

Shock Persistence and Shock Frequency in VIX

A Quantitative Analysis of Volatility Dynamics

CHONG Tin Tak (20920359)

CHOI Man Hou (20894196)

Vittorio Prana CHANDREAN (20896895)

HKUST - IEDA4000E

November 27, 2025

Outline

1 Introduction

2 Data

3 Methodology

4 Results

5 Discussion

6 Conclusion

What is VIX?

- **VIX** = CBOE Volatility Index, derived from S&P 500 option prices.
- Often called the “fear gauge” — rises when markets expect turbulence.
- Understanding VIX dynamics is crucial for:
 - Risk management and hedging
 - Derivatives pricing
 - Portfolio allocation

Research Questions

① **How persistent is volatility?**

How long does a VIX shock take to decay?

② **How frequently do large spikes occur?**

Can we model extreme events as a point process?

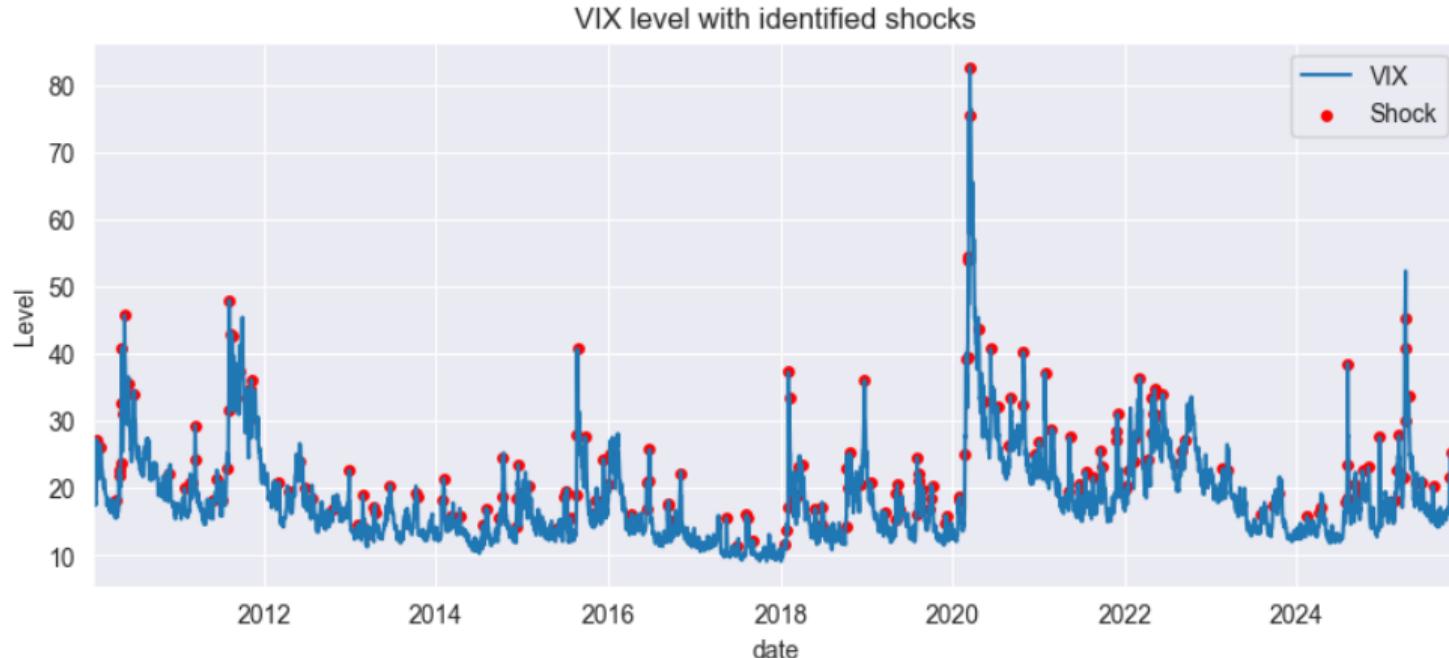
③ **Can we forecast VIX volatility?**

Do GARCH-type models beat simple baselines out-of-sample?

Data Overview

- **Source:** Yahoo Finance (ticker ^VIX)
- **Period:** January 2010 – November 2025
- **Observations:** 4,145 business days
- **Pre-processing:**
 - Forward-fill missing dates
 - 0.1% winsorization to limit outlier influence
 - Compute $\log(\text{VIX})$ and daily log-changes $\Delta \log(\text{VIX})$

VIX Time Series



- Red markers indicate identified shock days (top 5% of $\Delta \log VIX$).

Volatility Modeling: GARCH vs. EGARCH

GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

- Symmetric: positive and negative shocks have equal impact.
- Persistence = $\alpha + \beta$.

EGARCH(1,1):

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \alpha(|z_{t-1}| - \mathbb{E}|z|) + \gamma z_{t-1}$$

- Asymmetric via γ (leverage effect).
- Models log-variance \Rightarrow no positivity constraint.

Shock Identification & Arrival Process

- ① Define a **shock** as $\Delta \log(\text{VIX}) \geq 95\text{th percentile}$.
- ② Model inter-arrival times with:
 - **HPP** (Homogeneous Poisson Process): constant rate λ .
 - **NHPP** (Non-Homogeneous Poisson): time-varying λ_t via Poisson GLM with lagged covariates.
- ③ Covariates are *lagged* to avoid look-ahead bias (e.g., average log VIX from month $t-1$ predicts shocks in month t).

- **Out-of-Sample Design:**

- Train on first 75% of data.
- Monthly rolling re-estimation of GARCH.
- Forecast 1-step-ahead variance into the remaining 25%.

- **Baselines:**

- EWMA ($\lambda = 0.94$)
- 63-day rolling variance

- **Metrics:**

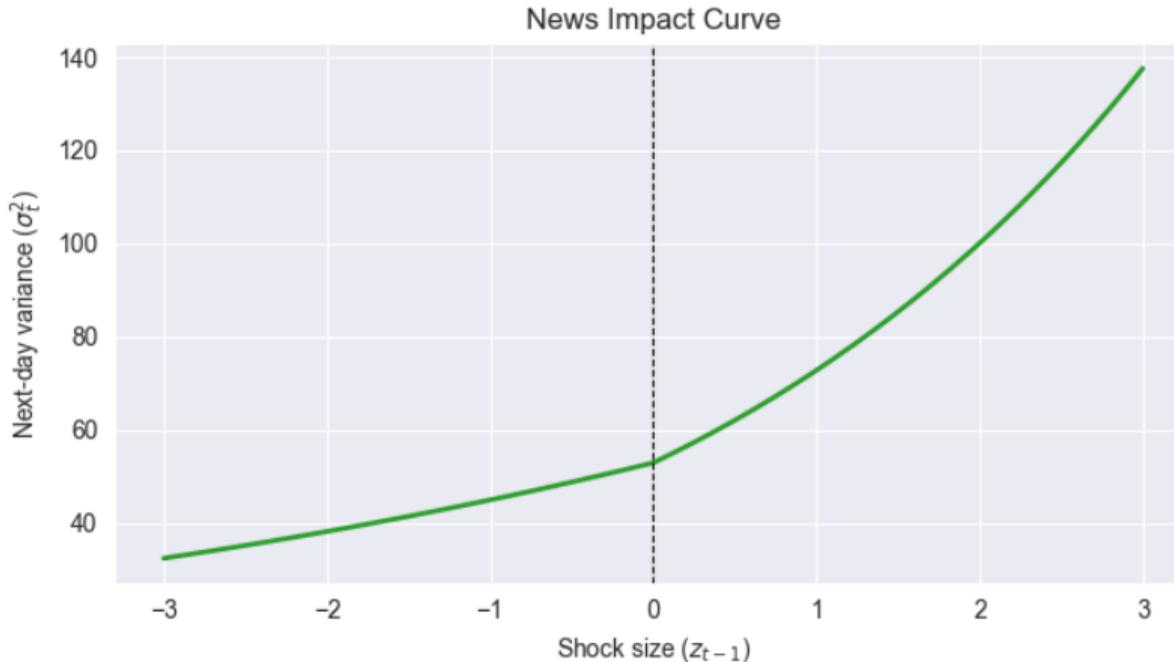
- Log-score (predictive density evaluation)
- 95% coverage rate
- PIT histogram (calibration diagnostic)
- Diebold–Mariano test (statistical significance)

Volatility Model Comparison

Model	Distribution	AIC	Persistence	Half-life (days)
GARCH(1,1)	GED	27,507	0.852	4.3
EGARCH(1,1)	GED	27,372	0.934	10.8

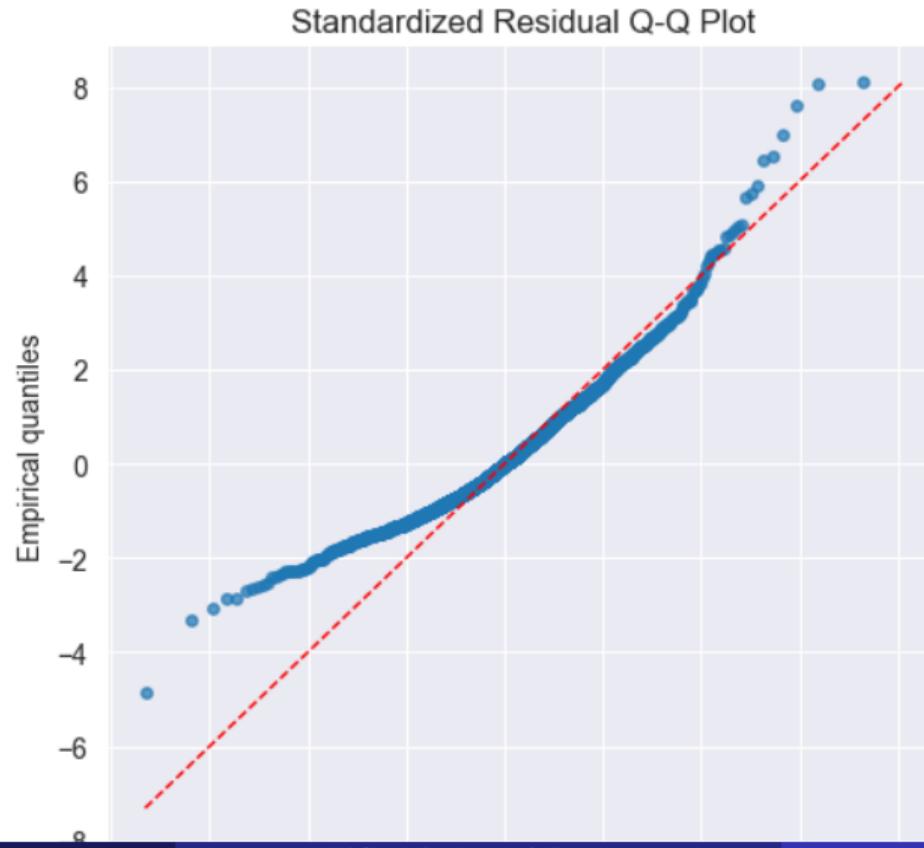
- EGARCH achieves lower AIC \Rightarrow better fit.
- Higher persistence in EGARCH \Rightarrow shocks decay more slowly (≈ 11 days half-life).
- GED distribution selected automatically via PIT uniformity diagnostics.

News Impact Curve (Asymmetry)

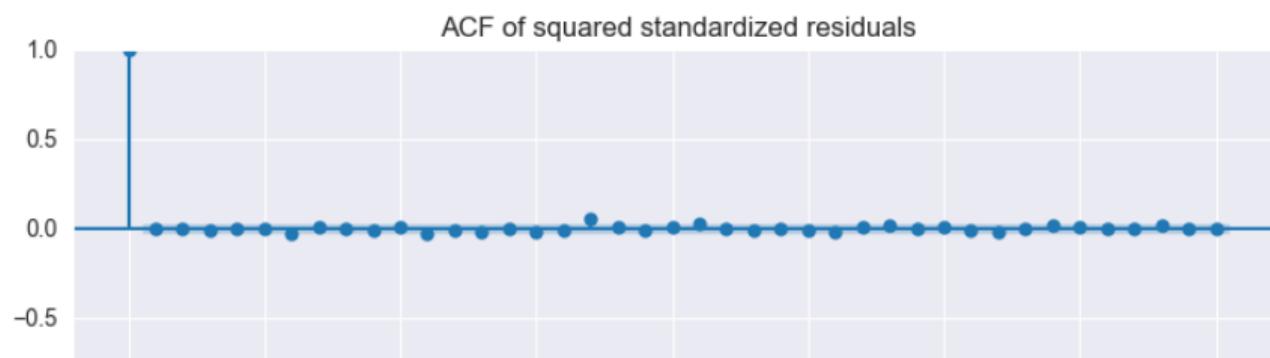
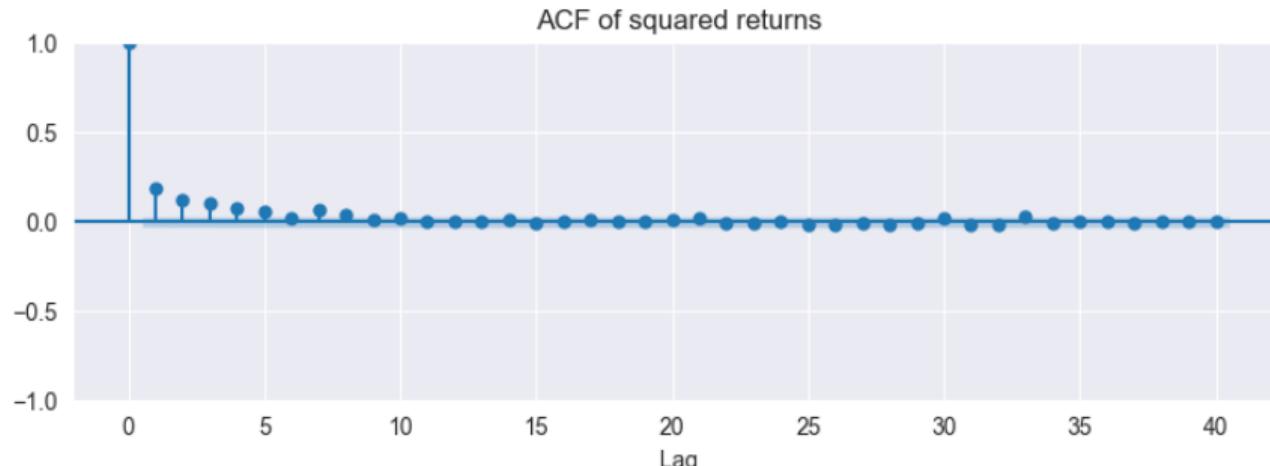


- Positive shocks (VIX spikes) increase future variance more than negative shocks of equal magnitude decrease it.

Q-Q Plot of Standardized Residuals



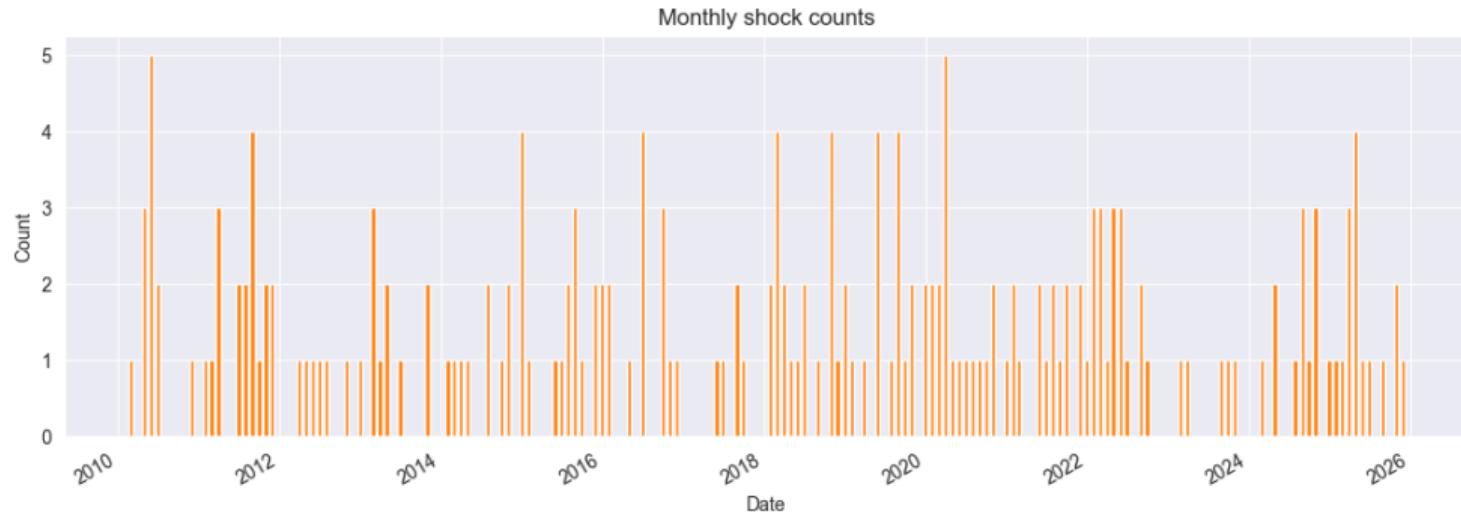
ACF Comparison: Before & After Filtering



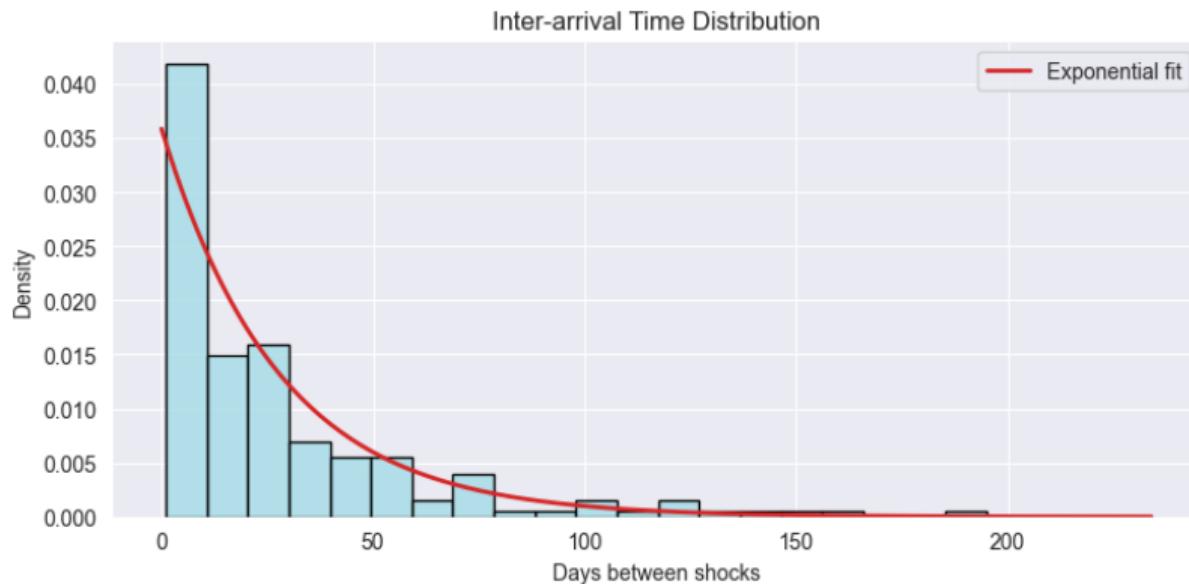
Shock Statistics

- **Threshold:** 95th percentile = 0.1267
- **Total shocks:** 208 events over 15 years
- **HPP rate:** \approx 9 shocks/year (95% CI: 7.8–10.4)
- **NHPP:** Lagged log VIX coefficient = -0.16 , indicating lower VIX levels predict fewer shocks next month.

Monthly Shock Counts



Inter-Arrival Time Distribution



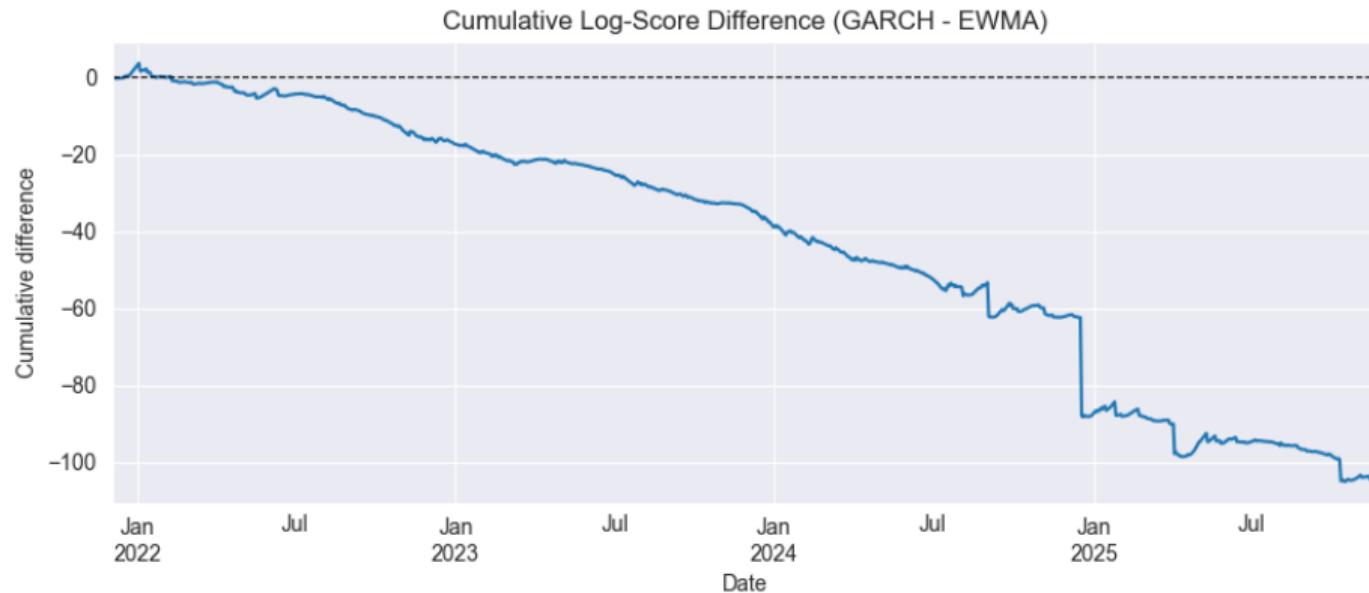
- Histogram vs. exponential PDF: reasonable fit validates Poisson assumption for shock arrivals.

Forecast Evaluation: Log-Scores

Model	Log-Score (higher = better)
GARCH	1.274
EWMA	1.375
Rolling Var	1.274

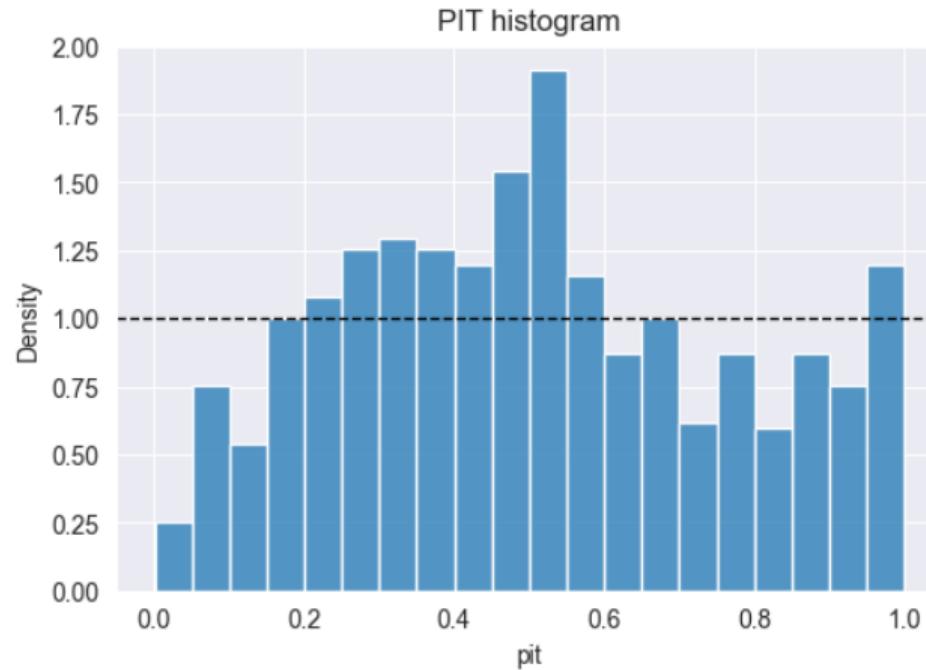
- EWMA slightly outperforms GARCH out-of-sample.
- Diebold–Mariano $p \approx 0 \Rightarrow$ difference is statistically significant.

Cumulative Log-Score Difference



- Downward trend: EWMA consistently accumulates better scores.
- GARCH rarely catches up, even briefly.

PIT Histogram (Calibration)



- Near-uniform distribution indicates well-calibrated density forecasts.
- Mean ≈ 0.51 , std ≈ 0.26 (ideal: 0.5, 0.29).
- 95% coverage: 95.0% (close to nominal).

Key Findings

- ① **Persistence:** VIX volatility shocks have a half-life of 4–11 days depending on model; EGARCH captures longer memory.
- ② **Asymmetry:** Positive shocks increase variance more than negative shocks reduce it (leverage effect confirmed).
- ③ **Shock Arrivals:** ~9 large spikes per year; inter-arrival times follow exponential distribution, supporting Poisson modeling.
- ④ **Forecasting:** Simple EWMA beats rolling GARCH on log-score, though calibration metrics are acceptable for both.

Limitations

- GARCH re-estimation is computationally expensive; fixed-parameter rolling used here.
- EWMA's superiority may reflect VIX's strong mean-reversion making elaborate models unnecessary.
- NHPP covariates limited to lagged VIX; macro/sentiment indicators could improve intensity modeling.
- Student-t/GED chosen automatically; skewed distributions not tested.

Future Work

- Incorporate EGARCH into the OOS forecasting loop.
- Test realized volatility or high-frequency data for improved variance proxies.
- Extend NHPP with external regressors (VIX term structure, credit spreads).
- Deploy model as a real-time monitoring dashboard.

- VIX exhibits **persistent, asymmetric volatility clustering** well captured by EGARCH.
- Large spikes arrive at $\sim 9/\text{year}$ and cluster around macro stress episodes.
- Out-of-sample, **EWMA remains a tough benchmark** to beat for density forecasting.
- The reproducible pipeline (`runall.py`) and diagnostic figures enable transparent, auditable research.

Thank you!

Questions?

Appendix: Project Structure

```
Shock-Persistence-and-Shock-Frequency-in-VIX/
+-- runall.py
+-- src/
|   +-- config.py
|   +-- data_pipeline.py
|   +-- volatility_models.py
|   +-- shock_modeling.py
|   +-- forecast_evaluation.py
|   +-- visualization.py
+-- figures/
+-- notebooks/
+-- SUMMARY.md
+-- requirements.txt
```

Appendix: Key Equations

Log-Score:

$$S_t = \log f(r_t | \mu_t, \sigma_t^2)$$

PIT:

$$u_t = F(r_t | \mu_t, \sigma_t^2) \quad \text{should be Uniform}(0, 1)$$

Diebold–Mariano Statistic:

$$\text{DM} = \frac{\bar{d}}{\sqrt{\widehat{\text{Var}}(\bar{d})}} \xrightarrow{d} N(0, 1)$$

where $d_t = L_t^{(A)} - L_t^{(B)}$ is the loss differential.

Appendix: References

- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*.
- Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*.
- Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics*.
- Whaley, R. E. (2000). The investor fear gauge. *Journal of Portfolio Management*.