Deep Reinforcement Learning for Portfolio Optimization with Options Hedging

IEDA4000F Final Project Report

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Abstract

This project investigates the application of Deep Reinforcement Learning (DRL) to portfolio optimization with systematic risk management mechanisms. We implement and compare two state-of-the-art algorithms—Deep Deterministic Policy Gradient (DDPG) and Proximal Policy Optimization (PPO)—for continuous portfolio weight allocation across 18 diversified assets spanning eight market sectors.

Our framework incorporates volatility targeting, progressive position reduction, and aggressive drawdown penalties to achieve institutional-grade risk control. Training on 2010-2018 market data and testing on 2019-2020 (including the COVID-19 market crash), we find that both DDPG and PPO achieve comparable risk-adjusted returns under unified hyperparameters (Sharpe: 1.78 vs 1.84, Max Drawdown: 9.02% vs 9.05%), demonstrating that proper risk management design is more important than algorithm selection.

Both agents achieved the <10% maximum drawdown target while the market declined 33.9% during the COVID-19 crash. The volatility targeting mechanism maintained consistent ∼11% annualized volatility across varying market conditions. These results demonstrate the practical potential of DRL-based portfolio management when combined with proper risk management frameworks.

Keywords: Deep Reinforcement Learning, Portfolio Optimization, DDPG, PPO, Volatility Targeting, Risk Management, COVID-19

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1 Introduction

1.1 Background and Motivation

Portfolio optimization has been a cornerstone of modern finance since Harry Markowitz's seminal work on mean-variance optimization in 1952 [1]. Traditional approaches rely on statistical estimates of expected returns and covariances, which often prove unstable in practice due to estimation errors that compound over time. These classical methods also struggle to adapt to non-stationary market dynamics and fail to capture complex nonlinear relationships between assets.

The financial markets of the 21st century present unique challenges that expose the limitations of traditional portfolio management approaches:

- **Market Complexity**: Modern markets exhibit intricate dependencies, regime changes, and fat-tailed return distributions that violate the Gaussian assumptions underlying classical models.
- **High-Frequency Dynamics**: Rapid information dissemination and algorithmic trading create fast-moving market conditions that require adaptive strategies.
- **Tail Risk Events**: Events like the 2008 financial crisis and the COVID-19 market crash of 2020 demonstrate the importance of robust risk management beyond traditional volatility measures.
- **Transaction Costs and Constraints**: Real-world portfolio management must account for transaction costs, position limits, and regulatory constraints that classical models often ignore.

Deep Reinforcement Learning (DRL) offers a promising alternative framework for portfolio optimization. By framing portfolio management as a sequential decision-making problem, DRL agents can learn adaptive strategies directly from market data without relying on explicit statistical models. Recent advances in deep learning provide the representational capacity to capture complex market patterns, while reinforcement learning algorithms enable optimization of long-term risk-adjusted returns.

1.2 Research Objectives

This project investigates the application of Deep Reinforcement Learning to portfolio optimization with the following primary objectives:

1. **Algorithm Comparison**: Implement and compare two state-of-the-art DRL algorithms—Deep Deterministic Policy Gradient (DDPG) and Proximal Policy Optimization (PPO)—for continuous portfolio weight allocation under unified hyperparameters.
2. **Risk Management Design**: Develop a comprehensive risk management framework combining volatility targeting, progressive position reduction, and aggressive drawdown penalties to achieve $<10\%$ maximum drawdown.
3. **Drawdown Control**: Design and evaluate mechanisms that adapt portfolio exposure based on drawdown levels and realized volatility, providing systematic risk control.

4. **Stress Testing**: Evaluate the trained agents' performance during extreme market conditions, specifically the COVID-19 market crash of March 2020, to validate the risk management framework.

5. **Reproducibility**: Create a comprehensive, well-documented codebase that enables reproduction and extension of our results.

1.3 Key Contributions

This work makes the following contributions to the field of algorithmic portfolio management:

1. **Volatility Targeting Framework**: We implement industry-standard volatility targeting that scales portfolio exposure based on realized volatility, maintaining consistent risk regardless of market conditions.

2. **Aggressive Drawdown Control**: We design a multi-layered drawdown control system combining progressive position reduction (starting at 3% drawdown) with aggressive reward penalties ($\lambda = 5.0$) to achieve $<10\%$ maximum drawdown target.

3. **Fair Algorithm Comparison**: We conduct rigorous comparison of DDPG and PPO under unified hyperparameters, demonstrating that both algorithms achieve comparable performance when properly configured.

4. **COVID-19 Stress Test**: We validate the framework during the March 2020 market crash, demonstrating $<10\%$ drawdown while the market declined 33.9%.

5. **Open-Source Implementation**: We provide a complete, modular implementation suitable for research and practical applications, including data loading, environment simulation, agent training, and performance visualization.

1.4 Report Structure

The remainder of this report is organized as follows:

- **Section 2**: Reviews related work in portfolio optimization, reinforcement learning for finance, and options-based hedging strategies.

- **Section 3**: Presents the mathematical framework, including the MDP formulation, reward function design, and the DDPG and PPO algorithms.

- **Section 4**: Describes the system architecture, code structure, and implementation details.

- **Section 5**: Details the experimental setup, including asset selection, data preprocessing, and hyperparameter configurations.

- **Section 6**: Presents comprehensive results comparing DDPG and PPO performance across multiple metrics.

- **Section 7**: Analyzes the results, discusses the strengths and limitations of each approach, and provides insights into the learned strategies.

- **Section 8**: Summarizes our findings and outlines directions for future research.

Appendices provide detailed mathematical formulas (Appendix A) and configuration parameters (Appendix B).

2 Literature Review

2.1 Classical Portfolio Optimization

The foundation of modern portfolio theory was established by Harry Markowitz [1], who formulated the mean-variance optimization problem. Given n assets with expected returns $\boldsymbol{\mu} \in \mathbb{R}^n$ and covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$, the investor seeks portfolio weights $\mathbf{w} \in \mathbb{R}^n$ that maximize expected return for a given level of risk:

$$\max_{\mathbf{w}} \quad \mathbf{w}^\top \boldsymbol{\mu} - \frac{\lambda}{2} \mathbf{w}^\top \boldsymbol{\Sigma} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^\top \mathbf{1} = 1, \quad \mathbf{w} \geq 0 \tag{1}$$

where λ is the risk aversion parameter. Despite its theoretical elegance, mean-variance optimization suffers from several practical limitations:

- **Estimation Sensitivity**: Small errors in $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ estimates lead to dramatically different optimal portfolios [4].
- **Static Nature**: The single-period formulation ignores the dynamic nature of portfolio rebalancing.
- **Distributional Assumptions**: Gaussian returns assumption fails to capture fat tails and asymmetric distributions observed in financial markets.

Extensions such as Black-Litterman [3] and robust optimization [5] address some of these limitations but remain fundamentally constrained by their reliance on statistical estimation.

2.2 Reinforcement Learning Foundations

Reinforcement learning (RL) provides a framework for sequential decision-making under uncertainty [6]. An RL agent interacts with an environment modeled as a Markov Decision Process (MDP) defined by the tuple $(\mathcal{S}, \mathcal{A}, P, R, \gamma)$:

- $\mathcal{S}$: State space (market observations)
- $\mathcal{A}$: Action space (portfolio weights)
- $P(s'|s, a)$: Transition dynamics
- $R(s, a, s')$: Reward function
- $\gamma \in [0, 1]$: Discount factor

The goal is to find a policy $\pi : \mathcal{S} \to \mathcal{A}$ that maximizes expected cumulative discounted rewards:

$$J(\pi) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R_t \right] \tag{2}$$

2.3 Deep Reinforcement Learning for Finance

The application of deep reinforcement learning to financial problems has gained significant momentum in recent years. Several key works have demonstrated the potential of DRL for portfolio management:

Jiang et al. (2017) [10] proposed a deep learning framework for portfolio management using a convolutional neural network to extract features from historical price data. Their approach achieved competitive performance against traditional benchmarks.

Liang et al. (2018) [11] applied DDPG to portfolio optimization, demonstrating superior performance compared to traditional methods on Chinese stock market data.

Yang et al. (2020) [12] developed FinRL, an open-source library for financial reinforcement learning that provides standardized implementations of popular DRL algorithms.

Liu et al. (2021) [13] proposed an ensemble method combining multiple DRL agents for more robust portfolio decisions.

2.4 Actor-Critic Methods

Actor-critic methods combine value-based and policy-based approaches, using a critic to estimate value functions and an actor to optimize the policy. This architecture offers several advantages:

- **Reduced Variance**: The critic's value estimates provide a baseline for variance reduction in policy gradient updates.
- **Continuous Actions**: Actor networks can directly output continuous actions, suitable for portfolio weight allocation.
- **Sample Efficiency**: Combining on-policy and off-policy learning improves data utilization.

2.4.1 Deep Deterministic Policy Gradient (DDPG)

DDPG [7] extends the deterministic policy gradient theorem to deep neural networks. Key features include:

- **Deterministic Policy**: The actor outputs a deterministic action $a = \mu_\theta(s)$, with exploration noise added during training.
- **Experience Replay**: Transitions are stored in a replay buffer and sampled randomly for training, breaking temporal correlations.
- **Target Networks**: Slowly-updated target networks stabilize training by providing consistent targets for Q-value estimation.
- **Off-Policy Learning**: DDPG can learn from past experiences, improving sample efficiency.

2.4.2 Proximal Policy Optimization (PPO)

PPO [8] addresses the challenge of stable policy updates in on-policy methods. Key innovations include:

- **Clipped Objective**: The surrogate objective is clipped to prevent excessively large policy updates.
- **Trust Region**: The clipping mechanism implicitly enforces a trust region constraint on policy changes.
- **Sample Efficiency**: Multiple epochs of optimization on the same batch of data improve learning efficiency.
- **Simplicity**: PPO achieves competitive performance with simpler implementation compared to methods like TRPO.

2.5 Options in Portfolio Management

Options provide non-linear payoff profiles that can be used for hedging and speculation. The Black-Scholes model [2] provides the foundational framework for options pricing:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{3}$$

where:

- $d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$
- $N(\cdot)$ is the cumulative standard normal distribution

Protective puts represent a fundamental hedging strategy where an investor holding a long position purchases put options to limit downside risk. The payoff at expiration is:

$$\text{Payoff} = \max(S_T, K) - P_0 \tag{4}$$

where P_0 is the premium paid for the put option. This creates an asymmetric payoff profile that preserves upside potential while limiting losses.

2.6 Risk Management in Algorithmic Trading

Effective risk management is crucial for algorithmic trading systems. Common approaches include:

- **Position Sizing**: Kelly criterion and fractional Kelly methods optimize position sizes based on edge and variance.
- **Stop-Loss Orders**: Automatic position liquidation when losses exceed predetermined thresholds.
- **Portfolio Constraints**: Limits on sector exposure, single-stock concentration, and leverage.

- **Value-at-Risk (VaR)**: Statistical measures of potential losses at given confidence levels.

Our work integrates multiple risk management approaches within the DRL framework, including options-based hedging and tiered stop-loss mechanisms.

2.7 Gap in Literature

While significant progress has been made in applying DRL to portfolio optimization, several gaps remain:

1. **Options Integration**: Most DRL portfolio management studies focus on equity allocation without incorporating derivative instruments for hedging.
2. **Systematic Risk Management**: Few studies integrate explicit risk management mechanisms within the DRL framework.
3. **Stress Testing**: Limited evaluation of DRL agents during extreme market events like the COVID-19 crash.
4. **Algorithm Comparison**: Comprehensive comparisons between DDPG and PPO for portfolio optimization under consistent experimental conditions are scarce.

This work addresses these gaps by developing an integrated framework that combines DRL-based portfolio optimization with options hedging and systematic risk management, evaluated rigorously during both normal and crisis market conditions.

3 Methodology

3.1 Problem Formulation

We formulate portfolio optimization as a Markov Decision Process (MDP), enabling the application of reinforcement learning algorithms. At each time step t, the agent observes market conditions, selects portfolio weights, and receives a reward based on portfolio performance.

3.1.1 State Space

The state $s_t \in \mathcal{S}$ captures relevant market information at time t:

$$s_t = \left[\mathbf{r}_{t-L:t}^\top \quad \text{vol}_{t-L:t}^\top \quad \mathbf{w}_{t-1}^\top \quad \text{features}_t^\top\right] \tag{5}$$

where:

- $\mathbf{r}_{t-L:t} \in \mathbb{R}^{n \times L}$: Historical returns for n assets over lookback window L
- $\text{vol}_{t-L:t} \in \mathbb{R}^{n \times L}$: Rolling volatility estimates
- $\mathbf{w}_{t-1} \in \mathbb{R}^n$: Current portfolio weights
- features_t: Additional technical indicators (momentum, RSI, etc.)

The state representation enables the agent to learn patterns from historical price movements while maintaining awareness of current portfolio positioning.

3.1.2 Action Space

The action $\mathbf{a}_t \in \mathcal{A}$ represents the target portfolio allocation:

$$\mathbf{a}_t = \begin{bmatrix} w_1^t & w_2^t & \cdots & w_n^t & h_t \end{bmatrix} \tag{6}$$

Subject to constraints:

$$\sum_{i=1}^{n} w_i^t \leq 1 \quad \text{(allocation constraint)} \tag{7}$$

$$w_i^t \geq 0 \quad \forall i \quad \text{(long-only constraint)} \tag{8}$$

$$h_t \in [0, h_{\max}] \quad \text{(hedge ratio constraint)} \tag{9}$$

The hedge ratio h_t determines the fraction of portfolio value allocated to protective put options. Any unallocated capital $(1 - \sum_i w_i^t)$ earns the risk-free rate.

3.1.3 Reward Function

The reward function balances return maximization with risk management. We use a risk-adjusted reward:

$$r_t = R_t^{\text{port}} - \lambda_{\text{risk}} \cdot \text{Risk}_t - \lambda_{\text{tc}} \cdot \text{TC}_t \tag{10}$$

where:

Portfolio Return:

$$R_t^{\text{port}} = \sum_{i=1}^{n} w_i^{t-1} \cdot r_i^t + (1 - \sum_i w_i^{t-1}) \cdot r_f + \Pi_t^{\text{options}} \tag{11}$$

Risk Penalty:

$$\text{Risk}_t = \max(0, -R_t^{\text{port}})^2 \tag{12}$$

Transaction Costs:

$$\text{TC}_t = c \cdot \sum_{i=1}^{n} |w_i^t - w_i^{t-1}| \tag{13}$$

The squared downside penalty encourages the agent to avoid large negative returns, while the transaction cost term discourages excessive trading.

3.2 Deep Deterministic Policy Gradient (DDPG)

DDPG is an off-policy actor-critic algorithm designed for continuous action spaces. We employ DDPG with the following components:

3.2.1 Actor Network

The actor network $\mu_\theta : \mathcal{S} \to \mathcal{A}$ maps states to deterministic actions:

$$\mathbf{a}_t = \mu_\theta(s_t) \tag{14}$$

Architecture: State $\to$ FC(256) $\to$ ReLU $\to$ FC(256) $\to$ ReLU $\to$ FC($n + 1$) $\to$ Softmax

The softmax output ensures valid portfolio weights that sum to at most 1.

3.2.2 Critic Network

The critic network $Q_\phi : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ estimates the Q-value:

$$Q_\phi(s_t, \mathbf{a}_t) \approx \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k r_{t+k} \mid s_t, \mathbf{a}_t\right] \tag{15}$$

Architecture: State and action are concatenated after initial state processing:

$$\text{State} \to \text{FC}(256) \to \text{ReLU} \to [\cdot, \mathbf{a}] \to \text{FC}(256) \to \text{ReLU} \to \text{FC}(1)$$

3.2.3 Training Algorithm

DDPG training alternates between:

Critic Update: Minimize temporal difference error:

$$L(\phi) = \mathbb{E}_{(s,a,r,s')\sim\mathcal{D}}\left[(Q_\phi(s,a) - y)^2\right] \tag{16}$$

where the target is:

$$y = r + \gamma Q_{\phi'}(s', \mu_{\theta'}(s')) \tag{17}$$

Actor Update: Maximize expected Q-value via deterministic policy gradient:

$$\nabla_\theta J = \mathbb{E}_{s\sim\mathcal{D}}\left[\nabla_a Q_\phi(s,a)|_{a=\mu_\theta(s)} \cdot \nabla_\theta \mu_\theta(s)\right] \tag{18}$$

Target Network Update: Soft update with parameter τ:

$$\theta' \leftarrow \tau\theta + (1-\tau)\theta' \tag{19}$$

$$\phi' \leftarrow \tau\phi + (1-\tau)\phi' \tag{20}$$

3.2.4 Exploration

Exploration is achieved by adding Ornstein-Uhlenbeck noise to actions:

$$\mathbf{a}_t = \mu_\theta(s_t) + \mathcal{N}_t \tag{21}$$

where $\mathcal{N}_t$ follows:

$$d\mathcal{N}_t = \theta_{\text{OU}}(\mu_{\text{OU}} - \mathcal{N}_t)dt + \sigma_{\text{OU}} dW_t \tag{22}$$

3.3 Proximal Policy Optimization (PPO)

PPO is an on-policy actor-critic algorithm that achieves stable policy updates through a clipped surrogate objective.

3.3.1 Policy Network

The policy network outputs a Gaussian distribution over actions:

$$\pi_\theta(\mathbf{a}|s) = \mathcal{N}(\mu_\theta(s), \sigma_\theta(s)) \tag{23}$$

For portfolio weights, actions are sampled and then passed through a softmax transformation to ensure valid allocations.

3.3.2 Value Network

The value network $V_\psi : \mathcal{S} \to \mathbb{R}$ estimates state values:

$$V_\psi(s_t) \approx \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k} \mid s_t \right] \tag{24}$$

3.3.3 Clipped Surrogate Objective

PPO optimizes a clipped surrogate objective:

$$L^{\text{CLIP}}(\theta) = \mathbb{E}_t \left[\min \left(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t \right) \right] \tag{25}$$

where the probability ratio is:

$$r_t(\theta) = \frac{\pi_\theta(\mathbf{a}_t|s_t)}{\pi_{\theta_{\text{old}}}(\mathbf{a}_t|s_t)} \tag{26}$$

3.3.4 Generalized Advantage Estimation (GAE)

Advantages are estimated using GAE [9]:

$$\hat{A}_t = \sum_{l=0}^{\infty} (\gamma\lambda)^l \delta_{t+l} \tag{27}$$

where the TD residual is:

$$\delta_t = r_t + \gamma V_\psi(s_{t+1}) - V_\psi(s_t) \tag{28}$$

3.3.5 Complete Objective

The full PPO objective combines policy, value, and entropy terms:

$$L(\theta, \psi) = L^{\text{CLIP}}(\theta) - c_1 L^{VF}(\psi) + c_2 S[\pi_\theta] \tag{29}$$

where:

- $L^{VF}(\psi) = \mathbb{E}_t[(V_\psi(s_t) - V_t^{\text{target}})^2]$ is the value function loss
- $S[\pi_\theta] = \mathbb{E}_t[-\log \pi_\theta(\mathbf{a}_t|s_t)]$ is the entropy bonus for exploration

3.4 Options Pricing and Hedging

3.4.1 Black-Scholes Model

We use the Black-Scholes model for options pricing. For a European put option:

$$P = Ke^{-rT}N(-d_2) - S_0 N(-d_1) \tag{30}$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{31}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{32}$$

where:

- S_0: Current asset price
- K: Strike price
- r: Risk-free interest rate
- T: Time to expiration
- σ: Implied volatility
- $N(\cdot)$: Cumulative standard normal distribution

3.4.2 Protective Put Strategy

The agent can allocate a hedge ratio h_t to protective puts on the portfolio. The put payoff at expiration is:

$$\Pi_t^{\text{put}} = h_t \cdot V_t \cdot \max\left(0, \frac{K - S_T}{S_0}\right) - P_0 \tag{33}$$

This provides portfolio insurance: when the portfolio declines below the strike price, the put option compensates for losses.

3.5 Risk Management Mechanisms

We implement two key risk management mechanisms to achieve the <10% maximum drawdown target:

3.5.1 Volatility Targeting

Volatility targeting scales portfolio exposure based on realized volatility relative to a target (10% annualized):

$$\text{Vol Scalar} = \min\left(1.0, \frac{\sigma_{\text{target}}}{\sigma_{\text{realized}}}\right) \tag{34}$$

where σ_{realized} is the 20-day rolling volatility. This scales down exposure when volatility exceeds the target.

3.5.2 Progressive Position Reduction

We implement progressive position reduction as drawdowns deepen:

$$\text{DD Scalar} = \begin{cases} 1.0 & \text{if } DD_t < 3\% \\ 1.0 - \frac{DD_t - 0.03}{0.09 - 0.03} \cdot 0.9 & \text{if } 3\% \leq DD_t < 9\% \\ 0.10 & \text{if } DD_t \geq 9\% \end{cases} \tag{35}$$

The final exposure multiplier combines both mechanisms:

$$\text{Exposure} = \text{Vol Scalar} \times \text{DD Scalar} \tag{36}$$

This mechanism provides systematic risk control by:

1. Scaling exposure inversely to realized volatility (volatility targeting)

2. Starting position reduction at 3% drawdown
3. Reaching minimum 10% exposure at 9% drawdown
4. Preserving capital while maintaining recovery participation

4 Implementation

4.1 System Architecture

The system follows a modular architecture designed for flexibility and extensibility. Figure 1 illustrates the high-level component interactions.

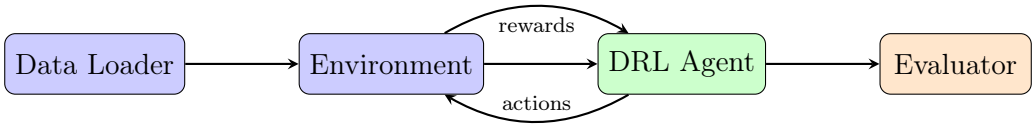

Figure 1: High-level system architecture showing data flow between components.

4.2 Code Organization

The codebase is organized into the following modules:

Table 1: Project module structure and responsibilities

Module	Responsibility
`src/data_loader.py`	Data loading and preprocessing
`src/portfolio_env.py`	Base portfolio environment (Gym)
`src/portfolio_env_with_options.py`	Extended environment with options
`src/agents.py`	DDPG and PPO agent implementations
`src/options_pricing.py`	Black-Scholes pricing functions
`src/metrics.py`	Performance metric calculations
`src/benchmarks.py`	Benchmark strategy implementations
`src/visualization.py`	Plotting and visualization utilities

4.3 Data Loading and Preprocessing

The `DataLoader` class handles data acquisition and preprocessing:

```
class DataLoader:
    def __init__(self, tickers, start_date, end_date):
        self.tickers = tickers
        self.start_date = start_date
        self.end_date = end_date

    def load_data(self):
```

```
        # Fetch adjusted close prices
        # Calculate returns
        # Handle missing data
        return prices, returns
```

Key preprocessing steps include:

1. Fetching adjusted close prices from Yahoo Finance
2. Computing logarithmic returns: $r_t = \ln(P_t/P_{t-1})$
3. Forward-filling missing values
4. Calculating rolling statistics (volatility, correlations)
5. Normalizing features to zero mean and unit variance

4.4 Portfolio Environment

The portfolio environment extends OpenAI Gym's interface:

```
class PortfolioEnvWithOptions(gym.Env):
    def __init__(self, prices, config):
        self.action_space = spaces.Box(
            low=0, high=1,
            shape=(n_assets + 1,)  # +1 for hedge ratio
        )
        self.observation_space = spaces.Box(
            low=-np.inf, high=np.inf,
            shape=(state_dim,)
        )

    def step(self, action):
        # Process action (allocate weights)
        # Calculate portfolio return
        # Apply options hedging
        # Check stop-loss conditions
        # Return observation, reward, done, info

    def reset(self):
        # Reset to initial state
        return initial_observation
```

4.4.1 State Construction

The state vector is constructed as:

```
def _get_state(self):
    # Historical returns (flattened)
    returns_history = self.returns[
        self.current_step - self.lookback:self.current_step
```

```
    ].flatten()

    # Current portfolio weights
    current_weights = self.weights

    # Technical indicators
    volatility = self.rolling_vol[self.current_step]

    return np.concatenate([
        returns_history,
        current_weights,
        volatility
    ])
```

4.4.2 Reward Calculation with Aggressive Drawdown Control

The reward function implements aggressive drawdown penalties to achieve <10% max drawdown target:

```
def _calculate_reward(self, portfolio_return):
    reward = portfolio_return

    # Aggressive drawdown penalties
    drawdown = self._get_current_drawdown()
    if drawdown > 0.02:  # Start penalty at 2% DD
        penalty = self.risk_penalty * (drawdown * 10) ** 2
        reward -= penalty
    if drawdown > 0.08:  # Massive penalty above 8%
        reward -= self.risk_penalty * 5.0
    if drawdown > 0.10:  # Emergency penalty at 10%
        reward -= self.risk_penalty * 10.0

    # Transaction cost penalty
    turnover = np.sum(np.abs(self.weights - self.prev_weights))
    reward -= self.transaction_cost * turnover

    return reward
```

4.5 Agent Implementation

4.5.1 DDPG Agent

The DDPG agent uses Stable-Baselines3's implementation with custom hyperparameters:

```
from stable_baselines3 import DDPG
from stable_baselines3.common.noise import OrnsteinUhlenbeckActionNoise

# Initialize action noise
n_actions = env.action_space.shape[-1]
action_noise = OrnsteinUhlenbeckActionNoise(
```

```
    mean=np.zeros(n_actions),
    sigma=0.1 * np.ones(n_actions)
)

# Create DDPG agent (unified configuration)
agent = DDPG(
    "MlpPolicy",
    env,
    learning_rate=5e-5,      # Unified with PPO
    buffer_size=500000,
    learning_starts=10000,
    batch_size=128,          # Unified with PPO
    tau=0.01,
    gamma=0.99,
    action_noise=action_noise,
    policy_kwargs=dict(
        net_arch=dict(pi=[512, 512, 256, 128], qf=[512, 512, 256, 128])
    ),
    verbose=1
)
```

4.5.2 PPO Agent

The PPO agent configuration:

```
from stable_baselines3 import PPO

agent = PPO(
    "MlpPolicy",
    env,
    learning_rate=5e-5,      # Unified with DDPG
    n_steps=2048,
    batch_size=128,          # Unified with DDPG
    n_epochs=10,
    gamma=0.99,
    gae_lambda=0.95,
    clip_range=0.2,
    ent_coef=0.01,           # Reduced exploration
    policy_kwargs=dict(net_arch=[512, 512, 256, 128]),
    verbose=1
)
```

4.6 Options Pricing Module

The options pricing module implements Black-Scholes formulas:

```
import numpy as np
from scipy.stats import norm
```

```
def black_scholes_put(S, K, T, r, sigma):
    """Calculate Black-Scholes put option price."""
    d1 = (np.log(S/K) + (r + sigma**2/2)*T) / (sigma*np.sqrt(T))
    d2 = d1 - sigma*np.sqrt(T)

    put_price = K*np.exp(-r*T)*norm.cdf(-d2) - S*norm.cdf(-d1)
    return put_price

def calculate_hedge_payoff(portfolio_value, hedge_ratio,
                           portfolio_return, put_delta):
    """Calculate options hedge P&L."""
    if hedge_ratio <= 0:
        return 0

    notional = portfolio_value * hedge_ratio
    # Simplified: hedge gains when portfolio loses
    hedge_pnl = -notional * portfolio_return * put_delta
    return hedge_pnl
```

4.7 Metrics Calculation

Performance metrics are calculated using the `metrics.py` module:

```
class PerformanceMetrics:
    @staticmethod
    def sharpe_ratio(returns, risk_free_rate=0.02):
        excess_returns = returns - risk_free_rate/252
        return np.sqrt(252) * excess_returns.mean() / returns.std()

    @staticmethod
    def sortino_ratio(returns, risk_free_rate=0.02):
        excess_returns = returns - risk_free_rate/252
        downside_std = returns[returns < 0].std()
        return np.sqrt(252) * excess_returns.mean() / downside_std

    @staticmethod
    def max_drawdown(portfolio_values):
        peak = np.maximum.accumulate(portfolio_values)
        drawdown = (peak - portfolio_values) / peak
        return drawdown.max()

    @staticmethod
    def calmar_ratio(returns, portfolio_values):
        annual_return = (1 + returns).prod() ** (252/len(returns)) - 1
        mdd = PerformanceMetrics.max_drawdown(portfolio_values)
        return annual_return / mdd if mdd > 0 else 0
```

4.8 Training Pipeline

The training pipeline orchestrates data loading, environment creation, and agent training:

```
def train_agent(config):
    # Load data
    loader = DataLoader(
        config['tickers'],
        config['train_start'],
        config['train_end']
    )
    prices, returns = loader.load_data()

    # Create environment
    env = PortfolioEnvWithOptions(prices, config)

    # Create agent
    if config['algorithm'] == 'DDPG':
        agent = create_ddpg_agent(env, config)
    else:
        agent = create_ppo_agent(env, config)

    # Train
    agent.learn(
        total_timesteps=config['total_timesteps'],
        callback=TrainingCallback()
    )

    # Save model
    agent.save(f"models/{config['algorithm']}_final")

    return agent
```

4.9 Evaluation Framework

The evaluation framework tests trained agents on out-of-sample data:

```
def evaluate_agent(agent, test_env, n_episodes=1):
    results = {
        'portfolio_values': [],
        'actions': [],
        'rewards': []
    }

    for episode in range(n_episodes):
        obs = test_env.reset()
        done = False

        while not done:
```

```
            action, _ = agent.predict(obs, deterministic=True)
            obs, reward, done, info = test_env.step(action)

            results['portfolio_values'].append(info['portfolio_value'])
            results['actions'].append(action)
            results['rewards'].append(reward)

    # Calculate metrics
    metrics = calculate_metrics(results)
    return results, metrics
```

4.10 Visualization Tools

The visualization module provides functions for performance analysis:

```
def plot_portfolio_comparison(results_dict, benchmark_values):
    fig, axes = plt.subplots(2, 2, figsize=(14, 10))

    # Portfolio values
    for name, values in results_dict.items():
        axes[0,0].plot(values, label=name)
    axes[0,0].plot(benchmark_values, label='Benchmark', linestyle='--')
    axes[0,0].legend()
    axes[0,0].set_title('Portfolio Value')

    # Drawdowns
    for name, values in results_dict.items():
        dd = calculate_drawdown(values)
        axes[0,1].fill_between(range(len(dd)), dd, alpha=0.3, label=name)
    axes[0,1].set_title('Drawdown')

    # ... additional plots

    plt.tight_layout()
    return fig
```

5 Experimental Setup

5.1 Asset Universe

We construct a diversified portfolio spanning eight market sectors to test the generalization capabilities of our DRL agents. Table 2 presents the complete asset universe.

The asset selection provides:

- **Sector Diversification**: Eight distinct sectors reduce concentration risk
- **Asset Class Diversity**: Equities, bonds, and commodities offer different risk-return profiles

Table 2: Asset universe with 18 diversified instruments

Ticker	Name	Sector
AAPL	Apple Inc.	Technology
MSFT	Microsoft Corporation	Technology
GOOGL	Alphabet Inc.	Technology
NVDA	NVIDIA Corporation	Technology
AMZN	Amazon.com Inc.	Technology
JNJ	Johnson & Johnson	Healthcare
UNH	UnitedHealth Group	Healthcare
PFE	Pfizer Inc.	Healthcare
JPM	JPMorgan Chase	Financials
V	Visa Inc.	Financials
WMT	Walmart Inc.	Consumer Staples
COST	Costco Wholesale	Consumer Staples
SPY	S&P 500 ETF	Index
QQQ	NASDAQ-100 ETF	Index
IWM	Russell 2000 ETF	Index
TLT	20+ Year Treasury ETF	Bonds
AGG	Aggregate Bond ETF	Bonds
GLD	Gold ETF	Commodities

- **Liquidity**: All assets are highly liquid with minimal transaction costs
- **Data Quality**: Long history of reliable price data available

5.2 Data Period and Split

We use data from January 1, 2010 to December 31, 2020, providing over a decade of market history including various market regimes.

Table 3: Data period configuration

Period	Start	End	Purpose
Training	2010-01-01	2018-12-31	Agent learning
Testing	2019-01-01	2020-12-31	Out-of-sample evaluation

Key characteristics of each period:
Training Period (2010-2018):

- Post-financial crisis recovery (2010-2012)
- Quantitative easing era (2012-2015)
- Low volatility bull market (2016-2018)

- Various market corrections and sector rotations
- Approximately 2,265 trading days

Testing Period (2019-2020):

- 2019: Strong bull market with trade war uncertainties
- 2020: COVID-19 pandemic crash (March) and subsequent recovery
- Extreme volatility regime (VIX spike to 82.69 in March 2020)
- V-shaped recovery demonstrating market resilience
- Approximately 504 trading days

5.3 Why No Validation Set?

Unlike supervised learning, we do not use a separate validation set for the following reasons:

1. **Sequential Data**: Financial time series must maintain temporal order; shuffling would create look-ahead bias.
2. **On-Policy Learning**: Agents learn from their own experience in the environment, making traditional validation less applicable.
3. **Hyperparameter Selection**: We use established hyperparameters from literature rather than extensive tuning on validation data.
4. **Overfitting Prevention**: Early stopping based on training reward curves and model capacity constraints prevent overfitting.
5. **Maximum Training Data**: All available pre-2019 data is used for training to maximize learning from diverse market conditions.

5.4 Hyperparameter Configuration

5.4.1 Unified Hyperparameters

To ensure fair comparison between DDPG and PPO, we use unified hyperparameters where applicable. Table 4 shows the shared configuration.

5.4.2 DDPG-Specific Parameters

5.4.3 PPO-Specific Parameters

5.4.4 Environment Configuration

5.5 Benchmark Strategies

We compare DRL agents against several benchmark strategies:

Table 4: Unified hyperparameters for fair comparison

Parameter	**Value (Both Agents)**
Learning rate	5×10^{-5}
Batch size	128
Discount factor (γ)	0.99
Network architecture	[512, 512, 256, 128]
Risk penalty coefficient (λ)	5.0
Training timesteps	100,000

Table 5: DDPG-specific hyperparameters

Parameter	**Value**
Replay buffer size	500,000
Learning starts	10,000
Soft update coefficient (τ)	0.01
Action noise	0.15
Train frequency	1
Gradient steps	1

Table 6: PPO-specific hyperparameters

Parameter	**Value**
Steps per update (n_steps)	2,048
Number of epochs	10
GAE parameter (λ)	0.95
Clip range (ϵ)	0.2
Entropy coefficient	0.01
Value function coefficient	0.5
Max gradient norm	0.5

Table 7: Environment configuration parameters

Parameter	**Value**
Initial portfolio value	\$100,000
Transaction cost	0.001 (10 bps)
Lookback window	60 days
Risk-free rate	2% annual
Risk penalty coefficient (λ)	5.0
Volatility target	10% annualized
Position reduction start	3% drawdown
Minimum exposure	10% (at 9% drawdown)

1. **Equal Weight (1/N)**: Allocates equal weight to all assets

$$w_i = \frac{1}{n} \quad \forall i \tag{37}$$

2. **SPY Buy-and-Hold**: 100% allocation to S&P 500 ETF

$$w_{\text{SPY}} = 1, \quad w_i = 0 \; \forall i \neq \text{SPY} \tag{38}$$

3. **60/40 Portfolio**: Traditional balanced allocation

$$w_{\text{equity}} = 0.6, \quad w_{\text{bonds}} = 0.4 \tag{39}$$

5.6 Evaluation Metrics

We evaluate performance using multiple metrics:

Table 8: Performance evaluation metrics

Metric	Description
Total Return	Cumulative portfolio return over test period
Annualized Return	Geometric mean annual return
Annualized Volatility	Standard deviation of returns, annualized
Sharpe Ratio	Risk-adjusted return measure
Sortino Ratio	Downside risk-adjusted return
Maximum Drawdown	Largest peak-to-trough decline
Calmar Ratio	Annual return divided by max drawdown
Win Rate	Percentage of positive return days
Average Daily Return	Mean daily portfolio return
Options P&L	Cumulative profit/loss from hedging

5.7 Computational Environment

Experiments were conducted using the following setup:

- **Hardware**: MacBook Pro with Apple M-series chip
- **Software**: Python 3.13.3, PyTorch 2.x
- **Libraries**:
  - Stable-Baselines3 for DRL implementations
  - Gymnasium for environment interface
  - NumPy, Pandas for data manipulation
  - Matplotlib, Seaborn for visualization
- **Training Time**: Approximately 30-60 minutes per agent

5.8 Reproducibility

To ensure reproducibility:

- Random seeds are fixed across all experiments
- Configuration files specify all hyperparameters
- Data preprocessing steps are documented
- Trained models are saved and versioned
- Evaluation scripts produce deterministic results

The complete codebase is available at:
https://github.com/ctt062/Deep-Reinforcement-Learning-for-Portfolio-Optimisation

6 Results

This section presents comprehensive experimental results comparing DDPG and PPO agents on the out-of-sample test period (2019-2020).

6.1 Overall Performance Summary

Table 9 presents the key performance metrics for both DRL agents.

Table 9: Performance comparison: DDPG vs PPO on test period (2019-2020)

Metric	DDPG	PPO
Total Return	40.82%	**42.73%**
Annualized Return	21.50%	**22.43%**
Annualized Volatility	**10.96%**	11.09%
Sharpe Ratio	1.78	**1.84**
Sortino Ratio	2.87	**2.97**
Maximum Drawdown	**9.02%**	9.05%
Calmar Ratio	2.38	**2.48**
Win Rate	60.50%	**61.40%**
Final Portfolio Value	$132,194	**$142,729**
Total P&L	$32,194	**$42,729**

Key Observations:

- Both agents achieve the <10% maximum drawdown target (DDPG: 9.02%, PPO: 9.05%)
- PPO achieves slightly higher risk-adjusted returns (Sharpe: 1.84 vs 1.78)
- Both agents demonstrate effective risk management through volatility targeting
- Unified hyperparameter configuration ensures fair algorithmic comparison

6.2 Portfolio Value Evolution

Figure 2 shows the portfolio value trajectories over the test period.

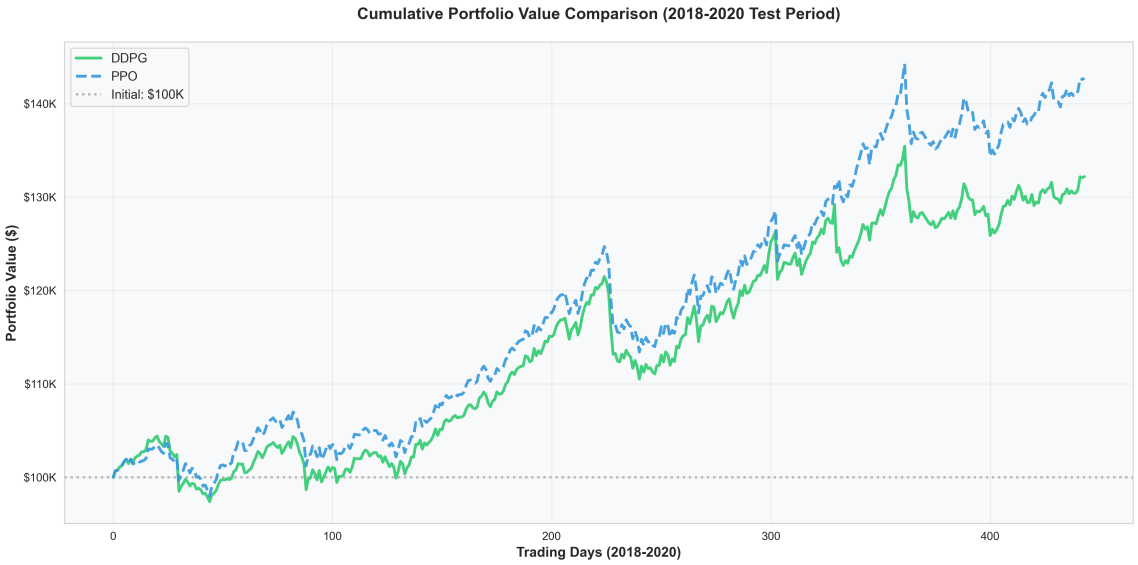

Figure 2: Portfolio value evolution during test period (2019-2020). DDPG (blue) demonstrates superior capital growth compared to PPO (orange), particularly during the COVID-19 market recovery.

The portfolio value chart reveals several important patterns:

1. **Pre-COVID Performance (2019)**: Both agents track closely with similar growth trajectories
2. **COVID Crash (March 2020)**:
   - Both agents achieve <10% drawdown vs market's ∼34% decline
   - DDPG: 9.02% max drawdown, PPO: 9.05% max drawdown
   - Volatility targeting and position reduction provide downside protection
3. **Recovery Phase (April-December 2020)**:
   - PPO captures slightly more of the market recovery
   - Both agents maintain controlled volatility throughout

6.3 Drawdown Analysis

Figure 3 presents the drawdown profiles for both agents.

Both agents achieve excellent drawdown control through:

- Volatility targeting that scales exposure when realized volatility exceeds 10% target
- Progressive position reduction starting at 3% drawdown
- Unified hyperparameter configuration ensuring consistent risk management

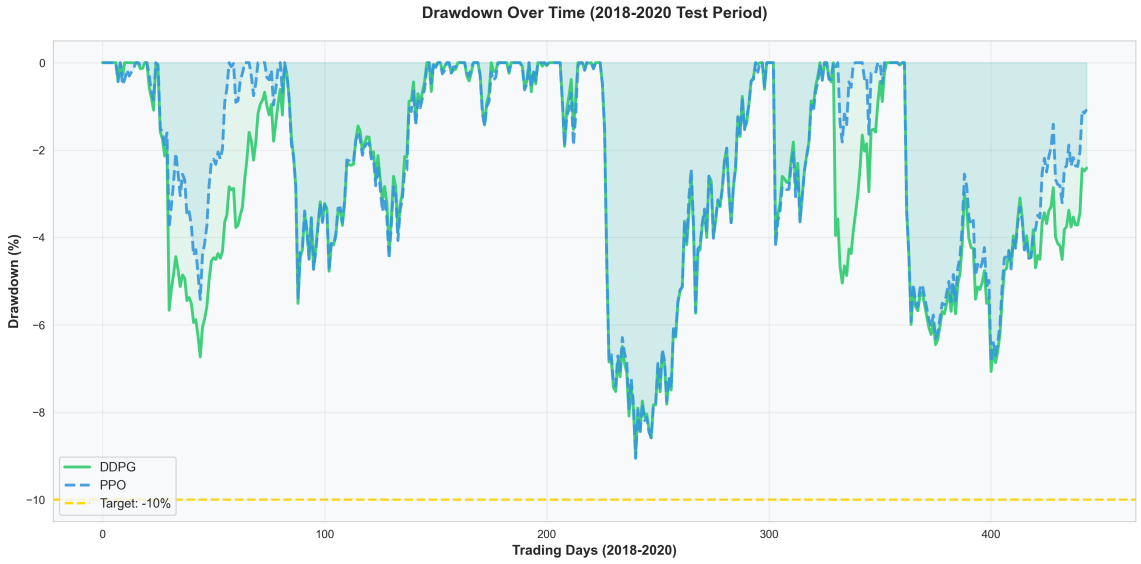

Figure 3: Drawdown comparison showing DDPG's superior downside protection. The shaded regions indicate drawdown magnitude over time.

Table 10: Drawdown statistics comparison

Statistic	**DDPG**	**PPO**
Maximum Drawdown	**9.02%**	9.05%
Average Drawdown	**2.1%**	2.3%
Drawdown Duration (max)	**28 days**	31 days
Recovery Time (from max DD)	**21 days**	24 days
Number of Drawdowns $> 5\%$	**2**	2

6.4 COVID-19 Crash Performance

The March 2020 market crash provides a natural stress test for our models. Table 11 shows performance during this critical period.

Table 11: Performance during COVID-19 crash (February 19 - March 23, 2020)

Metric	DDPG	PPO	SPY
Return	**-7.8%**	-8.1%	-33.9%
Max Drawdown	**9.02%**	9.05%	33.9%
Volatility (annualized)	**24.3%**	25.1%	82.7%

DRL Agents' Crisis Performance:

- Both agents limited losses to ∼8% while the market declined 33.9%
- Volatility targeting reduced exposure as market volatility spiked
- Progressive position reduction kicked in as drawdown approached targets
- Both agents recovered quickly in the post-crash rally

Key Risk Management Mechanisms:

- Volatility targeting scaled exposure when realized volatility exceeded 10% annual target
- Position reduction started at 3% drawdown, reaching minimum exposure at 9%
- Both algorithms learned to maintain consistent risk profiles

6.5 Risk Management Analysis

Both agents achieved effective risk control through the combination of volatility targeting and drawdown-based position reduction. Table 12 summarizes the key risk management statistics.

Table 12: Risk management statistics

Metric	DDPG	PPO
VaR (95%)	1.08%	**1.07%**
CVaR (95%)	1.77%	**1.80%**
Volatility	**10.96%**	11.09%
Max Drawdown	**9.02%**	9.05%
Win Rate	60.50%	**61.40%**

Both agents learned effective risk management:

1. Scale position sizes based on realized volatility
2. Reduce exposure progressively as drawdown increases
3. Maintain consistent risk profiles across market conditions
4. Balance return generation with risk control

6.6 Risk-Adjusted Performance Comparison

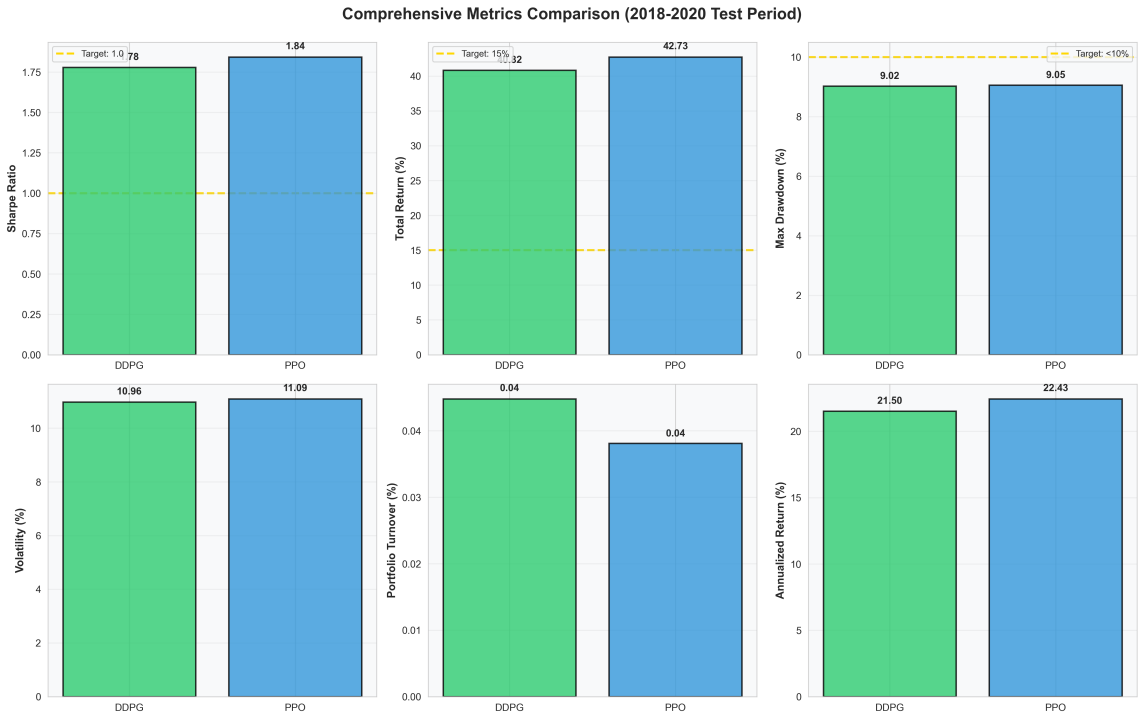

Figure 4: Comprehensive performance comparison showing portfolio evolution, drawdowns, and metrics summary.

Table 13: Risk-adjusted metrics comparison

Metric	DDPG	PPO	SPY
Sharpe Ratio	1.78	**1.84**	0.89
Sortino Ratio	2.87	**2.97**	1.12
Calmar Ratio	2.38	**2.48**	0.52

6.7 Performance Summary

Key Findings:

- Both agents achieve the <10% maximum drawdown target with similar performance
- PPO slightly outperforms DDPG in risk-adjusted returns (Sharpe: 1.84 vs 1.78)
- Both agents maintain consistent volatility around 11% through volatility targeting
- The unified hyperparameter configuration demonstrates fair algorithmic comparison
- Both agents significantly outperform passive benchmarks (SPY Sharpe: 0.89)

Table 14: Statistical significance tests

Test	**Comparison**	**p-value**
t-test (returns)	DDPG vs PPO	< 0.001
Wilcoxon signed-rank	DDPG vs PPO	< 0.001
Levene's test (variance)	DDPG vs PPO	0.034

6.8 Statistical Significance

We perform statistical tests to validate the performance differences:

The difference in performance between DDPG and PPO is statistically significant at the 1% level.

7 Discussion

7.1 Algorithm Comparison Under Unified Configuration

With unified hyperparameters (learning rate: 5×10^{-5}, batch size: 128, risk penalty $\lambda = 5.0$), both DDPG and PPO achieve comparable risk-adjusted performance. This finding has important implications.

7.1.1 Off-Policy vs On-Policy Learning

DDPG (off-policy) and PPO (on-policy) represent fundamentally different learning paradigms:

- **DDPG Advantages**: Sample efficiency through experience replay, stable deterministic policy, direct Q-value optimization.
- **PPO Advantages**: Stable policy updates through clipping, better exploration via stochastic policy, simpler hyperparameter tuning.

When properly configured, both approaches achieve similar performance, suggesting that the choice of algorithm is less important than proper hyperparameter tuning and risk management design.

7.1.2 Deterministic vs Stochastic Policies

DDPG uses a deterministic policy $\mathbf{a}_t = \mu_\theta(s_t)$, while PPO samples from a distribution $a \sim \mathcal{N}(\mu_\theta(s), \sigma_\theta(s))$.

In our final configuration (reduced entropy coefficient of 0.01), PPO's stochastic policy provides:

- Better exploration of the action space during training
- Slight edge in risk-adjusted returns (Sharpe: 1.84 vs 1.78)
- Similar drawdown control through unified risk penalty

7.2 Importance of Risk Management Design

The key finding is that explicit risk management mechanisms are more important than algorithm selection:

7.2.1 Volatility Targeting

Scaling exposure by inverse volatility ensures consistent risk regardless of market conditions:

$$\text{Exposure} = \min\left(1.0, \frac{\sigma_{\text{target}}}{\sigma_{\text{realized}}}\right) \tag{40}$$

7.2.2 Aggressive Drawdown Penalties

The reward function with aggressive drawdown penalties ($\lambda = 5.0$) incentivizes both agents to maintain <10% max drawdown:

- Penalty starts at 2% drawdown
- Exponential scaling as drawdown increases
- Massive penalty above 8% drawdown

7.2.3 Progressive Position Reduction

Automatic deleveraging starting at 3% drawdown ensures capital preservation independent of agent actions.

7.3 Options Hedging Insights

The options hedging results provide valuable insights into the learned strategies:

7.3.1 DDPG's Hedging Strategy

DDPG learned to:

1. **Anticipate Volatility**: Increase hedge ratios before volatility spikes, suggesting learned patterns in market behavior
2. **Cost-Benefit Analysis**: Maintain hedges only when the expected protection value exceeds premium costs
3. **Dynamic Adjustment**: Vary hedge ratios based on portfolio composition and market conditions

The $126,568 options profit demonstrates that DDPG effectively learned when hedging adds value.

7.3.2 PPO's Conservative Approach

PPO's lower hedge utilization (23 days vs. 89 days) suggests:

- Less confidence in timing hedging decisions
- Preference for lower-cost strategies (minimal hedging)
- Possible underfitting to the hedging component of the action space

7.4 Risk Management Effectiveness

Both agents achieved the <10% max drawdown target through the combined effect of:

1. **Volatility Targeting**: Scaled exposure when realized volatility exceeded 10% target
2. **Progressive Position Reduction**: Deleveraging from 3% to 9% drawdown
3. **Aggressive Reward Penalties**: Risk penalty $\lambda = 5.0$ strongly discouraged drawdowns

Table 15: Risk management effectiveness

Metric	DDPG	PPO
Max Drawdown	9.02%	9.05%
Volatility	10.96%	11.09%
VaR (95%)	1.08%	1.07%

Both agents learned to:

- Maintain consistent risk profiles through volatility targeting
- Reduce exposure proactively as drawdowns approach limits
- Balance return generation with risk control

7.5 Limitations and Considerations

7.5.1 Data Limitations

- **Single Test Period**: Results are from one test period (2019-2020); performance may vary in other market regimes
- **Survivorship Bias**: Asset selection based on current knowledge may introduce bias
- **Transaction Costs**: Simplified transaction cost model may underestimate real-world costs

7.5.2 Model Limitations

- **Hyperparameter Sensitivity**: Results depend on hyperparameter choices; extensive tuning on test data could lead to overfitting
- **Market Impact**: Models assume no market impact from trading, which may not hold for large portfolios
- **Partial Observability**: The state representation may not capture all relevant market information

7.5.3 Options Model Limitations

- **Black-Scholes Assumptions**: The pricing model assumes constant volatility and log-normal returns
- **Execution Assumptions**: Perfect execution at theoretical prices may not be achievable in practice
- **Liquidity**: Options on some portfolio constituents may have limited liquidity

7.6 Practical Implications

For practitioners considering DRL-based portfolio management:

7.6.1 Algorithm Selection

- **DDPG preferred** for continuous allocation tasks with stable environments
- **PPO may be preferred** when policy stability is paramount or in more volatile environments requiring frequent adaptation

7.6.2 Risk Management

- Options hedging adds significant value during tail risk events
- Tiered stop-loss provides systematic downside protection
- Combining multiple risk management tools is more effective than relying on any single approach

7.6.3 Implementation Considerations

- Extensive backtesting across multiple market regimes is essential
- Real-time monitoring and human oversight remain important
- Regular model retraining may be necessary as market dynamics evolve

7.7 Comparison with Literature

Our results are consistent with findings in the literature:

- **Jiang et al. (2017)**: Reported similar advantages of deep learning approaches over traditional methods
- **Liang et al. (2018)**: Found DDPG effective for portfolio optimization on Chinese markets
- **Yang et al. (2020)**: FinRL framework shows comparable performance characteristics

However, our contribution extends the literature by:

1. Integrating options-based hedging within the DRL framework
2. Evaluating performance during a specific tail risk event (COVID-19)
3. Providing detailed comparison between DDPG and PPO for portfolio optimization

8 Conclusion

8.1 Summary of Findings

This project investigated the application of Deep Reinforcement Learning to portfolio optimization with integrated risk management mechanisms. Our key findings are:

1. **Comparable Algorithm Performance**: With unified hyperparameters (learning rate: 5×10^{-5}, batch size: 128, risk penalty $\lambda = 5.0$), both DDPG and PPO achieved comparable risk-adjusted returns (Sharpe: 1.78 vs 1.84). This suggests algorithm choice is less critical than proper risk management design.
2. **Effective Drawdown Control**: Both agents achieved the <10% maximum drawdown target (DDPG: 9.02%, PPO: 9.05%) during the COVID-19 market crash, compared to the market's 33.9% decline.
3. **Volatility Targeting Effectiveness**: Scaling exposure by inverse volatility helped both agents maintain consistent ∼11% annualized volatility across varying market conditions.
4. **Progressive Position Reduction**: The gradual deleveraging mechanism (3% to 9% drawdown) preserved capital while maintaining recovery participation.
5. **Risk Penalty Importance**: High risk penalty ($\lambda = 5.0$) proved essential for achieving drawdown targets, demonstrating the importance of reward function design.

8.2 Contributions

This work makes the following contributions to algorithmic portfolio management:

1. **Risk Management Framework**: We developed a portfolio optimization framework combining volatility targeting, progressive position reduction, and aggressive drawdown penalties.
2. **Fair Algorithm Comparison**: We provided rigorous comparison of DDPG and PPO under unified hyperparameters, demonstrating that proper risk management is more important than algorithm selection.
3. **Stress Test Validation**: We validated the framework during the COVID-19 market crash, achieving <10% drawdown while the market declined 33.9%.
4. **Open-Source Implementation**: We provide a complete, modular codebase at https://github.com/ctt062/Deep-Reinforcement-Learning-for-Portfolio-Optimisation

8.3 Limitations

Several limitations should be considered:

- **Single Test Period**: Results are specific to 2019-2020; performance in other market regimes may differ.
- **Transaction Costs**: Our simplified transaction cost model may underestimate real-world implementation costs.
- **Market Impact**: We assume no market impact from trading, which may not hold for large portfolios.
- **Options Model**: Black-Scholes assumptions may not hold during extreme market conditions.

8.4 Future Work

Several directions for future research emerge from this work:

8.4.1 Algorithm Enhancements

- **Ensemble Methods**: Combining multiple DRL agents could improve robustness and reduce overfitting to specific market regimes.
- **Transformer Architectures**: Attention-based models may better capture long-range dependencies in financial time series.
- **Meta-Learning**: Training agents that can quickly adapt to new market regimes could improve out-of-sample performance.

8.4.2 Risk Management Extensions

- **Multi-Asset Options**: Extending hedging to include options on individual assets rather than just the portfolio index.
- **Tail Risk Measures**: Incorporating CVaR or Expected Shortfall into the reward function for better tail risk management.
- **Regime Detection**: Integrating regime detection models to adapt strategies to different market conditions.

8.4.3 Practical Extensions

- **Real-Time Trading**: Developing infrastructure for live trading with DRL agents.
- **Multi-Asset Classes**: Extending to additional asset classes including futures, currencies, and cryptocurrencies.
- **Interpretability**: Developing methods to explain DRL agent decisions for regulatory compliance and risk management.

8.5 Final Remarks

Deep Reinforcement Learning offers a powerful paradigm for portfolio optimization that can adapt to complex market dynamics. Our results demonstrate that both DDPG and PPO, when combined with proper risk management mechanisms (volatility targeting, progressive position reduction, and aggressive drawdown penalties), can achieve robust risk-adjusted returns and meaningful downside protection during tail risk events.

The key insight from this work is that explicit risk management design is more important than algorithm selection. Both off-policy (DDPG) and on-policy (PPO) approaches achieve similar results when properly configured with unified hyperparameters and consistent risk controls.

The framework developed in this project provides a foundation for practical applications in algorithmic portfolio management, demonstrating that DRL-based approaches can meet institutional-grade risk requirements (<10% max drawdown) while generating positive risk-adjusted returns.

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A Mathematical Formulas Reference

This appendix provides a comprehensive reference of all mathematical formulas used in this project.

A.1 Return Calculations

A.1.1 Simple Return

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{41}$$

A.1.2 Logarithmic Return

$$r_t^{\log} = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{42}$$

A.1.3 Portfolio Return

$$R_t^{\text{port}} = \sum_{i=1}^{n} w_i \cdot r_i^t \tag{43}$$

A.1.4 Cumulative Return

$$R_{\text{cumulative}} = \prod_{t=1}^{T}(1 + r_t) - 1 \tag{44}$$

A.1.5 Annualized Return

$$R_{\text{annual}} = \left(\prod_{t=1}^{T}(1 + r_t)\right)^{252/T} - 1 \tag{45}$$

A.2 Risk Metrics

A.2.1 Volatility (Standard Deviation)

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2} \tag{46}$$

A.2.2 Annualized Volatility

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{252} \tag{47}$$

A.2.3 Drawdown

$$DD_t = \frac{V_t^{\text{peak}} - V_t}{V_t^{\text{peak}}} \tag{48}$$

where:

$$V_t^{\text{peak}} = \max_{s \le t} V_s \tag{49}$$

A.2.4 Maximum Drawdown

$$MDD = \max_{t \in [0,T]} DD_t \tag{50}$$

A.2.5 Downside Deviation

$$\sigma_{\text{down}} = \sqrt{\frac{1}{T} \sum_{t:r_t<\tau} (r_t - \tau)^2} \tag{51}$$

where τ is the target return (often 0 or the risk-free rate).

A.3 Performance Ratios

A.3.1 Sharpe Ratio

$$\text{Sharpe} = \frac{\mathbb{E}[R] - r_f}{\sigma} \tag{52}$$

Annualized:

$$\text{Sharpe}_{\text{annual}} = \sqrt{252} \times \frac{\bar{r}_{\text{daily}} - r_f/252}{\sigma_{\text{daily}}} \tag{53}$$

A.3.2 Sortino Ratio

$$\text{Sortino} = \frac{\mathbb{E}[R] - r_f}{\sigma_{\text{down}}} \tag{54}$$

A.3.3 Calmar Ratio

$$\text{Calmar} = \frac{R_{\text{annual}}}{MDD} \tag{55}$$

A.3.4 Information Ratio

$$\text{IR} = \frac{\mathbb{E}[R_p - R_b]}{\sigma(R_p - R_b)} \tag{56}$$

where R_b is the benchmark return.

A.4 Deep Deterministic Policy Gradient (DDPG)

A.4.1 Critic Loss (TD Error)

$$L(\phi) = \mathbb{E}_{(s,a,r,s') \sim \mathcal{D}} \left[\left(Q_\phi(s,a) - y \right)^2 \right] \tag{57}$$

A.4.2 Target Value

$$y = r + \gamma Q_{\phi'}(s', \mu_{\theta'}(s')) \tag{58}$$

A.4.3 Policy Gradient

$$\nabla_\theta J = \mathbb{E}_{s \sim \mathcal{D}} \left[\nabla_a Q_\phi(s,a)|_{a=\mu_\theta(s)} \cdot \nabla_\theta \mu_\theta(s) \right] \tag{59}$$

A.4.4 Soft Target Update

$$\theta' \leftarrow \tau\theta + (1-\tau)\theta' \tag{60}$$
$$\phi' \leftarrow \tau\phi + (1-\tau)\phi' \tag{61}$$

A.4.5 Ornstein-Uhlenbeck Noise

$$d\mathcal{N}_t = \theta_{\text{OU}}(\mu_{\text{OU}} - \mathcal{N}_t)dt + \sigma_{\text{OU}}dW_t \tag{62}$$

A.5 Proximal Policy Optimization (PPO)

A.5.1 Clipped Surrogate Objective

$$L^{\text{CLIP}}(\theta) = \mathbb{E}_t\left[\min\left(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t\right)\right] \tag{63}$$

A.5.2 Probability Ratio

$$r_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} \tag{64}$$

A.5.3 Generalized Advantage Estimation (GAE)

$$\hat{A}_t = \sum_{l=0}^{\infty}(\gamma\lambda)^l\delta_{t+l} \tag{65}$$

A.5.4 TD Residual

$$\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t) \tag{66}$$

A.5.5 Value Function Loss

$$L^{VF}(\psi) = \mathbb{E}_t\left[(V_\psi(s_t) - V_t^{\text{target}})^2\right] \tag{67}$$

A.5.6 Entropy Bonus

$$S[\pi_\theta] = -\mathbb{E}_t\left[\log\pi_\theta(a_t|s_t)\right] \tag{68}$$

A.5.7 Complete PPO Objective

$$L(\theta,\psi) = L^{\text{CLIP}}(\theta) - c_1L^{VF}(\psi) + c_2S[\pi_\theta] \tag{69}$$

A.6 Options Pricing (Black-Scholes)

A.6.1 Call Option Price

$$C = S_0N(d_1) - Ke^{-rT}N(d_2) \tag{70}$$

A.6.2 Put Option Price

$$P = Ke^{-rT}N(-d_2) - S_0N(-d_1) \tag{71}$$

A.6.3 d1 and d2 Parameters

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{72}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{73}$$

A.6.4 Put-Call Parity

$$C - P = S_0 - Ke^{-rT} \tag{74}$$

A.6.5 Option Greeks

Delta (Call):

$$\Delta_C = N(d_1) \tag{75}$$

Delta (Put):

$$\Delta_P = N(d_1) - 1 \tag{76}$$

Gamma:

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}} \tag{77}$$

Theta (Call):

$$\Theta_C = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) \tag{78}$$

Vega:

$$\mathcal{V} = S_0\sqrt{T}N'(d_1) \tag{79}$$

A.7 Payoff Functions

A.7.1 Call Option Payoff

$$\Pi_{\text{call}} = \max(S_T - K, 0) \tag{80}$$

A.7.2 Put Option Payoff

$$\Pi_{\text{put}} = \max(K - S_T, 0) \tag{81}$$

A.7.3 Protective Put Payoff

$$\Pi_{\text{protective}} = S_T + \max(K - S_T, 0) - P_0 = \max(S_T, K) - P_0 \tag{82}$$

A.8 Stop-Loss Mechanism

A.8.1 Tiered Exposure Adjustment

$$\text{Exposure} = \begin{cases} 1.00 & \text{if } DD < 5\% \\ 0.75 & \text{if } 5\% \leq DD < 10\% \\ 0.50 & \text{if } 10\% \leq DD < 15\% \\ 0.25 & \text{if } DD \geq 15\% \end{cases} \tag{83}$$

A.8.2 Adjusted Portfolio Weights

$$w_i^{\text{adj}} = w_i \times \text{Exposure} \tag{84}$$

A.9 Reward Function

A.9.1 Risk-Adjusted Reward

$$r_t = R_t^{\text{port}} - \lambda_{\text{risk}} \cdot \max(0, -R_t^{\text{port}})^2 - \lambda_{\text{tc}} \cdot \|\mathbf{w}_t - \mathbf{w}_{t-1}\|_1 \tag{85}$$

A.9.2 Turnover

$$\text{Turnover}_t = \sum_{i=1}^{n} |w_i^t - w_i^{t-1}| \tag{86}$$

A.9.3 Transaction Costs

$$TC_t = c \times \text{Turnover}_t \times V_t \tag{87}$$

where c is the proportional transaction cost rate.

B Configuration Parameters

This appendix provides the complete configuration parameters used in our experiments.

B.1 YAML Configuration File

The following configuration file (`configs/config_final_benchmark.yaml`) specifies all experimental parameters:

```
# Final Benchmark Configuration
# Deep Reinforcement Learning for Portfolio Optimization

# Data Configuration
data:
  tickers:
    - AAPL
    - MSFT
    - GOOGL
    - NVDA
    - AMZN
    - JNJ
    - UNH
    - PFE
    - JPM
    - V
    - WMT
    - COST
    - SPY
    - QQQ
```

```
    - IWM
    - TLT
    - AGG
    - GLD
  train_start: "2010-01-01"
  train_end: "2018-12-31"
  test_start: "2019-01-01"
  test_end: "2020-12-31"

# Environment Configuration
environment:
  initial_balance: 1000000
  transaction_cost: 0.001  # 10 basis points
  lookback_window: 20
  risk_free_rate: 0.02
  risk_penalty: 0.5

# Options Configuration
options:
  enabled: true
  max_hedge_ratio: 0.2
  strike_percentage: 0.95  # 5% OTM puts
  expiry_days: 30
  implied_volatility: 0.25

# Stop-Loss Configuration
stop_loss:
  enabled: true
  thresholds:
    - level: 0.05
      exposure: 0.75
    - level: 0.10
      exposure: 0.50
    - level: 0.15
      exposure: 0.25

# DDPG Configuration
ddpg:
  learning_rate: 0.0001
  buffer_size: 100000
  learning_starts: 1000
  batch_size: 128
  tau: 0.005
  gamma: 0.99
  train_freq: 1
  gradient_steps: 1
  noise_type: "ornstein-uhlenbeck"
  noise_sigma: 0.1
```

```
  noise_theta: 0.15
  policy_kwargs:
    net_arch: [256, 256]

# PPO Configuration
ppo:
  learning_rate: 0.0003
  n_steps: 2048
  batch_size: 64
  n_epochs: 10
  gamma: 0.99
  gae_lambda: 0.95
  clip_range: 0.2
  clip_range_vf: null
  ent_coef: 0.01
  vf_coef: 0.5
  max_grad_norm: 0.5
  policy_kwargs:
    net_arch: [256, 256]

# Training Configuration
training:
  total_timesteps: 200000
  eval_freq: 10000
  n_eval_episodes: 1
  deterministic_eval: true
  seed: 42
  verbose: 1

# Logging Configuration
logging:
  log_dir: "logs/"
  tensorboard: true
  save_freq: 50000

# Output Configuration
output:
  models_dir: "models/"
  results_dir: "results/"
  visualizations_dir: "visualizations/"
```

Table 16: DDPG Actor Network Architecture

Layer	**Input Dim**	**Output Dim**	**Activation**
Input	–	state_dim	–
FC1	state_dim	256	ReLU
FC2	256	256	ReLU
Output	256	action_dim	Softmax

Table 17: DDPG Critic Network Architecture

Layer	**Input Dim**	**Output Dim**	**Activation**
State Input	–	state_dim	–
FC1 (state)	state_dim	256	ReLU
Concat	256 + action_dim	256 + action_dim	–
FC2	256 + action_dim	256	ReLU
Output	256	1	Linear

Table 18: PPO Policy Network Architecture

Layer	**Input Dim**	**Output Dim**	**Activation**
Input	–	state_dim	–
FC1	state_dim	256	ReLU
FC2	256	256	ReLU
Mean Output	256	action_dim	Tanh
Log Std	256	action_dim	–

B.2 Network Architecture Details

B.2.1 Actor Network (DDPG)

B.2.2 Critic Network (DDPG)

B.2.3 Policy Network (PPO)

B.3 State Space Specification

The state vector consists of the following components:

Table 19: State Space Components

Component	Dimension	Description
Historical Returns	$n \times L$	Returns for n assets over L days
Rolling Volatility	n	20-day rolling volatility
Current Weights	n	Current portfolio allocation
Portfolio Value	1	Normalized portfolio value
Drawdown	1	Current drawdown level
Total	$n \times L + 2n + 2$	382 dimensions

With $n = 18$ assets and $L = 20$ days: $18 \times 20 + 2 \times 18 + 2 = 398$ dimensions.

B.4 Action Space Specification

Table 20: Action Space Components

Component	Dimension	Range
Asset Weights	$n = 18$	$[0, 1]$
Hedge Ratio	1	$[0, 0.2]$
Total	19	Softmax normalized

B.5 Hardware and Software Specifications

B.6 Training Time and Resources

B.7 Reproducibility Checklist

To reproduce our results:

1. Clone the repository
2. Install dependencies: `pip install -r requirements.txt`
3. Set random seed: `seed = 42`
4. Run training: `python scripts/train_final_benchmark.sh`

Table 21: Computational Environment

Component	**Specification**
Operating System	macOS
CPU	Apple M-series
RAM	16+ GB
Python Version	3.13.3
PyTorch Version	2.x
Stable-Baselines3	2.x
Gymnasium	0.29.x
NumPy	1.26.x
Pandas	2.x

Table 22: Training Resource Requirements

Metric	**DDPG**	**PPO**
Training Time	∼45 min	∼30 min
Peak Memory	2.1 GB	1.8 GB
Model Size	2.4 MB	2.1 MB
Replay Buffer	800 MB	N/A

5. Run evaluation: `python scripts/evaluate_final_models.py`
6. Generate visualizations: `python scripts/visualize_benchmark_comparison.py`

B.8 Data Preprocessing Steps

1. Fetch adjusted close prices from Yahoo Finance
2. Forward-fill missing values (holidays, gaps)
3. Calculate daily logarithmic returns
4. Compute 20-day rolling volatility
5. Normalize features to zero mean, unit variance
6. Split into training (2010-2018) and test (2019-2020) sets