

Shock Persistence and Shock Frequency in VIX

A Quantitative Analysis of Volatility Dynamics

CHONG Tin Tak (20920359)
CHOI Man Hou (20894196)
Vittorio Prana CHANDREAN (20896895)

HKUST - IEDA4000E

November 27, 2025

Outline

- 1 Introduction
- 2 Data
- 3 Methodology
- 4 Results
- 5 Discussion
- 6 Conclusion

What is VIX?

- **VIX** = CBOE Volatility Index, derived from S&P 500 option prices.
- Often called the “fear gauge” — rises when markets expect turbulence.
- Understanding VIX dynamics is crucial for:
 - Risk management and hedging
 - Derivatives pricing
 - Portfolio allocation

Research Questions

① **How persistent is volatility?**

How long does a VIX shock take to decay?

② **How frequently do large spikes occur?**

Can we model extreme events as a point process?

③ **Do shocks cluster?**

Is there self-excitation in shock arrivals?

④ **How do regimes affect shock dynamics?**

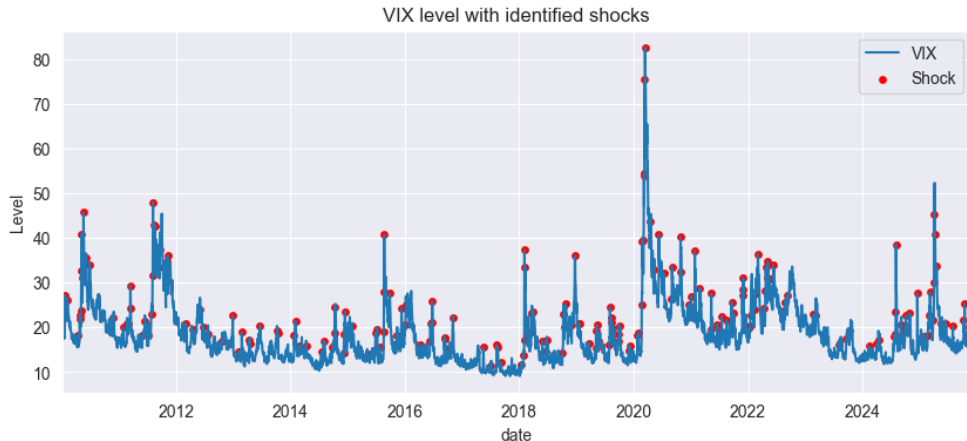
Do crisis periods show different behavior?

⑤ **Can we forecast VIX volatility?**

Do GARCH-type models beat simple baselines out-of-sample?

- **Source:** Yahoo Finance (ticker ^VIX)
- **Period:** January 2010 – November 2025
- **Observations:** 4,148 business days
- **Pre-processing:**
 - Forward-fill missing dates
 - 0.1% winsorization to limit outlier influence
 - Compute $\log(\text{VIX})$ and daily log-changes $\Delta \log(\text{VIX})$

VIX Time Series



- Red markers indicate identified shock days (top 5% of $\Delta \log \text{VIX}$).

Volatility Models: GARCH Family

GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

EGARCH(1,1):

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \alpha (|z_{t-1}| - \mathbb{E}|z|) + \gamma z_{t-1}$$

GJR-GARCH(1,1):

$$\sigma_t^2 = \omega + (\alpha + \gamma \mathbf{1}_{\varepsilon_{t-1} < 0}) \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

- GARCH: Symmetric; EGARCH/GJR: Asymmetric (leverage effect via γ).
- Distribution: Auto-selected (Normal, Student-t, GED) via PIT diagnostics.

HAR-RV: Realized Volatility Model

Heterogeneous Autoregressive Realized Volatility (Corsi, 2009):

$$RV_{t+1} = \beta_0 + \beta_d RV_t + \beta_w \overline{RV}_t^{(w)} + \beta_m \overline{RV}_t^{(m)} + \varepsilon_{t+1}$$

where:

- $RV_t = r_t^2$ (proxy for realized variance)
- $\overline{RV}_t^{(w)} = \frac{1}{5} \sum_{i=0}^4 RV_{t-i}$ (weekly average)
- $\overline{RV}_t^{(m)} = \frac{1}{22} \sum_{i=0}^{21} RV_{t-i}$ (monthly average)

Use: Baseline model capturing heterogeneous investor horizons.

Shock Identification: Two Approaches

Method 1: Quantile-Based (Fixed Threshold)

$$\text{Shock}_t = \mathbf{1}\{\Delta \log(\text{VIX})_t \geq Q_{0.95}\}$$

- Simple, interpretable: top 5% of daily changes.

Method 2: Volatility-Relative (Surprise-Based)

$$\text{Shock}_t = \mathbf{1}\{|r_t| > k \cdot \sigma_t\}, \quad k = 2$$

- Captures “surprises” relative to expected volatility.
- More meaningful in high-volatility regimes.

Point Process Models for Shock Arrivals

Homogeneous Poisson Process (HPP):

$$\lambda(t) = \lambda \quad (\text{constant rate})$$

Non-Homogeneous Poisson Process (NHPP):

$$\lambda(t) = \exp(\beta_0 + \beta_1 \cdot \text{lagged_log_VIX}_t)$$

Hawkes Self-Exciting Process:

$$\lambda(t) = \mu + \sum_{t_i < t} \alpha \cdot e^{-\beta(t-t_i)}$$

- Each shock temporarily increases intensity \Rightarrow **clustering**.
- **Branching ratio** $= \alpha/\beta < 1 \Rightarrow$ stationary.

- **Out-of-Sample Design:**

- Train on first 75% of data (through Dec 2021).
- Monthly rolling re-estimation of GARCH.
- Forecast 1-step-ahead variance into the remaining 25%.

- **Baselines:**

- EWMA ($\lambda = 0.94$)
- 63-day rolling variance
- HAR-RV model

- **Metrics:**

- Log-score (predictive density evaluation)
- 95% coverage rate
- PIT histogram (calibration diagnostic)
- Diebold–Mariano test (statistical significance)

Volatility Model Comparison

Model	Distribution	AIC	Persistence	Half-life	Leverage (γ)
GARCH(1,1)	GED	27,531	0.852	4.3 days	—
EGARCH(1,1)	GED	27,395	0.934	10.2 days	Yes
GJR-GARCH(1,1)	GED	27,447	0.866	4.8 days	−0.27

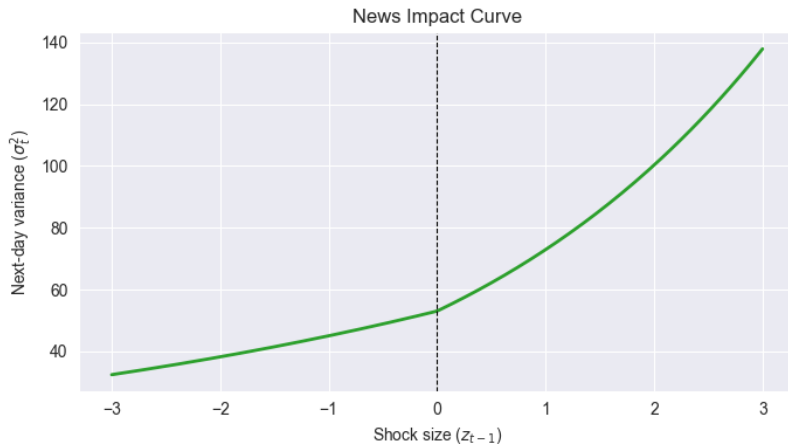
- EGARCH achieves lowest AIC \Rightarrow best in-sample fit.
- Higher persistence in EGARCH \Rightarrow shocks decay more slowly (≈ 10 days half-life).
- GJR-GARCH $\gamma < 0$: negative returns increase volatility more.

HAR-RV Results

Component	Coefficient	Interpretation
β_{daily}	0.117	Short-term (1-day) effect
β_{weekly}	0.231	Medium-term (5-day) effect
β_{monthly}	0.042	Long-term (22-day) effect
R^2	0.049	Explanatory power

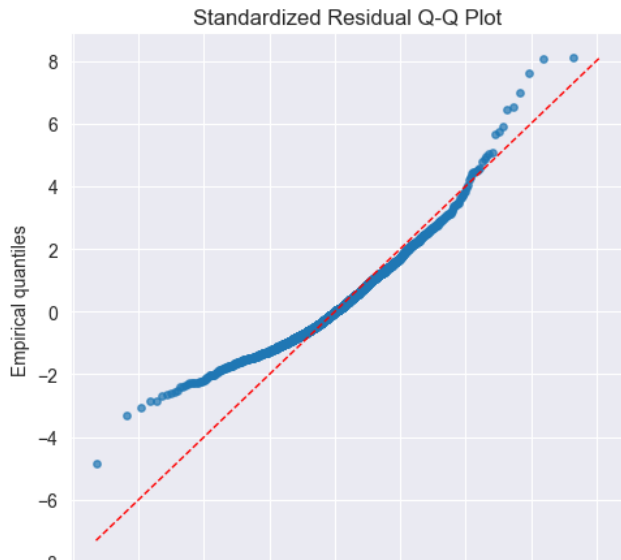
- Weekly component dominates \Rightarrow swing traders' horizon matters most.
- Low R^2 expected: squared returns are noisy proxies for true variance.
- HAR-RV provides parsimonious baseline for comparison.

News Impact Curve (Asymmetry)



- Positive shocks (VIX spikes) increase future variance more than negative shocks of equal magnitude.

Q-Q Plot of Standardized Residuals



Shock Statistics: Two Methods Compared

Metric	Quantile (95%)	Vol-Relative (2σ)
Threshold (avg)	0.127	0.147
Total shocks	208	207
Rate (shocks/year)	9.0	9.0

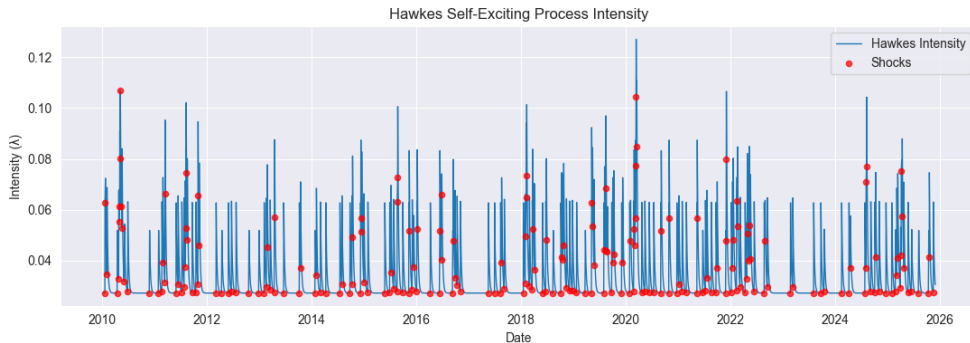
- Both methods identify ~ 9 shocks/year.
- Vol-relative threshold adapts to market conditions.
- NHPP: Lagged log VIX coefficient = $-0.16 \Rightarrow$ low VIX predicts fewer shocks.

Hawkes Self-Exciting Process

Parameter	Value	Interpretation
Baseline μ	0.027/day	Background intensity
Excitation α	0.043	Jump after each shock
Decay β	0.183	Rate of decay
Branching ratio	0.23	Fraction triggered by past shocks
Half-life	3.8 days	Excitation halving time

- Branching ratio $< 1 \Rightarrow$ process is **stationary**.
- $\approx 23\%$ of shocks are “triggered” by previous shocks.
- Excitation decays with ~ 4 -day half-life.

Hawkes Intensity Over Time



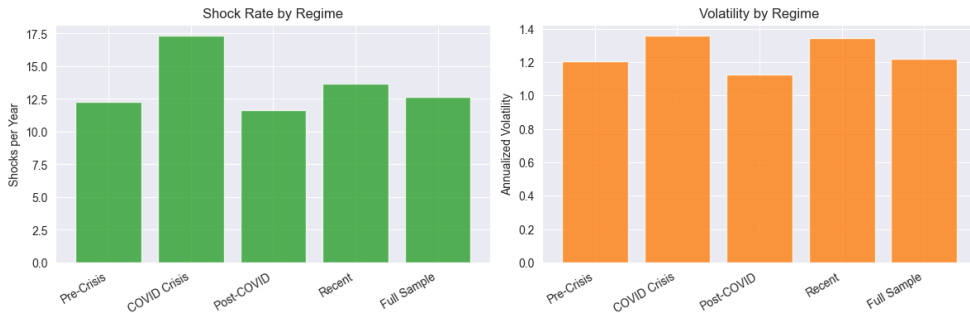
- Intensity spikes after each shock, then decays exponentially.
- Clustering visible during crisis periods.

Regime Analysis: Shock Rates Across Periods

Regime	Period	Obs	Shocks	Rate/Year	Ann. Vol
Pre-Crisis	2010–2019	2,606	127	12.3	1.21
COVID	2020	262	18	17.3	1.36
Post-COVID	2021–2023	781	36	11.6	1.12
Recent	2024–2025	499	27	13.6	1.34
Full Sample	2010–2025	4,148	208	12.6	1.22

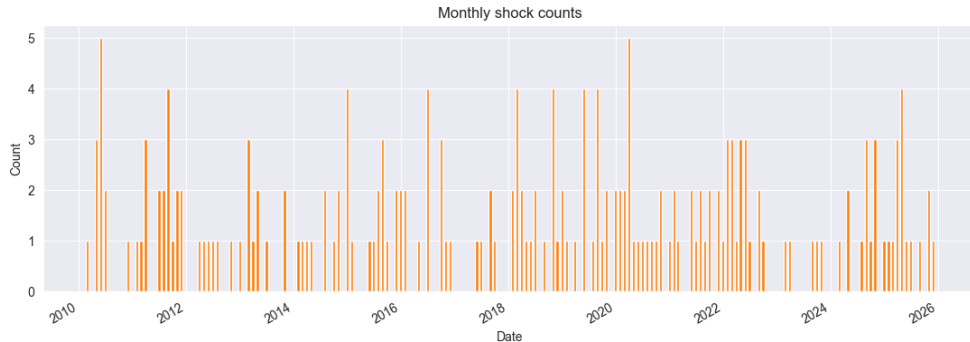
- COVID period: **41% higher** shock rate (17.3 vs 12.3/year).
- Volatility elevated across all crisis periods.

Regime Comparison Visualization



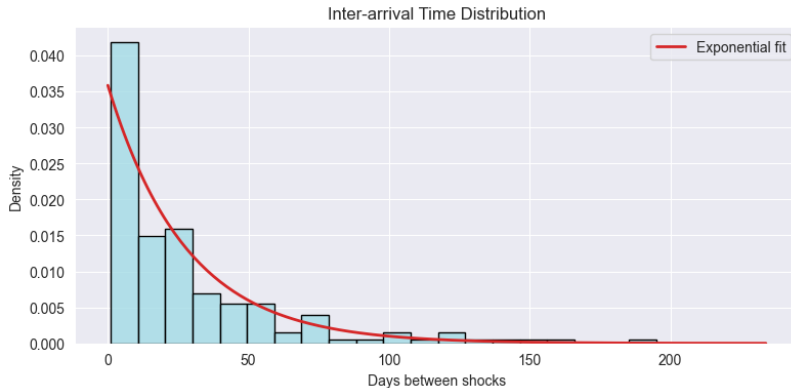
- Bar chart shows clear elevation during COVID and recent volatility.

Monthly Shock Counts



- Notable clustering: 2011–12 (Euro crisis), 2018 (Volmageddon), 2020 (COVID), 2022 (rate hikes).

Inter-Arrival Time Distribution



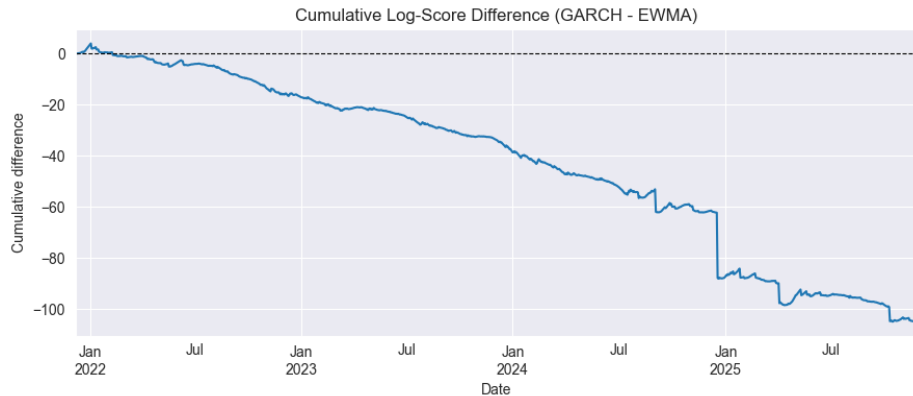
- Histogram vs. exponential PDF: reasonable fit validates Poisson assumption.

Forecast Evaluation: Log-Scores

Model	Log-Score (higher = better)
GARCH	1.275
EWMA	1.376
Rolling Var	1.276
HAR-RV	1.271

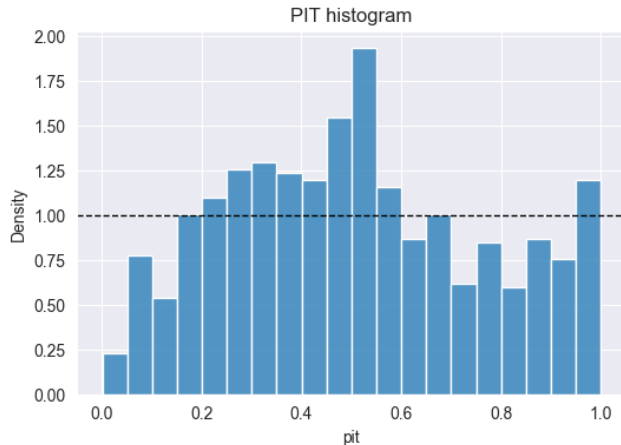
- EWMA slightly outperforms GARCH out-of-sample.
- Diebold–Mariano $p < 0.001 \Rightarrow$ difference is statistically significant.
- HAR-RV comparable to rolling variance baseline.

Cumulative Log-Score Difference



- Downward trend: EWMA consistently accumulates better scores.
- GARCH rarely catches up, even briefly.

PIT Histogram (Calibration)



- Near-uniform distribution indicates well-calibrated density forecasts.
- Mean ≈ 0.51 , std ≈ 0.26 (ideal: 0.5, 0.29).
- 95% coverage: 95.0% (at nominal).

Key Findings

- ① **Persistence:** VIX volatility shocks have a half-life of 4–10 days; EGARCH captures longer memory.
- ② **Leverage Effect:** GJR-GARCH $\gamma = -0.27$ confirms asymmetric response.
- ③ **Self-Excitation:** Hawkes branching ratio = 0.23 \Rightarrow shocks trigger more shocks.
- ④ **Regime Dependence:** COVID showed 41% higher shock rate than baseline.
- ⑤ **Forecasting:** EWMA beats GARCH on log-score; both well-calibrated.

- **Risk Management:**

- Use EGARCH/GJR-GARCH for asymmetric VaR calculations.
- Hawkes intensity can provide real-time “shock alert” signals.

- **Option Pricing:**

- Leverage effect matters: adjust for skewed vol-of-vol.
- Shock clustering affects term structure of implied volatility.

- **Portfolio Allocation:**

- Regime detection enables adaptive hedging strategies.
- 4-day Hawkes half-life suggests momentum in shock risk.

Limitations

- GARCH re-estimation is computationally expensive; rolling used here.
- EWMA's superiority may reflect VIX's strong mean-reversion.
- NHPP covariates limited to lagged VIX; macro indicators could improve.
- Hawkes estimation assumes exponential decay; other kernels possible.
- Daily data only; intraday dynamics not captured.

- Incorporate regime-switching GARCH (Markov-switching) models.
- Test realized volatility from high-frequency data.
- Extend NHPP/Hawkes with external regressors (credit spreads, VIX term structure).
- Multivariate Hawkes for cross-asset shock contagion.
- Deploy real-time monitoring dashboard with Hawkes intensity tracking.

- VIX exhibits **persistent, asymmetric volatility clustering** well captured by EGARCH.
- Large spikes arrive at ~ 9 /year and exhibit **self-excitation** (Hawkes branching ratio 0.23).
- **COVID regime** showed significantly elevated shock rates (+41%).
- Out-of-sample, **EWMA remains a tough benchmark** to beat for density forecasting.
- The reproducible pipeline (`runall.py`) with **23 unit tests** enables transparent research.

Thank you!

Questions?

Appendix: Project Structure

```
Shock-Persistence-and-Shock-Frequency-in-VIX/
+-- runall.py                # Main pipeline
+-- src/
|   +-- config.py           # Parameters
|   +-- data_pipeline.py    # Data loading
|   +-- volatility_models.py # GARCH/EGARCH/GJR/HAR-RV
|   +-- shock_modeling.py   # HPP/NHPP/Hawkes
|   +-- forecast_evaluation.py # OOS evaluation
|   +-- visualization.py    # Plotting
+-- tests/
|   +-- test_models.py      # 23 unit tests
+-- figures/                # Generated plots
```

Appendix: Key Equations

Log-Score:

$$S_t = \log f(r_t \mid \mu_t, \sigma_t^2)$$

PIT:

$$u_t = F(r_t \mid \mu_t, \sigma_t^2) \text{ should be Uniform}(0, 1)$$

Hawkes Intensity:

$$\lambda(t) = \mu + \sum_{t_i < t} \alpha \cdot e^{-\beta(t-t_i)}$$

Diebold–Mariano Statistic:

$$DM = \frac{\bar{d}}{\sqrt{\widehat{\text{Var}}(\bar{d})}} \xrightarrow{d} N(0, 1)$$

Appendix: References

- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *J. Econometrics*.
- Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns. *Econometrica*.
- Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between expected value and volatility. *J. Finance*.
- Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *J. Financial Econometrics*.
- Hawkes, A. G. (1971). Spectra of some self-exciting and mutually exciting point processes. *Biometrika*.
- Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *JBES*.