# Bayesian biomass dynamic model with kobe advice: the **bdm** and **kobe** packages

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## 1 The **bdm** package

The package fits a state space biomass dynamic model using Bayesian methods, specifically the Hamiltonian MCMC implemented in the package **rstan**. To a large extent, the package is an external wrapper for **rstan**, providing functionality relevant to the intended application. The package is generalisable, meaning that any number of model formulations can be specified by the user. The default implements the Fletcher-Schaefer hybrid model [1], formulated in terms of the depletion $x$, which is the biomass relative to the carrying capcity $K$ (equation 1b). The hybrid model is generic, in the sense that it allows a range of values for the depletion at MSY, or shape of the production curve relative to the biomass depletion. This is specified by the parameter $\phi$, which is given as an input value with a default of $\phi = 0.5$ (i.e. $n = 2$), making it equivalent to the logistic model.

$$x_{t+1} = x_t + g(x_t) - \frac{C_t}{K} \tag{1a}$$

$$g(x_t) = \begin{cases} r.x_t. \left(1 - \dfrac{x_t}{2\phi}\right) & \text{if } x \leq \phi \\ \gamma.m.\left(x_t - x_t^n\right) & \text{if } x > \phi \end{cases} \tag{1b}$$

$$\phi = \left(\frac{1}{n}\right)^{(1/(n-1))} \tag{2a}$$

$$\gamma = \frac{n^{n/(n-1)}}{n-1} \tag{2b}$$

$$m = \frac{r\phi}{4} \tag{2c}$$

## 1.1 Estimation framework

Parameters are estimated within a Bayesian state-space framework. This re-formulates the process equation to include a time-dependent error term (the process error, $\epsilon^p$) and a parallel observation process that relates an abundance index $I$ to the unobserved biomass state with some degree of error (the observation error, $\epsilon^o$), according to an estimated catchability scalar $q$.

$$x_{t+1} = [x_t + g(x_t) - H_t] \, . \epsilon_t^p \tag{3a}$$

$$I_{it} = [q_i x_t] . \epsilon_{it}^o \tag{3b}$$

The advantage of this class of models is that they allow both process and observation error to be represented simultaneously, which is important for effective precautionary or risk based management [2].

Parameters to be estimated in the model are therefore: $r$, $K$, $q$ for each index, and the error terms. The large number of parameters necessitates a Bayesian approach with appropriate priors. Parametric distributional assumptions for $\epsilon^p$ and $\epsilon^o$ are required, which unfortunately cannot be estimated in an hierarchical manner with fisheries data. We assume them to follow a log-normal distribution with an expectation of one, and therefore fix values of $\sigma_p$ and $\sigma_o$ on input, based on a subjective measure of model fit to the data, with minimum bounds $\sigma_p \geq 0.05$ and $\sigma_o \geq 0.15$.

$$r \sim LN(\mu_r, \sigma_r^2) \tag{4a}$$

$$ln(K) \sim U(.,.) \tag{4b}$$

$$\epsilon_{\cdot}^p \sim LN(-\sigma_p^2/2, \sigma_p^2) \tag{4c}$$

$$\epsilon_{\cdot\cdot}^o \sim LN(-\sigma_o^2/2, \sigma_o^2) \tag{4d}$$

The $r$ and $K$ parameters of the logistic model are highly correlated, and their estimation is helped through the use of an informative prior or priors. We assumed an uninformative log-uniform prior for $K$, but an informative log-normal prior for $r$. The expectation and variance for the prior on intrinsic growth, with $E[r] = \exp(\mu_r + \sigma_r^2/2)$ can be constructed from available life-history data using the function `bdm::rcalc`, which implements methods described by McAllister et al. [3].

Log-normal prior distributions for the error terms are specified to have an expection of one, which gives an intuitive interpretation of the expected quantities.

$$E[x_{t+1}] = x_t + g(x_t) - H_t \tag{5a}$$

$$E[I_{it}] = q_i x_t \tag{5b}$$

The catchability $q$ is estimated analytically from its maximum posterior density estimate assuming an uninformatinve uniform prior (i.e. $q \sim U(.,.)$).

$$\hat{q}_i = \exp\left[\frac{1}{n_t} \sum_t \{ln(I_{it}) - ln(x_t)\} + \frac{\sigma_o^2}{2}\right] \tag{6}$$
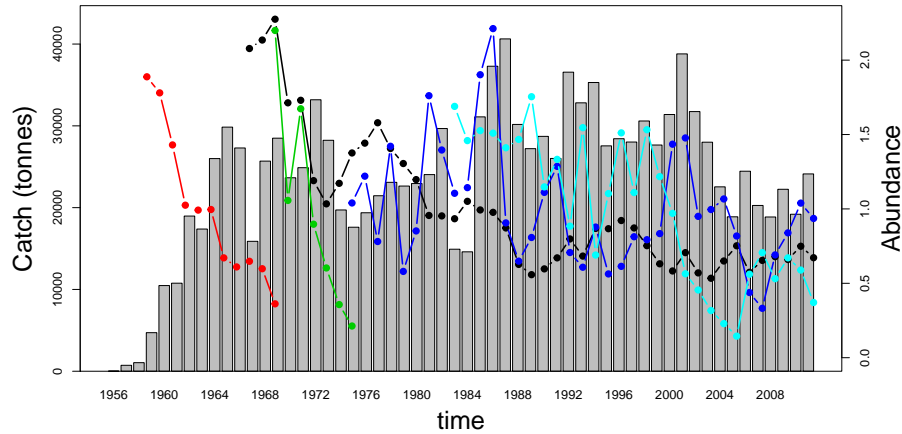
**Figure 1:** Data intputs for South Atlantic albacore

# 2 Application to albacore

## 2.1 Bayesian fit

We fit the default (logistic) model to albacore data (figure 1) from the South Atlantic. Trace and histogram plots for the posterior samples are shown in figures 2 and 3, and fits to the data in figure 4.

```
# load data into empirical data object
data(albsa)
dat <- edat(harvest = albsa$catch_tonnes, index = cbind(fleet1 = albsa$fleet1_cpue,
    fleet2 = albsa$fleet2_cpue, fleet3 = albsa$fleet3_cpue, fleet4 = albsa$fleet4_cpue,
    fleet8 = albsa$fleet8_cpue), time = rownames(albsa))

# initialise model
mdl <- bdm()

# update error terms and priors using values from Meyer and
# Millar (CJFAS 1999)
sigmao(dat) <- sqrt(0.0086/(1.71 - 1))
sigmap(dat) <- sqrt(0.0102/(3.79 - 1))
# intrinsic growth rate
mdl <- update_bdm(mdl, list(a = -1.38, b = 0.51, par = "r"))

# compile model
mdl <- compile_bdm(mdl)

# fit model
mdl <- fit(mdl, dat, iter = 2000)

traceplot(mdl, par = c("r", "logK"))
```
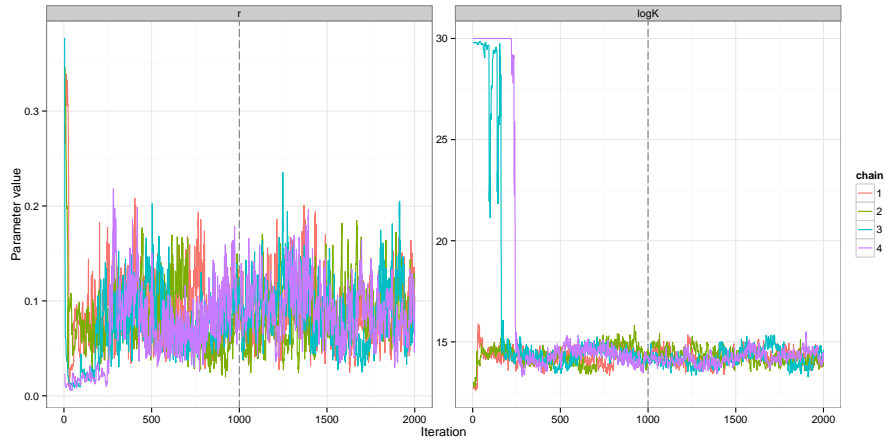
**Figure 2:** Trace plots for $r$ and $ln(K)$

```
histplot(mdl, par = c("r", "logK"))
```

## 2.2  Kobe advice

It is possible to produce kobe plots by using the `bdm::as.kobe` function, which creates a `data.frame` in the appropriate format for the **kobe** package. The kobe phase plot for albacore is shown in figure 5. We also perform some simple projections using `bdm::project` and plot the distribution of values from the final year in figure 6.

```
# projections
harvest.time <- 30
harvest.scenarios <- c(0.01, 0.05, 0.1)
mdl.projections <- project(mdl, harvest.scenarios, harvest.time,
    harvest_rate = TRUE)
```
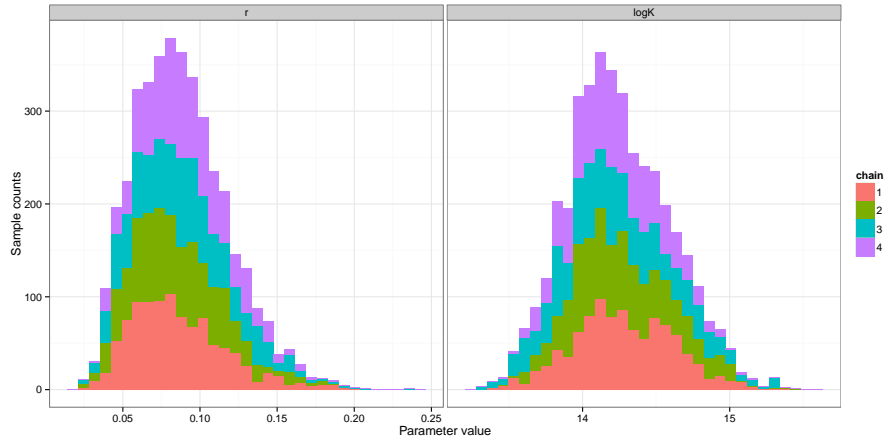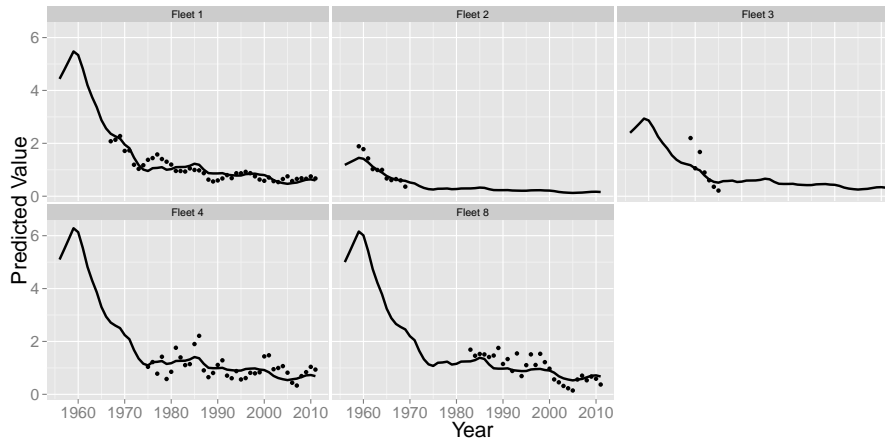
**Figure 3:** Posterior histograms for $r$ and $ln(K)$



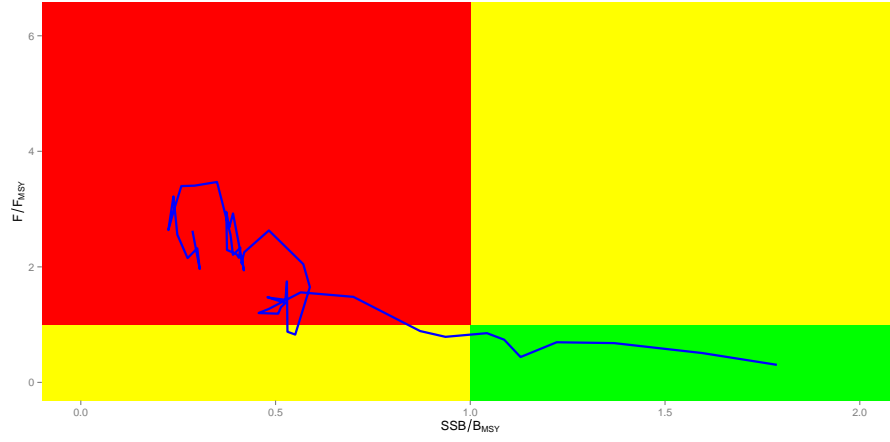**Figure 4:** observed and predicted abundance indices for each fleet

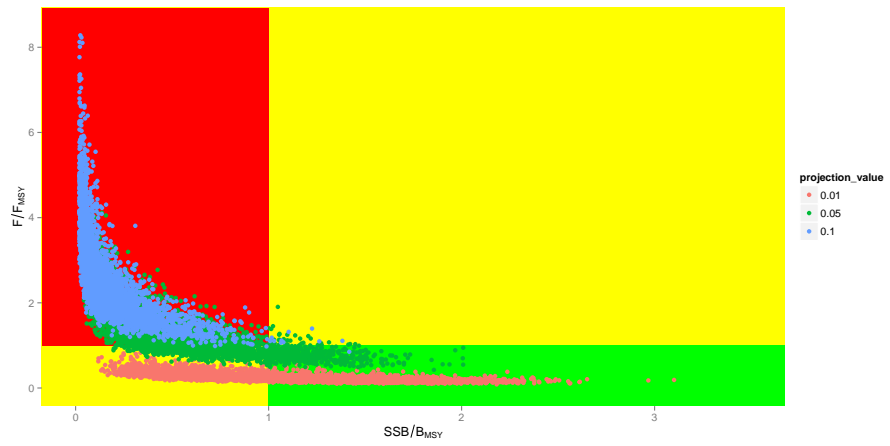**Figure 5:** Kobe trace of stock trajectory



**Figure 6:** Distribution of posterior biomass samples in the final year of the projection

# References

[1] M. K. McAllister, E. A. Babcock, E. K. Pikitch, and M. H. Prager. Application of a non-equilibrium generalized production model to south and north atlantic swordfish: combining bayesian and demographic methods for parameter estimation. *Collected Volume of Scientific Papers ICCAT*, 51(5):1253–1550, 2000.

[2] J. Harwood and K. Stokes. Coping with uncertainty in ecological advice: lessons from fisheries. *Trends in Ecology & Evolution*, 18(12):617–622, 2003.

[3] M. K. McAllister, E. K. Pikitch, and E. A. Babcock. Using demographic methods to construct bayesian priors for the intrinsic rate of increase in the schaefer model and implications for stock rebuilding. *Canadian Journal of Fisheries and Aquatic Sciences*, 58(9):1871–1890, 2001.