Homework 4– Due May 1, 2020

Please turn in a PDF that includes any written solutions and/or code used to complete the following problems. (Note that the PDF will be the only version of your code that you turn in.) Questions and deliverables that should be included with your submission are shown in **bold**.

1. (35 pts) Make an Armijo line search function that takes as arguments a function f, its derivative Df, a test point x, and a descent direction z and returns the new x that satisfies the sufficient decrease property. Using the algorithm box in Section 7.3, a starting guess of $x = [10, 10]^T$, $\alpha = 0.4$, and $\beta = 0.7$, minimize $f(x, y) = x^2 + 100y^2$. Solve for the decent direction z_i by analytically determining the minimum of the *quadratic* function $Df(x_i) \circ z + \langle z, z \rangle$. **Turn in your solution for (x, y) and a plot the evaluated function $f(x, y)$ for every iteration.**

2. (30 pts) Let $f : \mathbb{R}^{2\times 2} \to \mathbb{R}$ where

$$f(X) = v_1^T X^T X v_1 + 10 v_2^T X^T X v_2$$

with

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

   (a) Compute the **directional derivative** of $f(X)$ in the direction Z (i.e., $Df(X) \cdot Z$).

   (b) Compute the **gradient** $\nabla f \in \mathbb{R}^{2\times 2}$. Hint: One way of doing this is to write f as a function of the individual elements of the X matrix, then take the partial derivative with respect to each element.

3. (35 pts) Use the Armijo line search to compute the minimizer in $\mathbb{R}^{2\times 2}$ of f starting from an initial guess of $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.[27] **Turn in your solution for X and a plot the evaluated function $f(x, y)$ for every iteration.**

[27] Hint: make sure you are careful implementing $Df(X) \cdot Z$ in your line search.