

# Shock Persistence and Shock Frequency in VIX

## A Quantitative Analysis of Volatility Dynamics

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# Outline

- 1 Introduction
- 2 Data
- 3 Methodology
- 4 Results
- 5 Discussion
- 6 Conclusion

# What is VIX?

- **VIX** = CBOE Volatility Index, derived from S&P 500 option prices.
- Often called the “fear gauge” — rises when markets expect turbulence.
- Understanding VIX dynamics is crucial for:
  - Risk management and hedging
  - Derivatives pricing
  - Portfolio allocation

① **How persistent is volatility?**

How long does a VIX shock take to decay?

② **How frequently do large spikes occur?**

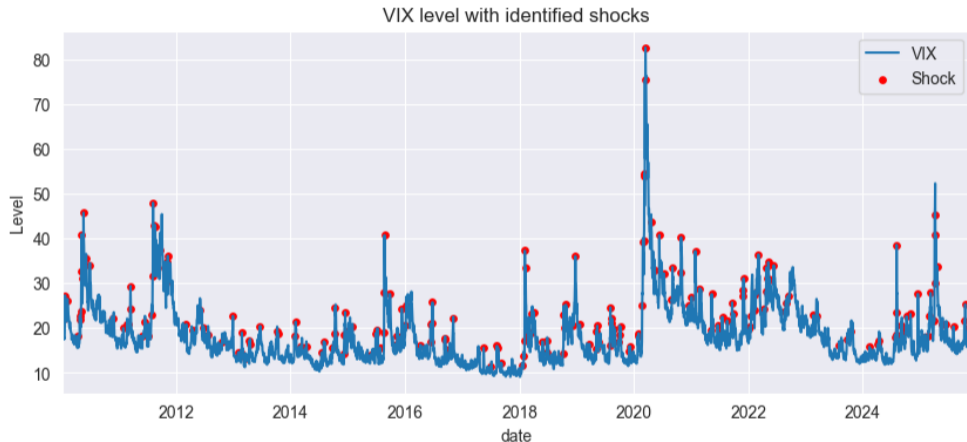
Can we model extreme events as a point process?

③ **Can we forecast VIX volatility?**

Do GARCH-type models beat simple baselines out-of-sample?

- **Source:** Yahoo Finance (ticker ^VIX)
- **Period:** January 2010 – November 2025
- **Observations:** 4,145 business days
- **Pre-processing:**
  - Forward-fill missing dates
  - 0.1% winsorization to limit outlier influence
  - Compute  $\log(\text{VIX})$  and daily log-changes  $\Delta \log(\text{VIX})$

# VIX Time Series



- Red markers indicate identified shock days (top 5% of  $\Delta \log \text{VIX}$ ).

# Volatility Modeling: GARCH vs. EGARCH

## GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

- Symmetric: positive and negative shocks have equal impact.
- Persistence =  $\alpha + \beta$ .

## EGARCH(1,1):

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \alpha (|z_{t-1}| - \mathbb{E}|z|) + \gamma z_{t-1}$$

- Asymmetric via  $\gamma$  (leverage effect).
- Models log-variance  $\Rightarrow$  no positivity constraint.

# Shock Identification & Arrival Process

- ① Define a **shock** as  $\Delta \log(\text{VIX}) \geq 95\text{th percentile}$ .
- ② Model inter-arrival times with:
  - **HPP** (Homogeneous Poisson Process): constant rate  $\lambda$ .
  - **NHPP** (Non-Homogeneous Poisson): time-varying  $\lambda_t$  via Poisson GLM with lagged covariates.
- ③ Covariates are *lagged* to avoid look-ahead bias (e.g., average log VIX from month  $t-1$  predicts shocks in month  $t$ ).

- **Out-of-Sample Design:**

- Train on first 75% of data.
- Monthly rolling re-estimation of GARCH.
- Forecast 1-step-ahead variance into the remaining 25%.

- **Baselines:**

- EWMA ( $\lambda = 0.94$ )
- 63-day rolling variance

- **Metrics:**

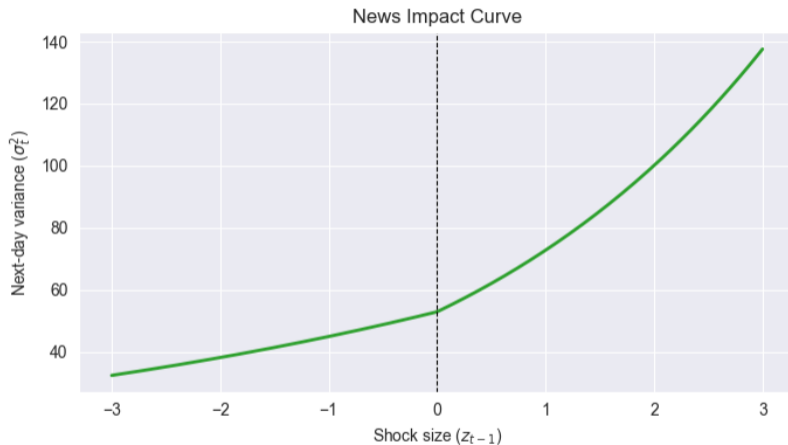
- Log-score (predictive density evaluation)
- 95% coverage rate
- PIT histogram (calibration diagnostic)
- Diebold–Mariano test (statistical significance)

# Volatility Model Comparison

Model	Distribution	AIC	Persistence	Half-life (days)
GARCH(1,1)	GED	27,507	0.852	4.3
EGARCH(1,1)	GED	<b>27,372</b>	0.934	10.8

- EGARCH achieves lower AIC  $\Rightarrow$  better fit.
- Higher persistence in EGARCH  $\Rightarrow$  shocks decay more slowly ( $\approx 11$  days half-life).
- GED distribution selected automatically via PIT uniformity diagnostics.

# News Impact Curve (Asymmetry)

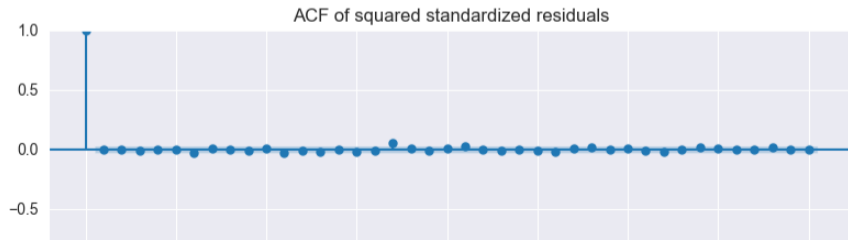
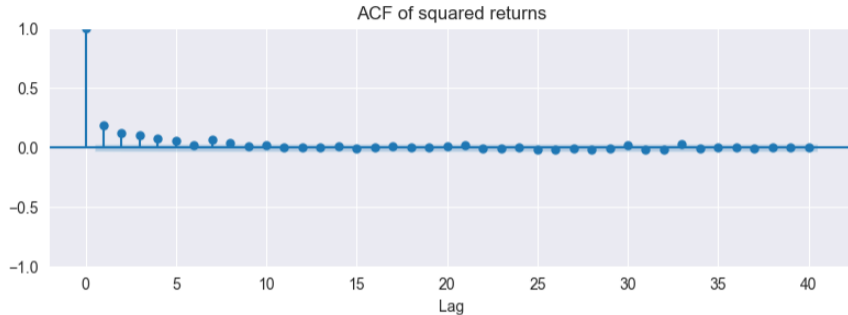


- Positive shocks (VIX spikes) increase future variance more than negative shocks of equal magnitude decrease it.

# Q-Q Plot of Standardized Residuals

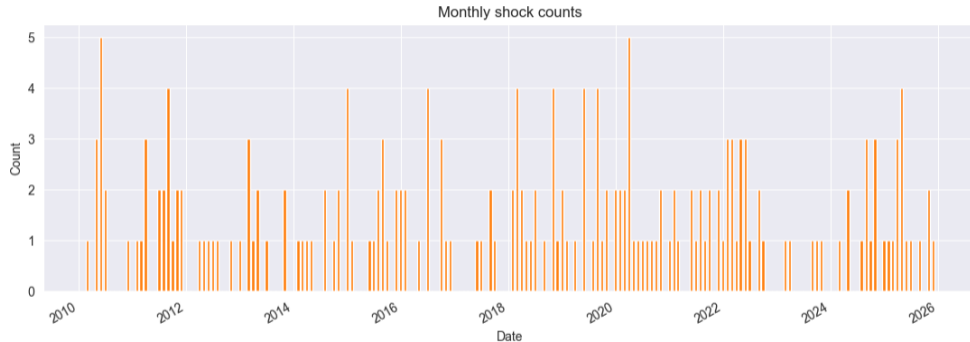


# ACF Comparison: Before & After Filtering



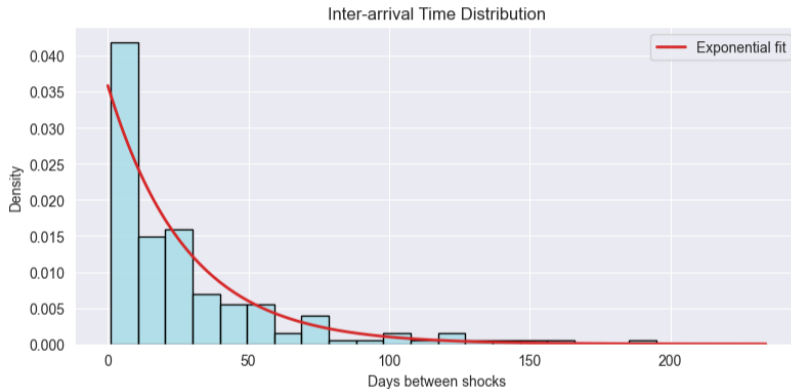
- **Threshold:** 95th percentile = 0.1267
- **Total shocks:** 208 events over 15 years
- **HPP rate:**  $\approx 9$  shocks/year (95% CI: 7.8–10.4)
- **NHPP:** Lagged log VIX coefficient =  $-0.16$ , indicating lower VIX levels predict fewer shocks next month.

# Monthly Shock Counts



- Notable clustering: 2011–12 (Euro crisis), 2018 (Volmageddon), 2020 (COVID), 2022 (rate hikes).

# Inter-Arrival Time Distribution



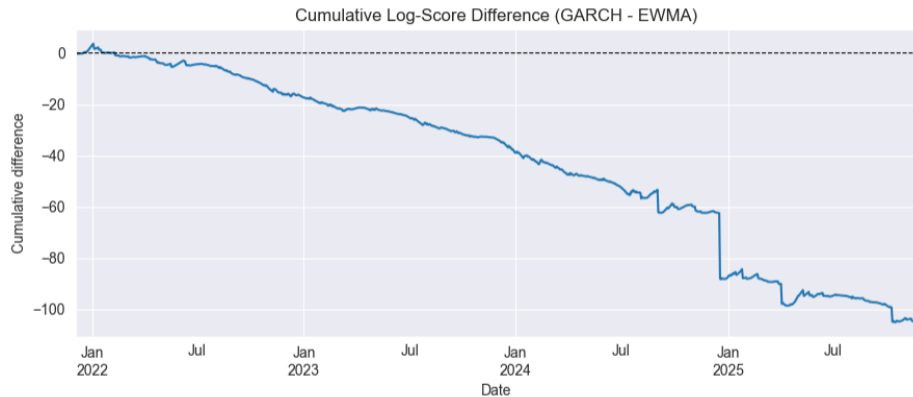
- Histogram vs. exponential PDF: reasonable fit validates Poisson assumption for shock arrivals.

# Forecast Evaluation: Log-Scores

Model	Log-Score (higher = better)
GARCH	1.274
EWMA	<b>1.375</b>
Rolling Var	1.274

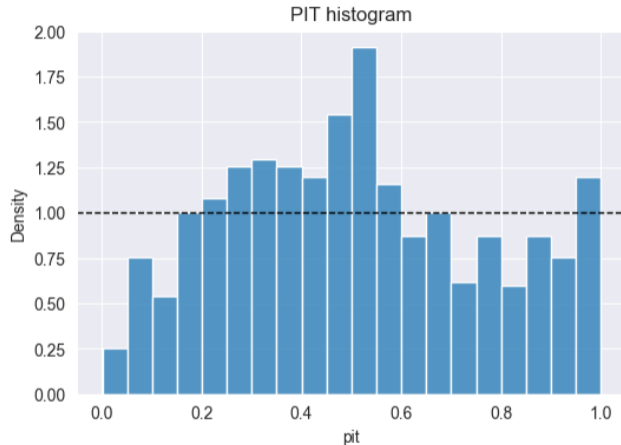
- EWMA slightly outperforms GARCH out-of-sample.
- Diebold–Mariano  $p \approx 0 \Rightarrow$  difference is statistically significant.

# Cumulative Log-Score Difference



- Downward trend: EWMA consistently accumulates better scores.
- GARCH rarely catches up, even briefly.

# PIT Histogram (Calibration)



- Near-uniform distribution indicates well-calibrated density forecasts.
- Mean  $\approx 0.51$ , std  $\approx 0.26$  (ideal: 0.5, 0.29).
- 95% coverage: 95.0% (close to nominal).

# Key Findings

- ➊ **Persistence:** VIX volatility shocks have a half-life of 4–11 days depending on model; EGARCH captures longer memory.
- ➋ **Asymmetry:** Positive shocks increase variance more than negative shocks reduce it (leverage effect confirmed).
- ➌ **Shock Arrivals:**  $\sim 9$  large spikes per year; inter-arrival times follow exponential distribution, supporting Poisson modeling.
- ➍ **Forecasting:** Simple EWMA beats rolling GARCH on log-score, though calibration metrics are acceptable for both.

# Limitations

- GARCH re-estimation is computationally expensive; fixed-parameter rolling used here.
- EWMA's superiority may reflect VIX's strong mean-reversion making elaborate models unnecessary.
- NHPP covariates limited to lagged VIX; macro/sentiment indicators could improve intensity modeling.
- Student-t/GED chosen automatically; skewed distributions not tested.

- Incorporate EGARCH into the OOS forecasting loop.
- Test realized volatility or high-frequency data for improved variance proxies.
- Extend NHPP with external regressors (VIX term structure, credit spreads).
- Deploy model as a real-time monitoring dashboard.

- VIX exhibits **persistent, asymmetric volatility clustering** well captured by EGARCH.
- Large spikes arrive at  $\sim 9$ /year and cluster around macro stress episodes.
- Out-of-sample, **EWMA remains a tough benchmark** to beat for density forecasting.
- The reproducible pipeline (`runall.py`) and diagnostic figures enable transparent, auditable research.

## Thank you!

Questions?

# Appendix: Project Structure

```
Shock-Persistence-and-Shock-Frequency-in-VIX/  
+-- runall.py  
+-- src/  
|   +-- config.py  
|   +-- data_pipeline.py  
|   +-- volatility_models.py  
|   +-- shock_modeling.py  
|   +-- forecast_evaluation.py  
|   +-- visualization.py  
+-- figures/  
+-- notebooks/  
+-- SUMMARY.md  
+-- requirements.txt
```

# Appendix: Key Equations

**Log-Score:**

$$S_t = \log f(r_t \mid \mu_t, \sigma_t^2)$$

**PIT:**

$$u_t = F(r_t \mid \mu_t, \sigma_t^2) \quad \text{should be Uniform}(0, 1)$$

**Diebold–Mariano Statistic:**

$$DM = \frac{\bar{d}}{\sqrt{\widehat{\text{Var}}(\bar{d})}} \xrightarrow{d} N(0, 1)$$

where  $d_t = L_t^{(A)} - L_t^{(B)}$  is the loss differential.

# Appendix: References

- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*.
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- Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics*.
- Whaley, R. E. (2000). The investor fear gauge. *Journal of Portfolio Management*.