Homework 7– Due May 22, 2015

`(Last Update: May 20, 2020)`

Please turn in a PDF that includes any written solutions and/or code used to complete the following problems. (Note that the PDF will be the only version of your code that you turn in.) Questions and deliverables that should be included with your submission are shown in **bold**.

For this assignment, you will be working with a differential drive vehicle for a length of time $T = 2\pi$ sec using $(x_d, y_d, \theta_d) = (\frac{4}{2\pi}t, 0, \pi/2)$ subject to the dynamics,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta)u_1 \\ \sin(\theta)u_1 \\ u_2 \end{bmatrix}, \quad (x(0), y(0), \theta(0)) = (0, 0, \pi/2).$$

Use the semi-circle you obtained in Homework 2 defined by $u_1(t) = 1$, $u_2(t) = -1/2$ as the initial trajectory and $(x_d, y_d, \theta_d) = (\frac{4}{2\pi}t, 0, \pi/2)$ as a reference trajectory. Note this corresponds to an infeasible trajectory for parallel parking. Use a cost function in the form:

$$J = \frac{1}{2}\int_0^T (x(t) - x_d(t))^T Q(t)(x(t) - x_d(t)) + u(t)^T R(t)u(t)dt + \frac{1}{2}(x(T) - x_d(T))^T P_1(x(T) - x_d(T))$$

1. (20 pts) Create a **function** that calculates the directional derivative of a cost function of the form described above for any given $\xi = (x(t), u(t))$ and $\zeta = (z(t), v(t))$ where $x_d(t)$ is the desired trajectory. **Evaluate the directional derivative** along the initial trajectory in the perturbation direction corresponding to $v_1(t) = -0.5\sin(t) - 0.1$, $v_2(t) = 0.5\cos(t)$. For this problem, use the following values for the cost function parameters:

$$Q = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

   Although your answer will slightly vary based on how you solve for the desired trajectory and the step size that you choose, you should get a value roughly around -4300.

2. (20 pts) Create a **function** that performs the Armijo line search for the differential drive vehicle. Use this function to determine the optimal step size for the descent direction defined by $v_1(t) = -0.5\sin(t) - 0.1$, $v_2(t) = 0.5\cos(t)$ and the initial semi-circle trajectory defined by $u_1(t) = 1$, $u_2(t) = -1/2$ as $\xi_i = (x_i, u_i)$. For $\gamma = \beta^n$, a choice of $\alpha = 0.4$, $\beta = 0.7$, and the Q, R, and P_1 values given in Problem 1, it should take around 3 iterations for your function to converge. **Turn in:** A plot of the perturbed trajectory and your initial trajectory with x on the x-axis and y on the y-axis.

3. (60 pts) Apply iLQR to the differential drive vehicle described at the top of this assignment. **Turn in:** A plot of the optimal trajectory using iLQR and your initial trajectory with x on the x-axis and y on the y-axis, a plot of you control signals u_1 and u_2, and tuned values for Q, R, and P_1.