	> 10	2.4645	0.7976	4.3400	> 10	6.7300	0.6696	0.7664	3.1338
GARCH	1000	GAS-T	0.3323	0.0182	0.0333	0.0250	0.0441	0.4436	0.1500	0.1276	0.1675
	II	MS-N	2.2681	0.1139	0.1699	0.1471	0.4476	1.0159	0.3090	0.2764	0.4223
DGP:	Z	MS-T	1.9027	0.0972	0.1494	0.1272	0.3523	0.9658	0.2956	0.2639	0.3918
_		SV-N	1.1118	0.0622	0.0982	0.0793	0.2137	0.6791	0.2167	0.1896	0.2749
		SV-T	0.3140	0.0184	0.0330	0.0245	0.0465	0.4010	0.1365	0.1157	0.1529
		GARCH-N	2.6449	0.1410	0.2203	0.1836	0.4778	1.3199	0.3980	0.3583	0.5390
		GARCH-T	2.6555	0.1429	0.2228	0.1853	0.4874	1.3283	0.4009	0.3608	0.5438
	00	GAS-N	> 10	1.2223	0.9560	1.9592	> 10	4.6304	0.7861	0.8607	2.0082
	2500	GAS-T	0.3220	0.0181	0.0330	0.0244	0.0434	0.4429	0.1515	0.1282	0.1690
	II	MS-N	2.0581	0.1143	0.1783	0.1453	0.3938	1.0940	0.3419	0.3024	0.4560
	Z	MS-T	2.0453	0.1096	0.1714	0.1419	0.3795	1.0995	0.3378	0.3013	0.4470
		SV-N	1.0545	0.0628	0.1016	0.0793	0.1981	0.7094	0.2280	0.1991	0.2887
		SV-T	0.3410	0.0204	0.0366	0.0267	0.0515	0.4218	0.1442	0.1220	0.1632

**Table 7:** Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	Me	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	2.3374	0.1265	0.1979	0.1592	0.4280	1.1736	0.3661	0.3223	0.4919
		GARCH-T	2.2496	0.1293	0.2022	0.1591	0.4346	1.1698	0.3681	0.3230	0.4971
	9	GAS-N	> 10	0.5658	0.4565	0.8755	> 10	2.4650	0.5154	0.5090	1.0801
	200	GAS-T	0.3279	0.0183	0.0331	0.0242	0.0449	0.4211	0.1467	0.1224	0.1646
	∥ Z	MS-N	1.4286	0.0842	0.1324	0.1015	0.2885	0.8304	0.2749	0.2350	0.3573
	2	MS-T	1.3299	0.0694	0.1088	0.0871	0.2532	0.7555	0.2462	0.2120	0.3146
		SV-N	0.7552	0.0474	0.0793	0.0581	0.1415	0.5895	0.2047	0.1711	0.2423
		SV-T	0.2182	0.0142	0.0259	0.0177	0.0342	0.3384	0.1237	0.1009	0.1360
		GARCH-N	2.4847	0.1440	0.2245	0.1760	0.4877	1.2687	0.3990	0.3503	0.5430
		GARCH-T	2.4037	0.1442	0.2251	0.1740	0.4859	1.2633	0.4004	0.3504	0.5446
GAS	00	GAS-N	> 10	0.6541	0.6814	0.9031	> 10	2.9270	0.6593	0.6602	1.3132
	1000	GAS-T	0.2777	0.0167	0.0305	0.0216	0.0404	0.4003	0.1423	0.1176	0.1588
DGP:	II	MS-N	1.4652	0.0981	0.1544	0.1127	0.3298	0.9060	0.3073	0.2602	0.4046
ă	Z	MS-T	1.1638	0.0768	0.1237	0.0904	0.2454	0.8100	0.2739	0.2324	0.3499
		SV-N	0.8344	0.0564	0.0924	0.0663	0.1723	0.6341	0.2187	0.1838	0.2705
		SV-T	0.2195	0.0150	0.0271	0.0184	0.0371	0.3332	0.1222	0.0996	0.1360
		GARCH-N	2.4426	0.1515	0.2367	0.1810	0.5081	1.3132	0.4175	0.3652	0.5690
		GARCH-T	2.4032	0.1516	0.2369	0.1798	0.5081	1.3093	0.4180	0.3649	0.5695
	00	GAS-N	> 10	0.8662	0.9124	1.1191	8.7939	3.5148	0.7818	0.7963	1.6479
	2500	GAS-T	0.2436	0.0160	0.0292	0.0200	0.0383	0.3867	0.1401	0.1148	0.1561
	II	MS-N	1.5815	0.1109	0.1753	0.1257	0.3644	0.9950	0.3378	0.2863	0.4486
	Z	MS-T	1.4069	0.0912	0.1471	0.1086	0.2873	0.9248	0.3086	0.2637	0.3996
		SV-N	0.8587	0.0620	0.1008	0.0708	0.1911	0.6581	0.2282	0.1915	0.2883
		SV-T	0.2146	0.0162	0.0291	0.0190	0.0403	0.3349	0.1247	0.1010	0.1404

**Table 8:** Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	М	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_{L}$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	2.3405	0.1669	0.2518	0.1810	0.6163	1.1962	0.4074	0.3438	0.5685
		GARCH-T	2.4600	0.1786	0.2682	0.1924	0.6637	1.2478	0.4235	0.3583	0.5971
	200	GAS-N	9.1394	0.3148	0.3923	0.3864	2.1378	1.6785	0.4840	0.4360	0.7924
		GAS-T	0.6880	0.0609	0.1036	0.0635	0.1748	0.6623	0.2568	0.2037	0.3083
	∥ Z	MS-N	1.3548	0.1124	0.1775	0.1166	0.3800	0.9082	0.3345	0.2721	0.4364
	~	MS-T	1.3345	0.1042	0.1659	0.1111	0.3481	0.8894	0.3246	0.2650	0.4190
		SV-N	0.9202	0.0695	0.1257	0.0812	0.1940	0.7007	0.2725	0.2157	0.2999
		SV-T	0.5707	0.0504	0.0886	0.0535	0.1379	0.5877	0.2329	0.1828	0.2683
		GARCH-N	2.3598	0.1754	0.2641	0.1874	0.6468	1.2409	0.4237	0.3575	0.5955
		GARCH-T	2.4155	0.1828	0.2745	0.1940	0.6760	1.2714	0.4337	0.3663	0.6133
SV	1000	GAS-N	10 غ	0.4004	0.4537	0.5307	6.7466	1.9746	0.5362	0.4885	0.9334
S	10	GAS-T	0.6289	0.0565	0.0965	0.0587	0.1597	0.6332	0.2473	0.1955	0.2956
DGP:	II	MS-N	1.3620	0.1172	0.1839	0.1195	0.3989	0.9275	0.3420	0.2782	0.4506
	Z	MS-T	1.2798	0.1058	0.1677	0.1103	0.3529	0.8911	0.3264	0.2663	0.4247
		SV-N	1.0067	0.0784	0.1337	0.0878	0.2468	0.7289	0.2771	0.2219	0.3210
		SV-T	0.5215	0.0463	0.0816	0.0491	0.1245	0.5656	0.2244	0.1760	0.2573
		GARCH-N	2.3625	0.1828	0.2751	0.1924	0.6745	1.2698	0.4357	0.3671	0.6160
		GARCH-T	2.4527	0.1899	0.2848	0.1998	0.7046	1.3023	0.4450	0.3758	0.6327
	9	GAS-N	8.1638	0.4008	0.5167	0.4575	2.2197	2.0234	0.5905	0.5340	0.9894
	2500	GAS-T	0.5982	0.0553	0.0944	0.0567	0.1563	0.6225	0.2445	0.1928	0.2920
	II	MS-N	1.5132	0.1330	0.2056	0.1336	0.4676	0.9959	0.3649	0.2980	0.4922
	Z	MS-T	1.3460	0.1144	0.1796	0.1174	0.3907	0.9322	0.3416	0.2788	0.4504
		SV-N	1.0485	0.0838	0.1396	0.0917	0.2693	0.7564	0.2848	0.2292	0.3400
		SV-T	0.5036	0.0456	0.0801	0.0478	0.1235	0.5575	0.2216	0.1736	0.2547

**Table 9:** Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	М	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	0.1912	0.0959	0.1429	0.0386	0.3809	0.2621	0.2713	0.1313	0.3647
		GARCH-T	0.1789	0.0997	0.1517	0.0390	0.3680	0.2726	0.2862	0.1379	0.3840
	0	GAS-N	ر 10	1.7069	0.4714	0.9703	ر10	1.4028	0.4055	0.2584	2.1100
	200	GAS-T	0.0316	0.0212	0.0378	0.0083	0.0551	0.1224	0.1472	0.0665	0.1652
	11	MS-N	0.1143	0.0625	0.0947	0.0243	0.2428	0.1921	0.2115	0.0994	0.2718
	Z	MS-T	0.1218	0.0612	0.0921	0.0244	0.2422	0.1918	0.2081	0.0984	0.2670
		SV-N	0.0457	0.0316	0.0572	0.0123	0.0851	0.1357	0.1725	0.0757	0.1814
		SV-T	0.0296	0.0204	0.0369	0.0079	0.0531	0.1092	0.1368	0.0606	0.1463
-		GARCH-N	0.1700	0.0980	0.1489	0.0381	0.3568	0.2653	0.2774	0.1340	0.3742
MSGARCH		GARCH-T	0.1550	0.0962	0.1500	0.0368	0.3275	0.2704	0.2890	0.1383	0.3841
Ä.	0	GAS-N	3.7793	0.5344	0.5033	0.2460	8.2986	0.6975	0.4706	0.2680	1.0039
Š	1000	GAS-T	0.0246	0.0182	0.0326	0.0069	0.0459	0.1121	0.1371	0.0615	0.1531
		MS-N	0.1083	0.0640	0.0972	0.0243	0.2418	0.1932	0.2148	0.1006	0.2780
DGP:	Z	MS-T	0.1204	0.0686	0.1032	0.0264	0.2631	0.2046	0.2232	0.1055	0.2908
ă		SV-N	0.0420	0.0312	0.0550	0.0116	0.0863	0.1345	0.1699	0.0749	0.1842
		SV-T	0.0241	0.0179	0.0325	0.0068	0.0451	0.1022	0.1286	0.0569	0.1380
-		GARCH-N	0.1627	0.0995	0.1525	0.0381	0.3495	0.2630	0.2763	0.1333	0.3753
		GARCH-T	0.1506	0.0972	0.1526	0.0368	0.3238	0.2721	0.2926	0.1398	0.3896
	00	GAS-N	2.6077	0.5596	0.5983	0.2486	4.9391	0.7648	0.5448	0.3080	1.1039
	2500	GAS-T	0.0223	0.0168	0.0302	0.0063	0.0418	0.1056	0.1300	0.0582	0.1451
	II	MS-N	0.0979	0.0639	0.0997	0.0237	0.2231	0.2011	0.2296	0.1064	0.2932
	Z	MS-T	0.1238	0.0745	0.1140	0.0284	0.2703	0.2221	0.2438	0.1150	0.3178
		SV-N	0.0420	0.0314	0.0549	0.0117	0.0864	0.1372	0.1714	0.0760	0.1887
		SV-T	0.0237	0.0178	0.0322	0.0067	0.0450	0.1028	0.1284	0.0570	0.1394

**Table 10:** Forecast evaluation under **contaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

	Model		MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	1.0683	0.0919	0.1420	0.0934	0.3209	0.6657	0.2707	0.2097	0.3605
		GARCH-T	1.0718	0.0948	0.1445	0.0947	0.3387	0.6541	0.2645	0.2056	0.3581
	200	GAS-N	> 10	0.5513	0.4396	0.7427	> 10	1.7941	0.4144	0.3859	0.9636
		GAS-T	0.1926	0.0197	0.0350	0.0198	0.0512	0.2964	0.1377	0.1002	0.1553
	∥ Z	MS-N	0.7403	0.0672	0.1042	0.0667	0.2345	0.5161	0.2166	0.1653	0.2816
	~	MS-T	0.7028	0.0612	0.0958	0.0620	0.2098	0.5031	0.2103	0.1608	0.2693
		SV-N	0.2893	0.0311	0.0538	0.0302	0.0873	0.3450	0.1627	0.1174	0.1815
		SV-T	0.1848	0.0198	0.0356	0.0196	0.0506	0.2849	0.1357	0.0975	0.1476
		GARCH-N	0.9879	0.0905	0.1403	0.0900	0.3093	0.6383	0.2608	0.2019	0.3508
_		GARCH-T	0.9925	0.0920	0.1416	0.0908	0.3190	0.6327	0.2575	0.1998	0.3492
GARCH	00	GAS-N	> 10	0.5204	0.5124	0.6299	7.1201	1.8003	0.4716	0.4321	0.9912
3AF	1000	GAS-T	0.1772	0.0189	0.0335	0.0186	0.0488	0.2809	0.1306	0.0950	0.1479
	II	MS-N	0.6571	0.0638	0.1002	0.0621	0.2140	0.4990	0.2128	0.1614	0.2757
DGP:	Z	MS-T	0.7404	0.0685	0.1069	0.0677	0.2348	0.5282	0.2217	0.1693	0.2887
		SV-N	0.2807	0.0306	0.0527	0.0295	0.0854	0.3433	0.1615	0.1167	0.1826
		SV-T	0.2111	0.0228	0.0402	0.0223	0.0603	0.2974	0.1405	0.1013	0.1561
		GARCH-N	0.9891	0.0925	0.1431	0.0913	0.3159	0.6334	0.2579	0.2001	0.3503
		GARCH-T	0.9898	0.0936	0.1444	0.0919	0.3220	0.6328	0.2578	0.2000	0.3513
	00	GAS-N	> 10	0.5666	0.5944	0.6420	5.2625	1.9374	0.5267	0.4822	1.0928
	2500	GAS-T	0.1752	0.0189	0.0334	0.0185	0.0488	0.2723	0.1268	0.0922	0.1445
	II	MS-N	0.7056	0.0693	0.1091	0.0673	0.2294	0.5264	0.2235	0.1699	0.2924
	Z	MS-T	0.8006	0.0765	0.1196	0.0749	0.2577	0.5605	0.2336	0.1792	0.3093
		SV-N	0.2854	0.0314	0.0538	0.0301	0.0884	0.3482	0.1629	0.1180	0.1868
		SV-T	0.2437	0.0268	0.0466	0.0258	0.0729	0.3215	0.1508	0.1092	0.1712

**Table 11:** Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed** markets

	М	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	1.0568	0.0988	0.1511	0.0936	0.3623	0.6491	0.2827	0.2107	0.3801
		GARCH-T	1.0268	0.1024	0.1564	0.0951	0.3655	0.6464	0.2808	0.2099	0.3824
	200	GAS-N	> 10	0.9756	0.4697	1.1284	> 10	2.1705	0.4428	0.4002	1.4168
		GAS-T	0.1729	0.0210	0.0365	0.0189	0.0564	0.2679	0.1355	0.0943	0.1557
	 Z	MS-N	0.5787	0.0613	0.0956	0.0553	0.2185	0.4579	0.2144	0.1545	0.2746
	~	MS-T	0.6544	0.0586	0.0912	0.0558	0.2070	0.4538	0.2080	0.1511	0.2655
-		SV-N	0.2208	0.0295	0.0516	0.0258	0.0812	0.3183	0.1684	0.1146	0.1855
		SV-T	0.1554	0.0210	0.0370	0.0183	0.0563	0.2607	0.1378	0.0939	0.1521
		GARCH-N	1.0954	0.1080	0.1641	0.1004	0.3882	0.6671	0.2876	0.2157	0.3951
		GARCH-T	1.0536	0.1073	0.1632	0.0986	0.3819	0.6564	0.2838	0.2127	0.3908
GAS	8	GAS-N	> 10	0.8242	0.6147	0.9185	> 10	2.1632	0.5295	0.4797	1.3530
	1000	GAS-T	0.1561	0.0197	0.0343	0.0176	0.0525	0.2472	0.1259	0.0874	0.1453
DGP:	II	MS-N	0.5677	0.0640	0.1011	0.0568	0.2165	0.4699	0.2210	0.1590	0.2844
ă	Z	MS-T	0.5654	0.0604	0.0963	0.0551	0.1962	0.4637	0.2159	0.1561	0.2754
		SV-N	0.2338	0.0321	0.0548	0.0274	0.0912	0.3244	0.1693	0.1160	0.1922
		SV-T	0.1723	0.0240	0.0416	0.0205	0.0658	0.2731	0.1431	0.0980	0.1615
		GARCH-N	1.0651	0.1107	0.1680	0.1011	0.3961	0.6648	0.2868	0.2153	0.3974
		GARCH-T	1.0506	0.1108	0.1680	0.1007	0.3963	0.6616	0.2859	0.2145	0.3966
	0	GAS-N	> 10	0.8064	0.7435	0.8791	> 10	2.2819	0.5954	0.5434	1.4013
	2500	GAS-T	0.1492	0.0198	0.0344	0.0173	0.0526	0.2378	0.1215	0.0842	0.1412
	II	MS-N	0.6244	0.0732	0.1151	0.0641	0.2443	0.5057	0.2365	0.1708	0.3094
	Z	MS-T	0.6361	0.0697	0.1101	0.0629	0.2299	0.4970	0.2285	0.1664	0.2979
		SV-N	0.2392	0.0340	0.0574	0.0285	0.0980	0.3277	0.1703	0.1170	0.1970
		SV-T	0.1910	0.0275	0.0470	0.0231	0.0767	0.2904	0.1519	0.1041	0.1740

**Table 12:** Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed** markets

	M	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	1.7035	0.2143	0.3061	0.1660	0.9247	0.9706	0.4411	0.3206	0.6421
		GARCH-T	2.1332	0.2660	0.3613	0.2026	1.2964	1.0746	0.4723	0.3489	0.7283
	200	GAS-N	> 10	0.9192	0.5236	1.5360	> 10	2.6725	0.5231	0.4422	1.4309
		GAS-T	0.5958	0.1019	0.1640	0.0736	0.3384	0.6164	0.3212	0.2192	0.4058
	∥ Z	MS-N	1.0378	0.1570	0.2325	0.1140	0.6370	0.7631	0.3749	0.2629	0.5169
	~	MS-T	1.0500	0.1541	0.2307	0.1142	0.6040	0.7766	0.3782	0.2663	0.5175
		SV-N	0.6464	0.0878	0.1598	0.0758	0.2644	0.5870	0.3105	0.2102	0.3329
		SV-T	0.5342	0.0863	0.1468	0.0658	0.2703	0.5650	0.3007	0.2031	0.3531
		GARCH-N	1.8119	0.2300	0.3218	0.1767	1.0447	0.9958	0.4460	0.3266	0.6635
		GARCH-T	2.1239	0.2630	0.3589	0.2023	1.2628	1.0774	0.4701	0.3485	0.7227
>	0	GAS-N	> 10	0.6912	0.5914	0.7223	> 10	1.8911	0.5569	0.4632	1.2376
S	1000	GAS-T	0.5854	0.0999	0.1613	0.0723	0.3299	0.6105	0.3185	0.2172	0.4013
DGP: SV	II	MS-N	1.0032	0.1575	0.2342	0.1135	0.6250	0.7655	0.3774	0.2644	0.5198
	Z	MS-T	1.0130	0.1561	0.2305	0.1130	0.6436	0.7684	0.3756	0.2642	0.5178
		SV-N	0.6907	0.0945	0.1666	0.0801	0.3113	0.6013	0.3139	0.2139	0.3440
		SV-T	0.5127	0.0815	0.1398	0.0629	0.2508	0.5473	0.2912	0.1966	0.3394
		GARCH-N	1.8262	0.2388	0.3302	0.1807	1.1230	0.9991	0.4466	0.3275	0.6738
		GARCH-T	2.0609	0.2644	0.3597	0.2005	1.2920	1.0676	0.4688	0.3467	0.7230
	0	GAS-N	> 10	0.6324	0.6026	0.5729	> 10	1.7504	0.5737	0.4688	1.1966
	2500	GAS-T	0.5652	0.0992	0.1602	0.0710	0.3280	0.6071	0.3180	0.2166	0.4011
	П	MS-N	1.1017	0.1746	0.2535	0.1242	0.7540	0.8041	0.3921	0.2763	0.5524
	Z	MS-T	0.9973	0.1607	0.2354	0.1140	0.6929	0.7713	0.3791	0.2661	0.5258
		SV-N	0.6786	0.0961	0.1667	0.0795	0.3315	0.5991	0.3129	0.2132	0.3473
		SV-T	0.4942	0.0801	0.1368	0.0611	0.2481	0.5396	0.2873	0.1940	0.3357

**Table 13:** Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed** markets

	Me	odel	MSE	QLIKE	$MSE_L$	$MSE_{Sd}$	$MSE_P$	MAE	$MAE_L$	$MAE_{Sd}$	$MAE_P$
		GARCH-N	0.1129	0.3905	0.4421	0.0478	2.8826	0.1783	0.4410	0.1333	0.8227
		GARCH-T	0.1088	0.3466	0.3775	0.0437	2.6597	0.1566	0.3599	0.1126	0.6975
	200	GAS-N	5.1360	> 10	> 10	5.0656	> 10	> 10	> 10	1.2573	1.9577
		GAS-T	0.0263	0.0768	0.1173	0.0114	0.2938	0.0809	0.2330	0.0650	0.2941
	 Z	MS-N	0.0731	0.2190	0.2491	0.0282	1.6653	0.1164	0.2916	0.0871	0.5026
	~	MS-T	0.0723	0.2254	0.2508	0.0285	1.7606	0.1145	0.2816	0.0850	0.5005
MSGARCH		SV-N	0.0322	0.1360	0.2243	0.0178	0.5179	0.1155	0.3813	0.0994	0.4257
		SV-T	0.0316	0.1216	0.2010	0.0167	0.4601	0.1094	0.3506	0.0928	0.3903
		GARCH-N	0.1053	0.3888	0.4332	0.0466	2.8596	0.1712	0.4169	0.1272	0.7988
		GARCH-T	0.1041	0.3541	0.3845	0.0436	2.7213	0.1551	0.3570	0.1119	0.7026
	8	GAS-N	0.8070	> 10	> 10	7.8892	> 10	> 10	> 10	1.6321	3.1176
	1000	GAS-T	0.0198	0.0708	0.1098	0.0097	0.2566	0.0748	0.2274	0.0620	0.2844
	II	MS-N	0.0626	0.2200	0.2433	0.0266	1.7278	0.1058	0.2662	0.0797	0.4815
DGP:	Z	MS-T	0.0667	0.2379	0.2616	0.0285	1.8585	0.1078	0.2673	0.0806	0.5032
ă		SV-N	0.0301	0.1376	0.2258	0.0174	0.5189	0.1146	0.3823	0.0993	0.4315
		SV-T	0.0296	0.1333	0.2177	0.0169	0.5047	0.1123	0.3710	0.0969	0.4210
		GARCH-N	0.1063	0.3931	0.4326	0.0471	2.9347	0.1697	0.4039	0.1247	0.7905
		GARCH-T	0.1058	0.3649	0.3934	0.0446	2.8559	0.1571	0.3591	0.1130	0.7154
	0	GAS-N	1.5851	> 10	> 10	> 10	> 10	> 10	> 10	2.0218	4.2721
	2500	GAS-T	0.0186	0.0683	0.1061	0.0093	0.2455	0.0732	0.2254	0.0612	0.2811
	II	MS-N	0.0656	0.2273	0.2473	0.0274	1.8238	0.1012	0.2531	0.0760	0.4766
	Z	MS-T	0.0681	0.2443	0.2663	0.0290	1.9376	0.1032	0.2566	0.0774	0.5000
		SV-N	0.0300	0.1400	0.2283	0.0175	0.5321	0.1148	0.3840	0.0997	0.4371
		SV-T	0.0296	0.1384	0.2258	0.0173	0.5237	0.1140	0.3812	0.0990	0.4340

**Table 14:** Forecast evaluation under **contaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed** markets

• The previously mentioned volatility models were applied to the daily return time series of the constituents of the Dow Jones Industrial Average Index.

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- Only time series without missing values over the considered period were analyzed, resulting in a final sample of 29 stocks.

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- Only time series without missing values over the considered period were analyzed, resulting in a final sample of 29 stocks.
- Volatility forecasting performance for each series was assessed using a rolling window scheme with a window size of 2,500 and 1,000 observations.

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- Only time series without missing values over the considered period were analyzed, resulting in a final sample of 29 stocks.
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- For these series, realized volatility measures were freely obtained from capire.stat.unipd.it and uses as a benchmark.

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- Only the MSE and QLIKE are implemented, as they are the only loss functions considered robust in the sense of Patton (2011).
- The Model Confidence Set of Hansen et al. (2011) was used to select the best set of models

	Min	$Q_1$	Med	Mean	$Q_3$	Max	Skew	Kurt	Sd	$ACF_1$
MMM	-12.9450	-0.5931	0.0540	0.0394	0.7338	22.9906	0.6217	26.4331	1.4631	-0.0438
AMZN	-14.0494	-0.9025	0.0938	0.1136	1.1906	15.7457	0.2606	9.2383	2.0610	-0.0190
AXP	-14.8187	-0.7041	0.0777	0.0751	0.9279	21.8823	0.8175	22.6249	1.8245	-0.0533
AMGN	-9.5846	-0.7233	0.0317	0.0614	0.8461	11.8180	0.3778	8.8610	1.5232	-0.075
AAPL	-12.8647	-0.7397	0.1004	0.1129	1.0357	11.9808	-0.0434	8.1919	1.7551	-0.0400
BA	-23.8484	-0.9198	0.0679	0.0635	1.0459	24.3186	0.1755	21.1000	2.2543	0.054
CAT	-14.2822	-0.8497	0.0590	0.0760	1.0278	10.3321	-0.1333	6.7862	1.8298	0.003
CVX	-22.1248	-0.7318	0.0703	0.0468	0.8295	22.7407	-0.2220	26.8621	1.6872	-0.067
CSCO	-16.2107	-0.6509	0.0520	0.0481	0.7960	15.9505	-0.4466	18.6962	1.6295	-0.063
KO	-9.6725	-0.4607	0.0566	0.0388	0.5729	6.4796	-0.6067	11.8785	1.0721	-0.034
HD	-19.7938	-0.5921	0.0940	0.0891	0.8301	13.7508	-0.6515	18.0384	1.4627	-0.042
HON	-12.0868	-0.6024	0.0713	0.0655	0.7542	15.0684	0.0125	12.3544	1.4405	-0.030
INTC	-26.0585	-0.9213	0.0566	0.0320	1.0257	19.5213	-0.7099	18.7537	2.0229	-0.070
IBM	-12.8507	-0.6128	0.0556	0.0376	0.7120	11.3010	-0.4608	12.7177	1.4006	-0.033
TNT	-10.0379	-0.4471	0.0312	0.0386	0.5695	7.9977	-0.1134	12.0442	1.0521	-0.068
JPM	-14.9649	-0.7608	0.0586	0.0719	0.9074	18.0125	0.2216	12.8850	1.7483	-0.097
MCD	-15.8753	-0.4841	0.0750	0.0585	0.6014	18.1255	0.3910	33.9165	1.1706	-0.100
MRK	-9.8630	-0.6197	0.0310	0.0484	0.7436	10.4080	-0.0100	9.6857	1.3077	-0.063
MSFT	-14.7390	-0.7078	0.0698	0.0900	0.9262	14.2169	0.0265	10.6409	1.6110	-0.104
NKE	-19.9809	-0.7633	0.0508	0.0607	0.9280	15.5314	0.0250	17.1611	1.7571	-0.037
PG	-8.7373	-0.4713	0.0568	0.0444	0.5846	12.0090	0.1688	14.9902	1.0764	-0.076
GS	-12.7910	-0.8520	0.0549	0.0555	0.9845	17.5803	0.0073	11.5460	1.8007	-0.064
TRV	-20.8004	-0.5772	0.1044	0.0612	0.7408	13.2902	-1.1312	25.3817	1.4064	-0.154
UNH	-17.2769	-0.6889	0.0957	0.0932	0.8602	12.7989	-0.0642	12.5191	1.6006	-0.065
VZ	-7.4978	-0.5835	0.0495	0.0317	0.6449	9.2705	0.0711	8.3111	1.1627	-0.035
V	-13.5472	-0.6974	0.1312	0.0861	0.8665	14.9973	0.1765	13.3194	1.5741	-0.096
WMT	-11.3758	-0.5144	0.0682	0.0589	0.6375	11.7085	0.1154	18.9946	1.2087	-0.057
DIS	-13.1632	-0.6788	0.0445	0.0503	0.8230	14.4123	0.2035	13.5132	1.6423	-0.051
CRM	-19.7371	-0.9626	0.0860	0.1029	1.1906	26.0449	0.4990	14.0377	2.2822	-0.027

			1	MSE					QLIKE								
	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T	
MMM	3.811	4.367	4.460	2.095	2.036	2.916	0.705	0.656	0.340	0.347	0.374	0.285		0.298			
AMZN	6.598	6.922	4.303	7.349	5.648	7.229	2.896	2.386	0.274	0.272	0.284	0.272	0.252	0.268	0.385	0.300	
AXP	7.114	6.763	4.254	5.992	4.613	7.500	1.747	1.446	0.319	0.308	0.336	0.308	0.280	0.307	0.369	0.31	
AMGN	1.466	1.710	1.533	1.240	1.392	1.433	0.893	0.796	0.251	0.266	0.277	0.248	0.243	0.261	0.440	0.379	
AAPL	3.255	3.915	2.605	4.532	2.773	3.606	1.437	1.258	0.285	0.296	0.290	0.308	0.274	0.292	0.354	0.309	
BA	12.445	17.245	7.899	15.445	10.850	19.435	9.753	8.852	0.235	0.257	0.221	0.257	0.220	0.261	0.676	0.595	
CAT	3.485	4.365	3.281	4.187	2.955	3.982	1.539	1.349	0.233	0.254	0.240	0.252	0.221	0.250	0.325	0.283	
CVX	3.395	3.965	2.909	3.306	2.950	3.594	1.439	1.345	0.237	0.246	0.237	0.239	0.225	0.223	0.348	0.344	
CSCO	2.043	2.289	1.848	1.807	1.973	2.108	0.578	0.488	0.339	0.315	0.385	0.289	0.316	0.315	0.361	0.28	
KO	0.297	0.350	0.315	0.333	0.325	0.399	0.217	0.210	0.189	0.183	0.214	0.195	0.194	0.184	0.313	0.273	
HD	1.209	1.343	1.290	1.246	1.080	1.266	0.889	0.828	0.194	0.200	0.203	0.203	0.189	0.197	0.335	0.30	
HON	1.316	1.597	1.069	1.473	1.128	1.509	0.619	0.554	0.222	0.233	0.227	0.229	0.212	0.223	0.297	0.27	
INTC	12.367	15.986	13.572	12.255	9.763	10.668	4.192	3.204	0.339	0.353	0.359	0.308	0.294	0.306	0.600	0.413	
$_{\rm IBM}$	1.421	1.958	1.241	1.424	1.349	1.621	0.430	0.409	0.282	0.297	0.309	0.269	0.281	0.288	0.328	0.26	
JNJ	0.288	0.313	0.248	0.286	0.276	0.289	0.211	0.206	0.193	0.194	0.202	0.194	0.191	0.191	0.270	0.248	
$_{ m JPM}$	2.824	3.619	3.770	3.072	2.607	4.017	0.925	0.809	0.255	0.272	0.303	0.260	0.246	0.258	0.296	0.27	
MCD	0.419	0.443	0.353	0.486	0.458	0.607	0.301	0.281	0.247	0.250	0.271	0.258	0.259	0.258	0.441	0.36	
MRK	0.745	0.918	0.685	0.739	0.819	0.763	0.476	0.425	0.222	0.226	0.260	0.219	0.225	0.220	0.352	0.289	
MSFT	2.444	3.168	1.941	3.368	2.308	2.724	0.934	0.836	0.264	0.277	0.279	0.285	0.266	0.263	0.324	0.277	
NKE	3.548	4.266	2.955	3.881	4.349	4.682	1.299	0.997	0.326	0.332	0.382	0.305	0.328	0.335	0.475	0.333	
PG	0.332	0.337	0.298	0.324	0.270	0.314	0.226	0.215	0.181	0.180	0.195	0.188	0.176	0.180	0.316	0.27	
GS	3.024	3.197	3.362	2.719	2.400	3.504	1.350	1.256	0.227	0.230	0.251	0.219	0.208	0.228	0.294	0.28	
TRV	1.376	1.961	1.193	1.488	1.243	2.065	0.823	0.703	0.209	0.230	0.230	0.211	0.204	0.221	0.342	0.27	
UNH	1.807	2.239	1.939	1.613	1.651	1.892	1.153	0.996	0.234	0.251	0.243	0.230	0.228	0.238	0.408	0.318	
VZ	0.574	0.672	0.617	0.480	0.518	0.638	0.282	0.273	0.320	0.315	0.354	0.302	0.319	0.320	0.475	0.42	
V	1.980	2.007	1.432	1.852	1.541	1.848	0.731	0.649	0.304	0.308	0.307	0.305	0.303	0.296	0.392	0.35	
WMT	0.726	0.628	0.721	0.580	0.608	0.628	0.338	0.311	0.312	0.307	0.356	0.262	0.292	0.277	0.403	0.32	
DIS	4.821	5.702	3.879	3.880	3.505	4.395	1.772	1.534	0.307	0.312	0.318	0.275	0.275	0.289	0.421	0.34	
CRM	12.925	15.807	17.502	10.292	8.529	9.766	3.461	2.858	0.311	0.329	0.401	0.288	0.290	0.289	0.444	0.33	

**Figure 1:** Out-of-sample average MSE (left panel) and QLIKE (right panel) forecasting performance of assets in the Dow Jones Average Index

			1	MSE				QLIKE								
	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	$_{ m MS-T}$	SV-N	SV-T	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T
MMM	3.897	3.890	3.466	2.487	2.685	3.133	0.711	0.653	0.365	0.339	0.373	0.301	0.322	0.339	0.354	0.282
AMZN	7.220	8.361	6.581	7.460	7.977	8.077	2.600	2.344	0.271	0.279	0.294	0.263	0.270	0.272	0.335	0.296
AXP	8.524	8.094	7.569	7.448	7.246	7.715	1.634	1.436	0.358	0.332	0.442	0.326	0.354	0.338	0.369	0.306
AMGN	1.309	1.463	1.422	1.331	1.336	1.452	0.933	0.796	0.240	0.250	0.273	0.245	0.237	0.236	0.476	0.378
AAPL	3.886	4.487	3.295	4.875	3.377	4.350	1.369	1.248	0.293	0.304	0.299	0.303	0.288	0.296	0.333	0.299
BA	16.314	24.239	13.741	24.389	16.806	23.132	9.124	8.255	0.277	0.304	0.281	0.306	0.268	0.286	0.597	0.538
CAT	3.899	4.616	3.895	4.700	4.011	4.448	1.462	1.298	0.252	0.267	0.264	0.269		0.256		
CVX	3.826	4.798	3.319	4.537	3.415	5.463	1.465	1.309	0.266	0.279	0.266	0.275	0.249	0.274	0.454	0.356
CSCO	2.397	2.588	2.262	2.000	2.543	2.263	0.558	0.508	0.346	0.340	0.401	0.298	0.352	0.315	0.328	0.289
CO.	0.707	0.476	0.545	0.462	0.481	0.462	0.207	0.211	0.247	0.201	0.247	0.207	0.212	0.198	0.292	0.264
ID	1.534	1.616	1.537	1.721	1.550	1.451	0.804	0.761	0.218	0.222	0.227	0.228	0.225	0.212	0.300	0.290
HON	1.258	1.628	1.233	1.548	1.294	1.425	0.623	0.561	0.234	0.237	0.259	0.233	0.237	0.233	0.329	0.287
NTC	11.618	20.523	10.189	15.221	12.837	14.006	4.012	3.111	0.347	0.381	0.358	0.343	0.341	0.351	0.538	0.383
BM	1.973	2.695	1.824	1.829	1.759	2.354	0.424	0.417	0.328	0.336	0.364	0.287	0.318	0.329	0.314	0.246
INJ	0.335	0.348	0.317	0.357	0.343	0.278	0.221	0.210	0.211	0.204	0.230	0.210	0.217	0.192	0.297	0.242
IPM	2.770	3.501	3.272	3.172	2.645	3.538	0.945	0.851	0.269	0.281	0.308	0.265	0.260	0.274	0.355	0.333
MCD	0.454	0.502	0.432	0.552	0.610	0.557	0.289	0.279	0.242	0.257	0.264	0.256	0.270	0.268	0.388	0.344
MRK	0.884	0.961	0.837	0.926	0.841	0.893	0.472	0.419	0.250	0.234	0.328	0.230	0.234	0.224	0.370	0.279
MSFT	3.105	3.807	2.827	3.491	2.924	3.239	0.858	0.846	0.274	0.284	0.286	0.281	0.269	0.273	0.288	0.274
NKE	5.484	6.642	4.922	4.871	5.811	6.573	1.161	0.992	0.417	0.397	0.464	0.345	0.379	0.365	0.434	0.359
PG	0.389	0.387	0.357	0.390	0.320	0.365	0.221	0.212	0.188	0.187	0.201	0.193	0.190	0.187	0.283	0.254
3S	2.805	2.869	2.846	3.105	2.427	2.597	1.337	1.237	0.235	0.237	0.240	0.238	0.219	0.224	0.319	0.285
$\Gamma RV$	1.923	2.223	1.520	1.994	1.575	2.161	0.763	0.680	0.234	0.241	0.257	0.235	0.219	0.231	0.334	0.281
JNH	1.886	2.016	1.825	1.755	1.676	1.596	1.125	0.991	0.243	0.247	0.248	0.238	0.238	0.228	0.365	0.302
/Z	0.830	0.998	0.975	0.690	0.643	0.826	0.284	0.280	0.335	0.336	0.396	0.314	0.339	0.332	0.492	0.405
7	2.225	2.363	2.001	2.264	1.817	1.968	0.697	0.657	0.304	0.309	0.312	0.306	0.304	0.298	0.381	0.359
VMT	1.103	0.752	0.933	0.707	0.786	0.779	0.320	0.308	0.327	0.316	0.383	0.263	0.325	0.281	0.355	0.30
OIS	6.441	7.183	5.424	5.695	5.158	8.076	1.716		0.384	0.366	0.389	0.320	0.339	0.355	0.419	0.340
CRM	14.985	12.765	11.633	8.845	10.617				0.338	0.315	0.425	0.270		0.296		

**Figure 2:** Out-of-sample average MSE (left panel) and QLIKE (right panel) forecasting performance of assets in the Dow Jones Average Index. Sample size **1000** observations

• For series contaminated by outliers, the SV-t and GAS-t models outperform their competitors.

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- This work highlights that a deep understanding of the models is more important than simply running "horse races" or relying on computational power, underscoring the crucial role of well-trained specialists in statistics and data science over untrained users.

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# THANK YOU.