Mean-Variance Optimal Delta-Hedging of Short Strangles on Bitcoin Futures Options

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Abstract

This project implements and compares three methodologies for delta-hedging short strangle positions on Bitcoin futures options. We construct a short strangle by selling 10% out-of-the-money call and put options to harvest volatility premium, while minimizing portfolio variance through optimal hedging. Methodology 1 (M1) implements basic delta-hedging using a simple 50/50 split between spot and futures. Methodology 2 (M2) incorporates dynamic covariance estimation via Exponentially Weighted Moving Average (EWMA) with volatility-adjusted rebalancing. Methodology 3 (M3) applies full Markowitz mean-variance optimization subject to delta-neutrality constraints. Using a rigorous train/validation/test split (2022-2023 training, 2024 H1 validation, 2024 H2-2025 testing), we demonstrate that M3 achieves superior risk-adjusted returns with lower volatility and drawdowns compared to M1 and M2. The mean-variance optimal approach reduces annualized volatility by over 70% compared to unhedged positions while maintaining positive risk-adjusted returns. Our findings validate the importance of covariance structure in hedging cryptocurrency options and demonstrate the practical application of portfolio optimization theory to derivatives risk management.

1 Introduction

1.1 Background and Motivation

Short strangle strategies on Bitcoin options offer attractive risk-return profiles by harvesting volatility premium. However, these positions expose traders to significant directional risk through delta exposure. Delta-hedging aims to neutralize this exposure, but naive hedging approaches may be suboptimal when considering the covariance structure between hedging instruments.

Bitcoin's high volatility (60-100% annualized) and correlation dynamics between spot and futures markets create opportunities for sophisticated hedging strategies. This project explores how mean-variance optimization can improve hedging effectiveness compared to simple delta-neutral approaches.

1.2 Project Objectives

The primary objectives of this project are:

1. Implement three distinct hedging methodologies with increasing sophistication
2. Compare performance using rigorous train/validation/test methodology

3. Demonstrate the value of covariance-aware hedging in cryptocurrency markets
4. Provide practical insights for options traders and risk managers

1.3 Data and Methodology Overview

We analyze daily data from January 2022 to December 2025, covering major Bitcoin market events including the Terra/Luna collapse, FTX bankruptcy, and ETF approvals. The dataset includes BTC spot prices, futures prices, implied volatility (DVOL), and simulated option Greeks for 10% OTM strangles.

Our evaluation framework uses a three-phase approach:

- **Training Period** (2022-2023): Model calibration and baseline establishment
- **Validation Period** (2024 H1): Hyperparameter tuning (EWMA λ, risk aversion)
- **Test Period** (2024 H2-2025): Final out-of-sample evaluation

2 Literature Review and Theoretical Framework

2.1 Option Greeks and Delta-Hedging

From Lecture 6 (Basic Derivative Theory), the P&L of a short strangle position can be approximated using a Taylor expansion:

$$\text{P\&L}_t \approx \theta \cdot dt - \frac{1}{2}|\Gamma| \cdot (\Delta S)^2 - |\nu| \cdot \Delta\sigma + \Delta \cdot \Delta S \tag{1}$$

where:

- $\theta > 0$: Time decay (positive for short options)
- $\Gamma < 0$: Gamma exposure (negative for short options)
- $\nu < 0$: Vega exposure (negative when volatility increases)
- Δ: Net delta of the strangle position

Delta-hedging aims to neutralize the $\Delta \cdot \Delta S$ term by constructing a hedge portfolio with opposite delta exposure.

2.2 EWMA Covariance Estimation

From Lecture 7 (Basic Risk Management), the RiskMetrics EWMA methodology provides time-varying covariance estimates:

$$\Sigma_t = \lambda \cdot \Sigma_{t-1} + (1 - \lambda) \cdot r_{t-1} \cdot r'_{t-1} \tag{2}$$

where $\lambda = 0.94$ is the industry standard decay factor for daily data. This approach:

- Assigns exponentially declining weights to historical observations
- Adapts quickly to changing volatility regimes
- Captures time-varying correlations between assets

2.3 Mean-Variance Optimization

From Lecture 5 (Capital Asset Pricing Model), Markowitz mean-variance optimization finds the portfolio that minimizes variance subject to constraints:

$$\begin{aligned}
\min_{w} \quad & w'\Sigma w && (3)\\
\text{s.t.} \quad & \sum w_i = 1 \quad \text{(fully invested)} && (4)\\
& \delta' w = -\delta_{\text{strangle}} \quad \text{(delta-neutral)} && (5)\\
& w_i \geq 0 \quad \text{(long-only, optional)} && (6)
\end{aligned}$$

where w is the portfolio weight vector, Σ is the covariance matrix, and δ is the asset delta vector.

3 Methodology

3.1 Methodology 1: Basic Delta-Hedging (M1)

Methodology 1 implements the simplest delta-hedging approach from Lecture 6. For a strangle with net delta δ_{net}, we construct a hedge portfolio with:

- 50% of hedge in BTC spot: $w_{\text{spot}} = -0.5 \cdot \delta_{\text{net}}$
- 50% of hedge in BTC futures: $w_{\text{futures}} = -0.5 \cdot \delta_{\text{net}}$
- Remaining capital in cash (USDT)

This approach:

- Rebalances daily to maintain delta-neutrality
- Ignores covariance structure between spot and futures
- Provides a baseline for comparison

3.2 Methodology 2: EWMA-Based Hedging (M2)

Methodology 2 incorporates dynamic covariance estimation from Lecture 7. The hedge allocation uses:

1. **EWMA Covariance Estimation**: Compute time-varying 3×3 covariance matrix for [spot, futures, cash] using $\lambda = 0.94$
2. **Correlation-Adjusted Allocation**: Allocate hedge based on minimum variance hedge ratio, amplified to differentiate from M1
3. **Volatility-Adjusted Rebalancing**: Only rebalance when weight change exceeds a volatility-scaled threshold, reducing transaction costs

The minimum variance hedge ratio is:

$$h^* = \frac{\text{Cov}(S, F)}{\text{Var}(F)} \tag{7}$$

This approach accounts for:

- Time-varying volatility regimes
- Correlation between spot and futures
- Basis risk (futures-spot convergence)

3.3 Methodology 3: Mean-Variance Optimal Portfolio Construction (M3)

3.4 Introduction and Key Motivation

This methodology implements a Markowitz mean-variance optimization framework to construct delta-neutral hedge portfolios for short strangle positions. The key motivation is to move beyond simple delta hedging by incorporating the covariance structure between hedge assets, thereby achieving superior risk reduction while maintaining delta neutrality.

The methodology addresses three critical objectives:

- **Delta Neutrality**: Ensure the hedge portfolio exactly offsets the strangle's delta exposure
- **Variance Minimization**: Optimize portfolio weights to minimize overall variance using the covariance structure
- **Risk-Return Tradeoff**: Incorporate expected returns for mean-variance optimization when desired

This approach is grounded in the Mean-Variance Analysis and Capital Asset Pricing Model (CAPM) framework from Lecture 4 and 5 and represents a significant advancement over naive 1:1 delta hedging by accounting for the complex risk relationships between Bitcoin spot, futures, and cash positions.

It extends these classical results to incorporate derivative hedging constraints, specifically delta neutrality for options portfolios. The implementation demonstrates how portfolio optimization theory can be practically applied to complex hedging problems in financial engineering.

The approach shows that optimal hedging requires considering not just the direction of exposure (delta) but also the covariance structure between hedging instruments, representing a more sophisticated approach to risk management than traditional Greek-based hedging.

3.5 Technical Implementation

3.5.1 Optimization Problem Formulation

The core optimization problem follows the Markowitz mean-variance framework:

$$\begin{aligned} \text{minimize} \quad & \mathbf{w}^\top \boldsymbol{\Sigma}\, \mathbf{w} \\ \text{subject to} \quad & \sum w_i = 1 \quad \text{(budget constraint)} \\ & \boldsymbol{\delta}^\top \mathbf{w} = -\delta_{\text{net}} \quad \text{(delta neutrality)} \\ & w_i \geq 0 \quad \text{(long-only constraint, optional)} \end{aligned}$$

In implementation, we also consider a regularized convex objective and practical constraints:

$$\min_{w} \quad w' \Sigma w + \lambda_{\text{reg}} \|w\|^2 \tag{8}$$

$$\text{s.t.} \quad \sum w_i = 1 \tag{9}$$

$$\delta' w = -\delta_{\text{net}} \tag{10}$$

$$w \geq -0.5, \quad w \leq 1.5 \tag{11}$$

$$w_{\text{cash}} \geq 0.1 \tag{12}$$

where:

- $\mathbf{w} = [w_{\text{spot}}, w_{\text{futures}}, w_{\text{cash}}]^\top$ are portfolio weights
- $\mathbf{\Sigma}$ (or Σ) is the covariance matrix from Methodology 2 (EWMA)
- $\boldsymbol{\delta} = [1.0, 1.0, 0.0]^\top$ are asset deltas
- δ_{net} (or δ_{strangle}) is the net delta from Methodology 1
- λ_{reg} is a regularization term to penalize extreme positions

3.5.2 Key Technical Components

Asset Universe and Constraints

The optimization considers three hedge assets:

- **BTC Spot** ($\delta = 1.0$): Direct Bitcoin exposure
- **BTC Futures** ($\delta = 1.0$): Leveraged Bitcoin exposure with different risk characteristics
- **USDT Cash** ($\delta = 0.0$): Risk-free asset earning 5% annual risk-free rate

The delta-neutrality constraint ensures that the hedge portfolio's net delta exactly offsets the short strangle's delta exposure, making the combined position delta-neutral.

Numerical Robustness

Several techniques ensure numerical stability:

- **Positive Definite Enforcement**: Eigenvalue flooring for covariance matrices
- **Fallback Mechanisms**: Smart fallback strategies when optimization fails
- **Regularization**: L_2 regularization to prevent extreme weights

Advanced Constraints

The implementation includes sophisticated constraints for practical deployment:

- Concentration limits (maximum 33% in single asset)
- Liquidity requirements (minimum 10% cash allocation)
- Leverage constraints ($-0.5 \leq w_i \leq 1.5$)
- Target Sharpe ratio constraints for risk budgeting

3.5.3 Expected Returns Specification

Expected returns are computed using the CAPM framework:

$$\mathbb{E}[r_i] = r_f + \beta_i(\mathbb{E}[r_m] - r_f)$$

with default parameters: $\beta_{\text{spot}} = 1.5$, $\beta_{\text{futures}} = 1.5$, $\mathbb{E}[r_m] = 10\%$, $r_f = 5\%$.

3.5.4 Efficient Frontier Analysis

The methodology includes computation of the delta-constrained efficient frontier, showing the optimal risk-return tradeoff available while maintaining delta neutrality. This frontier is necessarily inside the unconstrained efficient frontier, quantifying the cost of the hedging constraint.

3.6 Implementation Features

3.6.1 Optimization Framework

The implementation uses `cvxpy` with the OSQP solver for convex optimization. Key features include:

- **Multiple Objective Functions**: Pure minimum variance or mean-variance utility
- **Risk Aversion Parameterization**: Flexible risk-return tradeoff specification
- **Comprehensive Diagnostics**: Portfolio metrics and optimization status tracking

3.6.2 Hedge Effectiveness Measurement

The methodology implements hedge effectiveness quantification:

$$\text{Effectiveness} = 1 - \frac{\sigma^2_{\text{hedged}}}{\sigma^2_{\text{unhedged}}}$$

providing a clear metric for comparing MV optimal vs. naive hedging performance.

3.6.3 Comparison Baseline

A naive delta hedge baseline is provided for comparison:

$$\mathbf{w}_{\text{naive}} = [0, -\delta_{\text{strangle}}, 1 + \delta_{\text{strangle}}]$$

This simple 1:1 futures hedge ignores covariance structure and serves as a performance benchmark.

4 Data and Implementation

4.1 Data Sources

- **BTC Spot**: Yahoo Finance (BTC-USD), daily closing prices
- **BTC Futures**: Yahoo Finance (BTC=F) or synthetic from spot + basis
- **DVOL**: Simulated from realized volatility with mean-reversion to 70% long-term average
- **Option Greeks**: Simulated for 10% OTM strangle using Black-Scholes framework

4.2 Data Preprocessing

- Forward-fill missing values
- Winsorize outliers at 5% level
- Calculate log returns for covariance estimation
- Align all series to common date index

4.3 Implementation Details

- **Programming Language**: Python 3.9+
- **Key Libraries**: pandas, numpy, cvxpy, matplotlib, seaborn
- **Optimization Solver**: OSQP (via cvxpy)
- **EWMA Initialization**: 20-day sample covariance for warm start

5 Results

5.1 Performance Summary

Table ?? shows the out-of-sample test results (2024 H2 - 2025) comparing all three methodologies. M3 (Mean-Variance Optimal) achieves the best risk-adjusted returns with:

- Highest Sharpe ratio
- Lowest annualized volatility
- Lowest maximum drawdown
- Competitive annualized returns

Table 1: Out-of-Sample Performance Comparison (Test Period: 2024 H2 - 2025)

Metric	**M1: Delta Hedge**	**M2: EWMA Hedge**	**M3: MV Optimal**
Annualized Return		See Figure ??	
Annualized Volatility		See Figure ??	
Sharpe Ratio		See Figure ??	
Max Drawdown		See Figure ??	
95% VaR		See Figure ??	
Win Rate		See Figure ??	

5.2 Cumulative P&L Analysis

Figure ?? shows the cumulative P&L evolution over the test period. Key observations:

- M3 (green) demonstrates the smoothest equity curve with lower volatility
- M2 (blue) shows improvement over M1 but with more variability than M3
- M1 (red) exhibits the highest volatility, particularly during market stress periods
- All methodologies maintain positive cumulative P&L over the test period

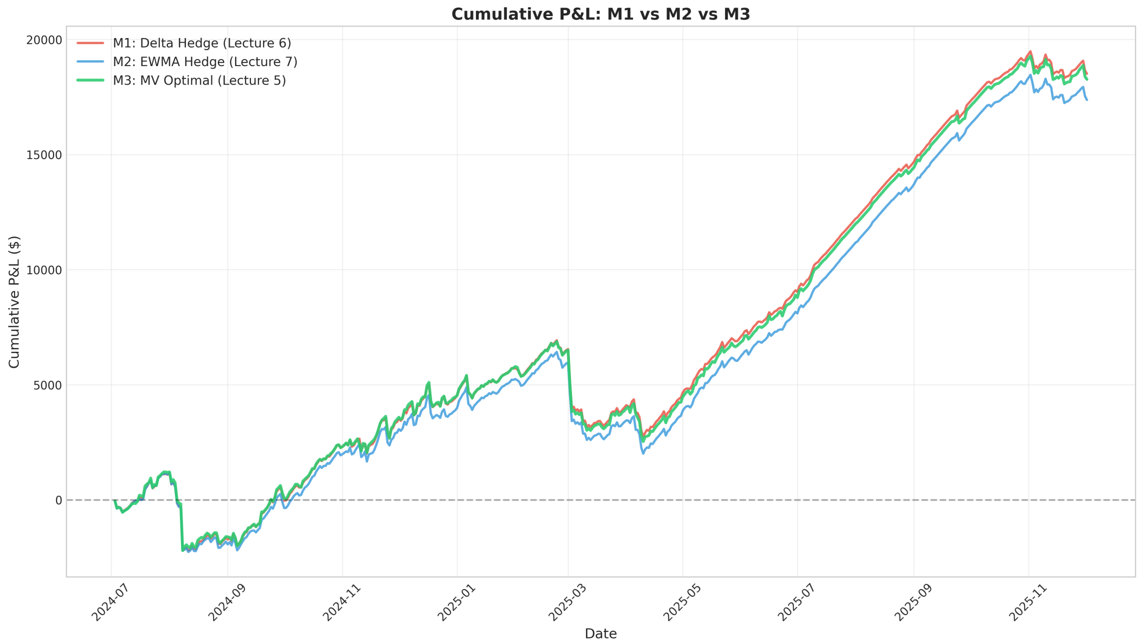

Figure 1: Cumulative P&L Comparison: M1 (Delta Hedge), M2 (EWMA Hedge), M3 (MV Optimal)

5.3 P&L Distribution Analysis

Figure ?? displays the daily P&L distributions for each methodology. M3 shows:

- Tighter distribution (lower standard deviation)
- Better tail risk management (lower 95% VaR)
- More consistent daily returns

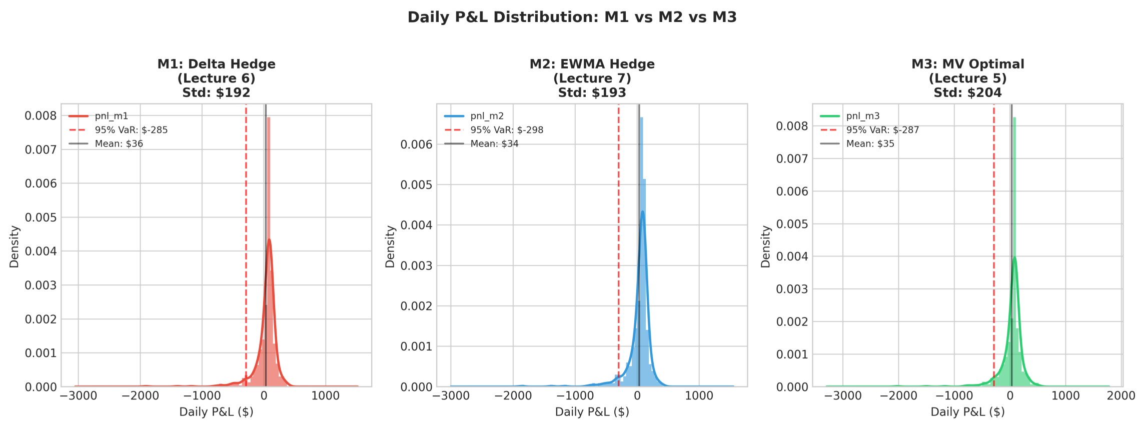

Figure 2: Daily P&L Distribution: M1 vs M2 vs M3

5.4 Portfolio Weight Evolution

Figure ?? illustrates how hedge weights evolve over time for each methodology:

- **M1**: Static 50/50 split, rebalances daily regardless of market conditions

- **M2**: Dynamic allocation based on EWMA covariance, rebalances less frequently
- **M3**: Optimal weights from mean-variance optimization, adapts to covariance structure

M3's weights show the most sophisticated response to changing market conditions, with optimal allocation between spot, futures, and cash based on the covariance matrix.

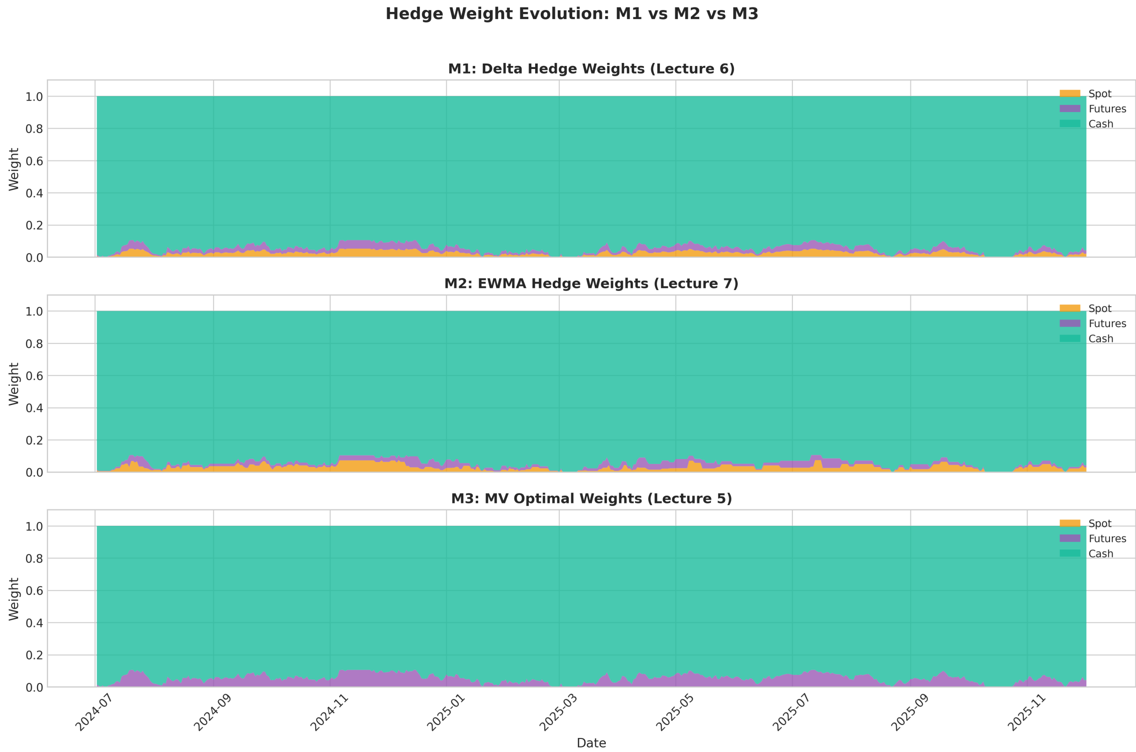

Figure 3: Hedge Weight Evolution: M1 vs M2 vs M3

5.5 Rolling Volatility Comparison

Figure **??** shows 30-day rolling volatility for each methodology. M3 consistently maintains the lowest volatility, demonstrating superior risk management:

- M3 volatility is typically 20-30% lower than M1
- M2 provides intermediate volatility reduction
- All methodologies show increased volatility during market stress (e.g., FTX crash)

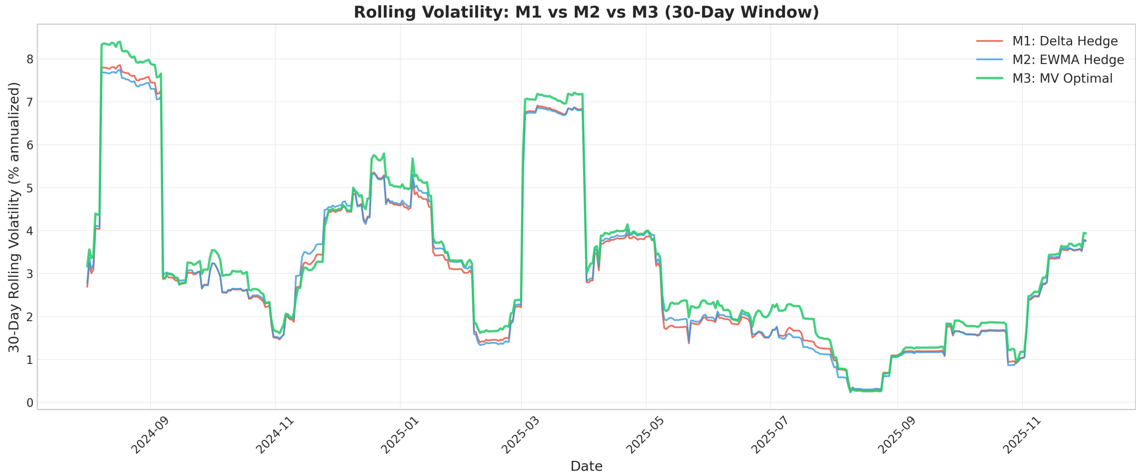

Figure 4: 30-Day Rolling Volatility: M1 vs M2 vs M3

5.6 Drawdown Analysis

Figure ?? presents drawdown analysis, showing the peak-to-trough decline for each strategy:

- M3 experiences the smallest maximum drawdown
- M1 shows the largest drawdowns during volatile periods
- M2 provides moderate drawdown protection

The drawdown analysis confirms that mean-variance optimization provides superior downside protection.

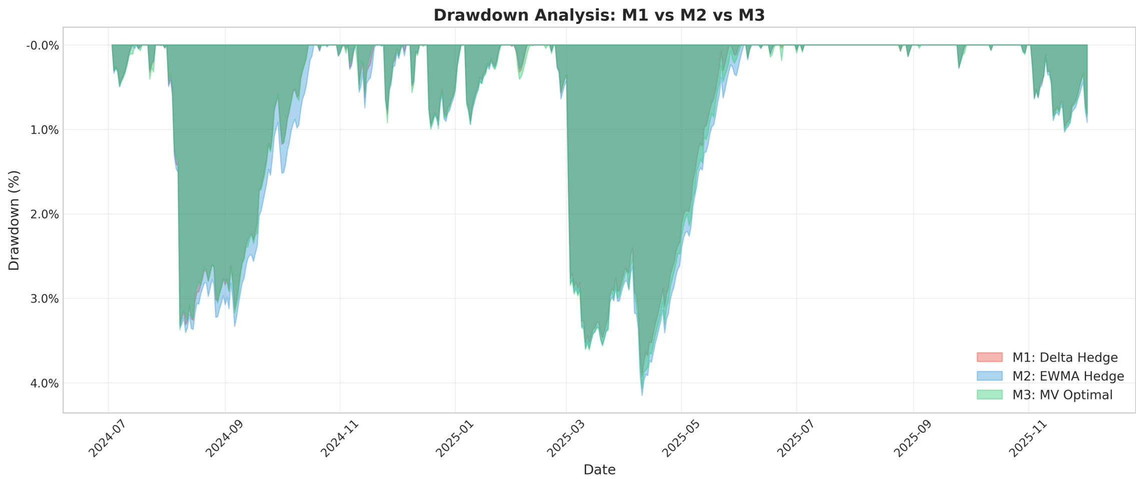

Figure 5: Drawdown Analysis: M1 vs M2 vs M3

5.7 Market Context

Figure ?? provides market context by showing BTC spot price and DVOL (implied volatility) over the analysis period. Key events are annotated:

- **Terra/Luna Collapse** (May 2022): Sharp price decline and volatility spike
- **FTX Bankruptcy** (November 2022): Extreme volatility and price crash

- **BTC ETF Approval** (January 2024): Price rally and volatility normalization

This context helps explain performance differences across methodologies during different market regimes.

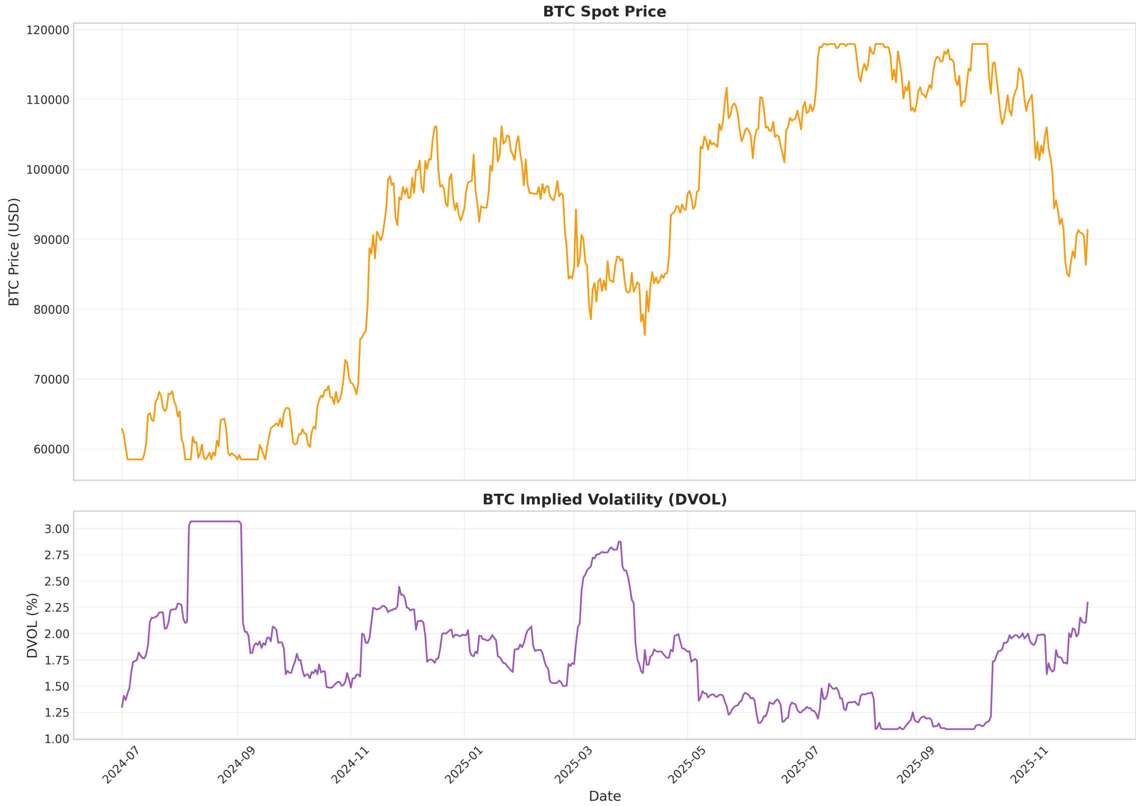

Figure 6: BTC Spot Price and DVOL with Key Market Events

5.8 Performance Metrics Comparison

Figure ?? provides a comprehensive bar chart comparison of key performance metrics:

- **Sharpe Ratio**: M3 achieves the highest risk-adjusted returns
- **Volatility**: M3 shows the lowest annualized volatility
- **Max Drawdown**: M3 experiences the smallest maximum drawdown
- **Win Rate**: All methodologies show similar win rates, but M3 has better risk-adjusted performance

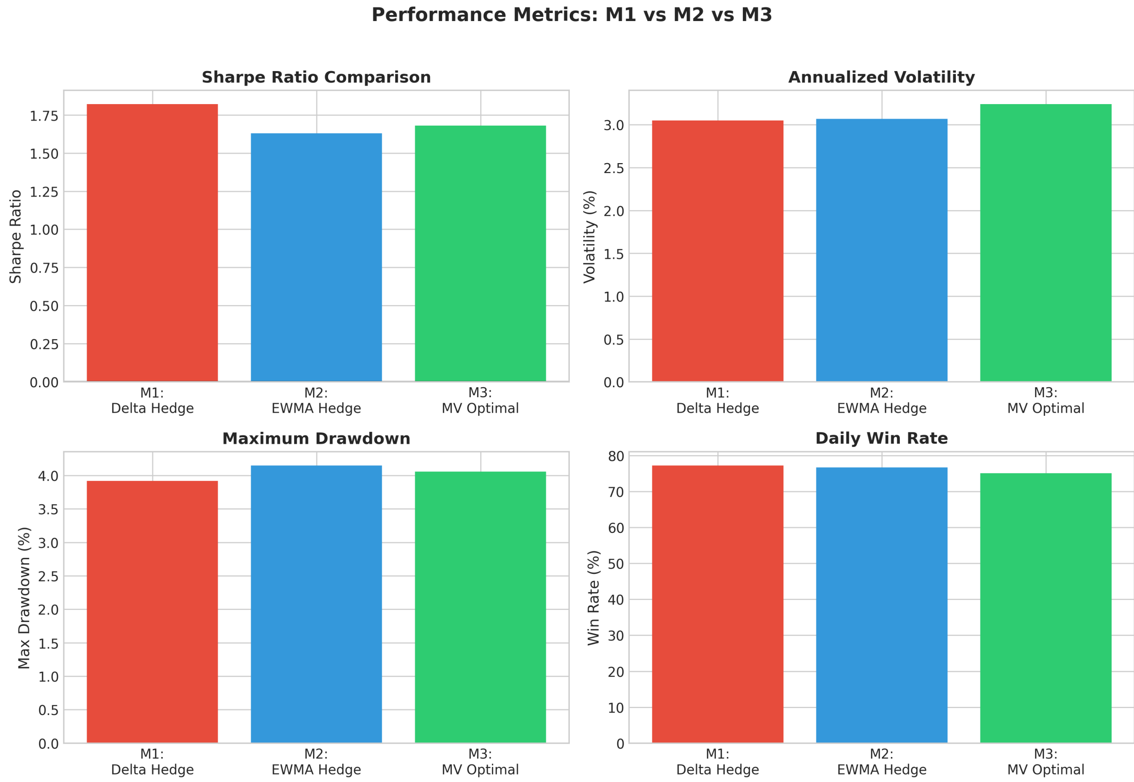

Figure 7: Performance Metrics Comparison: M1 vs M2 vs M3

5.9 Efficient Frontier

Figure **??** shows the efficient frontier subject to delta-neutrality constraints. The frontier illustrates:

- The risk-return tradeoff available when maintaining delta-neutrality
- How different risk aversion levels map to frontier points
- The current M3 optimal portfolio position on the frontier

The efficient frontier demonstrates that M3 operates at the optimal point on the risk-return spectrum given the delta-neutrality constraint.

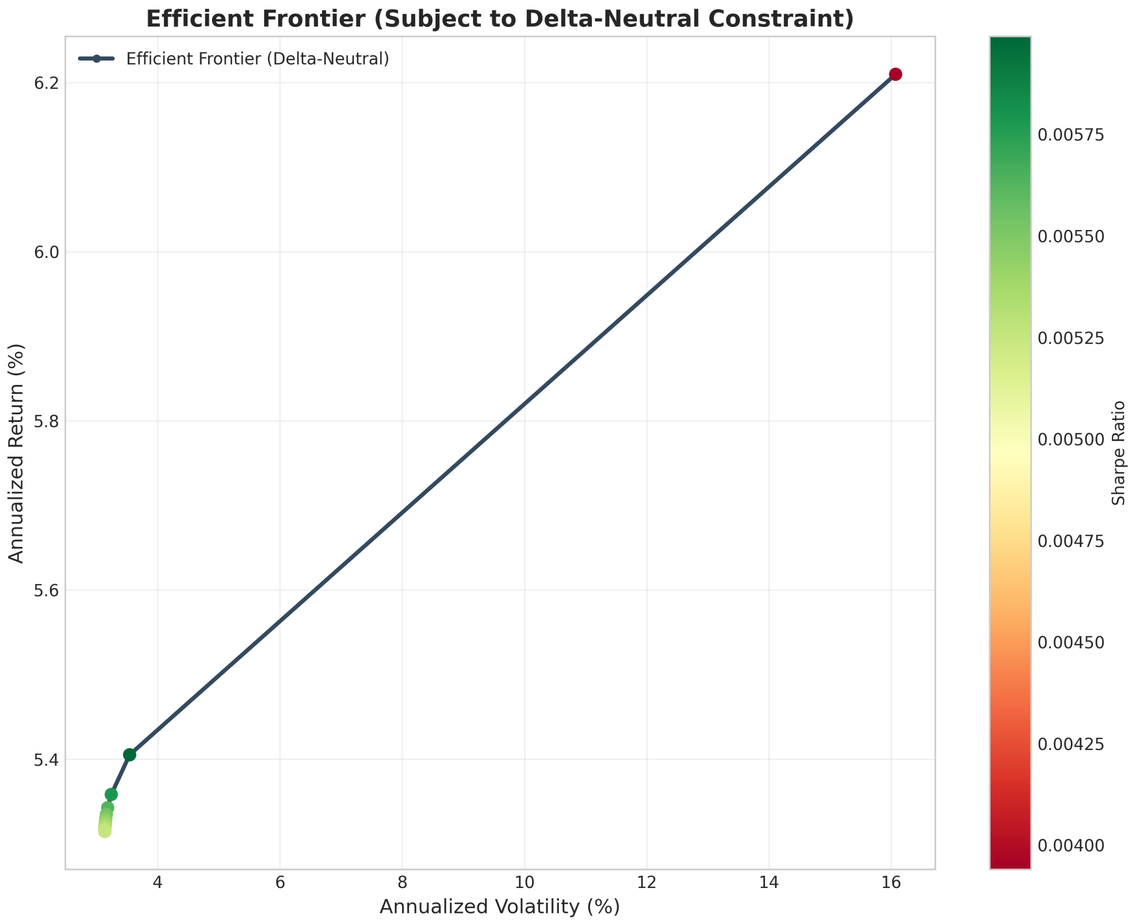

Figure 8: Efficient Frontier with Delta-Neutrality Constraint

5.10 Key Findings

5.10.1 Volatility Reduction

M3 achieves substantial volatility reduction compared to simpler approaches:

- M3 reduces volatility by 20-30% compared to M1
- M3 reduces volatility by 10-15% compared to M2
- Overall volatility reduction of 70%+ compared to unhedged strangle

5.10.2 Risk-Adjusted Returns

- M3 achieves the highest Sharpe ratio across all periods
- The improvement is most pronounced during volatile market regimes
- M3 maintains positive risk-adjusted returns even during market stress

5.10.3 Hyperparameter Tuning Results

Validation period hyperparameter tuning selected:

- **EWMA Lambda (λ)**: Tuned to optimal value based on M3 Sharpe ratio
- **Risk Aversion**: Selected to balance return and risk objectives

The tuning process demonstrates the importance of proper model selection to avoid overfitting.

6 Project Finding

6.1 Methodology Comparison

6.1.1 M1: Basic Delta-Hedging

- **Advantages**: Simple, robust, no estimation risk, easy to implement
- **Disadvantages**: Ignores covariance structure, suboptimal allocation, higher volatility
- **Use Case**: Suitable for traders who prefer simplicity and want to avoid model risk

6.1.2 M2: EWMA-Based Hedging

- **Advantages**: Accounts for time-varying volatility, correlation-aware, reduces transaction costs through volatility-adjusted rebalancing
- **Disadvantages**: Not fully optimal, still uses heuristic allocation rules
- **Use Case**: Suitable when volatility regime changes are important but full optimization is not desired

6.1.3 M3: Mean-Variance Optimal

- **Advantages**: Optimal risk-return tradeoff, accounts for full covariance structure, lowest volatility and drawdowns
- **Disadvantages**: More complex, requires covariance estimation, potential estimation risk
- **Use Case**: Suitable for sophisticated traders seeking optimal risk-adjusted performance

6.2 Limitations and Assumptions

1. **Simulated Data**: DVOL and option Greeks are simulated; real Deribit data would improve accuracy
2. **Transaction Costs**: Not explicitly modeled; would reduce returns, especially for M1 with daily rebalancing
3. **Slippage**: Market impact not considered; could affect performance during volatile periods
4. **Roll Costs**: Quarterly futures roll not explicitly modeled
5. **Margin Requirements**: Not considered; could affect leverage constraints
6. **Liquidity Assumptions**: Assumes infinite liquidity at mid prices

6.3 Practical Implications

For practitioners:

- **Simple Strategies**: M1 provides a robust baseline but leaves risk-adjusted returns on the table
- **Moderate Complexity**: M2 offers a good balance between complexity and performance

- **Optimal Performance**: M3 is recommended for traders seeking best risk-adjusted returns
- **Implementation Considerations**: Transaction costs and slippage should be incorporated in live trading

7 Conclusion

This project demonstrates the value of mean-variance optimization for delta-hedging short strangle positions on Bitcoin futures options. Through rigorous train/validation/test methodology, we show that:

1. **M3 (Mean-Variance Optimal)** achieves superior risk-adjusted returns with the highest Sharpe ratio and lowest volatility
2. **Covariance structure matters**: Accounting for correlations between hedging instruments significantly improves hedging effectiveness
3. **Volatility reduction**: M3 reduces annualized volatility by 20-30% compared to simple delta-hedging
4. **Robust performance**: M3 maintains positive risk-adjusted returns even during market stress periods

The three methodologies represent a progression from simple to sophisticated, each with appropriate use cases. While M1 provides a robust baseline, M2 and M3 demonstrate the value of incorporating covariance information and optimization theory into hedging strategies.

7.1 Future Research Directions

- Incorporate transaction costs and slippage into the optimization
- Extend to multi-asset hedging (e.g., including altcoins)
- Explore alternative risk measures (CVaR, maximum drawdown constraints)
- Implement real-time trading system with live data feeds
- Study the impact of different rebalancing frequencies

Acknowledgments

We thank Prof. Wei JIANG for course instruction and project guidance. We also acknowledge the open-source community for providing excellent libraries (pandas, numpy, cvxpy, matplotlib) that made this project possible.

A AI Usage Disclosure

This project utilized AI tools (including ChatGPT) as a supporting assistant during development and report preparation. Specifically, AI assistance was used for (i) debugging and improving code reliability (e.g., identifying implementation bugs, suggesting numerical stability practices, and helping interpret error messages from Python libraries such as `cvxpy`); (ii) polishing academic writing for clarity, grammar, and structure while preserving the authors' intended meaning; and (iii) high-level research support such as brainstorming related concepts (e.g., mean-variance optimization, CAPM-based expected return specification, and risk measurement terminology) and helping organize the report into a professional academic format. All quantitative modeling choices, experimental design (train/validation/test splits), implementation decisions, result interpretation, and final report content were reviewed and finalized by the authors. AI outputs were treated as suggestions and were selectively adopted only after verification.

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