

GARCH, GAS, SV, and MSGARCH models:

Do we really need all of them for forecasting daily risk measures?

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Introduction

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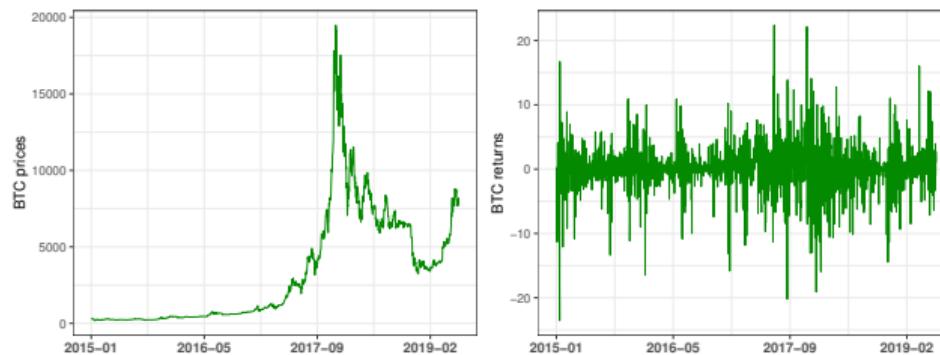


Figure 1: Prices (left panel) and returns (right panel) of Bitcoin

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- Other widely used risk measures include Value-at-Risk (VaR) and Expected Shortfall (ES).
- Estimating ES requires first estimating VaR.
- Estimating VaR, in turn, requires volatility estimation.

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- Given its importance, numerous approaches have been developed for forecasting daily volatility.
- While several options benefit researchers and experienced practitioners, they pose significant challenges for (untrained) practitioners, who must choose among these models for their daily tasks, often with limited or no information to guide their decisions.

Introduction

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Main goals and contribution

Offer insights to help researchers and practitioners in selecting the most appropriate volatility model for their data (based on user-friendly implementations).

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Main goals and contribution

Offer insights to help researchers and practitioners in selecting the most appropriate volatility model for their data (based on user-friendly implementations).

Our focus will be on easy-to-use, user-friendly implementations available in the open-source R environment.

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- **GAS**: Generalized Autoregressive Score model, introduced independently by Creal et al. (2013) and Harvey (2013).

Hereafter, let $r_t = (P_t - P_{t-1})/P_{t-1} \approx \log(P_t/P_{t-1})$ denote the return at time t , where P_t represents the closing price at time t . We assume $\mathbb{E}(r_t | \mathcal{F}_{t-1}) = 0$

GARCH model

Assumes that the conditional variance at time t is fully determined by past squared returns and its own past values. In its simplest form, the model is specified as:

$$r_t = \sigma_t \epsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where $\omega > 0$ and $\alpha, \beta \geq 0$ are model parameters, σ_t^2 represents the conditional variance (or squared volatility) at time t , and the innovation term ϵ_t has zero-mean and unit-variance.

In this study we considered the standard Normal and Student-t innovation distributions.

SV model

Assumes that the log-conditional variance evolves stochastically following an AR(1) process.
Its dynamics can be described as follows:

$$r_t = \exp(h_t/2)\epsilon_t, \quad (3)$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma\eta_t, \quad (4)$$

where h_t is the log conditional variance at time t , μ , ϕ and σ are parameters to be estimated, $\eta_t \sim N(0, 1)$. In this study, ϵ_t follows either a standardized Normal or Student-t distribution.

Models

MSGARCH model

This specification allows for multiple volatility regimes. In its simplest form, the dynamics can be described as follows:

$$r_t = \sigma_t^{(k)} \epsilon_t, \quad (5)$$

$$\sigma_t^{2(k)} = \omega^{(k)} + \alpha^{(k)} r_{t-1}^2 + \beta^{(k)} \sigma_{t-1}^{2(k)}, \quad (6)$$

where $\omega^{(k)} > 0$ and $\alpha^{(k)}, \beta^{(k)} \geq 0$ are the model parameters in regime k , $\sigma_t^{2(k)}$ denotes the conditional variance in regime k at time t , and ϵ_t follows either a standardized Normal or a standardized Student- t distribution.

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The regime-switching mechanism is governed by the latent process $\{S_t\}$, assumed to be a first-order Markov chain with transition probability matrix Π . Its elements are given by

$$\pi_{ij} = \mathbb{P}(S_t = j | S_{t-1} = i), \quad (7)$$

representing the probability of moving from state i at time $t - 1$ to state j at time t .

GAS

Its central idea is that the dynamic behaviour of time-varying parameters depends on their own past values and the score of the conditional density function (hence the name *score model*).

Let $r_t | \mathcal{F}_{t-1} \sim p(r_t; \theta_t)$ with $\theta_t \in \mathbb{R}^p$ being a vector of time-varying parameters fully characterising $p(\cdot)$. Then, in the general, unrestricted, GAS specification, the dynamics of θ_t is given by

$$\theta_{t+1} = \kappa + As_t + B\theta_t, \quad (8)$$

where $s_t = S_t(\theta_t) \nabla_t(r_t, \theta_t)$, with $\nabla_t(r_t, \theta_t)$ being the score of the conditional density function and $S_t(\theta_t) = I_t(\theta_t)^{-\gamma}$ with typical values of $\gamma \in \{0, 1/2, 1\}$, and $\kappa_{p \times 1}$, $A_{p \times p}$ and $B_{p \times p}$.

Models

GAS

When the parameter space is restricted, it is common to use a mapping function $\Lambda(\cdot)$ such that

$$\theta_{t+1} = \Lambda(\tilde{\theta}_{t+1}), \quad (9)$$

$$\tilde{\theta}_{t+1} = \tilde{\kappa} + \tilde{A}s_t + \tilde{B}\tilde{\theta}_t. \quad (10)$$

In particular, setting $\gamma = 0$ and using an exponential function for the time-varying scale parameter under a Student-t distribution assumption, we obtain the Beta-t-EGARCH model of Harvey and Sucarrat (2014).

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In particular, setting $\gamma = 0$ and using an exponential function for the time-varying scale parameter under a Student-t distribution assumption, we obtain the Beta-t-EGARCH model of Harvey and Sucarrat (2014).

$$r_t = \sigma_t \epsilon_t, \quad (11)$$

$$\log(\sigma_t) = \delta + \phi \log(\sigma_{t-1}) + \kappa \left(\frac{(\nu + 1)r_{t-1}^2}{\nu \sigma_{t-1}^2 + r_{t-1}^2} - 1 \right) \quad (12)$$

Models

- Parameters are estimated by Maximum Likelihood
- For SV, the procedure of Wahl (2018) is used.
- In all cases, we are interested in $\mathbb{V}(r_{T+1}|\mathcal{F}_T)$, where \mathcal{F}_T is the information available up to time T

Monte Carlo Simulations

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- The four models previously described are used both as true DGPs and for generating one-step-ahead volatility forecasts

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Parameters

More than 20 parameter vector configurations were designed to closely replicate patterns observed in real data.

Monte Carlo Simulations

Model	Parameter values 1	Parameter values 2
GARCH	$\omega = 0.18, \alpha = 0.09, \beta = 0.89$	$\omega = 0.37, \alpha = 0.14, \beta = 0.77$
GAS	$\kappa = 0.03, A = 0.22, B = 0.98$	$\kappa = 0.06, A = 0.34, B = 0.92$
SV	$\mu = 1.74, \phi = 0.97, \sigma_\eta = 0.17$	$\mu = 1.15, \phi = 0.90, \sigma_\eta = 0.36$
MSGARCH	$\omega_1 = 0.005, \alpha_1 = 0.025, \beta_1 = 0.95$ $\omega_2 = 0.1, \alpha_2 = 0.25, \beta_2 = 0.70$ $P = \begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix}$	$\omega_1 = 0.01, \alpha_1 = 0.16, \beta_1 = 0.30$ $\omega_2 = 0.18, \alpha_2 = 0.46, \beta_2 = 0.20$ $P = \begin{bmatrix} 0.98 & 0.05 \\ 0.02 & 0.95 \end{bmatrix}$

Table 1: Two parameter configurations (over 20) used in the Monte Carlo experiment

Monte Carlo Simulations

Loss Function	Formula	Loss Function	Formula
MSE	$R^{-1} \sum_{i=1}^R (\hat{\sigma}_i^2 - \sigma_i^2)^2$	MAE	$R^{-1} \sum_{i=1}^R \hat{\sigma}_i^2 - \sigma_i^2 $
QLIKE	$R^{-1} \sum_{i=1}^R \left(\frac{\sigma_i^2}{\hat{\sigma}_i^2} - \log \frac{\sigma_i^2}{\hat{\sigma}_i^2} - 1 \right)$	MAE _L	$R^{-1} \sum_{i=1}^R \log \hat{\sigma}_i^2 - \log \sigma_i^2 $
MSE _L	$R^{-1} \sum_{i=1}^R (\log \hat{\sigma}_i^2 - \log \sigma_i^2)^2$	MAE _{Sd}	$R^{-1} \sum_{i=1}^R \hat{\sigma}_i - \sigma_i $
MSE _{Sd}	$R^{-1} \sum_{i=1}^R (\hat{\sigma}_i - \sigma_i)^2$	MAE _P	$R^{-1} \sum_{i=1}^R \left \frac{\hat{\sigma}_i}{\sigma_i} - 1 \right $
MSE _P	$R^{-1} \sum_{i=1}^R \left(\frac{\hat{\sigma}_i}{\sigma_i} - 1 \right)^2$		

Table 2: Loss functions employed in the evaluation of volatility forecasts.

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MSE	$R^{-1} \sum_{i=1}^R (\hat{\sigma}_i^2 - \sigma_i^2)^2$	MAE	$R^{-1} \sum_{i=1}^R \hat{\sigma}_i^2 - \sigma_i^2 $
QLIKE	$R^{-1} \sum_{i=1}^R \left(\frac{\sigma_i^2}{\hat{\sigma}_i^2} - \log \frac{\sigma_i^2}{\hat{\sigma}_i^2} - 1 \right)$	MAE _L	$R^{-1} \sum_{i=1}^R \log \hat{\sigma}_i^2 - \log \sigma_i^2 $
MSE _L	$R^{-1} \sum_{i=1}^R (\log \hat{\sigma}_i^2 - \log \sigma_i^2)^2$	MAE _{Sd}	$R^{-1} \sum_{i=1}^R \hat{\sigma}_i - \sigma_i $
MSE _{Sd}	$R^{-1} \sum_{i=1}^R (\hat{\sigma}_i - \sigma_i)^2$	MAE _P	$R^{-1} \sum_{i=1}^R \left \frac{\hat{\sigma}_i}{\sigma_i} - 1 \right $
MSE _P	$R^{-1} \sum_{i=1}^R \left(\frac{\hat{\sigma}_i}{\sigma_i} - 1 \right)^2$		

Table 2: Loss functions employed in the evaluation of volatility forecasts.

To select the best model (or set of best models) the model confidence set of Hansen et al. (2011) was used

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	0.0608	0.0034	0.0068	0.0048	0.0070	0.1670	0.0608	0.0497	0.0607
	GARCH-T	0.0538	0.0027	0.0055	0.0039	0.0054	0.1569	0.0565	0.0463	0.0563
	GAS-N	2.4896	0.0064	0.0107	0.0247	0.0236	0.2422	0.0691	0.0606	0.0716
	GAS-T	0.0730	0.0035	0.0073	0.0053	0.0068	0.1851	0.0651	0.0541	0.0644
	MS-N	0.1189	0.0062	0.0124	0.0089	0.0128	0.2402	0.0863	0.0709	0.0859
	MS-T	0.0992	0.0049	0.0100	0.0073	0.0096	0.2187	0.0773	0.0640	0.0764
	SV-N	0.1443	0.0086	0.0186	0.0122	0.0149	0.2838	0.1100	0.0871	0.1015
	SV-T	0.1114	0.0062	0.0133	0.0090	0.0109	0.2398	0.0904	0.0726	0.0845
	GARCH-N	0.0282	0.0014	0.0028	0.0020	0.0028	0.1136	0.0406	0.0335	0.0405
	GARCH-T	0.0245	0.0012	0.0023	0.0017	0.0023	0.1048	0.0374	0.0308	0.0374
DGP: GARCH	N = 1000	1.9691	0.0045	0.0071	0.0197	0.0171	0.1910	0.0513	0.0461	0.0536
	GAS-N	0.0617	0.0023	0.0046	0.0038	0.0044	0.1498	0.0507	0.0429	0.0502
	GAS-T	0.0817	0.0039	0.0077	0.0058	0.0081	0.1896	0.0666	0.0554	0.0669
	MS-N	0.0566	0.0028	0.0056	0.0041	0.0055	0.1611	0.0568	0.0471	0.0564
	MS-T	0.1209	0.0072	0.0153	0.0099	0.0128	0.2703	0.1046	0.0828	0.0974
	SV-N	0.0999	0.0050	0.0106	0.0074	0.0091	0.2254	0.0839	0.0676	0.0789
	SV-T	0.0100	0.0006	0.0012	0.0008	0.0012	0.0720	0.0262	0.0214	0.0262
	GARCH-N	0.0080	0.0005	0.0010	0.0007	0.0010	0.0660	0.0243	0.0197	0.0243
	GARCH-T	0.0841	0.0018	0.0035	0.0036	0.0040	0.1243	0.0397	0.0343	0.0402
	GAS-N	0.0590	0.0016	0.0033	0.0030	0.0031	0.1246	0.0418	0.0354	0.0414
N = 2500	MS-N	0.0355	0.0019	0.0038	0.0027	0.0039	0.1339	0.0479	0.0395	0.0482
	MS-T	0.0303	0.0011	0.0023	0.0019	0.0023	0.1034	0.0352	0.0297	0.0352
	SV-N	0.1098	0.0065	0.0137	0.0088	0.0116	0.2644	0.1025	0.0810	0.0960
	SV-T	0.1034	0.0044	0.0093	0.0068	0.0080	0.2176	0.0803	0.0649	0.0759

Table 3: Forecast evaluation under **uncontaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	0.0758	0.0045	0.0089	0.0059	0.0094	0.1868	0.0714	0.0567	0.0719
	GARCH-T	0.0744	0.0042	0.0082	0.0056	0.0089	0.1803	0.0684	0.0546	0.0692
	GAS-N	0.1552	0.0075	0.0144	0.0106	0.0168	0.2396	0.0889	0.0716	0.0907
	GAS-T	0.0419	0.0029	0.0058	0.0036	0.0059	0.1488	0.0581	0.0457	0.0580
	MS-N	0.1412	0.0088	0.0173	0.0114	0.0184	0.2678	0.1033	0.0819	0.1039
	MS-T	0.0959	0.0062	0.0125	0.0080	0.0122	0.2214	0.0861	0.0679	0.0852
	SV-N	0.1325	0.0098	0.0210	0.0125	0.0172	0.2933	0.1212	0.0929	0.1121
	SV-T	0.0797	0.0058	0.0121	0.0073	0.0105	0.2215	0.0896	0.0694	0.0845
DGP: GAS	GARCH-N	0.0589	0.0030	0.0059	0.0042	0.0063	0.1590	0.0593	0.0476	0.0597
	GARCH-T	0.0530	0.0028	0.0054	0.0038	0.0058	0.1514	0.0565	0.0454	0.0570
	GAS-N	0.1198	0.0057	0.0107	0.0082	0.0130	0.2040	0.0743	0.0605	0.0761
	GAS-T	0.0221	0.0015	0.0030	0.0019	0.0029	0.1052	0.0408	0.0322	0.0405
	MS-N	0.1003	0.0069	0.0134	0.0086	0.0152	0.2258	0.0870	0.0691	0.0887
	MS-T	0.0696	0.0044	0.0087	0.0056	0.0089	0.1827	0.0703	0.0557	0.0701
	SV-N	0.1119	0.0082	0.0173	0.0103	0.0146	0.2808	0.1155	0.0887	0.1075
	SV-T	0.0651	0.0046	0.0096	0.0059	0.0084	0.2099	0.0844	0.0656	0.0799
N = 2500	GARCH-N	0.0465	0.0022	0.0043	0.0032	0.0047	0.1337	0.0487	0.0396	0.0495
	GARCH-T	0.0425	0.0020	0.0039	0.0030	0.0044	0.1277	0.0465	0.0379	0.0472
	GAS-N	0.1219	0.0049	0.0089	0.0076	0.0120	0.1851	0.0650	0.0539	0.0674
	GAS-T	0.0079	0.0005	0.0010	0.0007	0.0010	0.0639	0.0245	0.0195	0.0244
	MS-N	0.0745	0.0049	0.0092	0.0060	0.0116	0.1828	0.0706	0.0559	0.0727
	MS-T	0.0377	0.0023	0.0045	0.0030	0.0046	0.1354	0.0509	0.0408	0.0510
	SV-N	0.0982	0.0072	0.0152	0.0090	0.0131	0.2743	0.1127	0.0866	0.1057
	SV-T	0.0533	0.0036	0.0076	0.0047	0.0068	0.2000	0.0796	0.0621	0.0760

Table 4: Forecast evaluation under **uncontaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	0.5510	0.0434	0.0784	0.0487	0.1153	0.5507	0.2178	0.1708	0.2396
	GARCH-T	0.5214	0.0420	0.0759	0.0468	0.1098	0.5436	0.2150	0.1687	0.2381
	GAS-N	0.7431	0.0475	0.0842	0.0561	0.1320	0.5795	0.2235	0.1770	0.2495
	GAS-T	0.4303	0.0362	0.0671	0.0403	0.0887	0.5136	0.2061	0.1606	0.2242
	MS-N	0.5349	0.0444	0.0813	0.0494	0.1153	0.5611	0.2242	0.1751	0.2443
	MS-T	0.5231	0.0413	0.0765	0.0472	0.1026	0.5594	0.2219	0.1739	0.2406
	SV-N	0.5815	0.0391	0.0843	0.0527	0.0726	0.5548	0.2278	0.1755	0.2128
	SV-T	0.4341	0.0322	0.0646	0.0397	0.0685	0.4977	0.2016	0.1564	0.2028
DGP: SV	GARCH-N	0.4757	0.0380	0.0693	0.0427	0.0975	0.5223	0.2067	0.1621	0.2269
	GARCH-T	0.4715	0.0383	0.0694	0.0426	0.0985	0.5239	0.2076	0.1627	0.2295
	GAS-N	0.6814	0.0444	0.0769	0.0511	0.1383	0.5464	0.2129	0.1679	0.2389
	GAS-T	0.4005	0.0339	0.0628	0.0376	0.0824	0.4996	0.2004	0.1562	0.2181
	MS-N	0.4892	0.0413	0.0763	0.0456	0.1038	0.5415	0.2181	0.1697	0.2375
	MS-T	0.4728	0.0387	0.0713	0.0433	0.0971	0.5340	0.2128	0.1664	0.2314
	SV-N	0.5298	0.0353	0.0761	0.0479	0.0650	0.5376	0.2190	0.1695	0.2048
	SV-T	0.3978	0.0291	0.0581	0.0360	0.0617	0.4767	0.1920	0.1494	0.1937
N = 2500	GARCH-N	0.4618	0.0369	0.0671	0.0414	0.0942	0.5115	0.2025	0.1587	0.2227
	GARCH-T	0.4654	0.0377	0.0682	0.0419	0.0976	0.5170	0.2048	0.1605	0.2270
	GAS-N	0.7886	0.0451	0.0772	0.0534	0.1437	0.5494	0.2118	0.1675	0.2395
	GAS-T	0.3921	0.0334	0.0617	0.0369	0.0815	0.4920	0.1975	0.1539	0.2155
	MS-N	0.4681	0.0410	0.0742	0.0438	0.1069	0.5297	0.2144	0.1664	0.2370
	MS-T	0.4307	0.0361	0.0662	0.0398	0.0909	0.5072	0.2028	0.1583	0.2221
	SV-N	0.5254	0.0347	0.0743	0.0471	0.0646	0.5330	0.2160	0.1675	0.2031
	SV-T	0.3888	0.0284	0.0565	0.0351	0.0608	0.4688	0.1886	0.1468	0.1910

Table 5: Forecast evaluation under **uncontaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	0.0095	0.0056	0.0116	0.0025	0.0105	0.0610	0.0786	0.0343	0.0775
	GARCH-T	0.0076	0.0050	0.0101	0.0021	0.0099	0.0584	0.0754	0.0329	0.0755
	GAS-N	0.0115	0.0060	0.0121	0.0027	0.0121	0.0642	0.0809	0.0356	0.0807
	GAS-T	0.0090	0.0055	0.0112	0.0024	0.0104	0.0620	0.0794	0.0347	0.0785
	MS-N	0.0076	0.0052	0.0106	0.0021	0.0100	0.0591	0.0773	0.0335	0.0762
	MS-T	0.0095	0.0054	0.0112	0.0024	0.0102	0.0611	0.0780	0.0341	0.0764
	SV-N	0.0201	0.0150	0.0332	0.0061	0.0250	0.1051	0.1490	0.0620	0.1341
	SV-T	0.0161	0.0112	0.0245	0.0047	0.0189	0.0880	0.1220	0.0513	0.1111
DGP: MSGARCH	GARCH-N	0.0049	0.0031	0.0063	0.0013	0.0060	0.0444	0.0580	0.0251	0.0577
	GARCH-T	0.0044	0.0030	0.0060	0.0012	0.0058	0.0437	0.0569	0.0247	0.0569
	GAS-N	0.0066	0.0038	0.0075	0.0016	0.0078	0.0505	0.0638	0.0281	0.0644
	GAS-T	0.0067	0.0039	0.0080	0.0017	0.0075	0.0522	0.0665	0.0292	0.0660
	MS-N	0.0053	0.0033	0.0068	0.0014	0.0066	0.0479	0.0613	0.0268	0.0610
	MS-T	0.0057	0.0030	0.0060	0.0013	0.0060	0.0454	0.0570	0.0251	0.0568
	SV-N	0.0166	0.0133	0.0288	0.0052	0.0226	0.1022	0.1450	0.0603	0.1317
	SV-T	0.0133	0.0092	0.0199	0.0038	0.0159	0.0813	0.1118	0.0472	0.1027
N = 2500	GARCH-N	0.0027	0.0018	0.0037	0.0007	0.0035	0.0348	0.0457	0.0198	0.0455
	GARCH-T	0.0026	0.0018	0.0037	0.0007	0.0036	0.0360	0.0467	0.0203	0.0468
	GAS-N	0.0294	0.0039	0.0071	0.0026	0.0105	0.0505	0.0580	0.0264	0.0593
	GAS-T	0.0058	0.0031	0.0063	0.0014	0.0058	0.0471	0.0592	0.0262	0.0587
	MS-N	0.0021	0.0015	0.0030	0.0006	0.0030	0.0321	0.0415	0.0181	0.0415
	MS-T	0.0021	0.0014	0.0028	0.0006	0.0028	0.0306	0.0390	0.0171	0.0390
	SV-N	0.0161	0.0127	0.0273	0.0050	0.0218	0.1026	0.1455	0.0605	0.1329
	SV-T	0.0124	0.0084	0.0180	0.0035	0.0146	0.0792	0.1084	0.0459	0.1000

Table 6: Forecast evaluation under **uncontaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	3.0883	0.1346	0.2001	0.1837	0.5500	1.2254	0.3537	0.3240	0.4873
	GARCH-T	3.1632	0.1421	0.2104	0.1914	0.5799	1.2572	0.3629	0.3326	0.5047
	GAS-N	> 10	0.7279	0.5480	1.1870	> 10	2.9836	0.5240	0.5635	1.2506
	GAS-T	0.3745	0.0195	0.0355	0.0274	0.0477	0.4594	0.1529	0.1310	0.1710
	MS-N	> 10	0.4670	0.1858	0.8437	> 10	1.7158	0.2822	0.2763	0.7470
	MS-T	2.0181	0.0850	0.1237	0.1144	0.3986	0.8429	0.2521	0.2266	0.3355
	SV-N	1.2415	0.0617	0.0946	0.0808	0.2407	0.6494	0.2054	0.1800	0.2581
	SV-T	0.2877	0.0164	0.0300	0.0223	0.0398	0.3886	0.1333	0.1126	0.1464
DGP: GARCH	GARCH-N	2.8981	0.1419	0.2165	0.1879	0.5286	1.2992	0.3838	0.3484	0.5255
	GARCH-T	2.9288	0.1456	0.2216	0.1914	0.5453	1.3150	0.3895	0.3532	0.5351
	GAS-N	> 10	2.4645	0.7976	4.3400	> 10	6.7300	0.6696	0.7664	3.1338
	GAS-T	0.3323	0.0182	0.0333	0.0250	0.0441	0.4436	0.1500	0.1276	0.1675
	MS-N	2.2681	0.1139	0.1699	0.1471	0.4476	1.0159	0.3090	0.2764	0.4223
	MS-T	1.9027	0.0972	0.1494	0.1272	0.3523	0.9658	0.2956	0.2639	0.3918
	SV-N	1.1118	0.0622	0.0982	0.0793	0.2137	0.6791	0.2167	0.1896	0.2749
	SV-T	0.3140	0.0184	0.0330	0.0245	0.0465	0.4010	0.1365	0.1157	0.1529
N = 2500	GARCH-N	2.6449	0.1410	0.2203	0.1836	0.4778	1.3199	0.3980	0.3583	0.5390
	GARCH-T	2.6555	0.1429	0.2228	0.1853	0.4874	1.3283	0.4009	0.3608	0.5438
	GAS-N	> 10	1.2223	0.9560	1.9592	> 10	4.6304	0.7861	0.8607	2.0082
	GAS-T	0.3220	0.0181	0.0330	0.0244	0.0434	0.4429	0.1515	0.1282	0.1690
	MS-N	2.0581	0.1143	0.1783	0.1453	0.3938	1.0940	0.3419	0.3024	0.4560
	MS-T	2.0453	0.1096	0.1714	0.1419	0.3795	1.0995	0.3378	0.3013	0.4470
	SV-N	1.0545	0.0628	0.1016	0.0793	0.1981	0.7094	0.2280	0.1991	0.2887
	SV-T	0.3410	0.0204	0.0366	0.0267	0.0515	0.4218	0.1442	0.1220	0.1632

Table 7: Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

	Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	2.3374	0.1265	0.1979	0.1592	0.4280	1.1736	0.3661	0.3223	0.4919
	GARCH-T	2.2496	0.1293	0.2022	0.1591	0.4346	1.1698	0.3681	0.3230	0.4971
	GAS-N	> 10	0.5658	0.4565	0.8755	> 10	2.4650	0.5154	0.5090	1.0801
	GAS-T	0.3279	0.0183	0.0331	0.0242	0.0449	0.4211	0.1467	0.1224	0.1646
	MS-N	1.4286	0.0842	0.1324	0.1015	0.2885	0.8304	0.2749	0.2350	0.3573
	MS-T	1.3299	0.0694	0.1088	0.0871	0.2532	0.7555	0.2462	0.2120	0.3146
	SV-N	0.7552	0.0474	0.0793	0.0581	0.1415	0.5895	0.2047	0.1711	0.2423
	SV-T	0.2182	0.0142	0.0259	0.0177	0.0342	0.3384	0.1237	0.1009	0.1360
DGP: GAS	GARCH-N	2.4847	0.1440	0.2245	0.1760	0.4877	1.2687	0.3990	0.3503	0.5430
	GARCH-T	2.4037	0.1442	0.2251	0.1740	0.4859	1.2633	0.4004	0.3504	0.5446
	GAS-N	> 10	0.6541	0.6814	0.9031	> 10	2.9270	0.6593	0.6602	1.3132
	GAS-T	0.2777	0.0167	0.0305	0.0216	0.0404	0.4003	0.1423	0.1176	0.1588
	MS-N	1.4652	0.0981	0.1544	0.1127	0.3298	0.9060	0.3073	0.2602	0.4046
	MS-T	1.1638	0.0768	0.1237	0.0904	0.2454	0.8100	0.2739	0.2324	0.3499
	SV-N	0.8344	0.0564	0.0924	0.0663	0.1723	0.6341	0.2187	0.1838	0.2705
	SV-T	0.2195	0.0150	0.0271	0.0184	0.0371	0.3332	0.1222	0.0996	0.1360
N = 2500	GARCH-N	2.4426	0.1515	0.2367	0.1810	0.5081	1.3132	0.4175	0.3652	0.5690
	GARCH-T	2.4032	0.1516	0.2369	0.1798	0.5081	1.3093	0.4180	0.3649	0.5695
	GAS-N	> 10	0.8662	0.9124	1.1191	8.7939	3.5148	0.7818	0.7963	1.6479
	GAS-T	0.2436	0.0160	0.0292	0.0200	0.0383	0.3867	0.1401	0.1148	0.1561
	MS-N	1.5815	0.1109	0.1753	0.1257	0.3644	0.9950	0.3378	0.2863	0.4486
	MS-T	1.4069	0.0912	0.1471	0.1086	0.2873	0.9248	0.3086	0.2637	0.3996
	SV-N	0.8587	0.0620	0.1008	0.0708	0.1911	0.6581	0.2282	0.1915	0.2883
	SV-T	0.2146	0.0162	0.0291	0.0190	0.0403	0.3349	0.1247	0.1010	0.1404

Table 8: Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	2.3405	0.1669	0.2518	0.1810	0.6163	1.1962	0.4074	0.3438	0.5685
	GARCH-T	2.4600	0.1786	0.2682	0.1924	0.6637	1.2478	0.4235	0.3583	0.5971
	GAS-N	9.1394	0.3148	0.3923	0.3864	2.1378	1.6785	0.4840	0.4360	0.7924
	GAS-T	0.6880	0.0609	0.1036	0.0635	0.1748	0.6623	0.2568	0.2037	0.3083
	MS-N	1.3548	0.1124	0.1775	0.1166	0.3800	0.9082	0.3345	0.2721	0.4364
	MS-T	1.3345	0.1042	0.1659	0.1111	0.3481	0.8894	0.3246	0.2650	0.4190
	SV-N	0.9202	0.0695	0.1257	0.0812	0.1940	0.7007	0.2725	0.2157	0.2999
	SV-T	0.5707	0.0504	0.0886	0.0535	0.1379	0.5877	0.2329	0.1828	0.2683
DGP: SV	GARCH-N	2.3598	0.1754	0.2641	0.1874	0.6468	1.2409	0.4237	0.3575	0.5955
	GARCH-T	2.4155	0.1828	0.2745	0.1940	0.6760	1.2714	0.4337	0.3663	0.6133
	GAS-N	10	0.4004	0.4537	0.5307	6.7466	1.9746	0.5362	0.4885	0.9334
	GAS-T	0.6289	0.0565	0.0965	0.0587	0.1597	0.6332	0.2473	0.1955	0.2956
	MS-N	1.3620	0.1172	0.1839	0.1195	0.3989	0.9275	0.3420	0.2782	0.4506
	MS-T	1.2798	0.1058	0.1677	0.1103	0.3529	0.8911	0.3264	0.2663	0.4247
	SV-N	1.0067	0.0784	0.1337	0.0878	0.2468	0.7289	0.2771	0.2219	0.3210
	SV-T	0.5215	0.0463	0.0816	0.0491	0.1245	0.5656	0.2244	0.1760	0.2573
N = 2500	GARCH-N	2.3625	0.1828	0.2751	0.1924	0.6745	1.2698	0.4357	0.3671	0.6160
	GARCH-T	2.4527	0.1899	0.2848	0.1998	0.7046	1.3023	0.4450	0.3758	0.6327
	GAS-N	8.1638	0.4008	0.5167	0.4575	2.2197	2.0234	0.5905	0.5340	0.9894
	GAS-T	0.5982	0.0553	0.0944	0.0567	0.1563	0.6225	0.2445	0.1928	0.2920
	MS-N	1.5132	0.1330	0.2056	0.1336	0.4676	0.9959	0.3649	0.2980	0.4922
	MS-T	1.3460	0.1144	0.1796	0.1174	0.3907	0.9322	0.3416	0.2788	0.4504
	SV-N	1.0485	0.0838	0.1396	0.0917	0.2693	0.7564	0.2848	0.2292	0.3400
	SV-T	0.5036	0.0456	0.0801	0.0478	0.1235	0.5575	0.2216	0.1736	0.2547

Table 9: Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	0.1912	0.0959	0.1429	0.0386	0.3809	0.2621	0.2713	0.1313	0.3647
	GARCH-T	0.1789	0.0997	0.1517	0.0390	0.3680	0.2726	0.2862	0.1379	0.3840
	GAS-N	$\hat{\omega}_{10}$	1.7069	0.4714	0.9703	$\hat{\omega}_{10}$	1.4028	0.4055	0.2584	2.1100
	GAS-T	0.0316	0.0212	0.0378	0.0083	0.0551	0.1224	0.1472	0.0665	0.1652
	MS-N	0.1143	0.0625	0.0947	0.0243	0.2428	0.1921	0.2115	0.0994	0.2718
	MS-T	0.1218	0.0612	0.0921	0.0244	0.2422	0.1918	0.2081	0.0984	0.2670
	SV-N	0.0457	0.0316	0.0572	0.0123	0.0851	0.1357	0.1725	0.0757	0.1814
	SV-T	0.0296	0.0204	0.0369	0.0079	0.0531	0.1092	0.1368	0.0606	0.1463
DGP: MSGARCH	GARCH-N	0.1700	0.0980	0.1489	0.0381	0.3568	0.2653	0.2774	0.1340	0.3742
	GARCH-T	0.1550	0.0962	0.1500	0.0368	0.3275	0.2704	0.2890	0.1383	0.3841
	GAS-N	3.7793	0.5344	0.5033	0.2460	8.2986	0.6975	0.4706	0.2680	1.0039
	GAS-T	0.0246	0.0182	0.0326	0.0069	0.0459	0.1121	0.1371	0.0615	0.1531
	MS-N	0.1083	0.0640	0.0972	0.0243	0.2418	0.1932	0.2148	0.1006	0.2780
	MS-T	0.1204	0.0686	0.1032	0.0264	0.2631	0.2046	0.2232	0.1055	0.2908
	SV-N	0.0420	0.0312	0.0550	0.0116	0.0863	0.1345	0.1699	0.0749	0.1842
	SV-T	0.0241	0.0179	0.0325	0.0068	0.0451	0.1022	0.1286	0.0569	0.1380
N = 1000	GARCH-N	0.1627	0.0995	0.1525	0.0381	0.3495	0.2630	0.2763	0.1333	0.3753
	GARCH-T	0.1506	0.0972	0.1526	0.0368	0.3238	0.2721	0.2926	0.1398	0.3896
	GAS-N	2.6077	0.5596	0.5983	0.2486	4.9391	0.7648	0.5448	0.3080	1.1039
	GAS-T	0.0223	0.0168	0.0302	0.0063	0.0418	0.1056	0.1300	0.0582	0.1451
	MS-N	0.0979	0.0639	0.0997	0.0237	0.2231	0.2011	0.2296	0.1064	0.2932
	MS-T	0.1238	0.0745	0.1140	0.0284	0.2703	0.2221	0.2438	0.1150	0.3178
	SV-N	0.0420	0.0314	0.0549	0.0117	0.0864	0.1372	0.1714	0.0760	0.1887
	SV-T	0.0237	0.0178	0.0322	0.0067	0.0450	0.1028	0.1284	0.0570	0.1394
N = 2500	GARCH-N	0.1627	0.0995	0.1525	0.0381	0.3495	0.2630	0.2763	0.1333	0.3753
	GARCH-T	0.1506	0.0972	0.1526	0.0368	0.3238	0.2721	0.2926	0.1398	0.3896
	GAS-N	2.6077	0.5596	0.5983	0.2486	4.9391	0.7648	0.5448	0.3080	1.1039
	GAS-T	0.0223	0.0168	0.0302	0.0063	0.0418	0.1056	0.1300	0.0582	0.1451
	MS-N	0.0979	0.0639	0.0997	0.0237	0.2231	0.2011	0.2296	0.1064	0.2932
	MS-T	0.1238	0.0745	0.1140	0.0284	0.2703	0.2221	0.2438	0.1150	0.3178
	SV-N	0.0420	0.0314	0.0549	0.0117	0.0864	0.1372	0.1714	0.0760	0.1887
	SV-T	0.0237	0.0178	0.0322	0.0067	0.0450	0.1028	0.1284	0.0570	0.1394

Table 10: Forecast evaluation under contaminated series. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in emerging markets

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	1.0683	0.0919	0.1420	0.0934	0.3209	0.6657	0.2707	0.2097	0.3605
	GARCH-T	1.0718	0.0948	0.1445	0.0947	0.3387	0.6541	0.2645	0.2056	0.3581
	GAS-N	> 10	0.5513	0.4396	0.7427	> 10	1.7941	0.4144	0.3859	0.9636
	GAS-T	0.1926	0.0197	0.0350	0.0198	0.0512	0.2964	0.1377	0.1002	0.1553
	MS-N	0.7403	0.0672	0.1042	0.0667	0.2345	0.5161	0.2166	0.1653	0.2816
	MS-T	0.7028	0.0612	0.0958	0.0620	0.2098	0.5031	0.2103	0.1608	0.2693
	SV-N	0.2893	0.0311	0.0538	0.0302	0.0873	0.3450	0.1627	0.1174	0.1815
	SV-T	0.1848	0.0198	0.0356	0.0196	0.0506	0.2849	0.1357	0.0975	0.1476
DGP: GARCH	GARCH-N	0.9879	0.0905	0.1403	0.0900	0.3093	0.6383	0.2608	0.2019	0.3508
	GARCH-T	0.9925	0.0920	0.1416	0.0908	0.3190	0.6327	0.2575	0.1998	0.3492
	GAS-N	> 10	0.5204	0.5124	0.6299	7.1201	1.8003	0.4716	0.4321	0.9912
	GAS-T	0.1772	0.0189	0.0335	0.0186	0.0488	0.2809	0.1306	0.0950	0.1479
	MS-N	0.6571	0.0638	0.1002	0.0621	0.2140	0.4990	0.2128	0.1614	0.2757
	MS-T	0.7404	0.0685	0.1069	0.0677	0.2348	0.5282	0.2217	0.1693	0.2887
	SV-N	0.2807	0.0306	0.0527	0.0295	0.0854	0.3433	0.1615	0.1167	0.1826
	SV-T	0.2111	0.0228	0.0402	0.0223	0.0603	0.2974	0.1405	0.1013	0.1561
N = 2500	GARCH-N	0.9891	0.0925	0.1431	0.0913	0.3159	0.6334	0.2579	0.2001	0.3503
	GARCH-T	0.9898	0.0936	0.1444	0.0919	0.3220	0.6328	0.2578	0.2000	0.3513
	GAS-N	> 10	0.5666	0.5944	0.6420	5.2625	1.9374	0.5267	0.4822	1.0928
	GAS-T	0.1752	0.0189	0.0334	0.0185	0.0488	0.2723	0.1268	0.0922	0.1445
	MS-N	0.7056	0.0693	0.1091	0.0673	0.2294	0.5264	0.2235	0.1699	0.2924
	MS-T	0.8006	0.0765	0.1196	0.0749	0.2577	0.5605	0.2336	0.1792	0.3093
	SV-N	0.2854	0.0314	0.0538	0.0301	0.0884	0.3482	0.1629	0.1180	0.1868
	SV-T	0.2437	0.0268	0.0466	0.0258	0.0729	0.3215	0.1508	0.1092	0.1712

Table 11: Forecast evaluation under **contaminated series**. DGP: GARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed markets**

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	GARCH-N	1.0568	0.0988	0.1511	0.0936	0.3623	0.6491	0.2827	0.2107	0.3801
	GARCH-T	1.0268	0.1024	0.1564	0.0951	0.3655	0.6464	0.2808	0.2099	0.3824
	GAS-N	> 10	0.9756	0.4697	1.1284	> 10	2.1705	0.4428	0.4002	1.4168
	GAS-T	0.1729	0.0210	0.0365	0.0189	0.0564	0.2679	0.1355	0.0943	0.1557
	MS-N	0.5787	0.0613	0.0956	0.0553	0.2185	0.4579	0.2144	0.1545	0.2746
	MS-T	0.6544	0.0586	0.0912	0.0558	0.2070	0.4538	0.2080	0.1511	0.2655
	SV-N	0.2208	0.0295	0.0516	0.0258	0.0812	0.3183	0.1684	0.1146	0.1855
	SV-T	0.1554	0.0210	0.0370	0.0183	0.0563	0.2607	0.1378	0.0939	0.1521
DGP: GAS	GARCH-N	1.0954	0.1080	0.1641	0.1004	0.3882	0.6671	0.2876	0.2157	0.3951
	GARCH-T	1.0536	0.1073	0.1632	0.0986	0.3819	0.6564	0.2838	0.2127	0.3908
	GAS-N	> 10	0.8242	0.6147	0.9185	> 10	2.1632	0.5295	0.4797	1.3530
	GAS-T	0.1561	0.0197	0.0343	0.0176	0.0525	0.2472	0.1259	0.0874	0.1453
	MS-N	0.5677	0.0640	0.1011	0.0568	0.2165	0.4699	0.2210	0.1590	0.2844
	MS-T	0.5654	0.0604	0.0963	0.0551	0.1962	0.4637	0.2159	0.1561	0.2754
	SV-N	0.2338	0.0321	0.0548	0.0274	0.0912	0.3244	0.1693	0.1160	0.1922
	SV-T	0.1723	0.0240	0.0416	0.0205	0.0658	0.2731	0.1431	0.0980	0.1615
N = 2500	GARCH-N	1.0651	0.1107	0.1680	0.1011	0.3961	0.6648	0.2868	0.2153	0.3974
	GARCH-T	1.0506	0.1108	0.1680	0.1007	0.3963	0.6616	0.2859	0.2145	0.3966
	GAS-N	> 10	0.8064	0.7435	0.8791	> 10	2.2819	0.5954	0.5434	1.4013
	GAS-T	0.1492	0.0198	0.0344	0.0173	0.0526	0.2378	0.1215	0.0842	0.1412
	MS-N	0.6244	0.0732	0.1151	0.0641	0.2443	0.5057	0.2365	0.1708	0.3094
	MS-T	0.6361	0.0697	0.1101	0.0629	0.2299	0.4970	0.2285	0.1664	0.2979
	SV-N	0.2392	0.0340	0.0574	0.0285	0.0980	0.3277	0.1703	0.1170	0.1970
	SV-T	0.1910	0.0275	0.0470	0.0231	0.0767	0.2904	0.1519	0.1041	0.1740

Table 12: Forecast evaluation under **contaminated series**. DGP: GAS with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed markets**

Monte Carlo Simulations

		Model	MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P
N = 500	DGP: SV	GARCH-N	1.7035	0.2143	0.3061	0.1660	0.9247	0.9706	0.4411	0.3206	0.6421
		GARCH-T	2.1332	0.2660	0.3613	0.2026	1.2964	1.0746	0.4723	0.3489	0.7283
		GAS-N	> 10	0.9192	0.5236	1.5360	> 10	2.6725	0.5231	0.4422	1.4309
		GAS-T	0.5958	0.1019	0.1640	0.0736	0.3384	0.6164	0.3212	0.2192	0.4058
		MS-N	1.0378	0.1570	0.2325	0.1140	0.6370	0.7631	0.3749	0.2629	0.5169
		MS-T	1.0500	0.1541	0.2307	0.1142	0.6040	0.7766	0.3782	0.2663	0.5175
		SV-N	0.6464	0.0878	0.1598	0.0758	0.2644	0.5870	0.3105	0.2102	0.3329
		SV-T	0.5342	0.0863	0.1468	0.0658	0.2703	0.5650	0.3007	0.2031	0.3531
N = 1000	DGP: SV	GARCH-N	1.8119	0.2300	0.3218	0.1767	1.0447	0.9958	0.4460	0.3266	0.6635
		GARCH-T	2.1239	0.2630	0.3589	0.2023	1.2628	1.0774	0.4701	0.3485	0.7227
		GAS-N	> 10	0.6912	0.5914	0.7223	> 10	1.8911	0.5569	0.4632	1.2376
		GAS-T	0.5854	0.0999	0.1613	0.0723	0.3299	0.6105	0.3185	0.2172	0.4013
		MS-N	1.0032	0.1575	0.2342	0.1135	0.6250	0.7655	0.3774	0.2644	0.5198
		MS-T	1.0130	0.1561	0.2305	0.1130	0.6436	0.7684	0.3756	0.2642	0.5178
		SV-N	0.6907	0.0945	0.1666	0.0801	0.3113	0.6013	0.3139	0.2139	0.3440
		SV-T	0.5127	0.0815	0.1398	0.0629	0.2508	0.5473	0.2912	0.1966	0.3394
N = 2500	DGP: SV	GARCH-N	1.8262	0.2388	0.3302	0.1807	1.1230	0.9991	0.4466	0.3275	0.6738
		GARCH-T	2.0609	0.2644	0.3597	0.2005	1.2920	1.0676	0.4688	0.3467	0.7230
		GAS-N	> 10	0.6324	0.6026	0.5729	> 10	1.7504	0.5737	0.4688	1.1966
		GAS-T	0.5652	0.0992	0.1602	0.0710	0.3280	0.6071	0.3180	0.2166	0.4011
		MS-N	1.1017	0.1746	0.2535	0.1242	0.7540	0.8041	0.3921	0.2763	0.5524
		MS-T	0.9973	0.1607	0.2354	0.1140	0.6929	0.7713	0.3791	0.2661	0.5258
		SV-N	0.6786	0.0961	0.1667	0.0795	0.3315	0.5991	0.3129	0.2132	0.3473
		SV-T	0.4942	0.0801	0.1368	0.0611	0.2481	0.5396	0.2873	0.1940	0.3357

Table 13: Forecast evaluation under **contaminated series**. DGP: SV with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed markets**

Monte Carlo Simulations

Model		MSE	QLIKE	MSE_L	MSE_{Sd}	MSE_P	MAE	MAE_L	MAE_{Sd}	MAE_P	
N = 500	GARCH-N	0.1129	0.3905	0.4421	0.0478	2.8826	0.1783	0.4410	0.1333	0.8227	
	GARCH-T	0.1088	0.3466	0.3775	0.0437	2.6597	0.1566	0.3599	0.1126	0.6975	
	GAS-N	5.1360	> 10	> 10	5.0656	> 10	> 10	> 10	1.2573	1.9577	
	GAS-T	0.0263	0.0768	0.1173	0.0114	0.2938	0.0809	0.2330	0.0650	0.2941	
	MS-N	0.0731	0.2190	0.2491	0.0282	1.6653	0.1164	0.2916	0.0871	0.5026	
	MS-T	0.0723	0.2254	0.2508	0.0285	1.7606	0.1145	0.2816	0.0850	0.5005	
	SV-N	0.0322	0.1360	0.2243	0.0178	0.5179	0.1155	0.3813	0.0994	0.4257	
	SV-T	0.0316	0.1216	0.2010	0.0167	0.4601	0.1094	0.3506	0.0928	0.3903	
	GARCH-N	0.1053	0.3888	0.4332	0.0466	2.8596	0.1712	0.4169	0.1272	0.7988	
	GARCH-T	0.1041	0.3541	0.3845	0.0436	2.7213	0.1551	0.3570	0.1119	0.7026	
DGP: MSGARCH	N = 1000	GAS-N	0.8070	> 10	> 10	7.8892	> 10	> 10	> 10	1.6321	3.1176
	N = 1000	GAS-T	0.0198	0.0708	0.1098	0.0097	0.2566	0.0748	0.2274	0.0620	0.2844
	N = 1000	MS-N	0.0626	0.2200	0.2433	0.0266	1.7278	0.1058	0.2662	0.0797	0.4815
	N = 1000	MS-T	0.0667	0.2379	0.2616	0.0285	1.8585	0.1078	0.2673	0.0806	0.5032
	N = 1000	SV-N	0.0301	0.1376	0.2258	0.0174	0.5189	0.1146	0.3823	0.0993	0.4315
	N = 1000	SV-T	0.0296	0.1333	0.2177	0.0169	0.5047	0.1123	0.3710	0.0969	0.4210
	N = 1000	GARCH-N	0.1063	0.3931	0.4326	0.0471	2.9347	0.1697	0.4039	0.1247	0.7905
	N = 1000	GARCH-T	0.1058	0.3649	0.3934	0.0446	2.8559	0.1571	0.3591	0.1130	0.7154
	N = 2500	GAS-N	1.5851	> 10	> 10	> 10	> 10	> 10	> 10	2.0218	4.2721
	N = 2500	GAS-T	0.0186	0.0683	0.1061	0.0093	0.2455	0.0732	0.2254	0.0612	0.2811
N = 2500	MS-N	0.0656	0.2273	0.2473	0.0274	1.8238	0.1012	0.2531	0.0760	0.4766	
	MS-T	0.0681	0.2443	0.2663	0.0290	1.9376	0.1032	0.2566	0.0774	0.5000	
	SV-N	0.0300	0.1400	0.2283	0.0175	0.5321	0.1148	0.3840	0.0997	0.4371	
	SV-T	0.0296	0.1384	0.2258	0.0173	0.5237	0.1140	0.3812	0.0990	0.4340	

Table 14: Forecast evaluation under **contaminated series**. DGP: MSGARCH with standardized Student-t innovation distribution. Parameters values close to the ones obtained in **developed markets**

Empirical Application

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- The Model Confidence Set of Hansen et al. (2011) was used to select the best set of models

Empirical Application

	Min	Q_1	Med	Mean	Q_3	Max	Skew	Kurt	Sd	ACF ₁
MMM	-12.9450	-0.5931	0.0540	0.0394	0.7338	22.9906	0.6217	26.4331	1.4631	-0.0438
AMZN	-14.0494	-0.9025	0.0938	0.1136	1.1906	15.7457	0.2606	9.2383	2.0610	-0.0190
AXP	-14.8187	-0.7041	0.0777	0.0751	0.9279	21.8823	0.8175	22.6249	1.8245	-0.0533
AMGN	-9.5846	-0.7233	0.0317	0.0614	0.8461	11.8180	0.3778	8.8610	1.5232	-0.0755
AAPL	-12.8647	-0.7397	0.1004	0.1129	1.0357	11.9808	-0.0434	8.1919	1.7551	-0.0400
BA	-23.8484	-0.9198	0.0679	0.0635	1.0459	24.3186	0.1755	21.1000	2.2543	0.0540
CAT	-14.2822	-0.8497	0.0590	0.0760	1.0278	10.3321	-0.1333	6.7862	1.8298	0.0033
CVX	-22.1248	-0.7318	0.0703	0.0468	0.8295	22.7407	-0.2220	26.8621	1.6872	-0.0671
CSCO	-16.2107	-0.6509	0.0520	0.0481	0.7960	15.9505	-0.4466	18.6962	1.6295	-0.0638
KO	-9.6725	-0.4607	0.0566	0.0388	0.5729	6.4796	-0.6067	11.8785	1.0721	-0.0340
HD	-19.7938	-0.5921	0.0940	0.0891	0.8301	13.7508	-0.6515	18.0384	1.4627	-0.0424
HON	-12.0868	-0.6024	0.0713	0.0655	0.7542	15.0684	0.0125	12.3544	1.4405	-0.0309
INTC	-26.0585	-0.9213	0.0566	0.0320	1.0257	19.5213	-0.7099	18.7537	2.0229	-0.0703
IBM	-12.8507	-0.6128	0.0556	0.0376	0.7120	11.3010	-0.4608	12.7177	1.4006	-0.0331
JNJ	-10.0379	-0.4471	0.0312	0.0386	0.5695	7.9977	-0.1134	12.0442	1.0521	-0.0681
JPM	-14.9649	-0.7608	0.0586	0.0719	0.9074	18.0125	0.2216	12.8850	1.7483	-0.0978
MCD	-15.8753	-0.4841	0.0750	0.0585	0.6014	18.1255	0.3910	33.9165	1.1706	-0.1008
MRK	-9.8630	-0.6197	0.0310	0.0484	0.7436	10.4080	-0.0100	9.6857	1.3077	-0.0638
MSFT	-14.7390	-0.7078	0.0698	0.0900	0.9262	14.2169	0.0265	10.6409	1.6110	-0.1046
NKE	-19.9809	-0.7633	0.0508	0.0607	0.9280	15.5314	0.0250	17.1611	1.7571	-0.0378
PG	-8.7373	-0.4713	0.0568	0.0444	0.5846	12.0090	0.1688	14.9902	1.0764	-0.0766
GS	-12.7910	-0.8520	0.0549	0.0555	0.9845	17.5803	0.0073	11.5460	1.8007	-0.0649
TRV	-20.8004	-0.5772	0.1044	0.0612	0.7408	13.2902	-1.1312	25.3817	1.4064	-0.1548
UNH	-17.2769	-0.6889	0.0957	0.0932	0.8602	12.7989	-0.0642	12.5191	1.6006	-0.0654
VZ	-7.4978	-0.5835	0.0495	0.0317	0.6449	9.2705	0.0711	8.3111	1.1627	-0.0359
V	-13.5472	-0.6974	0.1312	0.0861	0.8665	14.9973	0.1765	13.3194	1.5741	-0.0965
WMT	-11.3758	-0.5144	0.0682	0.0589	0.6375	11.7085	0.1154	18.9946	1.2087	-0.0578
DIS	-13.1632	-0.6788	0.0445	0.0503	0.8230	14.4123	0.2035	13.5132	1.6423	-0.0512
CRM	-19.7371	-0.9626	0.0860	0.1029	1.1906	26.0449	0.4990	14.0377	2.2822	-0.0275

Empirical Application

	MSE										QLIKE									
	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T												
MMM	3.811	4.367	4.460	2.095	2.036	2.916	0.705	0.656	0.340	0.347	0.374	0.285	0.278	0.298	0.377	0.304	0.251	0.251	0.251	0.251
AMZN	6.598	6.922	4.303	7.349	5.648	7.229	2.896	2.386	0.274	0.272	0.284	0.272	0.252	0.268	0.385	0.300	0.251	0.251	0.251	0.251
AXP	7.114	6.763	4.254	5.992	4.613	7.500	1.747	1.446	0.319	0.308	0.336	0.308	0.280	0.307	0.369	0.310	0.251	0.251	0.251	0.251
AMGN	1.466	1.710	1.533	1.240	1.392	1.433	0.893	0.796	0.251	0.266	0.277	0.248	0.243	0.261	0.440	0.379	0.251	0.251	0.251	0.251
AAPL	3.255	3.915	2.605	4.532	2.773	3.606	1.437	1.258	0.285	0.296	0.290	0.308	0.274	0.292	0.354	0.309	0.251	0.251	0.251	0.251
BA	12.445	17.245	7.899	15.445	10.850	19.435	9.753	8.852	0.235	0.257	0.221	0.257	0.220	0.261	0.676	0.595	0.251	0.251	0.251	0.251
CAT	3.485	4.365	3.281	4.187	2.955	3.982	1.539	1.349	0.233	0.254	0.240	0.252	0.221	0.250	0.325	0.283	0.251	0.251	0.251	0.251
CVX	3.395	3.965	2.909	3.306	2.950	3.594	1.439	1.345	0.237	0.246	0.237	0.239	0.225	0.223	0.348	0.344	0.251	0.251	0.251	0.251
CSCO	2.043	2.289	1.848	1.807	1.973	2.108	0.578	0.488	0.339	0.315	0.385	0.289	0.316	0.315	0.361	0.284	0.251	0.251	0.251	0.251
KO	0.297	0.350	0.315	0.333	0.325	0.399	0.217	0.210	0.189	0.183	0.214	0.195	0.194	0.184	0.313	0.273	0.251	0.251	0.251	0.251
HD	1.209	1.343	1.290	1.246	1.080	1.266	0.889	0.828	0.194	0.200	0.203	0.203	0.189	0.197	0.335	0.308	0.251	0.251	0.251	0.251
HON	1.316	1.597	1.069	1.473	1.128	1.509	0.619	0.554	0.222	0.233	0.227	0.229	0.212	0.223	0.297	0.276	0.251	0.251	0.251	0.251
INTC	12.367	15.986	13.572	12.255	9.763	10.668	4.192	3.204	0.339	0.353	0.359	0.308	0.294	0.306	0.600	0.413	0.251	0.251	0.251	0.251
IBM	1.421	1.958	1.241	1.424	1.349	1.621	0.430	0.409	0.282	0.297	0.309	0.269	0.281	0.288	0.328	0.260	0.251	0.251	0.251	0.251
JNJ	0.288	0.313	0.248	0.286	0.276	0.289	0.211	0.206	0.193	0.194	0.202	0.194	0.191	0.191	0.270	0.248	0.251	0.251	0.251	0.251
JPM	2.824	3.619	3.770	3.072	2.607	4.017	0.925	0.809	0.255	0.272	0.303	0.260	0.246	0.258	0.296	0.274	0.251	0.251	0.251	0.251
MCD	0.419	0.443	0.353	0.486	0.458	0.607	0.301	0.281	0.247	0.250	0.271	0.258	0.259	0.258	0.441	0.367	0.251	0.251	0.251	0.251
MRK	0.745	0.918	0.685	0.739	0.819	0.763	0.476	0.425	0.222	0.226	0.260	0.219	0.225	0.220	0.352	0.289	0.251	0.251	0.251	0.251
MSFT	2.444	3.168	1.941	3.368	2.308	2.724	0.934	0.836	0.264	0.277	0.279	0.285	0.266	0.263	0.324	0.277	0.251	0.251	0.251	0.251
NKE	3.548	4.266	2.955	3.881	4.349	4.682	1.299	0.997	0.326	0.332	0.382	0.305	0.328	0.335	0.475	0.332	0.251	0.251	0.251	0.251
PG	0.332	0.337	0.298	0.324	0.270	0.314	0.226	0.215	0.181	0.180	0.195	0.188	0.176	0.180	0.316	0.276	0.251	0.251	0.251	0.251
GS	3.024	3.197	3.362	2.719	2.400	3.504	1.350	1.256	0.227	0.230	0.251	0.219	0.208	0.228	0.294	0.281	0.251	0.251	0.251	0.251
TRV	1.376	1.961	1.193	1.488	1.243	2.065	0.823	0.703	0.209	0.230	0.230	0.211	0.204	0.221	0.342	0.276	0.251	0.251	0.251	0.251
UNH	1.807	2.239	1.939	1.613	1.651	1.892	1.153	0.996	0.234	0.251	0.243	0.230	0.228	0.238	0.408	0.318	0.251	0.251	0.251	0.251
VZ	0.574	0.672	0.617	0.480	0.518	0.638	0.282	0.273	0.320	0.315	0.354	0.302	0.319	0.320	0.475	0.424	0.251	0.251	0.251	0.251
V	1.980	2.007	1.432	1.852	1.541	1.848	0.731	0.649	0.304	0.308	0.307	0.305	0.303	0.296	0.392	0.351	0.251	0.251	0.251	0.251
WMT	0.726	0.628	0.721	0.580	0.608	0.628	0.338	0.311	0.312	0.307	0.356	0.262	0.292	0.277	0.403	0.324	0.251	0.251	0.251	0.251
DIS	4.821	5.702	3.879	3.880	3.505	4.395	1.772	1.534	0.307	0.312	0.318	0.275	0.275	0.289	0.421	0.342	0.251	0.251	0.251	0.251
CRM	12.925	15.807	17.502	10.292	8.529	9.766	3.461	2.858	0.311	0.329	0.401	0.288	0.290	0.289	0.444	0.337	0.251	0.251	0.251	0.251

Figure 2: Out-of-sample average MSE (left panel) and QLIKE (right panel) forecasting performance of assets in the Dow Jones Average Index

Empirical Application

	MSE										QLIKE									
	GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T		GARCH-N	GARCH-T	GAS-N	GAS-T	MS-N	MS-T	SV-N	SV-T			
MMM	3.897	3.890	3.466	2.487	2.685	3.133	0.711	0.653	0.365	0.339	0.373	0.301	0.322	0.339	0.354	0.282				
AMZN	7.220	8.361	6.581	7.460	7.977	8.077	2.600	2.344	0.271	0.279	0.294	0.263	0.270	0.272	0.335	0.296				
AXP	8.524	8.094	7.569	7.448	7.246	7.715	1.634	1.436	0.358	0.332	0.442	0.326	0.354	0.338	0.369	0.306				
AMGN	1.309	1.463	1.422	1.331	1.336	1.452	0.933	0.796	0.240	0.250	0.273	0.245	0.237	0.236	0.476	0.378				
AAPL	3.886	4.487	3.295	4.875	3.377	4.350	1.369	1.248	0.293	0.304	0.299	0.303	0.288	0.296	0.333	0.299				
BA	16.314	24.239	13.741	24.389	16.806	23.132	9.124	8.255	0.277	0.304	0.281	0.306	0.268	0.286	0.597	0.538				
CAT	3.899	4.616	3.895	4.700	4.011	4.448	1.462	1.298	0.252	0.267	0.264	0.269	0.249	0.256	0.325	0.268				
CVX	3.826	4.798	3.319	4.537	3.415	5.463	1.465	1.309	0.266	0.279	0.266	0.275	0.249	0.274	0.454	0.356				
CSCO	2.397	2.588	2.262	2.000	2.543	2.263	0.558	0.508	0.346	0.340	0.401	0.298	0.352	0.315	0.328	0.289				
KO	0.707	0.476	0.545	0.462	0.481	0.462	0.207	0.211	0.247	0.201	0.247	0.207	0.212	0.198	0.292	0.264				
HD	1.534	1.616	1.537	1.721	1.550	1.451	0.804	0.761	0.218	0.222	0.227	0.228	0.225	0.212	0.300	0.290				
HON	1.258	1.628	1.233	1.548	1.294	1.425	0.623	0.561	0.234	0.237	0.259	0.233	0.237	0.233	0.329	0.287				
INTC	11.618	20.523	10.189	15.221	12.837	14.006	4.012	3.111	0.347	0.381	0.358	0.343	0.341	0.351	0.538	0.383				
IBM	1.973	2.695	1.824	1.829	1.759	2.354	0.424	0.417	0.328	0.336	0.364	0.287	0.318	0.329	0.314	0.246				
JNJ	0.335	0.348	0.317	0.357	0.343	0.278	0.221	0.210	0.211	0.204	0.230	0.210	0.217	0.192	0.297	0.242				
JPM	2.770	3.501	3.272	3.172	2.645	3.538	0.945	0.851	0.269	0.281	0.308	0.265	0.260	0.274	0.355	0.331				
MCD	0.454	0.502	0.432	0.552	0.610	0.557	0.289	0.279	0.242	0.257	0.264	0.256	0.270	0.268	0.388	0.344				
MRK	0.884	0.961	0.837	0.926	0.841	0.893	0.472	0.419	0.250	0.234	0.328	0.230	0.234	0.224	0.370	0.279				
MSFT	3.105	3.807	2.827	3.491	2.924	3.239	0.858	0.846	0.274	0.284	0.286	0.281	0.269	0.273	0.288	0.274				
NKE	5.484	6.642	4.922	4.871	5.811	6.573	1.161	0.992	0.417	0.397	0.464	0.345	0.379	0.365	0.434	0.359				
PG	0.389	0.387	0.357	0.390	0.320	0.365	0.221	0.212	0.188	0.187	0.201	0.193	0.190	0.187	0.283	0.254				
GS	2.805	2.869	2.846	3.105	2.427	2.597	1.337	1.237	0.235	0.237	0.240	0.238	0.219	0.224	0.319	0.285				
TRV	1.923	2.223	1.520	1.994	1.575	2.161	0.763	0.680	0.234	0.241	0.257	0.235	0.219	0.231	0.334	0.281				
UNH	1.886	2.016	1.825	1.755	1.676	1.596	1.125	0.991	0.243	0.247	0.248	0.238	0.238	0.228	0.365	0.302				
VZ	0.830	0.998	0.975	0.690	0.643	0.826	0.284	0.280	0.335	0.336	0.396	0.314	0.339	0.332	0.492	0.405				
V	2.225	2.363	2.001	2.264	1.817	1.968	0.697	0.657	0.304	0.309	0.312	0.306	0.304	0.298	0.381	0.359				
WMT	1.103	0.752	0.933	0.707	0.786	0.779	0.320	0.308	0.327	0.316	0.383	0.263	0.325	0.281	0.355	0.307				
DIS	6.441	7.183	5.424	5.695	5.158	8.076	1.716	1.517	0.384	0.366	0.389	0.320	0.339	0.355	0.419	0.340				
CRM	14.985	12.765	11.633	8.845	10.617	12.271	3.247	2.994	0.338	0.315	0.425	0.270	0.314	0.296	0.413	0.357				

Figure 3: Out-of-sample average MSE (left panel) and QLIKE (right panel) forecasting performance of assets in the Dow Jones Average Index. Sample size 1000 observations

Takeaways

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- For series contaminated by outliers, the SV-t and GAS-t models outperform their competitors.

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- GAS-t and SV-t should serve as new benchmarks for developing robust-to-outliers procedures.
- This work highlights that a deep understanding of the models is more important than simply running “horse races” or relying on computational power, underscoring the crucial role of well-trained specialists in statistics and data science over untrained users.

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References

- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.
- Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.
- Haas, M., Mittnik, S., and Paolella, M. S. (2004). A new approach to markov-switching GARCH models. *Journal of Financial Econometrics*, 2(4):493–530.
- Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The model confidence set. *Econometrica*, 79(2):453–497.
- Harvey, A. and Sucarrat, G. (2014). EGARCH models with fat tails, skewness and leverage. *Computational Statistics & Data Analysis*, 76:320–338.

- Harvey, A. C. (2013). *Dynamic Models for Volatility and Heavy Tails: with Applications to Financial and Economic Time Series*, volume 52. Cambridge University Press.
- Patton, A. J. (2011). Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics*, 160(1):246–256.
- Taylor, S. J. (1982). Financial returns modelled by the product of two stochastic processes-a study of the daily sugar prices 1961-75. *Time Series Analysis: Theory and Practice*, 1:203–226.
- Wahl, J. C. (2018). Parameter estimation of multivariate factor stochastic volatility models. Master's thesis, The University of Bergen.