Homework 1 - Due April 10, 2020

Please turn in a PDF that includes any code used to complete the following problems. (Note that the PDF will be the only version of your code that you turn in.) Questions and deliverables that should be included with your submission are shown in **bold**.

1. (25 pts) Think about the function

$$J(x(t), y(t)) = \int_0^1 \frac{1}{2}(x(t)^2 + (y(t) - 1)^2)dt.$$

   Does the minimizer of this function exist? If so, what is it?

2. (25 pts) Think about the function

$$J(x(t), y(t)) = \int_0^1 \frac{1}{2}(x(t)^2 - (y(t) - 1)^2)dt.$$

   Does the minimizer of this function exist? If so, what is it?

3. (50 pts) Write a *Python* function that takes the first derivative of an arbitrary function $f(x)$ where $x \in \mathbb{R}^n$. For instance, in *Python* you can create the function $f(x, y) = x^2 + ay^2$ by typing

```
import sympy as sym
x, y, a = sym.symbols('x, y, a')
f = x**2 + a * y**2
```

   and then differentiate it with respect to x using

```
sym.diff(f,x)
```

   You will want to set your function up so that it can evaluate and return multiple partial derivatives of a multivariate function. Depending on how you do it, this is potentially a very short problem. **Demonstrate that your function works by calculating the gradient of $f(x, y)$ defined above. Include both the python code and its output in the PDF you turn in.**