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A Model of Economic Evolution across Kondratieff Long Wave Cycles

Macroeconomic Agent Based Simulation

Using a wide range of papers to describe multiple facets of an overall economic system, this paper attempts to generate endogenously cyclicality, with varying levels of technology, through the asymmetric behavior of heterogeneous agents. It makes use of an ABS – Agent Based Simulation (yet to be fully programmed in Python) which aggregates the micro founded decisions of individual agents and firms. Ultimately, through myopic and backward-looking optimization decisions, from the behavior of agents, some noticeable macroeconomic patterns emerge. Most importantly, it sheds light on the formation of business cycles, elucidates a hidden relationship behind the Philips curve, and describes some effects of major innovations on markets and the overall economy.

Ciprian Tudor

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1. Introduction

The current state of the world economy is disappointing to say the least. While the most "developed" countries hide behind erroneous unemployment numbers (because zero-hour contracts and partial employment disguise the startling reality); their lack of investment in much needed infrastructure like education, healthcare, or transportation erodes the human capital of its population and jeopardizes a future of equitable and sustainable growth. At the same time, succumbing to avaricious lobbying efforts, governments put in place ludicrous protectionist policies, such as America's overpowering subsidies to the few already rich farmers in control of agri-business. This hinders the growth of developing countries by transforming too many of their products into virtually uncompetitive goods. Faced with all this, what we need is a clear view of how capitalist economies develop, starting from a base understanding of what its constituents need to grow and thrive. Only then can the right course of action, accounting for the needs of the many, be wriggled from the confusion that macroeconomic thought has currently become.

In this paper, I discuss how Kondratieff Cycles, the longest-term type of growth cycle indicative of a 50 year rhythm to the introduction of major innovations in markets, can arise from the individual, asymmetric decisions of workers and firms. In the Literature Review section, I go over a range of macroeconomic patterns that, when understood, paint an overall picture of the relevant interacting parts of economic systems which generate cyclicality. Firstly, I discuss different types of growth cycles and the choice to model the Kondratieff Cycles themselves. Next, I go over the benefits of adopting the view of certain parts of a Marxist framework, at the same time justifying how economic cycles can be deterministic and giving an example of similar ABS work. Lastly, I discuss information needed to form an understanding of how technological advancement and certain macroeconomic variables can be modelled.

In the Methodology section, I present the model as well justifications for its formulation which are based on the ideas explored in the Literature Review. Moreover, I summarize the up-to-date code work being done, with the project being almost fully implemented into Python.

In the final section, I analyze the model to reach various conclusions about an economy's cyclicality, the relationship of the Philips curve, and the impact of major innovations being introduced in the market. From this, policy prescriptions are provided which are aimed at decreasing the magnitude of an economic crisis and increasing the magnitude of an economic boom, ideally shifting the way in which cycles occur. This point in particular refers to the idea that once we fully understand how business cycles are manifested, we become aware of the variables which cause it and hence can affect these variables to in turn change the way economic growth transpires.

2. Literature Review

2.1 Types of Growth Cycles

When beginning to analyze the cyclicality of an economy, it is important to realize that its development over time can take multiple forms. As Reijinders (2009)[1] documents, and Korotayev, Tsirel (2010)[2] further describes, several cyclical patterns of the aggregate economy can be distinguished:

Cycle	Duration	Potential Cause
Kitchin	3 – 5 years	"time lags in information movements affecting the decision making of commercial firms"[3] (market information asymmetries)
Juglar	7 – 12 years	Patterns of investment in fixed capital
Kuznets	15 – 25 years	Demographic swings or Infrastructural investment cycles
Kondratieff	40 – 60 years	Introduction of major technical innovations in markets

In fact, it was Schumpeter (1939)[4] who initially theorized that "barring very few cases in which difficulties arise, it is possible to count off, historically as well as statistically, six Juglars to a Kondratieff and three Kitchins to a Juglar – not as an average but in every individual case".

Besides the different ranges these cycles, there are generally two major discussion points regarding all types of economic cycles: the first debates their existence, whereas the second their cause (endogenous or exogenous). With reference to the first, some economists "deny the presence of any economic cycles altogether, as the title of a respective section in a classical Principles of Economics textbook by N. Gregory Mankiw showcases – "Economic Fluctuations Are Irregular and Unpredictable" [Mankiw 2008: 740]". But, other economists such as DeGroot and Franses (2007)[5] explicitly make the point that "economies of industrialized countries show cyclical patterns. Recessions since WWII seem to emerge every 8 to 10 years, which is usually associated with the business cycle, and long swings like the well-known 55 year Kondratieff cycle can be observed for a variety of variables." For our purposes, this paper takes the view that while cycles might in fact be "irregular and unpredictable", the wealth of data accounting for their historical existence is hard to refute. Further, while real-life outcomes, often muddled by too many variables even the smartest economists cannot account for, are hard to pinpoint, a more regulated theoretical model can shed some light on economic cycle formation. In other words, what we see in the real-world is hard to comprehend as an overall cohesive result, due to the countless caveats that arise unsuspected, but that does not mean that underlying cycles do not manifest themselves. Moreover, even if we cannot fit data to the length of the above-mentioned cycles as precisely as we hoped to, this does not mean

[1] Reijinders (2009), *Trend movements and inverted Kondratieff waves in the Dutch economy, 1800–1913*
[2] Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*
[3] Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*
[4] Schumpeter (1939), *BUSINESS CYCLES: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*
[5] DeGroot and Franses (2007), *Stability through Cycles*

that understanding the theoretical causes for the cycles will not yield pertinent economic knowledge (from which policy prescriptions may arise).

Further, another debate emerging from these observations is more concerned with exogeneity. Namely, do these cyclical patterns arise out of possible pre-existing conditions that accumulate over time resulting in a shock, or, are they the result of unforeseeable forces? “Roughly speaking, there are two views on the presence of one or more economic cycles. The first is that cycles are caused by shocks that are exogenous and largely unpredictable. […] A second view on the presence of economic cycles […] is that, loosely speaking, there have always been multiple cycles and there always will be. Hence, these cycles are not fully stochastic and caused by external shocks, but are in fact partly deterministic.” This strikes at the heart of what the paper at hand attempts to do: elucidate the possible deterministic causes for economic cycles in hope to understand how economic evolution transpires. The choice of addressing the longest-ranging Kondratieff cycles stems simply out of a better understanding of technological propagation and an ease of modelling this type of situation.

2.2 Marxist Framework and ABS

Professor Shibata […] contends that the theory of general economic equilibrium, which has received its most precise and complete formulation in the works of the School of Lausanne, "is ineffectual in making clear systematically either the organisation of present-day capitalistic society or the laws of its development ", while the Marxian political economy, " though it is now shown to contain many defects, sets forth theories which are either intended to enunciate systematically the organisation of present-day capitalistic society and the laws governing its development, or have inseparable and necessary bearings on them."[6]

To analyze how these cycles, and in particular the Long-Waves may be formed, it is perhaps most useful to view the economy through some particular parts of the Marxist framework. While many principles of this such as the Labor Theory of Value or the development of capital progression are not fully accepted by economists or taught at universities, the world-view one gains from addressing variables in such a way is invaluable given the paradigm-shifting level of technological progress we are currently witnessing. As Chakravarty (1987)[7] so pertinently emphasizes, “Marx was a very keen student of technology” and so in his writings, attempted to understand what was happening around him from the point of view of the unrelenting technological progress that capitalism is accompanied by.

Moreover, as Lange (1935)[8] argues, with surprising pertinence to our contemporary issues, “the Marxist's claim to superiority […] is that […] economics has utterly failed to explain the fundamental tendencies of the development of the Capitalist system. These tendencies are:

[6] Lange (1935), *Marxian Economics and Modern Economic Theory*
[7] Chakravarty (1937), *Marxist economics and contemporary developing economies*
[8] Lange (1935), *Marxian Economics and Modern Economic Theory*

1. the constant increase of the scale of production (which has led to the present monopolistic / oligopolistic market structures in Capitalism)
2. the substitution of interventionism and "planning" for laisser-fair (which deregulation policy in part arising from lobbying efforts has compromised the stability of the economy)
3. the transition from free trade to high protectionism and economic nationalism in international relations (a fascinating prediction in line with current world trends)
4. the constant expansion of the capitalist method of production in non-capitalist countries
5. increase of economic instability in the capitalist system (the main study point of the paper at hand)

It is perhaps disappointing that almost a century onwards from this writing, capitalist societies are still plagued by issues one should have perhaps already found solutions to. Talking of how “experts” deal with the situation, “many of them denied this development until the phenomena apparently became so overwhelming as to be familiar to anybody but the professional economist who was always the last to recognise their existence”. “Thus the Marxian claim that "bourgeois" economists failed to grasp the fundamental tendencies of the evolution of the Capitalist system proves to be true”, and in fact, continues to be proven true with every crisis of increased magnitude collapsing atop us. “On the other hand, Marxian economics must be admitted to have anticipated these tendencies correctly and to have developed a theory which investigates the causal mechanism of this evolution and thus shows its inevitability”. Nevertheless, this isn’t to say that one should wholeheartedly accept the many (and often erroneous) premises that Marxist economics puts forth: “this superiority of Marxian economics is only a partial one. There are some problems before which Marxian economics is quite powerless, while "bourgeois" economics solves them easily.” Here, bourgeois economics refers to the trends precipitating through the western thought led by the likes of Pareto, Marshall, Ricardo, etc. Ultimately, as the Romans have realized long ago, but many more analysts disregard in their one-sided use of theories that fit their world view, “in medio stat virtus” – virtues stand in the middle. This phrase, one of my personal favorites, is indicative of the idea that a complete (and hopefully correct) understanding of what is going on around us can only be achieved by considering worthy ideas from all economic fronts.

In particular, as Lange goes on to discuss, the merits that can be subtracted from both schools of thought is that on one hand, the so called “bourgeois” economics is well-fitted in grasping ”the phenomena of the every-day life of a capitalist economy”, whereas Marxist economics is better suited towards understanding “the economic evolution of capitalist society into a consistent theory”. Specifically, “the anticipation of the future course of events deduced from the Marxian theory is not a mechanical extrapolation of a purely empirical trend, but an anticipation based on the recognition of a law of development”.

Moreover, typical economists are of the idea that the phenomena occurring across the aggregate economy are in fact too complicated to be accounted for by a single set of principles. To this, I would like to point them in the direction of a new trend emerging in macroeconomic analysis: agent-based simulations. Given the increasing powers of calculation our technology equips us with it, we have perhaps reached a point in time when even the most convoluted, interdependent, and previously incomputable relations can be modeled with relative ease. While the formulation of present-day ABSs may appear rough, as any fledgling field tends to be, it has clearly been shown by e.g. Riccetti, Russo, and Gallegati (2012)[9] that using computer simulations to account for these "too complicated phenomenae", fascinating results arise. As they discuss, "macroeconomic properties emerge such as endogenous business cycles, nominal GDP growth, unemployment rate fluctuations, the Phillips curve, leverage cycles and credit constraints, bank defaults and financial instability, and the importance of government as an acyclical sector which stabilize the economy. The model highlights that even extended crises can endogenously emerge." Taking a step back to consider this for a moment, one should be nothing short of amazed at how replicable our economic woes and successes can be. Keep in mind, this was done without the ludicrous assumption of homo economicus attempting to optimize an expected discounted future stream of income for his entire lifetime, but through single-minded rules of thumb that are more representative of what people in current society might use for their economic decision-making.

Grounding the discussion back into our study of economic cyclicality, at the time of Lange's writing, "if the contribution of Marxian and of "bourgeois" economics to the theory of the business cycle is considered […] neither of them can give a complete solution of the problem." With reference to current economic thought, while we are able to understand the impacts of a disturbance to equilibrium, and address the steps that must be taken to follow an adjustment process back to equilibrium, our thoughts fail in elucidating *why* these disturbances occur in a regular fashion. Disappointingly, they are most often assumed to be exogenous and inexplicable.

As such, what parts of Marxist economics can ease our understanding of economic evolution? Most importantly, Lange emphasizes, is that "the corner-stone of the Marxian analysis of Capitalism, is the division of the population into two parts, one of which owns the means of production while the other owns only labour power." This is a powerful concept to keep in mind as it is indicative of a class conflict which has the power to generate many of the cyclical dynamics that give rise to business fluctuations. For example, a beatifically expressed view of the world comes from Goodwin (1965, 2014)[10], whose formulation of "A Growth Cycle" makes use of the Lotka-Volterra predator-prey equations ("Theorie Mathematique de la Lutte pour la Vie", 1931) to describe the conflicting incomes of workers and capitalists. His explanation helps explain the following story:

"When profit is greatest, employment is average, and the high growth rate pushes employment to its maximum which squeezes the profit rate to its average value. The deceleration in growth lowers employment (relative) to its average value again, where profit and growth are again at their nadir. This low growth rate leads to a fall in output and employment to well below full employment, thus restoring profitability to its average value because productivity is now rising faster than wage rates. This is, I believe, essentially what Marx meant by the contradiction of capitalism and its transitory resolution in booms and slumps."

Overall, I would like to point out the fact that a real world economy is rarely (If ever) seen "in equilibrium". More precisely, it is always in a movement towards or away from equilibrium, with equilibrium occurring at a single moment in time before endogenous forces push it away down a

[9] Riccetti, Russo, and Gallegati (2012), *An Agent Based Decentralized Matching Macroeconomic Model*
[10] Goodwin (1965, 2014), *A Growth Cycle*

path of lengthy readjustment once again. Furthermore, in his assumptions, Goodwin takes a "steady technological progress" as given and exogenous. In reality, it is likely that the magnitude of technological progress (i.e. importance of a new innovation being brought to markets, or, how big a paradigm shift they provide) will affect the corresponding size of the boom and bust arising out of it. As such, in better understanding the cyclicality of economic evolution, it is worth discussing the means by which technological advancement is propagated.

2.3 Technological Advancement

Since a generous body of literature discusses many facets of technological advancement as it impacts the science of economics, I will specifically restrict this discussion to exploring what causes innovations to appear and how these impact the economy. Simply put, as Anderson and Tushman (1990)[11] describe "while there is a scarcity of models for understanding technological change, research from multiple disciplines suggests several themes that help get inside the black box of technological change", "Basalla's (1988) comprehensive review of technological evolution was anchored in the concepts of diversity, continuity, novelty, and selection".

Moreover, innovation results are undoubtedly also a result of investment. Namely, a paper by Courvisanos and Verspagen (2002)[12] "shows the relation between innovation and investment instability of business cycles and thus affecting the trend growth of these cycles. This way any strong upswing in a cycle must be related to the following downswing and its implication for new investment and further growth." In other words, since investment in past periods has an effect on the magnitude of technological advancement in future periods, it can be understood that the rise of major innovations across markets is in part deterministic as pre-existing conditions facilitate it.

"Research in innovation and investment has tended to be uncoupled, with linkage between the two becoming sporadic. Only economists examining the economy as a vast interconnected "open systems" canvas continued to maintain this link; notably in respect to the heritage of study we can identify Karl Marx, Rosa Luxemburg, Michal Kalecki and Joseph Schumpeter."

Most importantly, to maintain the view that a comprehensive understanding of economic evolution (at least in regards to Kondratieff cycles) can be achieved, it is important to take on the above mantra of the economy being a "vast interconnected open systems canvas", which can nevertheless be modeled to a debatable degree of accuracy. Moreover, it is worth differentiating between two types in which technological advance can occur. As Anderson and Tushman (1990) discuss, "Technological discontinuities (innovations that dramatically advance an industry's price vs. performance frontier) trigger a period of ferment that is closed by the emergence of a dominant design". They take on the view that technological advancements come about in one of two ways. Firstly, a major innovation which has the potential to revolutionize the market can be introduced (the technological discontinuity); this is normally brought about by highly innovative start-up firms

[11] Anderson and Tushman (1990), *Technological Discontinuities and Dominant Designs: A Cyclical Model of TechnologicalChange*
[12] Courvisanos and Verspagen (2002), *Innovation and Investment in Capitalist Economies: 1870-2000: Kaleckian Dynamics and Evolutionary Life Cycles*

and can be seen as a bursting increase in the level of technology. Secondly, gradual improvements are being added onto current designs; something that can be seen as a step-by-step increase in the level of technology. These two types of growth in the technology level are vital in generating the forces that could give rise to the cyclicality of macroeconomic variables.

Lastly, as Acemoglu (2003)[13] presents in a lecture series, the matter of human capital is strongly interlinked with technological progression. On one hand, it is expected that a population with higher human capital can generate more and better innovations: e.g. America's top education system (at least for those who can afford it) is often cited as one of the causes for the Silicon Valley environment. However, increasing human capital is not, as Acemoglu puts it, a "panacea" (cure for everything). The reason for this is due to the way in which technology affects the labor market:

- increase in the supply of *skilled* workers results in Skill-Biased Technical Change (SBTC)
- increase in the supply of *unskilled* workers results in skill replacing technologies

With reference to the latter skill replacing technologies, a simple example of this can be seen by looking at a hypothetical factory in the last several decades. Maybe 40-50 years ago, a manufacturing line job in a factory used to be a stable profession, where workers were valued for the important but (albeit) small part they played in the creation process of specific goods. We can imagine this factory to have had e.g. 100 workers, a few managers, and a few technicians. However, as SBTC transformed the working place, the routine and blueprint ready jobs of the workers were easily automated by a robot, so 100 lower skilled workers were displaced in favor of a few extra technicians. This engenders increased productivity at a firm level, with higher wages for its lesser amount of employees.

With reference to SBTC, because the new technologies require a certain expertise and can thus only be used by professionals, Acemoglu pertinently emphasizes that "low-skill workers [are] potentially excluded from benefits of new technologies [leading to] greater inequality". And, because as many papers, e.g. Chevalier et al. (2005)[14] show, human capital development in children is strongly dependent on parental income, this perpetuates a class of lower skilled workers across time with continually less opportunities for employment. As such, while improvement of population wide skills might not be a panacea, directed human capital improvements for the most vulnerable part of the population has the potential to decrease the technology-skill mismatch.

2.4 Macroeconomic Variables and Relations

Lastly, in addressing the cyclicality of economic evolution, it is paramount to understand how key macroeconomic variables behave in response to changes in output and the growth rate. Specifically, the discussion here will refer to 1) the relation between markup, inflation and unemployment, as well as 2) a brief overview of the intergenerational transmission of human capital.

[13] Acemoglu (2003), *Human Capital and the Nature of Technological Progress*
[14] Chevalier, Harmon, O'Sullivan, Walker (2005), *The Impact of Parental Income and Education on the Schooling of their Children*

With reference to the first discussion point, all of these quantities occur at a firm level, and noticeably, it is the firms' markup choice (i.e. how much profit they wish to make) which is most responsible in determining both inflation and unemployment on the aggregate level. However, as Blanchard (2008)[15] candidly puts it:

"How markups move, in response to what, and why, is however nearly terra incognita for macro. . . . [W]e are a long way from having either a clear picture or convincing theories, and this is clearly an area where research is urgently needed."

Nevertheless, a lot can be said on the matter. As Banerjee and Russel (2003)[16] discuss, "Empirical evidence in favour of a negative relationship between inflation and the markup has grown in recent years, including papers by Richards and Stevens (1987), Bénabou (1992), Franz and Gordon (1993), Cockerell and Russell (1995), [...] All the estimation undertaken in these papers has assumed that inflation and the markup are stationary variables." However, all these approaches use the same Neo-Keynesian models. As such, this hints at the issue of being stuck in the predominant paradigm of thought: using similar models will naturally yield similar results. In order to explore an ambiguous relationship, it is warranted to use many different approaches in hope of understanding where differences in results come from. One such alternative is provided by Nekarda and Ramey (2013)[17] who "show that frameworks for measuring markups that have produced the strongest evidence for countercyclicality produce the opposite result when we substitute new methods and data".

As such, we may understand that the ongoing debate regarding the cyclicality of the markup can be framed as follows. If the markup is countercyclical (as most estimations make it to be), inflation occurs because as output rises, firm costs increase by a proportionally larger amount, but, firms lower their markup by a lesser amount compared to the increase in costs (naturally, they want to lose as little as possible from their profits) which leads to an increase in the price level. Conversely, if the mark up is cyclical, as Nekarda and Ramey put it, this occurs conditional on a technology shock. More precisely, due to technological advancement, output is able to increase with a relatively lower rise in costs (increased productivity). In this case, as aggregate demand increases (and aggregate output follows), firms raise the markup because they see it as an opportunity to garner higher profits. Overall, seeing as the paper at hand is concerned with understanding the technological causes for business cycle fluctuations, it seems like a natural choice to adopt the latter view from above.

Lastly, and perhaps most importantly, it is important to discuss a relation which has been the main subject matter of numerous papers: namely, how human capital is shaped and transmitted. As Solon (1999)[18] candidly admits:

[15] Blanchard (2008), *The State of Macro*
[16] Banerjee and Russel (2003), *Inflation and Measures of the Markup*
[17] Nekarda and Ramey (2013), *The Cyclical Behavior of the Price-Cost Markup*
[18] Solon(1999), *The Causes and Consequences of Increasing Inequality*

"Unfortunately, we remain fairly ignorant about the causal processes underlying the intergenerational transmission of earnings…does parental income matter so much as it does because high-income parents are able to invest more in their children's human capital, or because the genetic or cultural traits that contributed to the parents high earnings are passed on to the children?"

Nevertheless, even without regard for the underlying cause, the empirical relation that is often remarked on is that the best indication of a child's future earnings is in fact his parents' earnings. This could be, as Heckman and Masterov (2004)[19] discuss, due to the fact that "children brought up in less favorable conditions obtain less education despite the large financial returns to schooling". Moreover, Chevalier et al. (2005)[20], utilizing a new approach to econometrically asses this situation, discover that in regards to human capital accumulation "the strong effects of parental education become insignificant and permanent income matters much more".

Overall, the relationship that can be obtained from this understanding is that familial income determines the rate at which the child's human capital changes. In other words, poorer families will be less able to provide the right opportunities and as a result, the human capital of their children will develop at a slower pace than that of a child in a richer family. This could be either because of a direct increased investment in the quality of schooling or because of the better genetic and cultural traits being passed on. Regardless of the reason, it is clear that a higher parental income leads to more human capital accumulation for one's offspring.

3. Methodology

3.1 Economic Model Overview

The model starts from the base constituents of the economy: workers and jobs. By applying a worker's human capital to a firm's capital, value (in terms of goods and services) is created in the economy. One of the most important assumptions moving forward is that both human capital and the quality of firm capital are normally distributed. Each worker and firm is characterized by a single value of capital H or K respectively.

$$H_i \sim N(u_H, \sigma_H^2)$$

$$K_i \sim N(u_K, \sigma_K^2)$$

3.1.1 Firm Details

In more detail, it is important to understand the relation between a firm's quality of capital K and the skill required to perform in the respective job. As an example, consider construction firms in poor and rich countries. On one hand, the representatively poor construction firm might own shovels, hammers and pickaxes as its repertoire of tools and technology. As such, workers do not

[19] Heckman and Masterov (2004), *The Productivity Argument for Investing in Young Children*
[20] Chevalier, Harmon, O'Sullivan, Walker (2005), *The Impact of Parental Income and Education on the Schooling of their Children*

necessitate high levels of skill to work in this job, but at the same time, the number of workers needed to complete a construction job would be much higher. Conversely, a representatively richer firm might own more technologically advanced capital comprising of trucks, cranes, and bulldozers. This requires more skilled workers (ones that have the ability to operate machinery that is more complex than a simple hammer), but also a fewer number of these workers to complete the same job. From this example, we may understand that the quality of a firm's capital is not only a direct measure of its *technological level*, but also a proxy for the *difficulty of a job*, i.e. skill required for a worker to be productive. As such, the production function within firms takes the Cobb-Douglas form and considers how a worker's human capital is applied to a firm's respective quality of capital.

$$Q = \mathrm{H}^{\alpha}\mathrm{K}^{1-\alpha}$$

Above, Q is the number of goods produced. These goods can be one of two types: "old" goods and "novel" goods. The market in which any given firm operates is determined by the relation of its technological advancement with other firms, such that:

- If $K < u_K$, old goods
- If $K > u_K$, novel goods

Next, firms face costs which include the maintenance of its workers' capital usage, number of goods produced, wages paid out, and new workers added to its roster, such that costs C become:

$$C = \varphi(\mathrm{m})^{\mathrm{scale}}\,\mathrm{K}^{\frac{\mathrm{K}}{u_K}} + \gamma\sum_{\mathrm{i}=1}^{\mathrm{m}}\mathrm{Q_i} + \sum_{\mathrm{i}=1}^{\mathrm{m}}\mathrm{W_i}(\mathrm{T_i})$$

Where **m** is the number of workers a firm has and **scale** is a settable parameter adjusting returns to scale. Further, W(T) are wages paid out based on a Tier Index Value **T**. This value keeps track of seniority at the firm $\mathbf{y_f}$ (i.e. how many years the worker has been there) and the value of his contribution in terms of human capital **H**:

$$T = (\mathrm{y_f})^{\mathrm{d}}(\mathrm{H})^{1-\mathrm{d}}$$

Based on this, wage payment is ordered in the firm. The person with the highest tier value, becomes the owner / CEO of the firm and receives the current market value of his productivity (current market value = (number of goods produced)*(price firm sets for its goods)). From this point, all remaining people are paid exponentially less with a settable parameter. For example, if the exponentiality rate is 2, next worker gets paid ½, then next ¼ and so on. The parameter can be adjusted to yield a more equal or unequal distribution of income within firms. Furthermore, this wage payment setup allows us to account for empirical inequality considerations, as well as the serious issue of a decoupling between wages and productivity for most workers in current capitalist society.

Moving on, the way in which firms set their prices occurs at an individual level and is only loosely affected by competition. Most importantly, even though specific consumers get randomly assigned prices from various firms, naturally, a large portion of the cheapest goods should be bought first. In this case, if there is a surplus of production, most of the goods which remain unsold are the most expensive ones. It is also important to note that there is no stock of goods or means to carry the products over to the next period. Each good can only be sold in the period it has been produced, otherwise it disappears. For the initial period, a price **p** is set with exogenously determined mark-up lambda (which is a settable variable)

$$\mathrm{p} = \left(\frac{\mathrm{C}}{\mathrm{Q}}\right)(1 + \lambda)$$

This is in line with realistic (and empirically supported) firm choices of setting price as a markup over average costs, because there is difficulty in ascertaining marginal costs. Next, in the following periods, firms follow a myopic, backward looking adjustment rule which depends on the quantity sold Qs compared to the quantity produced Q:

If Qs/Q = 1,

$$\lambda_t = 0.2 + \lambda_{t-1}$$

If Qs/Q < 1

$$\lambda_t = \lambda_{t-1}\left(\frac{Q_s}{Q}\right)$$

With the price set, a sales process (to be discussed later) determines the quantity sold by a firm Qs, which in turn determines the revenue in the current period

$$R_t = (Q_{s,t} * p_t) - C_t$$

Furthermore, it is important to keep in mind that each period, a firm uses the majority of its profits to either save at a global interest rate, or to pay a quantity of debt (which also increases at the same global interest rate). As such, profits become:

$$P_t = R + (S'D) - RD$$

Where (S’D) is savings or debt, i.e. a positive quantity if savings or a negative one if debt. Moreover, from these profits, firms invest part of them RD into technological advancement (research and development). The exact amount chosen to invest is determined by a firms “utility function” which places value both on profit retention and on technological progression.

$$U_f = (P)^{\alpha_f}(RD)^{1-\alpha_f}$$

The parameter α_f is a settable variable which describes a firm’s propensity to develop technologically. This type of growth accounts for the “era of ferment”, i.e. gradual improvements to the current level of technology discussed in the literature review. Ultimately, the amount chosen to invest in research and development RD determines the period-by-period increase in the quality of a firm’s capital:

$$K_{t+1} = K_t + \frac{RD}{K_t}$$

Next, when considering a hiring decision the firm checks if Q = Qs, i.e. all its produced goods have been sold. In that case, it is eligible to hire someone. To do this, they perform a simple rule of thumb calculation: if they were to add an extra employee, given his productivity, would he cover the additional average cost brought to the firm?

$$\frac{pQ}{C}_{with\ worker} > \frac{pQ}{C}_{without\ worker} \ ?$$

A maximum of one worker per firm per period can be hired.

Next, when considering a firing decision, if the firm has been in debt for **x** periods or has a debt greater than a value of **y** (settable parameters), it simply removes the worker with the lowest tier index, and continues to remove the next worker with lowest tier index in the next period if, and only if, its debt is increasing. If its debt is decreasing, i.e. accruing profits, the status quo prevails.

Lastly, if a firm is in debt for **d** or more periods, or has a debt larger than a value of **e** (settable parameters) it becomes classified as "bankrupt", becomes dissolved and all of its workers are unemployed.

3.1.2 Entrepreneurship

To account for the second type of technological advancement, bursting, the model allows workers to create their own firms. Every period, unemployed workers attempt to become entrepreneurs and create a business. This is done through a bank which is the decision-making center that selects which loans allow the creation of new firms. The following process applies

"High-Tech" Start-up

- If an unemployed worker's H is such that $H/u_K > 1.5$, allow creation of "high-tech" startup.
- Only one "high-tech" start up may be initiated once every **h** periods.
- This is awarded to the unemployed worker with the highest Human capital.

"Normal" Start-up

- Within +/- 10% of a worker's H, how many firms with corresponding K value are there? Establish variable n_K
- Within +/- 10% of a worker's H, how many unemployed workers with corresponding H are there? Establish variable n_u
- If $n_u/n_K > \omega$, allow creation of firm (ω is a settable parameter which will adjust the rate at which new firms are created)
- No two firms within the same +/- 10% range of eachother may be created in the same period

3.1.3 Worker Details

Workers are arranged by households such that income is pooled if there are two working people. New households are formed the moment a child becomes independent, which is considered to occur when he finishes education and gets employed. Afterwards, a household can gain additional members in two ways:

- Marriage
  - A marriage rate of **marriage_rate =** 80% is upheld in the model
  - Marriages occur between single-person households of the closest age difference
- Natality
  - A natality rate of **natality_rate =** 1.2 children per household is maintained
  - An average childbearing age of **childbearage =** 26 is predominant
  - When natality rate drops, the household with the closest average parent age of 26 has a child

It is important to note that these values are based on empirical observations from the U.S. economy. In particular, when attempting to simulate realistic aggregate behavior of a society, it is important to use reasonable quantities for the caveats of progression through time.

Next, given the wages received from their jobs and savings from the last period, households have an income Y:

$$hhY_t = W_{1,t} + W_{2,t} + S_{1,t-1} + S_{2,t-1}$$

From this, they pay off their yearly living costs. This is dependent on age such that young children have high costs (medicine, continually outgrowing clothes, different nutritional needs, etc.) and older people above 50 also have high costs (psychological woes, increased doctor visits, etc.)

$$L(a) = \frac{(30-a)^{1.6}}{8}$$

Where a is age. After accounting for these costs, each adult within a household receives ½ the household's disposable income to optimize his expenditure, i.e. adults are treated as independent consumers in all regards after they have jointly paid the living costs of their household (the other adults and any possible children).

$$DisY_t = \frac{1}{2}\, DishhY_t = hhY_t - \sum_{n=1}^{i} L_i(a)$$

Next, expenditure is optimized to maximize a utility function subject to the constraint of the available disposable income. The utility function places value on spending for both types of goods, as well as savings, but gives a higher importance to newer goods. This parallels people's wish for e.g. "the newest Iphone" and at the same time generates higher demand for novel goods so as to give firms an incentive for technological advancement (as discussed in the literature review).

$$U = (exp_o)^{\alpha}(exp_n)^{\beta}(S)^{c}$$

- If employed and $DisY/u_y < 1.25$
  - <u>And below age 50</u>, $\alpha + \beta + c = 1$, $\beta = \alpha + 0.1$, $c = 0.2$
  - <u>And above age 50</u>, $\alpha + \beta + c = 1$, $\beta = \alpha + 0.1$, c = [(a-50)^2]/300 + 0.2
- If employed and $DisY/u_y >= 1.25$
  - <u>And below age 50</u>, $\alpha + \beta + c = 1$, $\beta = \alpha + 0.1$, $c = 1 - u_y/DisY$
  - <u>And above age 50</u>, $\alpha + \beta + c = 1$, $\beta = \alpha + 0.1$, c = [(a-50)^2]/300 + 0.25
- If unemployed
  - $\alpha + \beta + c = 1$, $\beta = \alpha + 0.1$, $c = 0.6$

The exact parameters for maximizing the utility function depend on three things, the first of which is employment. When adults are employed they consume a larger portion of their income and save less. This occurs because their expected future stream of income is certain, i.e. determined by a wage. However, if unemployed, they severely decrease their consumption (value placed on savings increases from c = 0.2 to c = 0.6). Moreover, based on their age, agents can start saving for retirement. As they are above an age of 50, their savings parameter continually increases period by period. It starts at a value of 0.2 (which is also the value of the savings parameter when below 50) so as to prevent a discontinuity in progression across age and to minimize threshold effects.

Lastly, agents adjust their savings based on how their income compares to the average income. This happens in such a manner that agents with an income much above the average, save more and consume a lower proportion of their overall income. The justification for this arises from the empirical observations that the richest agents do not consume significantly more than the poorer ones by an amount proportional to their income difference. In other words, even if the richest people buy more expensive food, they still buy 3 meals a day. The way this is implemented is such that if an agents disposable income **DisY** divided by $\mathbf{u_y}$ (average income of agents in the economy) yields a ratio greater or equal to 1.25, a new condition takes hold. This condition states that the savings parameter $\mathbf{c} = 1 - \mathbf{u_y/DisY}$. By the form of this function, at a ratio of $\mathbf{DisY/u_y} = 1.25$, the ratio of $\mathbf{u_y/DisY} = 1/1.25 = 0.8$, so $c = 1 - 0.8 = 0.2$. This once again prevents a discontinuity in the change of the savings parameter and minimizes potential threshold effects. At the same time, as a person's DisY increases, $\mathbf{u_y/DisY}$ decreases so the saving parameter increases.

At this point, it is important to note that as children do not have their own income to spend on goods, they are simply considered to be dependents, as such, they do not have a utility function they are trying to maximize and do not generate any expenditure on the goods. They only place a burden on households in terms of living costs. As such, only the utility maximization problem of adults in a household is relevant to economic consumption.

Lastly, and perhaps most importantly, agents can gain human capital in the initial stages of their life. As the literature review has discussed, the rate at which this occurs is most noticeably dependent on familial income, and in particular, how that income compares to that of other households in the economy. As such, this relation can be phrased as follows:

$$H_{t+1} = H_t \; [1 + \chi(\frac{hhY_t}{AY_t})^y]$$

Where **χ** is a settable parameter, **AY** is aggregate income and **y** is years of education. Moreover, children pursue education from 6 years of age, until a minimum of 18 and a maximum of 25. The exact age at which education stops for a given agent depends on how his human capital H relates to labour market conditions. Specifically, children pursue education until their skill is as close as possible to the average level of technology u_K. The moment H >= u_K, education stops, or if an age of 25 is reached, education stops. This is meant to consider the idea that children pursue an education that is relevant to the current labor market (e.g. parents nudging their offspring to learn programming because it is a widely applicable skill).

3.1.4 Sales Process

Across all firms, a certain number of goods have been created, with each firm setting similar but different prices (depending on their costs) for their stock of produced goods. We can compute the total value (in monetary terms) of the goods produced in the two markets 0 and n by:

$$TS_0 = \sum_{n=1}^{i} p_i * Q_i \qquad (for\ firms\ producing\ in\ market\ of\ good\ 0)$$

$$TS_n = \sum_{n=1}^{i} p_i * Q_i \qquad (for\ firms\ producing\ in\ market\ of\ good\ n)$$

Which yields the amount of money that TS = total supply is worth in each market type.

Next, from an agent's perspective, in the process of optimizing their utility, each person decides the amount of expenditure they will attribute to either good 0, good n, or savings. As such, through a similar process, we can aggregate the demand that agents generate for each good:

$$TD_o = \sum_{n=1}^{i} exp_{o,i} \qquad (for\ goods\ demanded\ in\ market\ of\ good\ o)$$

$$TD_n = \sum_{n=1}^{i} exp_{n,i} \qquad (for\ goods\ demanded\ in\ market\ of\ good\ n)$$

From this, in each respective market, one of two cases can arise:

1. Excess Demand: TD > TS
2. Excess Supply: TS > TD

In the first case, agents have more money to spend than the value of the goods they can buy. As such, all goods that the firms produced are sold, and the value of excess demand is calculated as

$$ED = TD - TS$$

Following this, the agents receive an amount of money back **RS** = return spending which is proportional to their initial share of total demand

$$RS_i = \frac{exp_i}{TD} * ED$$

Moving on to the second case, the total value of the goods produced exceeds the total expenditure that consumers are willing to make. As such, this will lead to some firms selling less than the amount of goods they produced. The selling function prioritizes firms with the lowest price first. To calculate this, the firms are ordered in ascending value of prices for their respective markets from n_i to n_n. Following from this, each firm sells a certain **PSS** = proportion of stock sold from their goods such that:

$$PSS_i = 0.4 - 0.3 * \frac{n_i}{n}$$

This generates the behaviour that the firm with the lowest price sells 40% of its stock, then we move on to the firm with next lowest price which sells a bit below 40% and so on until the firm with the highest price sells 10% of its stock. If by the time the last firm is reached, the value of TD has not been fully expended, the process repeats iteratively until the entire TD is exhausted. It is important to note that this process gives all firms the opportunity to sell at least a proportion of their goods which also accounts for how much they produced (as a measure of market dominance). The reason for this is that a firm selling 10% of its stock might refer to a number of goods that is equal to a different firm selling 40% of its stock (because the stock size differs across firms dependent on their productivity.

3.1.4 Step-by-Step Timeline

Lastly, to understand how the model interactions are processed, I have laid out the following step-by-step timeline which occurs in every period t (meant to represent a year).

1. Age Check - agent status updated
2. Natality Rate and Marriage Rate check - household status updated
3. Entrepreneurship - firm status updated
4. FIRMS produce goods
5. FIRMS pay wages
6. FIRMS set price
7. AGENTS aggregate savings and wages in their households, paying living costs
8. AGENTS educate their children
9. AGENTS optimize utility to generate their demand for good 0 and good n

10. Sales process occur in markets
11. FIRMS gain revenue, and update price for next period
12. FIRMS pay debt
13. FIRMS optimize between profits and spending on research and development
14. FIRMS advance technologically
15. FIRMS hire or fire workers dependent on their situation

3.2 Code Documentation of the Model

The model best yields itself to analysis once fully implemented with adjustable parameters into Python. While the description above is somewhat straightforward, the code needed to generate the expressed interactions is rather complex. Nevertheless, it can be approached in the language of Python through the use of classes and extensive list comprehension operations. An overview of notable completed sections is provided below (a link to download the code is the appendix).

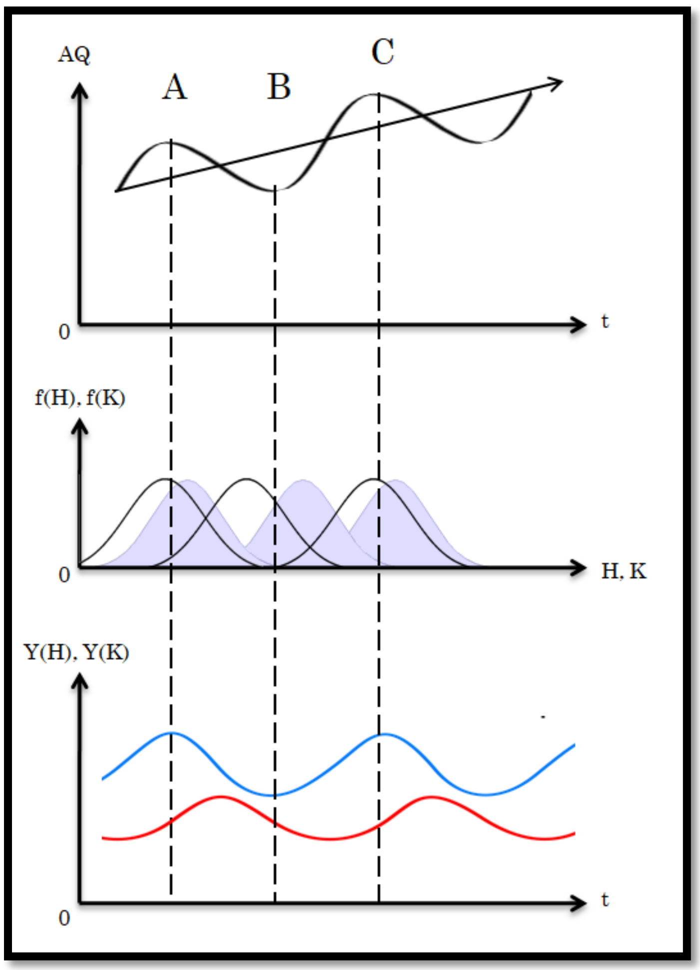
- **THREE CLASSES**: Humans, Households, Firms
    - Human Functions
        - Check age conditions
        - Optimize utility
        - Set and Transform income, savings, and living costs across time
        - Extract demand generated
    - Household Functions
        - Aggregate human classes into households of singles, married couples or couples with children
        - Pay living costs as a family
        - Split income between adults allowing them to optimize their behaviour
    - Firm Functions
        - Calculate productivity of all workers in firms
        - Calculate costs
        - Set and Transform price, and savings/debt over time
        - Optimize division of funds between profits and technological advancement
- **ARRAYS OF INFORMATION** needed to initialize
    - Initial Human Capital H list (np.random.normal)
    - Initial Firm Capital K list (np.random.normal)
    - Age lists (np.random.uniform)
    - Initial costs dependent on the above age lists
    - Initial savings list (three classes of income with np.random.normal)
- **LIST OPERATIONS** setting up initial conditions and enacting the passage of time
    - Organize initial humans in lists of variable length to create households
    - Organize initial humans into firms up to a settable proportion of employment
    - List comprehension to uphold marriage rate and natality rate
    - List comprehension to enact entrepreneurship rules
    - List comprehension to enact sales process
    - List comprehension to check if bankruptcy should be declared
    - List comprehension to fire or hire workers

4. Results Analysis and Conclusion

4.1 Results Analysis

4.1.1 Business Cycle Progression

Overall, from the above model set-up several considerations can be made to understand how the economy might progress through steady-state equilibriums over time. In particular, it is helpful to form a general view by starting with a graphical representation of relevant quantities:



In the above graph, the top panel shows the business cycle as tracked by aggregate production in the economy. This ideally showcases an endogenously cyclical behavior. The middle panel shows the progression of the two normally distributed quantities (Human Capital H and quality of Firm Capital K) at different points in time, with the blue bell curve showing K. Lastly, the bottom panel shows the income paths of workers (in blue) and firms (in red), unfolding in the typical predator-prey relation we have discussed in the literature review.

From the above graph, consider point A. This is the peak of a boom, i.e. a local maximum in aggregate production. At this point, the two normal curves of H and K are close to overlapping: this means that workers have skills which match the difficulty of the jobs in the economy. As such, many of them find suitable employment which greatly increases productivity of firms. Since many of them are employed and receiving a wage, this coincides with the highest point for the income of workers, which in turn generates a relatively higher level of demand than in previous periods. Now, since firms "prey" on the incomes of workers, by the mark-up adjustment formulation, inflation occurs over time as all (or most) of the goods produced can be sold (since more workers are employed and receiving wages). This begins to increase the income of the firms (as the red line in the bottom panel shows) and this income generates more investment into research and development on the firm's side. However, it is in fact this improvement in the firms' quality of capital which generates problems: at this point, due to the increasing income of the firms (and decreasing income of workers) because prices are rising to garner more profits for the firms, the rate of change of firm capital is higher than that of human capital:

$$\frac{dK}{dt} > \frac{dH}{dt}$$

As such, the gap between workers' skills and the difficulty of the jobs they must perform increases due to technological advancement. In fact Kondratieff himself makes a statement about this in saying: "During the recession of the long waves, an especially large number of important discoveries and inventions [...] are made, which, however, are usually applied on a large scale only at the beginning of the next long upswing". The time taken to "large-scale application" of improvements in technology can be understood as the periods needed to wait for human capita to attune itself to the new labour market conditions (i.e. higher skill requirements).

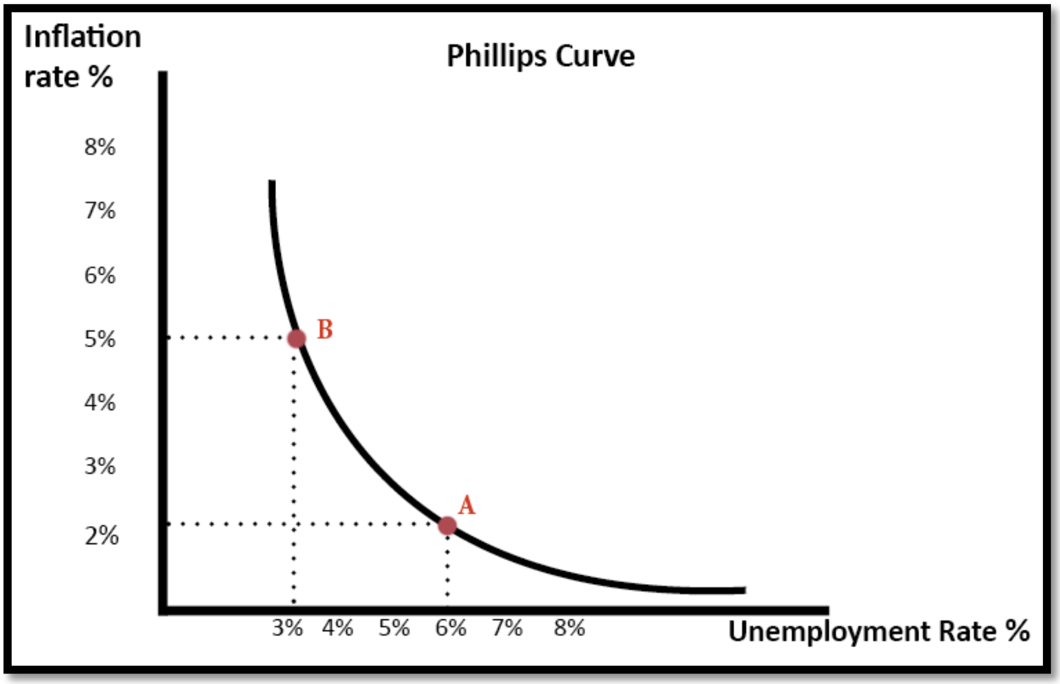
However, before that can occur, the bust itself has to happen. The way this occurs is that by the relation above, advancement in technology on an aggregate level leads to lower productivity over time and unemployment (because human capital cannot keep up with the skill requirements of the firms). Consequently, this generates lower income for workers which begins also eroding firms profits. From this lower income, lower demand eventually follows which also causes the firms' income to decrease. We have now reached point B, an economic bust, where the two normal curves show the least overlap (i.e. maximum skill mismatch and unemployment). For things to return on a path to growth, firms lower prices, and spend less on research and development. This leads to a higher rate of human capital accumulation:

$$\frac{dK}{dt} < \frac{dH}{dt}$$

As such, the slowing down in the rate at which technology develops allows workers' skills to catch up with technology, decreasing the skill-mismatch and beginning to set up the conditions for a boom at point C.

4.1.2 Philips Curve Extension

Next, I would like to remark on the interaction between unemployment and inflation that the Phillips Curve presents, and how the model rephrases this relation to gain a new understanding of the economy. A simple representation of the Philips curve can be seen below:



The trade-off relationship between inflation and the unemployment rate has long seemed to me to hint at an underlying process which I haven't much seen discussed. Specifically, one can adopt the view that the unemployment rate is a proxy for labour's aggregate income.

$$unemployment\ rate \cong labour's\ aggregate\ income \cong demand\ for\ goods\ \&\ services$$

In the model, unemployment varies across the business cycle such that the highest rates of employment occur when the skill-technology mismatch is the lowest (because most agents can find employment in a job that is suitable to their skills). This is also the point at which (because more agents are hired and receiving a wage) aggregate demand is potentially largest. In turn, due to the higher demand, firms notice over time that they can capitalize on it by increasing their prices, hence inflation. As such, this view of the interaction, stemming from the predator-prey formulation, suggests that firms' motivation for increasing prices arises out of a noticed availability to increase their prices without necessarily garnering fewer profits.

Ultimately, this is able to shed light on concepts such as stagflation by explaining them in a more straightforward way. For example, we may look at the USA's 1970s period of stagflation under the microscope. Following the Post-War Boom, the Fed's view, perhaps too reliant on Keynesian thought, was that they could use their monetary policy to increase demand for goods, while keeping unemployment low (with inflation rising in a safe and controlled manner).

As we have learned however, that turned out to be far from the case. As money was poured into the economy to increased demand (in line with the predictions of the model at hand), firms capitalized on this noticeable additional demand by increasing their prices. However, the rate at which prices were increasing was faster than the rate of wage increase. Ultimately this led to a wage-price spiral by which workers were unwilling to supply their labour at wage that did not sustain their needs. Ultimately, to rein in the situation, macroeconomic thought partially changed its mantra to account for the ideas of Friedman, most noticeably his understanding that inflation occurs when "too much money chase too few goods."

As we can see, stagflation had its roots in the artificial demand generated by the Fed, which did not properly reflect the amount of goods that labour's aggregate income could buy. Unemployment remained high (because workers were not accepting unsustainable wages) while prices kept rising because demand for goods was artificially propped up by the Fed's monetary policy. Instead of pumping more money into the economy when inflation was out of control, the Fed should have accepted that in order to emerge from a dire situation, a period of crisis is needed to fundamentally change labour market conditions.

4.1.3 High-Tech Entrepreneurship Effects

By allowing the introduction of a firm with significantly higher capital quality K (via the high-tech entrepreneurship process) at a settable interval across the process of economic evolution, the introduction of major innovations into the market can be simulated. This has two major effects on the economy as a whole: 1. Increased competition among firms and 2. Decrease costs.

With reference to the first, as the new "high-tech" firm solidifies its position, it will be able to produce significantly more goods than most other firms. As such, the price it will set has a notable bearing on competition in the market through the sales process. We can expect the introduction of these types of firms to be a catalyst to the bankruptcy of less competent ones in its respective market. This increases unemployment in that period but at the same time is able to set the conditions for more growth in the future.

With reference to the second, it is important to reiterate that firms' costs are defined by:

$$C = \varphi(\mathrm{m})^{\mathrm{scale}}\, \mathrm{K}^{\frac{\mathrm{K}}{u_K}} + \gamma \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{Q_i} + \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{W_i}(\mathrm{T_i})$$

In particular the term $\mathrm{K}^{\frac{\mathrm{K}}{u_K}}$ ties an individual firm's costs to the average level of technology in the economy u_K. As such, when a new "high-tech" firm is created, the costs of all other firms drop which will decrease the price at which their goods are sold.

4.2 Conclusions

Overall, the conclusions of the paper can be summarized into three main points:

1. *Business cycle formation can be modelled as endogenous process which arises from the interaction of human capital H with firm capital K. We may understand that an increase in total productivity of the economy (AQ = GDP) comes from either an improvement in the quality/quantity of the labour force (changes in H) or from an improvement in the quality of firm capital (changes in K).*
2. *The Philips curve, as currently formulated, is indicative of an underlying relationship between the unemployment rate and total aggregate consumer income. In other words, inflation arises not necessarily out of changes in the employment rate, but changes in total income of consumers which affects the amount of demand generated.*
3. *New highly innovative firms impact the market by changing intra-firm competition in two ways. Firstly, it impacts the price consumers receive by supplying goods at a much more efficient rate than other firms are able to. Secondly, though instant in the model (but over a certain period of time in reality), it decreases all other firms' costs by increasing the average level of technology available in the economy.*

4.3 Policy Prescriptions

As we can see from the result analysis section, specifically the business cycle formation, the human capital progression of workers (dH/dt) is what creates the slump in cyclicality, and also what prevents the economy from quickly readjusting on a path to growth. In particular, the main issue is the skill vs. technology mismatch of the workers with the lowest human capital. The reason for this is that as firms progress technologically (dK/dt), the first workers to be influenced by this advancement are in fact the ones with least matching skill, i.e. the lowest human capital.

Simply put, to minimize the magnitude of a business cycle slump, the government should invest *comparatively more* into human capital infrastructure as a boom is occurring (to diminish the skill-technology mismatch) and invest *comparatively less* during a crisis. As an extensive literature discusses in depth, human capital improvements on a national scale can be achieved by measures such as:

1. Enacting a more equal income distribution
2. Ensuring equality of opportunity prevails (this is probably the most important point when seeking to more wholly use the skill of a workforce)
3. Providing for basic needs: nutrition, healthcare, safe environment

4.4 Shortfalls and Possible Extensions

Some shortfalls in the analysis section can be noted for improvement. In the discussion of business cycle progression, an implicit assumption that the distribution of H and K across time retains its bell curve form is made. This is perhaps unrealistic and when the model is fully simulated by code, it may prove to be false.

Next, depending on how the costs to scale parameter is set, mark-ups may prove to either cyclical or counter-cyclical. Both of these options will influence the means by which a business cycle is formed, but at the moment, their exact effects are not fully understood. Once modelled on the computer, adjustment of these parameters may yield interesting information.

Lastly, despite the rough shape of the model, given additional work, it has the potential to deliver a wealth of results. Some possible extensions of the paper at hand are as follows:

4.4.0 Program

The model presented as a much more complex system to program than expected. At the moment, I have created all necessary classes, set up the initial conditions for their interaction and am finalizing the operations inside the time loop. The main issue was that the list comprehension operations needed to construct the time loop were hard to formulate and organize in a logical manner. Nevertheless, the exercise of programming the model allowed me to improve my economic understanding of the exact interactions taking place and consider a lot more caveats that would pose a problem to the ease of modelling.

4.4.1 Taxation Effects

Expanding on the policy prescription above, one way to adjust the rate at which the human capital of the lowest skilled workers progresses is through taxation and redistribution of income. Firstly, it is important to reiterate that the education of children is dependent on familial income and how this related to aggregate labour income:

$$H_{t+1} = H_t \; [1 + \chi(\frac{hhY}{AY})^{y}]$$

As such, decreasing taxation on low income households and increasing taxation on high income ones can equalize the rate at which human capital is accumulated across households. Most importantly, because AY would decrease (as a result of taxation), the ratio of hhY/AY would decrease, thus speeding up the rate of dH/dt.

4.4.2 Migration Studies

Discussing an additional extension, which is perhaps more complex than the previous one, the model can be integrated to study labour migration. In particular, by the formulation of human capital and technological availability of jobs, one can address the increasingly pressing issue of labour complementarity as described by Borjas. In a few words, it is suggested that countries may benefit from migration of workers which have complementary skills to the current workforce.

For example, an economy might have a labour force with high levels of human capital, but still have jobs which require a low human capital (low K). As such, migration from countries with comparatively lower human capital may satisfy the labour demand of these less technologically advanced firms.

5. Bibliography

NOTE: Bibliography is organized by order in which the paper was cited and used.

1. **Jan P.G. Reijnders**, *Trend movements and inverted Kondratieff waves in the Dutch economy, 1800–1913*, Structural Change and Economic Dynamics 20 (2009) 90–113
2. **Korotayev, Andrey V, Tsirel, Sergey V**., *A Spectral Analysis of World GDP Dynamics: Kondratieff Waves, Kuznets Swings, Juglar and Kitchin Cycles in Global Economic Development, and the 2008–2009 Economic Crisis,* Structure and Dynamics, 4(1), (2010)
3. **Joseph A. Schumpeter**, *BUSINESS CYCLES: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*, New York Toronto London : McGraw-Hill Book Company, 1939, 461 pp. Abridged, with an introduction, by Rendigs Fels
4. **Bert de Groot and Philip Hans Franses**, *Stability through Cycles*, Erasmus University Rotterdam Econometric Institute Report 2006-07
5. **O. Lange**, *Marxian Economics and Modern Economic Theory,* The Review of Economic Studies, Vol. 2, No. 3 (Jun., 1935), pp. 189-201
6. **Sukhamoy Chakravarty**, *Marxist economics and contemporary developing economies*, Cambridge Journal of Economics 1987,11, 3-22
7. **Luca Riccetti, Alberto Russo and Mauro Gallegati**, *An Agent Based Decentralized Matching, Macroeconomic Model*, Munich Personal RePEc Archive October 2012
8. **R. M. Goodwin**, *A Growth Cycle*, Republished 2014, Presented at the First World Congress of the Econometric Society held in Rome, 1965

9. **Philip Anderson and Michael L. Tushman**, *Technological Discontinuities and Dominant Designs: A Cyclical Model of Technological Change*, Administrative Science Quarterly, Vol. 35, No. 4 (Dec., 1990), pp. 604-633
10. **Jerry Courvisanos and Bart Verspagen**, *Innovation and Investment in Capitalist Economies: 1870-2000: Kaleckian Dynamics and Evolutionary Life Cycles*, Investigación Económica, Vol. 62, No. 242 (octubre-diciembre, 2002), pp. 33-80
11. **Daron Acemoglu**, *Human Capital and the Nature of Technological Progress*, AstraZeneca and StoraEnso Lecture Stockholm, December 11, 2003
12. **Arnaud Chevalier, Colm Harmon, Vincent O'Sullivan, Ian Walker**, *The Impact of Parental Income and Education on the Schooling of their Children*, THE INSTITUTE FOR FISCAL STUDIES WP05, (2005)
13. **Blanchard, Olivier J.** , *The State of Macro*, National Bureau of Economic Research Working Paper 14259, Cambridge, MA. (2008)
14. **Anindya Banerjee and Bill Russell**, *Inflation and Measures of the Markup*, Journal of Macroeconomics Volume 27, Issue 2, June 2005, Pages 289-306
15. **Christopher J. Nekarda, Valerie A. Ramey**, The Cyclical Behavior of the Price-Cost Markup, NBER Working Paper No. 19099, Issued in June 2013
16. **Gary Solon (1999)**, The Causes and Consequences of Increasing Inequality, The University of Chicago Press 2001, p. 164
17. **James J. Heckman and Dimitriy V. Masterov**, *The Productivity Argument for Investing in Young Children*, T.W. Schultz Award Lecture at the Allied Social Sciences Association annual meeting, Chicago, January 5-7, 2007

6. Appendix

The code to program the model, which is currently on the brink of being finished, can be found online at the following link:

https://drive.google.com/open?id=17F_3WXdopj3_NyU1NtAsGfyCVJM20p5I