Homework 5– Due May 8, 2020

Please turn in a PDF that includes any written solutions and/or code used to complete the following problems. (Note that the PDF will be the only version of your code that you turn in.) Questions and deliverables that should be included with your submission are shown in **bold**.

1. (25 pts) Demonstrate that the state transition matrix does in fact solve the ordinary differential equation $\dot{x} = Ax$ with $x(0) = x_0$ where $A = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ and $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Do this by numerically solving the ordinary differential equation for $x(t)$ and the ordinary differential equation for $\Phi(t, t_0)$ and compare $x(0.5)$ with $\Phi(0.5, 0)x_0$. **Turn in:** your solution for $x(0.5)$ and $\Phi(0.5, 0)x_0$.

2. (25 pts) Demonstrate that the state transition matrix does in fact *also* solve the ordinary differential equation $\dot{p} = Ap$ with $p(0.5) = p_T$ where $A = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ and $p_T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Do this by numerically solving the ordinary differential equation for $p(t)$ and the ordinary differential equation for $\Phi(t, t_0)$ and compare $p(0)$ with $\Phi(0.5, 0)^{-1}p_T$. **Turn in:** your solution for $p(0)$ and $\Phi(0.5, 0)^{-1}p_T$.

3. (25 pts) Compute the control $u(t)$ that minimizes

$$J = \frac{1}{2}\int_0^{10} x^T \begin{bmatrix} 2 & 0 \\ 0 & 0.01 \end{bmatrix} x + u^T[0.1]u dt + \frac{1}{2}x(10)^T \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} x(10)$$

subject to the constraint that

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1.6 & -0.4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad x(0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

by solving the Two Point Boundary Value Problem. **Plot your solution for $x(t)$ and $u(t)$ over time.**

Hint: If you use scipy.integrate.solve_bvp, the initial solution will be a 4xN matrix of values for x and p, possibly matrix of zeros (np.zeros((4,N))). Your function for the boundary conditions should return the *error* as a numpy array with length 4 between the boundary conditions and the given initial and final values of x and p.

4. (25 pts) Compute the control $u(t)$ that minimizes

$$J = \frac{1}{2}\int_0^{10} x^T \begin{bmatrix} 2 & 0 \\ 0 & 0.01 \end{bmatrix} x + u^T[0.1]u dt + \frac{1}{2}x(10)^T \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} x(10)$$

subject to the constraint that

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1.6 & -0.4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad x(0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

by solving the Riccati Equation. **Plot your solution for $x(t)$ and $u(t)$ over time. Plot the *difference*** between $x(t)$ and $u(t)$ computed using the Ricatti Equation and $x(t)$ and $u(t)$ computed using the TPBVP.