```
signed char *(*(*bar)[5])(long int );
int (*(*foo)(void ))[3];
typedef struct u U;
char (*(*x())[5])();
typedef char (*(*x[3])())[5];
x *(*(**foo[][8])())[];
struct u { long (*f)(U);};
struct v { x f; U a;};
double (*f)(int (U *, int ), int );
```

```
signed char *(*(*bar)[5])(long int );
int (*(*foo)(void ))[3];
typedef struct u U;
char (*(*x())[5])();
typedef char (*(*x[3])())[5];
x *(*(**foo[][8])())
struct u { long (*f)
struct v { x f; U a;;
double (*f)(int (U *, int ), int );
```

The boring basic types:

struct tag	int	double
union tag	float	long
enum tag	char	short
$typedef_name$	void	

The leftout *type modifiers*:

long	unsigned	
short	signed	
extern	static	
register		

The inevitable *derived types* (in order of precedence):

- ▶ •[] array of...
 - "Array of' can be undimensioned -[] or dimensioned -[10] but the sizes don't really play significantly into reading a declaration.
- ▶ •() function returning...

Denoted by a matched pair of parentheses -() – possibly containing an abstract formal parameter list. Parameters lists (if present) are read separately.

▶ *• pointer to...

This is denoted by the * character, and it always has to point to something.

We'll note that parens are also used for grouping: grouping parens surround the variable name, while "function returning" parens are always on the right.



```
char (*(*foo())[5])();
```

- 1. Always start with the variable name: foo is ...
- and always end with the basic type: foo is ... char
- 3. The "filling in the middle" part can be summarize with this rule:

"go right when you can, go left when you must"

Work your way out from the variable name and consume *derived-type* tokens *to the right as far as possible* without bumping into a grouping parenthesis. Then *go left to the matching parenthesis*. Rinse and repeat!

```
(*(* • ())[5])();
```

- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
- 3. The "filling in the middle" part can be summarize with this rule:

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Work your way out from the variable name and consume *derived-type* tokens *to the right as far as possible* without bumping into a grouping parenthesis. Then *go left to the matching parenthesis*. Rinse and repeat!

foo is ... char

```
(*(* • )[5])();
```

- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
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foo is a function returning ... char

```
(* • [5])();
```

- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
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foo is a function returning a pointer to ... char

```
(* • )();
```

- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
- 3. The "filling in the middle" part can be summarize with this rule:

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Work your way out from the variable name and consume *derived-type* tokens *to the right as far as possible* without bumping into a grouping parenthesis. Then *go left to the matching parenthesis*. Rinse and repeat!

foo is a function returning a pointer to an array of 5 ... char



();

- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
- 3. The "filling in the middle" part can be summarize with this rule:

"go right when you can, go left when you must"

Work your way out from the variable name and consume *derived-type* tokens *to the right as far as possible* without bumping into a grouping parenthesis. Then *go left to the matching parenthesis*. Rinse and repeat!

foo is a function returning a pointer to an array of 5 pointers to ... char



- 1. Always start with the variable name: foo is ...
- 2. and always end with the *basic type*: foo is ... char
- 3. The "filling in the middle" part can be summarize with this rule:

"go right when you can, go left when you must"

Work your way out from the variable name and consume *derived-type* tokens *to the right as far as possible* without bumping into a grouping parenthesis. Then *go left to the matching parenthesis*. Rinse and repeat!

foo is a function returning a pointer to an array of 5 pointers to functions returning char

```
typedef double (*F00)(int (*(*)())(), struct s *);
```

▶ typedef declares the variable name as a *typename*

```
(* • )(int (*(*)())(), struct s *);
```

▶ typedef declares the variable name as a *typename*

Declare typename F00 as ... double

- (int (*(*)())(), struct s *);
- ▶ typedef declares the variable name as a *typename*

Declare typename F00 as pointer to ... double

1

- typedef declares the variable name as a typename
- structs act like basic data types

Declare typename F00 as pointer to a function (int(*(*)())()), pointer to struct s) returning double



```
int (*(*)())()
```

- typedef declares the variable name as a typename
- structs act like basic data types
- abstract declarations for formal parameters where does go?
 - 1. to the right of all the "pointer to" derived type tokens
 - 2. to the left of all "array of" derived type tokens
 - 3. to the left of all "function returning" derived type tokens
 - 4. inside all the grouping parentheses

Declare typename F00 as pointer to a function (int(*(*)())()), pointer to struct s) returning double

```
int (*(*•)())()
```

- typedef declares the variable name as a typename
- structs act like basic data types
- abstract declarations for formal parameters
 - 1. to the right of all the "pointer to" derived type tokens
 - 2. to the left of all "array of" derived type tokens
 - 3. to the left of all "function returning" derived type tokens
 - 4. inside all the grouping parentheses

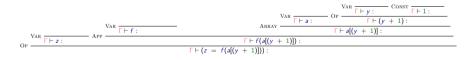
Declare typename F00 as pointer to a function (int(*(*)())()), pointer to struct s) returning double

$$\Gamma = \left\{ \begin{array}{c} & \text{int } y, \\ & \text{double } z, \\ & \text{int } f(\text{double}), \\ & \text{int } a[], \\ & \text{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

$$\Gamma = \left\{ \begin{array}{c} & \textbf{int } y, \\ & \textbf{double } z, \\ & \textbf{int } f(\textbf{double}), \\ & \textbf{int } a[], \\ & \textbf{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$



$$\Gamma = \left\{ \begin{array}{c} & \text{int } y, \\ & \text{double } z, \\ & \text{int } f(\text{double}), \\ & \text{int } a[], \\ & \text{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

```
Array -
                                                               \lceil \vdash a[(y + 1)]:
                       \Gamma \vdash f(a[(y + 1)]):
\Gamma \vdash (z = f(a[(y + 1)])):
```

$$\Gamma = \left\{ \begin{array}{c} & \textbf{int } y, \\ & \textbf{double } z, \\ & \textbf{int } f(\textbf{double}), \\ & \textbf{int } a[], \\ & \textbf{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

```
 \text{VAR} \frac{\text{VAR} \frac{\text{VAR} \frac{\text{VAR}}{\text{$\vdash$} \text{$\vdash$} \text{$\vdash$}
```

$$\Gamma = \left\{ \begin{array}{c} & \textbf{int } y, \\ & \textbf{double } z, \\ & \textbf{int } f(\textbf{double}), \\ & \textbf{int } a[], \\ & \textbf{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

$$\Gamma = \left\{ \begin{array}{c} & \textbf{int } y, \\ & \textbf{double } z, \\ & \textbf{int } f(\textbf{double}), \\ & \textbf{int } a[], \\ & \textbf{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

$$\sum_{\text{DP}} \frac{\text{Var} \frac{\text{Var}$$

$$\Gamma = \left\{ \begin{array}{c} & \textbf{int } y, \\ & \textbf{double } z, \\ & \textbf{int } f(\textbf{double}), \\ & \textbf{int } a[], \\ & \textbf{int } x, \end{array} \right\}$$

$$(z = f(a[(y + 1)]));$$

```
 \sum_{\text{OP}} \frac{\text{Var} \frac{\text{Var}
```

```
\Gamma = \left\{ \begin{array}{l} & \text{int } y(\text{int}), \\ & \text{double } z, \\ & \text{int } f(\text{int}), \\ & \text{struct } \{ \text{double } a1; \text{int} * \ a2; \} \ s, \\ & \text{int} * \ a, \\ & \text{int} (* \ arr[])(), \\ & \text{int } x, \end{array} \right\}
```

```
(s.a2 = f(a[(*y + 1)]));
```

```
Γ = 

    int y(int),

    double z,

    int f(int),

    struct {double a1; int * a2; } s,

    int * a,

    int(* arr[])(),

    int x,
    }
    (s.a2 = f(a[(*y + 1)])) ;
```

 $\Gamma \vdash (s.a2 = f(a[(*y + 1)]))$:

```
\Gamma = \left\{ \begin{array}{l} & \text{int } y(\text{int}), \\ & \text{double } z, \\ & \text{int } f(\text{int}), \\ & \text{struct } \{ \text{double } a1; \text{int} * a2; \} \ s, \\ & \text{int} * a, \\ & \text{int} (* \ arr[])(), \\ & \text{int } x, \end{array} \right\} 
(s.a2 = f(a[(*y + 1)])) ;
```

```
Γ = 

    int y(int),

    double z,

    int f(int),

    struct {double a1; int * a2; } s,

    int * a,

    int(* arr[])(),

    int x,
    }
    (s.a2 = f(a[(*y + 1)])) ;
```

```
 \begin{array}{c} V_{AR} \\ V_{AR} \\ \hline \\ STRUCT \\ \hline \\ CP \\ \hline \\ F + s. 2 \end{array} \begin{array}{c} V_{AR} \\ \hline \\ V_{AR} \\ \hline \\ F + g : \\ \hline \\ F + g
```

```
Γ = 

int y(int),

double z,

int f(int),

struct {double a1; int * a2; } s,

int * a,

int(* arr[])(),

int x,

(s.a2 = f(a[(*y + 1)])) ;
```

```
Γ = 

int y(int),

double z,

int f(int),

struct {double a1; int * a2; } s,

int * a,

int(* arr[])(),

int x,

(s.a2 = f(a[(*y + 1)])) ;
```

```
\frac{\text{Var}}{\text{F-}s: \text{struct } \{\text{double al; int * a2;}\}} \underbrace{\text{Var}}_{\text{F-}f: \text{int }\_\{\text{int}\}} \underbrace{\text{Var}}_{\text{F-}b: \text{int * }} \underbrace{\text{Var}}_{\text{F-}b: \text{int * }} \underbrace{\text{Non-simple index type:}}_{\text{F-}a[(*y+1)]: \text{int }\_} \underbrace{\text{F-}(*y+1): \text{error } f}_{\text{F-}a[(*y+1)]: \text{int }\_} \underbrace{\text{Const}}_{\text{F-}a[(*y+1)]: \text{const}} \underbrace{\text{Const}}_{\text{F-}a[(*y+1
```

```
Γ = 

    int y(int),

    double z,

    int f(int),

    struct {double a1; int * a2; } s,

    int * a,

    int(* arr[])(),

    int x,
    }
    (s.a2 = f(a[(*y + 1)])) ;
```

```
\frac{\text{Var}}{\text{F-}s: \text{struct } \{\text{double al; int * a2;}\}} \\ \text{STRUCT} \\ \frac{\text{Var}}{\text{F-}s: \text{a2: int * }} \\ \text{APP} \\ \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{APP} \\ \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{APP} \\ \frac{\text{Var}}{\text{F-}b: \text{int.} * f \text{ Non-simple index type:}} \\ \text{F-}s: \frac{\text{Var}}{\text{F-}b: \text{int.} (\text{int})} \\ \text{F-}s: \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{F-}s: \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{F-}s: \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{F-}f: \frac{\text{Var}}{\text{F-}f: \text{int.} (\text{int})} \\ \text{F-}f: \frac{\text{Var}}{\text{F-}b: \text{int.} (\text{int})} \\ \text{F-}f: \frac{\text{Var}}{\text{F-}f: \text{Int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{int.} (\text{in
```

```
Γ = 

    int y(int),

    double z,

    int f(int),

    struct {double a1; int * a2; } s,

    int * a,

    int(* arr[])(),

    int x,
    }
    (s.a2 = f(a[(*y + 1)])) ;
```

```
 \sum_{\text{FRUCT}} \frac{\text{Var}}{\text{F} \vdash s : \text{struct} \{ \text{double a1; int * a2; } \}}{\text{F} \vdash s : \text{struct} \{ \text{double a1; int * a2; } \}} \sum_{\text{APP}} \frac{\text{Var}}{\text{F} \vdash f : \text{int } \_(\text{int})} \sum_{\text{ARRAY}} \frac{\text{Var}}{\text{F} \vdash a : \text{int * } } \int_{\text{F}} \frac{\text{Non-simple index type:}}{\text{F} \vdash a[(*y + 1)] : \text{int } \_} \frac{\text{Var}}{\text{F} \vdash (*y + 1) : \text{error } f} = \frac{\text{Var}}{\text{F} \vdash a : \text{int * } } \sum_{\text{F} \vdash f : \text{int } \_(\text{int})} \frac{\text{Var}}{\text{F} \vdash a : \text{int * } } \frac{\text{Var}}{\text{F} \vdash a : \text{int * } } \int_{\text{F}} \frac{\text{Var}}{\text{F} \vdash a : \text{int * } } \frac{\text{Var}}{\text{F} \vdash
```

```
\Gamma = \left\{ egin{array}{ll} & 	ext{struct } t\{ 	ext{int } d; 	ext{int}(*\ f)(	ext{int}); \}\ s, \ & 	ext{double } d, \ & 	ext{struct } t*\ g(	ext{int}), \ & 	ext{int } i, \end{array} 
ight. 
ight.
```

```
((&(*g(i))).f)(i) ;
```

```
\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(*f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}  ((&(*g(i))).f)(i);
```



```
\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(*f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}  ((&(*g(i))).f)(i);
```

```
APP \xrightarrow{\text{VAR } \overline{\Gamma \vdash g : struct } t * \_(int)} \overline{\text{VAR } \overline{\Gamma \vdash i : int }}_{\underline{\Gamma} \vdash i : int }_{\underline{\Gamma} \vdash i : int }_{\underline{\Gamma} \vdash g(i) : \underline{\Gamma} \vdash (*g(i)) :}_{\underline{\Gamma} \vdash (*g(i)) : \underline{\Gamma} \vdash ((\&(*g(i))).f) :}_{\underline{\Gamma} \vdash ((\&(*g(i))).f)(i) :}_{\underline{\Gamma} \vdash ((\&(*g(i))).f)(i) :}_{\underline{\Gamma} \vdash (\&(*g(i))).f)(i) :}_{\underline{\Gamma} \vdash (\&(*g(i))).f}_{\underline{\Gamma} \vdash (\&(*g(i
```

```
\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(*f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}  ((&(*g(i))).f)(i);
```

```
\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(* f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}  ((&(*g(i))).f)(i);
```

```
APP = \frac{APP \frac{VAR}{\Gamma \vdash g : struct t * \_(int)} \frac{VAR}{\Gamma \vdash i : int}}{\Gamma \vdash g(i) : struct t *}

\frac{REF}{T} = \frac{APP}{T} \frac{VAR}{\Gamma \vdash g : struct t * \_(int)} \frac{VAR}{\Gamma \vdash i : int}

\frac{F \vdash (*g(i)) : struct t}{\Gamma \vdash (\&(*g(i))) : F}

\frac{F \vdash ((\&(*g(i))) \cdot f) :}{\Gamma \vdash ((\&(*g(i))) \cdot f)(i) :}

VAR}{\Gamma \vdash i :}
```

Type Checking

Type Checking
$$\Gamma = \left\{ \begin{array}{c} \text{struct } t\{\text{int } d; \text{int}(*f)(\text{int}); \} s, \\ \text{double } d, \\ \text{struct } t*g(\text{int}), \\ \text{int } i, \end{array} \right\} \tag{(\&(*g(i))).f)(i)};$$

```
VAR = \frac{1}{\Gamma \vdash g : struct t} * \_(int) VAR = \frac{1}{\Gamma \vdash i : int}
                              Deref
                                                   \lceil \vdash (*g(i)) : \text{struct } t
                Ref ·
                                          \Gamma \vdash (\&(*g(i))) : \text{struct } t *
     Struct
                                        \Gamma \vdash ((\&(*g(i))).f):
APP
                                                     \Gamma \vdash ((\&(*g(i))).f)(i):
```

Complete the type check! In case of a type error, mark the erroneous subexpression!

Type Checking

$$\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(*f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}$$
 ((&(*g(i))).f)(i);

Complete the type check! In case of a type error, mark the erroneous subexpression!

Type Checking

$$\Gamma = \left\{ \begin{array}{c} \operatorname{struct} t\{\operatorname{int} d; \operatorname{int}(*f)(\operatorname{int}); \} s, \\ \operatorname{double} d, \\ \operatorname{struct} t * g(\operatorname{int}), \\ \operatorname{int} i, \end{array} \right\}$$
 ((&(*g(i))).f)(i);

Complete the type check! In case of a type error, mark the erroneous subexpression!

```
a By
```

```
ia 🖳
```

 $B \le A$

 $B \le A$ |struct $s2\{\text{int } c: \text{int} * f(\text{int}*, \text{int}): A* s: \text{int} * a: \text{int} * b: \} \le A$

```
\begin{aligned} & B \leq A \\ & | \\ & \text{struct} \ s2\{\text{int} \ c; \text{int} \ast \ f(\text{int*}, \text{int}); A \ast \ s; \text{int} \ast \ a; \text{int} \ast \ b; \} \leq A \\ & | \\ & | \\ & \text{struct} \ s2\{\text{int} \ c; \text{int} \ast \ f(\text{int*}, \text{int}); A \ast \ s; \text{int} \ast \ a; \text{int} \ast \ b; \} \leq \text{struct} \ s1\{\text{int} \ast \ f(\text{int*}, \text{int}); A \ast \ s; \text{int} \ast \ a; \text{int} \ast \ b; \} \end{aligned}
```





$$B \le A$$

```
B \leq A
| struct s2\{ \textbf{int } c; \textbf{int } * f(\textbf{int } *, \textbf{int}); A * s; \textbf{int } * a; \textbf{int } * b; \} \leq A
| struct s2\{ \textbf{int } c; \textbf{int } * f(\textbf{int } *, \textbf{int}); A * s; \textbf{int } * a; \textbf{int } * b; \} \leq struct s1\{ \textbf{int } * f(\textbf{int } *, \textbf{int}); A * s; \textbf{int } * a; \textbf{int } * b; \}
| \textbf{int } * (\textbf{int } *, \textbf{int}) \leq \textbf{int } * (\textbf{int } *, \textbf{int}) \qquad A * \leq A * \quad \textbf{int } * \leq \textbf{int } * \quad \textbf{int } * \leq \textbf{int } *
| \textbf{int } * (\textbf{int } *, \textbf{int}) \leq \textbf{int } * \quad \textbf{int } * \leq \textbf{int } * \end{cases}
```



$$B \le A$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

Prove or disprove:

$$C \leq B$$



 $C \leq B$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g;};
struct s1{A f(B); A * g;};
struct s2{B f(A);};
```

$$C \leq B$$

$$C \leq B$$

 $struct s3{C f(C); C * g;} \le B$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq structs2\{B f(A); \}$$

$$C \leq B$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq structs2\{B f(A); \}$$

$$|$$

$$C _(C) \leq B _(A)$$

$$C \leq B$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq structs2\{B f(A); \}$$

$$|$$

$$C _{-}(C) \leq B _{-}(A)$$

$$C \leq B$$

$$A \leq C$$

$$C \leq B$$

```
typedef struct s1\ A;

typedef struct s2\ B;

typedef struct s3\ C;

struct s3\{C\ f(C); C*\ g;\};

struct s1\{A\ f(B); A*\ g;\};

struct s2\{B\ f(A);\};
```

$$C \leq B$$

$$C \leq B$$

$$|$$

$$struct s3\{C f(C); C * g; \} \leq B$$

$$|$$

$$struct s3\{C f(C); C * g; \} \leq struct s2\{B f(A); \}$$

$$|$$

$$C = C \leq B$$

$$A \leq C$$

$$|$$

$$struct s1\{A f(B); A * g; \} \leq C$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$C \leq B$$

$$|$$

$$structs3\{C f(C); C * g; \} \leq B$$

$$|$$

$$C _{-}(C) \leq B _{-}(A)$$

$$C \leq B$$

$$A \leq C$$

$$|$$

$$structs1\{A f(B); A * g; \} \leq C$$

$$|$$

$$structs3\{C f(C); C * g; \}$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$| \\
structs3{C f(C); C * g;} \leq B$$

$$| \\
structs3{C f(C); C * g;} \leq structs2{B f(A);} \\
| \\
C = (C) \leq B = (A)$$

$$C \leq B$$

$$A \leq C$$

$$| \\
structs1{A f(B); A * g;} \leq C$$

$$| \\
structs1{A f(B); A * g;} \leq structs3{C f(C); C * g;} \\
A = (B) \leq C = (C) = A * C$$

```
typedef struct s1 A;
typedef struct s2 B;
typedef struct s3 C;
struct s3{C f(C); C * g; };
struct s1{A f(B); A * g; };
struct s2{B f(A); };
```

$$C \leq B$$

$$C \leq B$$

$$structs3\{C \ f(C); C * \ g; \} \leq B$$

$$structs3\{C \ f(C); C * \ g; \} \leq structs2\{B \ f(A); \}$$

$$C \subseteq B \subseteq A \subseteq C$$

$$structs1\{A \ f(B); A * \ g; \} \leq C$$

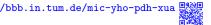
$$structs1\{A \ f(B); A * \ g; \} \leq structs3\{C \ f(C); C * \ g; \}$$

$$A \subseteq C \subseteq B \subseteq A \subseteq C$$

```
typedef struct s S;
typedef struct t T;
typedef T(* F)(S);
struct s{F f; T t; };
struct t{S(* f)(T); S t; };
```

```
typedef struct s S;
  typedef struct t T;
   typedef T(*F)(S);
  struct s{F f; T t; };
struct t\{S(*f)(T); S t; \};
```

Prove or disprove: $S \leq T$



S < T

```
typedef struct s S;
  typedef struct t T;
   typedef T(*F)(S);
  struct s{F f; T t; };
struct t\{S(*f)(T); S t; \};
```

Prove or disprove: $S \leq T$

https://bbb.in.tum.de/mic-yho-pdh-xua



```
typedef struct s S;
typedef struct t T;
typedef T(* F)(S);
struct s{F f; T t; };
struct t{S(* f)(T); S t; };
```

Prove or disprove: $S \leq T$

https://bbb.in.tum.de/mic-yho-pdh-xua



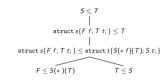
```
S \leq T \\ | \\ structs\{F f; T t; \} \leq T \\ | \\ structs\{F f; T t; \} \leq structt\{S(* f)(T); S t; \}
```

```
typedef struct s S;
typedef struct t T;
typedef T(* F)(S);
struct s{F f; T t; };
struct t{S(* f)(T); S t; };
```

Prove or disprove: $S \leq T$

https://bbb.in.tum.de/mic-yho-pdh-xua





```
typedef struct s S;

typedef struct t T;

typedef T(*F)(S);

struct s \{F f; T t; \};

struct t \{S(*f)(T); S t; \};
```



$$S \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq struct t\{S(*f)(T); S t;\}$$

$$F \leq S(*)(T)$$

$$T \leq S$$

$$|$$

$$T(*)(S) \leq S(*)(T)$$

```
typedef struct s S;
  typedef struct t T;
   typedef T(*F)(S);
  struct s{F f; T t;};
struct t\{S(*f)(T); S t; \};
```



$$S \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq structt\{S(* f)(T); S t;\}$$

$$F \leq S(*)(T)$$

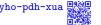
$$|$$

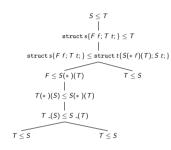
$$T(*)(S) \leq S(*)(T)$$

$$|$$

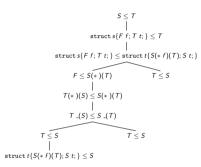
$$T_{*}(S) \leq S_{*}(T)$$

```
typedef struct s S;
  typedef struct t T;
   typedef T(*F)(S);
  struct s{F f; T t;};
struct t\{S(*f)(T); S t; \};
```





```
typedef struct s S;
  typedef struct t T;
   typedef T(*F)(S);
  struct s{F f; T t;};
struct t\{S(*f)(T); S t; \};
```



```
typedef struct s S;

typedef struct t T;

typedef T(*F)(S);

struct s{F f; T t;};

struct t{S(*f)(T); S t;};
```

$$S \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq T$$

$$|$$

$$structs\{F f; T t;\} \leq struct t\{S(* f)(T); S t;\}$$

$$F \leq S(*)(T)$$

$$|$$

$$T(*)(S) \leq S(*)(T)$$

$$|$$

$$T \leq S$$

$$|$$

$$T \leq S$$

$$|$$

$$T \leq S$$

$$|$$

$$Struct t\{S(* f)(T); S t;\} \leq S$$

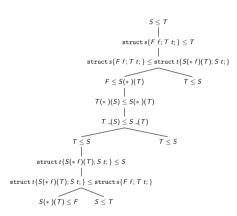
$$|$$

$$Struct t\{S(* f)(T); S t;\} \leq S$$

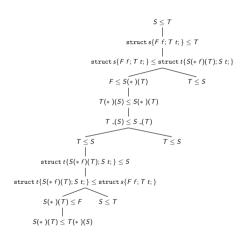
$$|$$

$$Struct t\{S(* f)(T); S t;\} \leq S$$

```
typedef struct s S;
typedef struct t T;
typedef T(* F)(S);
struct s{F f; T t; };
struct t{S(* f)(T); S t; };
```



```
typedef struct s S;
typedef struct t T;
typedef T(* F)(S);
struct s{F f; T t; };
struct t{S(* f)(T); S t; };
```



```
typedef struct s S;

typedef struct t T;

typedef T(*F)(S);

struct s{F f; T t;};

struct t{S(* f)(T); S t;};
```

```
typedef struct s S;

typedef struct t T;

typedef T(*F)(S);

struct s{F f; T t;};

struct t{S(*f)(T); S t;};
```

$$S \leq T \\ | \\ structs\{F \ f; T \ t;\} \leq T \\ | \\ structs\{F \ f; T \ t;\} \leq T \\ | \\ F \leq S(*)(T) \qquad T \leq S \\ | \\ T(*)(S) \leq S(*)(T) \\ | \\ T \leq S \qquad T \leq S \\ | \\ struct\ t\{S(* \ f)(T); S \ t;\} \leq S \\ | \\ struct\ t\{S(* \ f)(T); S \ t;\} \leq S \\ | \\ S(*)(T) \leq F \qquad S \leq T \\ | \\ S(*)(T) \leq T \leq S \\ | \\ S \leq T \leq T \\ | \\ S \leq T \\ | \\ S \leq T \leq T \\ |$$