Shock Persistence and Shock Frequency in VIX

A Quantitative Analysis of Volatility Dynamics

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Outline

What is VIX?

- **VIX** = CBOE Volatility Index, derived from S&P 500 option prices.
- Often called the "fear gauge" — rises when markets expect turbulence.
- Understanding VIX dynamics is crucial for:
    - Risk management and hedging
    - Derivatives pricing
    - Portfolio allocation

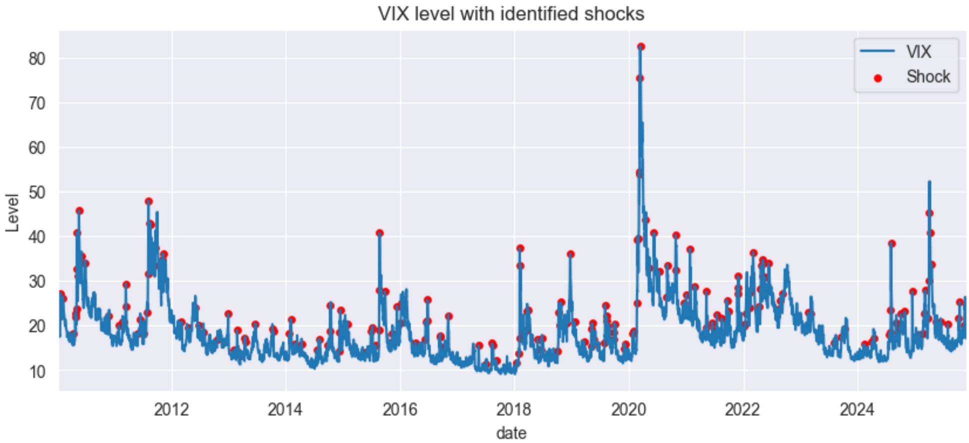
Research Questions

1. **How persistent is volatility?**
   How long does a VIX shock take to decay?
2. **How frequently do large spikes occur?**
   Can we model extreme events as a point process?
3. **Do shocks cluster?**
   Is there self-excitation in shock arrivals?
4. **How do regimes affect shock dynamics?**
   Do crisis periods show different behavior?
5. **Can we forecast VIX volatility?**
   Do GARCH-type models beat simple baselines out-of-sample?

Data Overview

- **Source:** Yahoo Finance (ticker `^VIX`)
- **Period:** January 2010 – November 2025
- **Observations:** 4,148 business days
- **Pre-processing:**
  - Forward-fill missing dates
  - 0.1% winsorization to limit outlier influence
  - Compute log(VIX) and daily log-changes $\Delta \log(\text{VIX})$

VIX Time Series



- Red markers indicate identified shock days (top 5% of $\Delta \log \text{VIX}$).

Volatility Models: GARCH Family

GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha\,\varepsilon_{t-1}^2 + \beta\,\sigma_{t-1}^2$$

EGARCH(1,1):

$$\ln\sigma_t^2 = \omega + \beta\ln\sigma_{t-1}^2 + \alpha\big(|z_{t-1}| - \mathbb{E}|z|\big) + \gamma\, z_{t-1}$$

GJR-GARCH(1,1):

$$\sigma_t^2 = \omega + (\alpha + \gamma\,\mathbf{1}_{\varepsilon_{t-1}<0})\,\varepsilon_{t-1}^2 + \beta\,\sigma_{t-1}^2$$

- GARCH: Symmetric; EGARCH/GJR: Asymmetric (leverage effect via γ).
- Distribution: Auto-selected (Normal, Student-t, GED) via PIT diagnostics.

HAR-RV: Realized Volatility Model

Heterogeneous Autoregressive Realized Volatility (Corsi, 2009):

$$RV_{t+1} = \beta_0 + \beta_d\, RV_t + \beta_w\, \overline{RV}_t^{(w)} + \beta_m\, \overline{RV}_t^{(m)} + \varepsilon_{t+1}$$

where:

- $RV_t = r_t^2$ (proxy for realized variance)
- $\overline{RV}_t^{(w)} = \frac{1}{5} \sum_{i=0}^{4} RV_{t-i}$ (weekly average)
- $\overline{RV}_t^{(m)} = \frac{1}{22} \sum_{i=0}^{21} RV_{t-i}$ (monthly average)

Use: Baseline model capturing heterogeneous investor horizons.

Shock Identification: Two Approaches

Method 1: Quantile-Based (Fixed Threshold)

$$\text{Shock}_t = \mathbf{1}\{\Delta \log(\text{VIX})_t \geq Q_{0.95}\}$$

- Simple, interpretable: top 5% of daily changes.

Method 2: Volatility-Relative (Surprise-Based)

$$\text{Shock}_t = \mathbf{1}\{|r_t| > k \cdot \sigma_t\}, \quad k = 2$$

- Captures "surprises" relative to expected volatility.
- More meaningful in high-volatility regimes.

Point Process Models for Shock Arrivals

Homogeneous Poisson Process (HPP):

$$\lambda(t) = \lambda \quad \text{(constant rate)}$$

Non-Homogeneous Poisson Process (NHPP):

$$\lambda(t) = \exp\big(\beta_0 + \beta_1 \cdot \text{lagged\_log\_VIX}_t\big)$$

Hawkes Self-Exciting Process:

$$\lambda(t) = \mu + \sum_{t_i < t} \alpha \cdot e^{-\beta(t - t_i)}$$

- Each shock temporarily increases intensity $\Rightarrow$ **clustering**.
- **Branching ratio** $= \alpha/\beta < 1 \Rightarrow$ stationary.

Forecast Evaluation Framework

- **Out-of-Sample Design:**
  - Train on first 75% of data (through Dec 2021).
  - Monthly rolling re-estimation of GARCH.
  - Forecast 1-step-ahead variance into the remaining 25%.
- **Baselines:**
  - EWMA ($\lambda = 0.94$)
  - 63-day rolling variance
  - HAR-RV model
- **Metrics:**
  - Log-score (predictive density evaluation)
  - 95% coverage rate
  - PIT histogram (calibration diagnostic)
  - Diebold–Mariano test (statistical significance)

Volatility Model Comparison

Model	Distribution	AIC	Persistence	Half-life	Leverage (γ)
GARCH(1,1)	GED	27,531	0.852	4.3 days	–
EGARCH(1,1)	GED	**27,395**	0.934	10.2 days	Yes
GJR-GARCH(1,1)	GED	27,447	0.866	4.8 days	-0.27

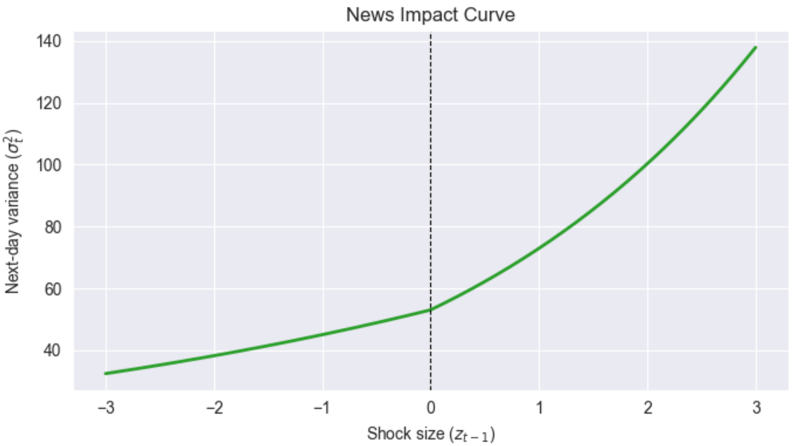
- EGARCH achieves lowest AIC $\Rightarrow$ best in-sample fit.
- Higher persistence in EGARCH $\Rightarrow$ shocks decay more slowly ($\approx$10 days half-life).
- GJR-GARCH $\gamma < 0$: negative returns increase volatility more.

HAR-RV Results

Component	Coefficient	Interpretation
β_{daily}	0.117	Short-term (1-day) effect
β_{weekly}	0.231	Medium-term (5-day) effect
β_{monthly}	0.042	Long-term (22-day) effect
R^2	0.049	Explanatory power

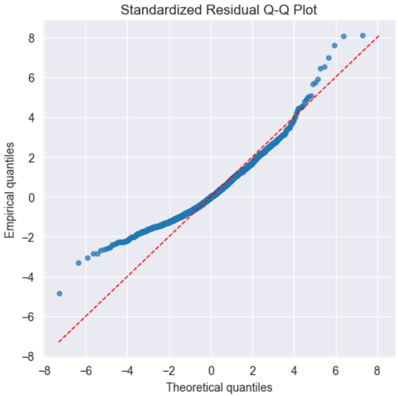
- Weekly component dominates $\Rightarrow$ swing traders' horizon matters most.
- Low R^2 expected: squared returns are noisy proxies for true variance.
- HAR-RV provides parsimonious baseline for comparison.

News Impact Curve (Asymmetry)



- Positive shocks (VIX spikes) increase future variance more than negative shocks of equal magnitude.

Q–Q Plot of Standardized Residuals



- Points hug the 45° line in the tails $\Rightarrow$ GED captures fat tails well.

Shock Statistics: Two Methods Compared

Metric	Quantile (95%)	Vol-Relative (2σ)
Threshold (avg)	0.127	0.147
Total shocks	208	207
Rate (shocks/year)	9.0	9.0

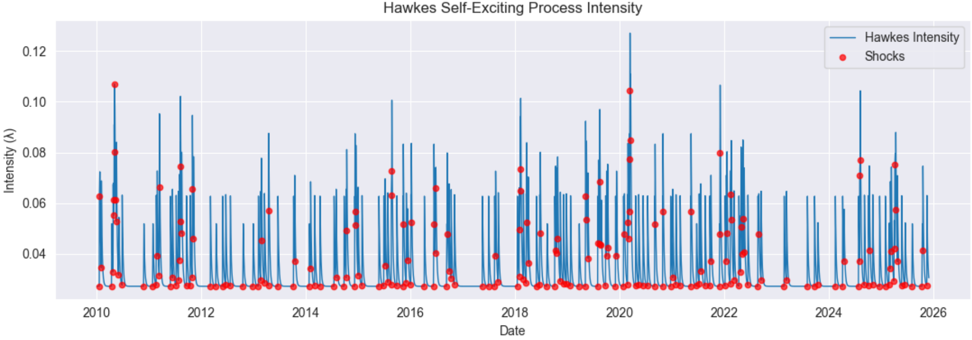
- Both methods identify $\sim$9 shocks/year.
- Vol-relative threshold adapts to market conditions.
- NHPP: Lagged log VIX coefficient $= -0.16 \Rightarrow$ low VIX predicts fewer shocks.

Hawkes Self-Exciting Process

Parameter	Value	Interpretation
Baseline μ	0.027/day	Background intensity
Excitation α	0.043	Jump after each shock
Decay β	0.183	Rate of decay
Branching ratio	0.23	Fraction triggered by past shocks
Half-life	3.8 days	Excitation halving time

- Branching ratio $< 1 \Rightarrow$ process is **stationary**.
- $\approx$23% of shocks are "triggered" by previous shocks.
- Excitation decays with $\sim$4-day half-life.

Hawkes Intensity Over Time



- Intensity spikes after each shock, then decays exponentially.
- Clustering visible during crisis periods.

Compound Poisson Process: Motivation

Problem: HPP/NHPP/Hawkes model *when* shocks occur, but not *how big*.

Compound Poisson Process (CPP):

$$S(T) = \sum_{i=1}^{N(T)} J_i$$

where:

- $N(T) \sim \mathrm{Poisson}(\lambda T)$: number of shocks by time T
- $J_i \sim F$: random jump sizes (iid)
- $S(T)$: cumulative shock impact over horizon T

Key Insight: Models *both* timing and magnitude $\Rightarrow$ risk quantification.

CPP: Jump Size Distribution Selection

Candidates for jump size distribution F:

- Exponential: $f(x) = \lambda e^{-\lambda x}$ (memoryless)
- Gamma: $f(x) \propto x^{k-1} e^{-x/\theta}$ (flexible shape)
- Lognormal: $\ln(J) \sim N(\mu, \sigma^2)$ (multiplicative)
- **Pareto**: $f(x) = \frac{\alpha x_m^\alpha}{x^{\alpha+1}}$ (heavy tails)
- Weibull: $f(x) \propto x^{k-1} e^{-(x/\lambda)^k}$ (hazard rate)

Selection via AIC + KS test:

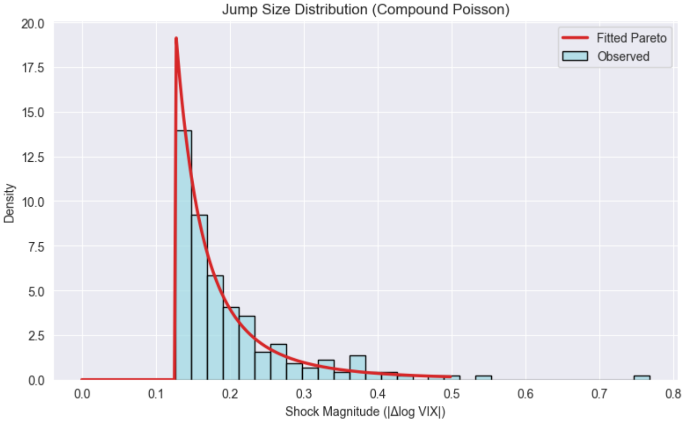
- Best fit: **Pareto** ($\alpha = 2.50$, $x_{\text{min}} = 0.127$)
- KS p-value $= 0.42 \Rightarrow$ cannot reject fit

CPP: Fitted Parameters

Parameter	Value	Interpretation
λ	$12.64/\text{year}$	Shock arrival rate
$\mathbb{E}[J]$	0.211	Mean jump size (21% log-move)
$\mathrm{Std}[J]$	0.189	Jump size volatility
$\mathbb{E}[S]$	$2.67/\text{year}$	Expected annual impact
VaR (95%)	4.24	95th percentile annual impact
CVaR (95%)	5.01	Expected Shortfall (tail avg)

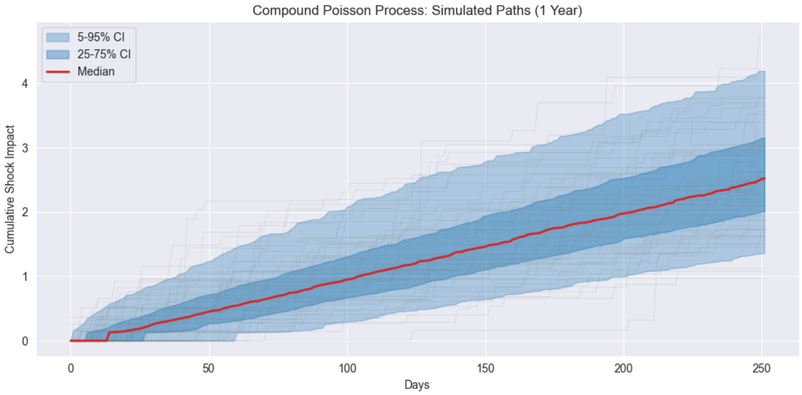
- $\mathbb{E}[S] = \lambda \cdot \mathbb{E}[J] = 12.64 \times 0.211 = 2.67$
- VaR/CVaR computed via 10,000 Monte Carlo simulations.

CPP: Jump Size Distribution



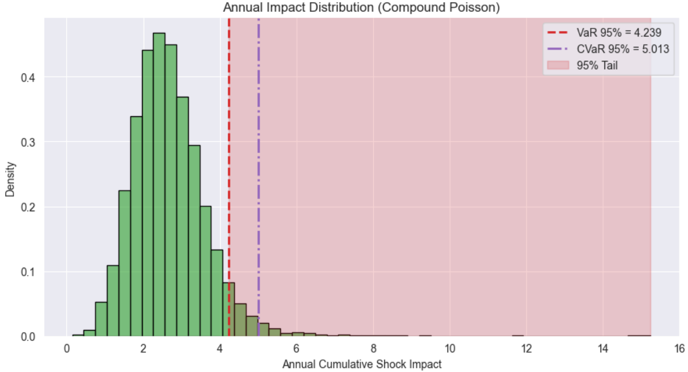
- Pareto distribution captures heavy right tail of shock magnitudes.
- Most shocks are moderate; a few are extreme.

CPP: Simulated Paths



- Gray: 50 sample paths of cumulative annual shock impact.
- Shaded: 5–95% and 25–75% confidence bands.
- Red: Median trajectory.

CPP: VaR and CVaR Distribution



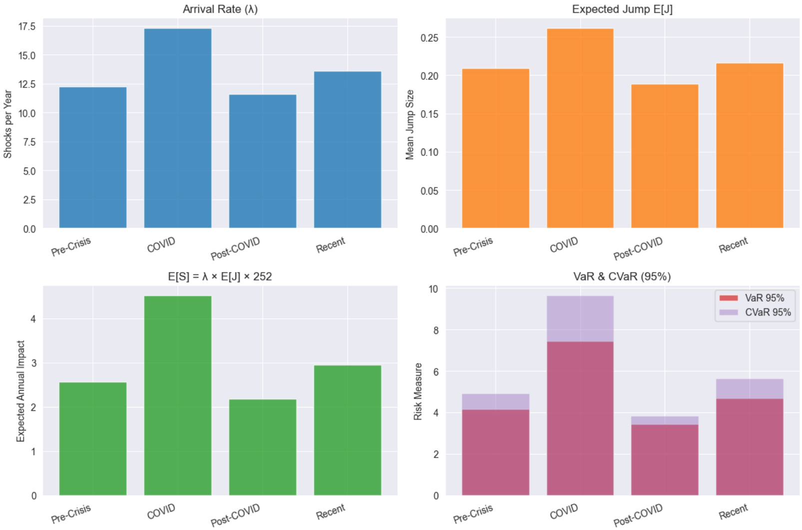
- Distribution of annual cumulative shock impact.
- VaR (95%) $= 4.24$: "In 95% of years, total impact ≤ 4.24."
- CVaR (95%) $= 5.01$: "Average impact in worst 5% of years."

CPP: Regime Comparison

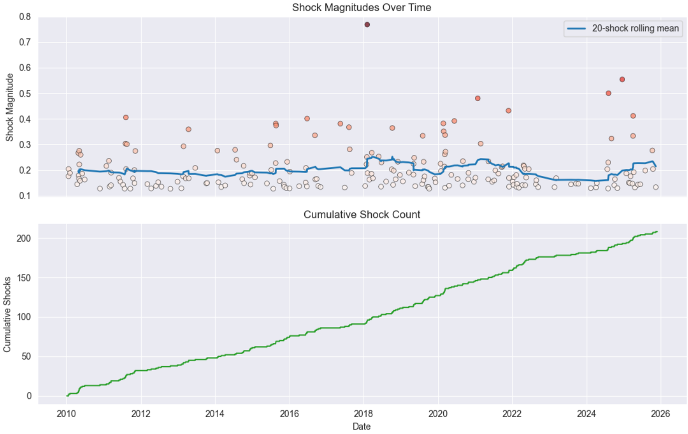
Regime	λ/Year	$\mathbb{E}[J]$	$\mathbb{E}[S]$/Year	VaR 95%	CVaR 95%
Pre-Crisis	12.3	0.209	2.57	4.15	4.92
COVID	**17.3**	**0.262**	**4.53**	**7.44**	**9.65**
Post-COVID	11.6	0.188	2.19	3.44	3.85
Recent	13.6	0.216	2.95	4.70	5.63

- COVID: Both higher arrival rate AND larger jumps $\Rightarrow$ 76% higher $\mathbb{E}[S]$.
- VaR nearly doubles during crisis periods.

CPP: Regime Risk Comparison



Shock Magnitudes Over Time



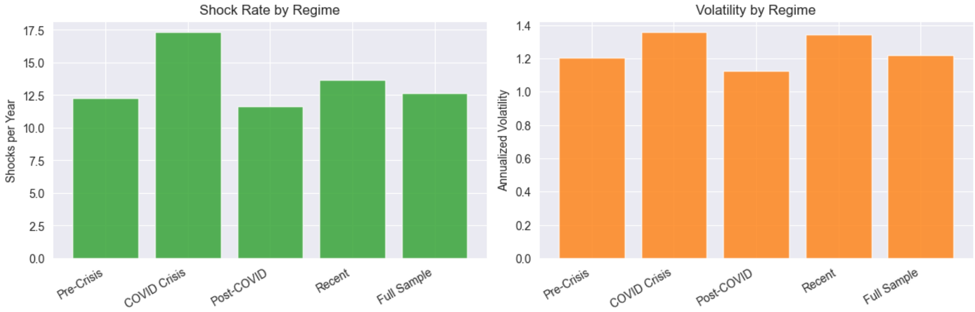
- Top: Individual shock magnitudes with 20-shock rolling mean.
- Bottom: Cumulative shock count shows acceleration during crises.

Regime Analysis: Shock Rates Across Periods

Regime	Period	Obs	Shocks	Rate/Year	Ann. Vol
Pre-Crisis	2010–2019	2,606	127	12.3	1.21
COVID	2020	262	18	**17.3**	**1.36**
Post-COVID	2021–2023	781	36	11.6	1.12
Recent	2024–2025	499	27	13.6	1.34
Full Sample	2010–2025	4,148	208	12.6	1.22

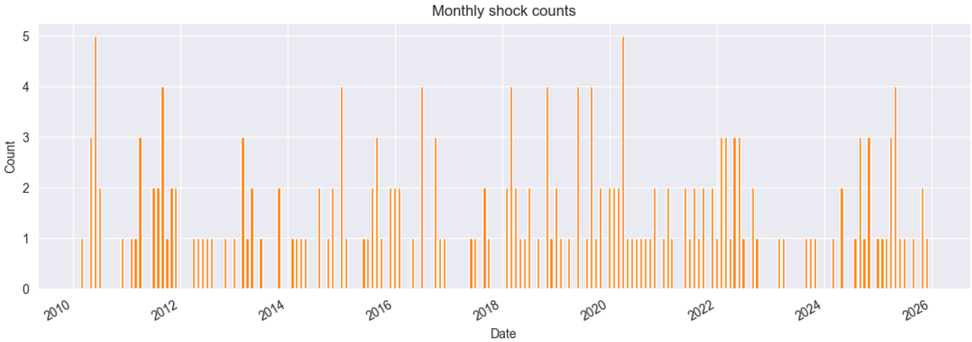
- COVID period: **41% higher** shock rate (17.3 vs 12.3/year).
- Volatility elevated across all crisis periods.

Regime Comparison Visualization



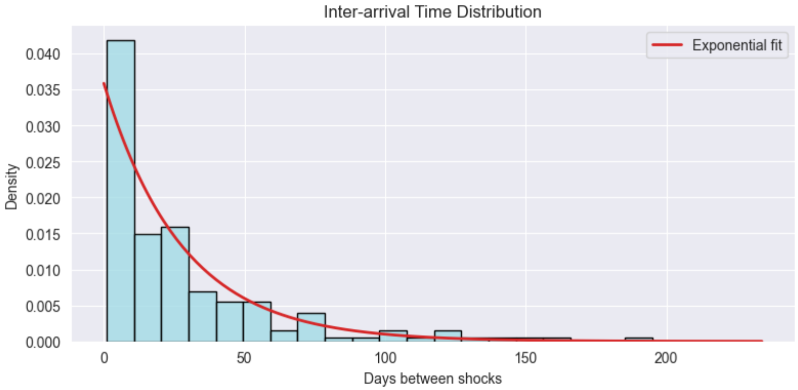
- Bar chart shows clear elevation during COVID and recent volatility.

Monthly Shock Counts



- Notable clustering: 2011–12 (Euro crisis), 2018 (Volmageddon), 2020 (COVID), 2022 (rate hikes).

Inter-Arrival Time Distribution



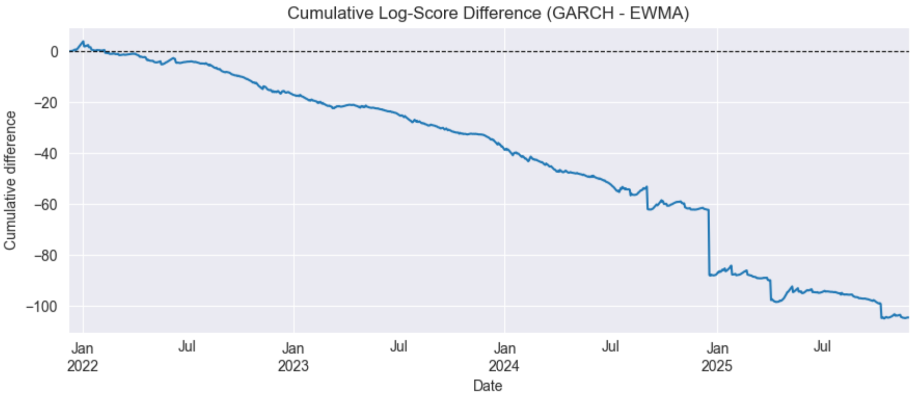
- Histogram vs. exponential PDF: reasonable fit validates Poisson assumption.

Forecast Evaluation: Log-Scores

Model	Log-Score (higher = better)
GARCH	1.275
EWMA	**1.376**
Rolling Var	1.276
HAR-RV	1.271

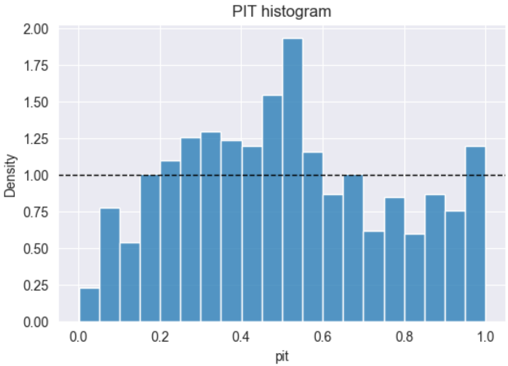
- EWMA slightly outperforms GARCH out-of-sample.
- Diebold–Mariano $p < 0.001 \Rightarrow$ difference is statistically significant.
- HAR-RV comparable to rolling variance baseline.

Cumulative Log-Score Difference



- Downward trend: EWMA consistently accumulates better scores.
- GARCH rarely catches up, even briefly.

PIT Histogram (Calibration)



- Near-uniform distribution indicates well-calibrated density forecasts.
- Mean ≈ 0.51, std ≈ 0.26 (ideal: 0.5, 0.29).
- 95% coverage: 95.0% (at nominal).

Key Findings

1. **Persistence:** VIX volatility shocks have a half-life of 4–10 days; EGARCH captures longer memory.
2. **Leverage Effect:** GJR-GARCH $\gamma = -0.27$ confirms asymmetric response.
3. **Self-Excitation:** Hawkes branching ratio $= 0.23 \Rightarrow$ shocks trigger more shocks.
4. **Compound Poisson:** Pareto-distributed jumps; VaR 95% $= 4.24$/year.
5. **Regime Dependence:** COVID showed 76% higher expected annual impact.
6. **Forecasting:** EWMA beats GARCH on log-score; both well-calibrated.

Practical Implications

- **Risk Management:**
  - Use EGARCH/GJR-GARCH for asymmetric VaR calculations.
  - Hawkes intensity provides real-time "shock alert" signals.
  - CPP gives VaR/CVaR for aggregate annual shock impact.
- **Option Pricing:**
  - Leverage effect matters: adjust for skewed vol-of-vol.
  - Pareto jumps justify heavy-tail adjustments in pricing.
- **Portfolio Allocation:**
  - Regime detection enables adaptive hedging strategies.
  - CPP regime analysis shows crisis periods need $2\times$ risk budget.

Limitations

- GARCH re-estimation is computationally expensive; rolling used here.
- EWMA's superiority may reflect VIX's strong mean-reversion.
- NHPP covariates limited to lagged VIX; macro indicators could improve.
- Hawkes estimation assumes exponential decay; other kernels possible.
- Daily data only; intraday dynamics not captured.

Future Work

- Incorporate regime-switching GARCH (Markov-switching) models.
- Test realized volatility from high-frequency data.
- Extend NHPP/Hawkes with external regressors (credit spreads, VIX term structure).
- Multivariate Hawkes for cross-asset shock contagion.
- Deploy real-time monitoring dashboard with Hawkes intensity tracking.

Conclusion

- VIX exhibits **persistent, asymmetric volatility clustering** well captured by EGARCH.
- Large spikes arrive at $\sim$13/year and exhibit **self-excitation** (Hawkes branching ratio 0.23).
- **Compound Poisson** quantifies aggregate shock risk: VaR 95% $=$ 4.24/year.
- **COVID regime** showed 76% higher expected annual impact than baseline.
- Out-of-sample, **EWMA remains a tough benchmark** to beat for density forecasting.
- The reproducible pipeline (`runall.py`) with **28 unit tests** enables transparent research.

Thank you!

Questions?

Appendix: Project Structure

```
Shock-Persistence-and-Shock-Frequency-in-VIX/
+-- runall.py                # Main pipeline
+-- src/
|   +-- config.py            # Parameters
|   +-- data_pipeline.py     # Data loading
|   +-- volatility_models.py # GARCH/EGARCH/GJR/HAR-RV
|   +-- shock_modeling.py    # HPP/NHPP/Hawkes/CPP
|   +-- forecast_evaluation.py # OOS evaluation
|   +-- visualization.py     # Plotting
+-- tests/
|   +-- test_models.py       # 28 unit tests
+-- figures/                 # Generated plots
```

Appendix: Key Equations

Log-Score:

$$S_t = \log f(r_t \mid \mu_t, \sigma_t^2)$$

Hawkes Intensity:

$$\lambda(t) = \mu + \sum_{t_i < t} \alpha \cdot e^{-\beta(t - t_i)}$$

Compound Poisson Process:

$$S(T) = \sum_{i=1}^{N(T)} J_i, \quad N(T) \sim \mathrm{Poisson}(\lambda T), \quad J_i \sim F$$

Expected Annual Impact:

$$\mathbb{E}[S] = \lambda \cdot \mathbb{E}[J] \cdot T$$

Appendix: References

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