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ACE Model of Technological Cycles

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Using Agent Based Computational Economics (ACE), this paper models the way in which Technological Cycles arise out of the individual, asymmetric behavior of multiple agents in a simulated economy. It is argued that while there are certain drawbacks to the ACE method, acknowledging and accounting for these can allow for a justified approach which still has economic merit. In the model, Workers, Households, Firms and the Bank make choices at a micro level which interact to aggregate into macroeconomic patterns such as endogenous cyclicality of GDP. When checking the emergence of these economic phenomena against targeted moments (or empirically observed stylized facts), I conclude the model performs very poorly in its ability to replicate reality. Nevertheless, based on the cyclical behavior observed, I am able to formulate the conclusion that technologically driven growth occurs out of the relative growth of both Human and Firm capital and that cyclicality stems out of an imbalance between the two forms of capital.

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1. Introduction

In order to make sense of an increasingly complex world, economic models may need to evolve and simultaneously account for multiple facets of a system, rather than promote the specificity that boxed-in approaches currently display. In an attempt of this, the paper at hand uses Agent Based Computational Economics (ACE) to model the way in which Technological Cycles can arise from the individual, asymmetric decision-making of multiple agents. This topic, while at first glance a complicated matter, may prove to be surprisingly pertinent to analyzing the main transformation that modern economies are currently going through: mechanization. As Ford (2015)[1] documents extensively, virtually all industries ranging from white-collar to blue-collar work will be deeply affected by the improvement of our technology. Evidence of this can already be seen in the employment patterns that developed countries are plagued by; in particular the increase in part-time and zero-hour contracts (especially among the population of young adults) signals that workers are having difficulties finding suitable and stable employment. To account for this, an approach is needed that seeks to understand not just the decisions and outcomes of a single representative agent, but also the development of fringe cases and how these can be addressed.

In the next section of the paper, I provide a discussion of the relevant literature. Beginning with an overview of the merits for Agent Based Computational Economics, I argue that much can be gained from adopting such a wide approach to understanding the economy. Further, I review the core focus of the paper, namely the cyclicality of an economy, and put the choice of addressing Technological Cycles into perspective. Lastly, I briefly review several macroeconomic phenomena that are relevant to the modeling process.

In the third section, I start by going over the paper's methodology, describing the general ACE approach. I list the targeted moments, which the model is trying to simulate, as well as the corresponding parameters which adjust the output to match said moments. Following that, I provide a detailed overview of the model and a list of the settings for initial conditions and parameter values. Lastly I provide a list of the implicit assumptions that come across from the model formulation.

In the fourth section, I firstly present and analyze the results of the model, discussing the patterns they showcase in further detail. Next, I perform a moment comparison exercise, where the model's output is checked against empirically observed stylized facts.

Lastly, in the fifth section, I summarize the main results into several conclusions and discuss potential policy recommendations as well as plans for future work. The sixth section showcases the used references and the seventh section is an annex providing further details.

[1] Ford (2015), *The Rise of the Robots*

2. Literature Review

2.1 Agent Based Computational Economics (ACE)

Perhaps the biggest question mark overlaying this paper is the choice of using an ACE approach. As such, it is directly needed to carefully review the merits (and drawbacks) of this method, in order to correctly assess the worth of the paper. In this section, I will begin with the motivation for using the ACE approach, then move on to discuss a few variants of ACE models, and lastly go over some strengths and weaknesses of this method.

Initially, ACE models were created out of dissatisfaction with the pitfalls of dynamic stochastic general equilibrium (DGSE) models. As Dilaver et al. (2016)[2] points out, "ACE models move away from DGSE models along the lines of heterogeneity, complexity and rationality". More specifically, they aim to accomplish the following: firstly and perhaps most importantly, challenge the representative agent assumption, and allow heterogeneous agents to interact in order to understand what happens across a spectrum of situations. Secondly, allow macroeconomic phenomena to emerge out of complexity, i.e. the aggregation of multiple interactions at a micro level. "Dosi et al. (2008), for example, argue that real business cycle models and New Keynesian DSGE models are both inadequate because a large part of the dynamics are driven by aggregate technology shocks", and "both streams of literature dramatically underestimate the role of endogenous technological shocks occurring at the microeconomic level""[3]. Lastly, ACE models differ from DGSEs in their use of bounded rationality: whereas agents in DGSE models may have perfect information and ideally optimize their decision making, agents in ACE models use decision heuristics and rules of thumb. With these differences, ACE models wish to move away from DGSEs and provide a more realistic alternative to the modeling of economic evolution.

At this point, it is perhaps useful to get a more concrete idea of several versions of ACE models and how they are applied. As Turell (2017)[4] at the Bank of England discusses, "Agent-Based Models explain the behaviour of a system by simulating the behaviour of each individual 'agent' within it". In other words, this is a bottom-up approach whereby different optimization choices at the micro level aggregate to create macro patterns. This "is in contrast to models which are 'top down', and which presume how agents' behaviour will combine together, sometimes by assuming that all agents are identical." As Turell points out, ACE models "show how even very simple behaviours can combine from the 'bottom up' to recreate more complex behaviours observed in the real world."

With reference to specific macroeconomic models, despite earlier formulations such as the K&S Model (Dosi et al. (2008)), the CATS Model (Gaffeo et al. (2008)) or the Eurace Model lacking rigorous validation, newer versions seem to fare better. As Dilaver et al. (2016)[5] assesses, an "attempt to combine the K&S and CATS frameworks is presented in Assenza et al. (2015) [...] this paper presents by far the most in depth moment comparison exercise, comparing [...] GDP,

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[2] Dilaver, Jump, Levine (2016). *ABM and DGSEs – where do we go from here?*
[3] Ibid.
[4] Turell (2017), *ABMs – Understanding the Economy From the Bottom Up*
[5] Dilaver, Jump, Levine (2016).

consumption, investment, and unemployment against equivalent US time series." "The model performs strikingly well along these dimensions". As Dilaver concludes: "macro ACE models are very rich [… but] their complexity and analytical intractability make them very difficult to validate empirically. […] However, the available evidence […] suggests that ACE is a fruitful modeling approach in macroeconomics." In order to see this, it is worth addressing the strengths and weakness of ACE.

As can be concluded from the statements above, one of the most attractive points of using this approach is the idea of "Emergent Behaviour"; or simply put, "the individual actions of the agents combine to produce macroscopic behaviour"[6]. This is an idea starting as early as "Adam Smith's metaphor of the invisible hand", whereby the self-interested actions of many different agents produce the economic outcomes witnessed around us. Most importantly, if accurately expressed, it allows the modeller to observe the source causes of e.g. economic shocks and understand their formation, rather than simply assume an exogenous origin.

Another strong suit of the ACE approach is Heterogeneity: "agents being different in some way, perhaps by income, preferences, education or productivity". Through this differentiation, we are allowed to observe how economic outcomes affect each particular group (those more or less productive, educated, rich, etc.). As such, the approach provides a very valuable insight into the fringe cases, from which pertinent policy prescriptions can be designed, keeping a (disadvantaged) targeted group in focus.

Other strengths are worth briefly going over as they round out the discussion. Firstly, these models may include "realistic behaviour", or simple decision heuristics which are well-suited to modelling individual decision making. Secondly, the ACE approach allows for the exploration of multiple possibilities due to the high customization available to such models. Lastly, these models are ideal for simulating "complexity, non-linearity and multiple equilibria". As is most likely the case in the real-world, an economy is never in equilibrium, but moving either to or from that said equilibrium. This is a powerful idea which puts into question the rigid view of conventional models. As Turell explains, "recent work on agent-based models of the macroeconomy has described phase transitions between low and high unemployment" (Gualdi et al (2015)). If we are able to understand what happens during economic transitions, rather than just seeing the endpoints, and also keeping in mind that most of the time, an economy is in fact in one of these transitions, a much deeper understanding of the correct policy approaches may arise.

Moving on, it is needless to say that any approach comes with its share of drawbacks, and ACE is no exception. As Turell so promptly puts it, "in many ways, the greatest strength – the flexibility to model such a vast range of scenarios – is also the greatest weakness." There is perhaps too much choice in selecting the correct parts, and so simulations will naturally greatly differ based on the assumptions used. "Further work is needed to develop objective means for choosing the most appropriate assumptions."

[6] Turell (2017)

Overall, while the formulation of present-day ABSs may appear rough, as any fledgling field tends to be, it has clearly been shown by e.g. Riccetti, Russo, and Gallegati (2012)[7] and many more, that using a computer simulation to model complex, emergent behaviour, indeed provides a possible avenue of economic inquiry. As they discuss, "macroeconomic properties emerge such as endogenous business cycles, nominal GDP growth, unemployment rate fluctuations, the Phillips curve, leverage cycles and credit constraints, bank defaults and financial instability, and the importance of government as an acyclical sector which stabilizes the economy. The model highlights that even extended crises can endogenously emerge." Taking a step back to consider this for a moment, one should be nothing short of amazed at how replicable our economic woes and successes can be. Keep in mind; this was done without the assumption of homo economicus attempting to optimize an expected discounted future stream of income for his entire lifetime, but through single-minded rules of thumb that are more representative of what people in current society might use for their economic decision-making. As researchers will improve their understanding and use of ACE models, much economic insight may come to be gained.

2.2 Economic Cycles

When beginning to analyze the cyclicality of an economy, it is important to realize that its development over time can take multiple forms. As Reijinders (2009)[8] documents, and Korotayev, Tsirel (2010)[9] further describes, several cyclical patterns of the aggregate economy can be distinguished:

TABLE 1 – Types of Economic Cycles

Cycle	Duration	Potential Cause
Kitchin	3 – 5 years	"time lags in information movements affecting the decision making of commercial firms"[10] (market information asymmetries)
Juglar	7 – 12 years	Patterns of investment in fixed capital
Kuznets	15 – 25 years	Demographic swings or Infrastructural investment cycles
Kondratieff	40 – 60 years	Introduction of major technical innovations in markets

In fact, it was Schumpeter (1939)[11] who initially showed that "barring very few cases in which difficulties arise, it is possible to count off, historically as well as statistically, six Juglars to a Kondratieff and three Kitchins to a Juglar – not as an average but in every individual case". As this discussion as well as more data seems to support the existence of Kondratieff waves, and the model attempts to describe technological evolution, it follows that this is the type of cycle most in our scope. As such, the paper will mostly be concerned with this kind of cyclicality.

[7] Riccetti, Russo, and Gallegati (2012), *An Agent Based Decentralized Matching Macroeconomic Model*
[8] Reijinders (2009), *Trend movements and inverted Kondratieff waves in the Dutch economy, 1800–1913*
[9] Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*
[10] Ibid.
[11] Schumpeter (1939), *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*

However, before delving into a more in-depth analysis of K-waves, there are generally two major discussion points regarding all types of economic cycles: whether they exist, and whether they are caused exogenously or endogenously. With reference to the first, some economists deny the presence of any economic cycles altogether, as the title of a respective section in a classical Principles of Economics textbook by N. Gregory Mankiw showcases – "Economic Fluctuations Are Irregular and Unpredictable" [Mankiw 2008: 740]". But, other economists such as DeGroot, Franses (2007)[12] explicitly make the point that "economies of industrialized countries show cyclical patterns. Recessions since WWII seem to emerge every 8 to 10 years, which is usually associated with the business cycle, and long swings like the well-known 55 year Kondratieff cycle can be observed for a variety of variables." For this paper's purposes, a view is adopted that while cycles might in fact seem to be "irregular and unpredictable", the wealth of data accounting for their historical existence is hard to refute. Further, while real-life outcomes, often muddled by too many variables even the smartest economists cannot account for, are hard to pinpoint, a more regulated theoretical model can shed some light on economic cycle formation. In other words, what we see in the real-world is hard to analyze as an overall cohesive result, due to the countless caveats that arise unsuspected, but that does not mean that underlying cycles do not manifest themselves. Moreover, even if we cannot fit data to the length of the above-mentioned cycles as precisely as we hoped to, this does not mean that understanding the theoretical causes for the cycles will not yield pertinent economic knowledge, from which policy prescriptions can arise.

Secondly, another debate emerging from these observations is more concerned with exogeneity. Namely, do these cyclical patterns arise out of possible pre-existing conditions that accumulate over time resulting in a shock, or, are they the result of unforeseeable forces? As DeGroot and Franses point out, "there have always been multiple cycles and there always will be. Hence, these cycles are not fully stochastic and caused by external shocks, but are in fact partly deterministic."[13] This strikes at the heart of what the paper at hand attempts to do: elucidate the possible deterministic causes for economic cycles in hope to better understand how economic evolution transpires. The choice of addressing the longest-ranging Kondratieff cycles stems simply out of an ease to model long-term average data rather than the volatile creation of shorter term cycles. As such, for the scope of the paper at hand, it will be assumed that economic cycles in fact do exist and are formed by endogenous causation.

One of the better expressed understandings of cyclicality comes from Goodwin (1965, 2014)[14], whose formulation of "A Growth Cycle" makes use of the Lotka-Volterra predator-prey equations ("Theorie Mathematique de la Lutte pour la Vie", 1931) to describe the conflicting incomes of workers and capitalists. His explanation puts forth the following story:

"When profit is greatest, employment is average, and the high growth rate pushes employment to its maximum which squeezes the profit rate to its average value. The deceleration in growth lowers employment (relative) to its average value again, where profit and growth are again at their nadir. This low growth rate leads to a fall in output and employment to well below full employment, thus

[12] DeGroot, Franses (2007), *Stability through Cycles*
[13] Ibid.
[14] Goodwin (1965, 2014), *A Growth Cycle*

restoring profitability to its average value because productivity is now rising faster than wage rates. This is, I believe, essentially what Marx meant by the contradiction of capitalism and its transitory resolution in booms and slumps."

Overall, I would like to point out the fact that a real world economy is rarely (If ever) seen "in equilibrium". More precisely, it is always in a movement towards or away from equilibrium, with equilibrium occurring at a single moment in time before endogenous forces push it away down a path of lengthy readjustment once again. Furthermore, in his assumptions, Goodwin takes a "steady technological progress" as given and exogenous. In reality, it is likely that the magnitude of technological progress (i.e. importance of a new innovation being brought to markets, or, how big a paradigm shift they provide) will affect the corresponding size of the boom and bust arising out of it. As such, in better understanding the cyclicality of economic evolution, it is worth discussing the means by which technological advancement is propagated.

2.3 Technological Advancement

Since a generous body of literature discusses many facets of technological advancement as it impacts the science of economics, I will specifically restrict this discussion to exploring what causes innovations to appear and how these impact the economy. Simply put, as Anderson and Tushman (1990)[15] describe "while there is a scarcity of models for understanding technological change, research from multiple disciplines suggests several themes that help get inside the black box of technological change", "Basalla's (1988) comprehensive review of technological evolution was anchored in the concepts of diversity, continuity, novelty, and selection".

One way to look at it is that the aforementioned are precipitated to different degrees as a result of investment, which in its turn varies across the period of a cycle. In particular, a paper by Courvisanos and Verspagen (2002)[16] "shows the relation between innovation and investment instability of business cycles and thus affecting the trend growth of these cycles. This way any strong upswing in a cycle must be related to the following downswing and its implication for new investment and further growth." In other words, since investment in past periods has an effect on the magnitude of technological advancement in future periods, it can be understood that the rise of major innovations across markets is in part deterministic as pre-existing conditions facilitate it.

"Research in innovation and investment has tended to be uncoupled, with linkage between the two becoming sporadic. Only economists examining the economy as a vast interconnected "open systems" canvas continued to maintain this link; notably in respect to the heritage of study we can identify Karl Marx, Rosa Luxemburg, Michal Kalecki and Joseph Schumpeter."[17]

[15] Anderson and Tushman (1990), *Technological Discontinuities and Dominant Designs: A Cyclical Model of TechnologicalChange*
[16] Courvisanos and Verspagen (2002), *Innovation and Investment in Capitalist Economies: 1870-2000: Kaleckian Dynamics and Evolutionary Life Cycles*
[17] Ibid

Most importantly, to maintain the view that a comprehensive understanding of economic evolution can be achieved(at least in regards to Kondratieff cycles) , it is important to take on the above mantra of the economy being a “vast interconnected open systems canvas”, which can nevertheless be modeled to a debatable degree of accuracy. Moreover, it is worth differentiating between two types in which technological advance can occur. As Anderson and Tushman (1990) discuss, “Technological discontinuities (innovations that dramatically advance an industry's price vs. performance frontier) trigger a period of ferment that is closed by the emergence of a dominant design”. They take on the view that technological advancements come about in one of two ways. Firstly, a major innovation which has the potential to revolutionize the market can be introduced (the technological discontinuity); this is normally brought about by highly innovative start-up firms and can be seen as a bursting increase in the level of technology. Secondly, gradual improvements are being added onto current designs; something that can be seen as a step-by-step increase in the level of technology. These two types of growth in the technology level are vital in generating the forces that could give rise to the cyclicality of macroeconomic variables.

Lastly, as Acemoglu (2003)[18] presents in a lecture series, the matter of human capital is strongly interlinked with technological progression. On one hand, it is expected that a population with higher human capital can generate more and better innovations: e.g. America’s top education system (at least for those who can afford it) is often cited as one of the causes for the Silicon Valley environment. However, increasing human capital is not, as Acemoglu puts it, a “panacea” (cure for everything). The reason for this is due to the way in which technology affects the labor market: a) increase in the supply of *skilled* workers results in Skill-Biased Technical Change (SBTC); b) increase in the supply of *unskilled* workers results in skill replacing technologies

With reference to the latter skill replacing technologies, a simple example of this can be seen by looking at a hypothetical factory in the last several decades. Maybe 40-50 years ago, a manufacturing line job in a factory used to be a stable profession, where workers were valued for the important but (albeit) small part they played in the creation process of specific goods. We can imagine this factory to have had e.g. 100 workers, a few managers, and a few technicians. However, as SBTC transformed the working place, the routine and blueprint ready jobs of the workers were easily automated by a robot, so 100 lower skilled workers were displaced in favor of a few extra technicians. This engenders increased productivity at a firm level, with higher wages for its lesser amount of employees.

With reference to SBTC, because the new technologies require a certain expertise and can thus only be used by professionals, Acemoglu pertinently emphasizes that “low-skill workers [are] potentially excluded from benefits of new technologies [leading to] greater inequality”. And, because as many papers, e.g. Chevalier et al. (2005)[19] show, human capital development in children is strongly dependent on parental income, this perpetuates a class of lower skilled workers across time with continually less opportunities for employment. As such, while improvement of population wide skills might not be a panacea, directed human capital improvements for the most vulnerable part of the population has the potential to decrease the technology-skill mismatch.

[18] Acemoglu (2003), *Human Capital and the Nature of Technological Progress*
[19] Chevalier, Harmon, O’Sullivan, Walker (2005), *The Impact of Parental Income and Education on the Schooling of their Children*

2.4 Macroeconomic Variables and Relations

Lastly, in addressing the cyclicality of economic evolution, it is paramount to understand how key macroeconomic variables behave in response to changes in output and the growth rate. Specifically, the discussion here will refer to 1) the relation between markup, inflation and unemployment, as well as 2) a brief overview of the intergenerational transmission of human capital.

With reference to the first discussion point, all of these quantities occur at a firm level, and noticeably, it is the firms' markup choice (i.e. how much profit they wish to make) which is most responsible in determining both inflation and unemployment on the aggregate level. However, as Blanchard (2008)[20] candidly puts it:

"How markups move, in response to what, and why, is however nearly terra incognita for macro. . . . [W]e are a long way from having either a clear picture or convincing theories, and this is clearly an area where research is urgently needed."

Nevertheless, a lot can be said on the matter. As Banerjee and Russel (2003)[21] discuss, "Empirical evidence in favour of a negative relationship between inflation and the markup has grown in recent years, including papers by Richards and Stevens (1987), Bénabou (1992), Franz and Gordon (1993), Cockerell and Russell (1995), […] All the estimation undertaken in these papers has assumed that inflation and the markup are stationary variables." However, all these approaches use the same Neo-Keynesian models. As such, this hints at the issue of being stuck in the predominant paradigm of thought: using similar models will naturally yield similar results. In order to explore an ambiguous relationship, it is warranted to use many different approaches in hope of understanding where differences in results come from. One such alternative is provided by Nekarda and Ramey (2013)[22] who "show that frameworks for measuring markups that have produced the strongest evidence for counter cyclicality produce the opposite result when we substitute new methods and data".

As such, we may understand that the ongoing debate regarding the cyclicality of the markup can be framed as follows. If the markup is countercyclical (as most estimations make it to be), inflation occurs because as output rises, firm costs increase by a proportionally larger amount, but, firms lower their markup by a lesser amount compared to the increase in costs (naturally, they want to lose as little as possible from their profits) which leads to an increase in the price level. Conversely, if the mark up is cyclical, as Nekarda and Ramey put it, this occurs conditional on a technology shock. More precisely, due to technological advancement, output is able to increase with a relatively lower rise in costs (increased productivity). In this case, as aggregate demand increases (and aggregate output follows), firms raise the markup because they see it as an opportunity to garner higher profits. Overall, for the model itself, I will adopt the rule of thumb used in Riccetti, Russo, and Gallegati (2012)[23], whereby the firms increase their mark-up only if they managed to

[20] Blanchard (2008), *The State of Macro*
[21] Banerjee and Russel (2003), *Inflation and Measures of the Markup*
[22] Nekarda and Ramey (2013), *The Cyclical Behavior of the Price-Cost Markup*
[23] Riccetti, Russo, and Gallegati (2012), *An Agent Based Decentralized Matching Macroeconomic Model*

sell their full stock, otherwise they decrease it. Through this self-adjusted rule, it will be interesting to note whether the mark-up turns out to be cyclical or acyclical.

Lastly, it is important to discuss a relation which has been the main subject matter of numerous papers: namely, how human capital is shaped and transmitted. As Solon (1999)[24] candidly admits: *"Unfortunately, we remain fairly ignorant about the causal processes underlying the intergenerational transmission of earnings…does parental income matter so much as it does because high-income parents are able to invest more in their children's human capital, or because the genetic or cultural traits that contributed to the parents high earnings are passed on to the children?"*

Nevertheless, even without regard for the underlying cause, the empirical relation that is often remarked is that the best indication of a child's future earnings is in fact his parents' earnings. This could be, as Heckman and Masterov (2004)[25] discuss, due to the fact that "children brought up in less favorable conditions obtain less education despite the large financial returns to schooling". Moreover, Chevalier et al. (2005)[26], utilizing a new approach to econometrically asses this situation, discover that in regards to human capital accumulation "the strong effects of parental education become insignificant and permanent income matters much more".

Overall, the relationship that can be obtained from this understanding is that familial income determines the rate at which the child's human capital changes. In other words, poorer families will be less able to provide the right opportunities and as a result, the human capital of their children will develop at a slower pace than that of a child in a richer family. This could be either because of a direct increased investment in the quality of schooling or because of the better genetic and cultural traits being passed on. Regardless of the reason, it is clear that a higher parental income leads to more human capital accumulation for one's offspring.

3. Methodology

3.1 Structure of Approach

To put the paper into perspective, I formulate the following question:

Can the agent-based simulation of technological cycles replicate the empirically observed, long-run macro-economic moments?

To answer the research question, a model of economic evolution (which is described later in this section), is programmed in Python. From this, the model's output is adjusted by calibrating a list of parameters to appropriate levels in order to replicate a list of moments, which appear in empirically observed patterns as detailed by data, graphs, papers, etc. The graphs supporting the moments can be found in the annex.

[24] Solon(1999), *The Causes and Consequences of Increasing Inequality*
[25] Heckman and Masterov (2004), *The Productivity Argument for Investing in Young Children*
[26] Chevalier, Harmon, O'Sullivan, Walker (2005), *The Impact of Parental Income and Education on the Schooling of their Children*

Overall, this approach provides a means of modeling technological cycles (first part of the research question) which then allows the causal interpretation of economic phenomena (second part of the research question). The choice of using Agent-Based Computational Economics is that if successful, it allows one to trace back how verified economic relations arise out of the aggregated, heterogeneous decision-making of agents. This in theory holds great explanatory power for how economic systems behave across a spectrum of individual situations, not just that of a single representative agent.

Taking a step back however, the worth of such model will ultimately depend on two major factors: firstly, are the targeted moments replicated *realistically* in the sense that their magnitude, period, relation etc. is correct. And secondly, are the base assumptions of the model economically justified such that the agents' behavior is realistic. In other words, were the right results obtained in the right way? By objectively discussing this matter, in the conclusion section, I will assess the strength of my results and provide ideas for the improvement of future work.

3.2 Targeted Moments and Model Parameters

The model is targeting to replicate the following list of moments (on the next page). The table below shows the targeted moment, corresponding stylized fact and a brief explanation of what parameters are calibrated to adjust the moment to match empirical data. Information regarding the value of parameters and initial conditions can be found after the model description.

By implementing the model into a Monte Carlo simulation (multiple iterations of the system with different normal seeds), graphs quantifying the targeted moments can be made. From this, unexpected behavior will be corrected by calibrating the parameters until the results match empirical observations. Ultimately, conclusions can be drawn from the individual behavior of agents as well as the overall economy; out of this, policy prescriptions can be formulated.

TABLE 2 – Targeted Moments

Moment	Stylized Fact	Calibrated With
1. Cyclical Behaviour	K- Waves empirically observed from the 1800s onwards	Relative rates of change between human capital and firm capital acquisition, χ, γ - determines employment/wages → demand → firm profitability and entry/exit → employment/wages once again, in a cyclical fashion
2. Cycle Amplitude	K-Waves associated with changes of 2-3% in GDP	Firms' costs to scale o, s, ω - the costlier it is to scale up, the less production can expand
3. Cycle Period	K-Waves period roughly 50 years	Bank leverage ratio Ξ – determines amount of new advanced firms created each new period

4. Down-skilling	Skill requirement for hiring new workers falls with the unemployment rate	Return to human capital θ which determines firms' cost of wages
5. Productivity-Wage Decoupling	Increases in worker compensation have not kept up with increases in firm productivity	Relative rates of change between human capital and firm capital acquisition, ⍰, γ
6. Phillips Curve	Negative relation between inflation and unemployment	Firms' costs to scale determine costs of additional workers (i.e. when there is more employment)
7. Pro-cyclical Mark-up	Positive relationship between output and price mark-up	Firms' price mark-up adjustment ψ
8. Two Types of Tech. Advancement	Majority of firms advance incrementally, but major technological discontinuities occur at certain intervals	Firm Capital Acquisition γ – determines rate of incremental advancement. Bank's leverage ratio Ξ and costs of entrepreneurship Z – determines how many new firms can be established
9. Timing of Tech. Discontinuity	Major discoveries made approx. every 50 years, discovered during downswing, widely implemented during upswing	Variance of Human Capital distribution in new workers $\sigma^2_{H,t=T}$ - timing of new "high skilled" entrepreneurs in the right conditions will introduce a technological discontinuity
10. Propensity for incremental Tech. advancement	Approx. 30% of firms innovate at all points of cycles	Firm costs – additional money (after paying production costs and debt) are invested in tech. Advancement
11. Skill-Biased Technical Change	SBTC is a change in production technology that favors more skilled over less skilled workers	Production Function weight φ which determines demand for labor relative to capital

3.3 Model Setup

TABLE 3 – Overview of Model Variables

Variable	Symbol	Comments
Worker Related		
Fertility Rate	G_t	Number of children per woman
Human Capital	$H_{i,t}$	Skill of worker i
Period of worker's life	$p_{i,t}$	age of the worker
Household Income	$Y_{HH\,k,t}$	Sum of all wages of earners in household k
Disposable Income	$Y_{dis\,i,t}$	Money agents optimize between consumption/savings
Living Costs	$L_{i,t}$	Based on the period of worker's life
Aggregate Income	AY_t	Total disposable income of workers in the economy
Employment Status	$e_{i,t}$	Dummy variable, 0 or 1
Utility (of consumer)	$U_{i,t}$	Utility to maximize
Expenditure on goods	$x_{i,t}$	Spending of worker on novel goods to maximize utility
Savings	$S_{i,t}$	Savings for next period
Firm Related		
Firm Capital	$K_{j,t}$	Level of Tech. Advancement, proxy for skill requirement
Number of workers firm j	$m_{j,t}$	
Productivity (of individual worker)	$Q_{i,t}$	Amount of goods produced by a worker at that firm
Firm Productivity	$\hat{Q}_{j,t}$	Total amount of goods produced by the firm
Wage (of individual worker)	$W_{i,t}$	Return to human capital
Total Costs	$C_{j,t}$	Costs including fixed and variable
Price	$p_{j,t}$	Price a firm sets for its goods
Mark-up	$\lambda_{j,t}$	Mark-up over average costs
Objective function (firm utility)	$U_{f,j,t}$	Objective function to maximize
Profits = Savings = Debt	$P_{j,t}$	Profits that are put into savings or used to pay debt
R&D Expenditure	$D_{j,t}$	Money invested in technological advancement
Processes		
Aggregate Demand	AD_t	Total "planned" expenditure of workers in the economy
Aggregate Supply	AQ_t	Total value of goods produced in the economy
Aggregate Income	AY_t	Total income of all workers in the economy
Market Share	$\xi_{j,t}$	Market dominance of a particular firm
Bank Related		
Interest Rate	r	Return on savings
Lending Rate	r_l	Costs of debt
Bond Rate	r_B	Return on bank savings (government bonds)
Entrepreneurship Costs	E_j	Costs to establish a new firm
Consumer Savings	$\bar{S}_{c,t}$	Total savings of workers in the economy
Firm Savings	$\bar{S}_{f,t}$	Total savings of firms in the economy
Firm Debt	$\bar{D}_{f,t}$	Total debt of firms in the economy
Investment	I_t	Money used by bank to invest in new firms
Target capital level of an investment	$\hat{K}_j$	Potential capital level of a firm that could be established

The economy consists of workers (i = 1, 2, ..., I), households (k = 1, 2, ..., K), firms (j = 1, 2, ...,J) and a central bank, which interact over a time span t = 1, 2, ..., T in the following markets:

- Labour Market: workers and firms
- Goods Market: workers and firms
- Credit Market: firms and the bank
- Deposit Market: workers and the bank

A. Workers and Households

Each worker lives up to 70 periods of time p. New workers are born as children that accumulate human capital. The fertility rate (children per woman), i.e. the population growth rate is:

$$G_t = \Phi_G + \frac{u_{H,t}}{u_{K,t}} \tag{1.0}$$

A general overview of a worker's life progression can be understood as follows:

- Up until period of life $p = (\frac{H_i}{u_H})^{1/2} * 18$ the worker is a child
  - accumulates Human Capital based on household income
  - has living costs paid out of household income
  - no expenditure on goods.
- Past period of life $p = (\frac{H_i}{u_H})^{1/2} * 18$, if and only if employment is found the worker becomes an adult
  - establishes own household,
  - has living costs,
  - works and saves
  - buys goods
- At period of life p=60, the worker is a senior
  - has living costs
  - consumes savings to buy goods

All workers are characterized by a specific Human Capital, which determines their productive capacity, the wage they ask for, and their costs of establishing new firms (if that is the case). The Human Capital H of all agents i is set as an initial condition and is normally distributed (bounded at 0 and 100) with mean u_H and variance σ_H^2 , as in equation (1.1):

$$H_i \sim N(u_{H,t=0}, \sigma^2_{H,t=0}) \tag{1.1}$$

Workers are divided into households for two purposes: firstly, to jointly pay living costs for all (since children cannot do so by themselves), and secondly, to accumulate human capital based on the household's income. Living costs depend on the period (p = 1, 2 … 70) of life of the worker, and vary as such:

$$\mathrm{L}_{i,t} = \Omega(\mathrm{p} - 30)^2 + \Phi \tag{1.2}$$

Moreover, the income of a particular household $Y_{HH,\mathrm{k},t}$ determines the growth in human capital of those members that can do so (children and young adults). This happens based on how the household's income relates to aggregate income in the economy AY_t such that:

$$H_{i,t+1} = H_{\mathrm{i},t} \left[1 + (0.1 * \left(\frac{Y_{HH,\mathrm{k},t}}{u_{yHH,t}}\right)^{\chi})^{\ln(p)}\right] \quad (1.3)$$

Once the living costs of all household members have been paid, and the calculation for human capital increase has been made, the earners of the household receive back disposable income $\mathrm{Y_{dis}}$, an amount proportional to what they have contributed. I.e. if the main earner provides 60% of the household's income, they get back 60% of the money after living costs have been paid. If one of the earners is unemployed, he receives unemployment benefits in value of the 0.5 the lowest wage paid out in the economy.

$$\mathrm{Y_{HH,t}} = \mathrm{S_{t-1}} + \textstyle\sum_1^2 \mathrm{W_{i,t}} \quad (1.4)$$

$$\mathrm{Y_{dis,i,t}} = \frac{\mathrm{W_{i,t}}}{\sum_1^2 \mathrm{W_{i,t}}} (\mathrm{Y_{HH,t}} - \textstyle\sum_1^n \mathrm{L_{i,t}}) \quad (1.5)$$

With this money, each worker then individually maximizes their utility according to their employment status and relative income.

$$U_{i,t} = (\mathrm{x_{i,t}})^{1-\alpha} (\mathrm{S_{i,t}})^{\alpha} \quad (1.6)$$

Where $\alpha + \beta + \gamma = 1$ and $\beta = 0.8 * \alpha$

- If employed e=1 $\quad \gamma = 0.1 + \left(\frac{\mathrm{Y_{dis,i,t}}}{u_{Y_{dis,i,t}}}\right) * 0.1 \quad \gamma_{\max} = 0.8$

- If unemployed e=0, $\quad \gamma = 0.5 + \left(\frac{\mathrm{Y_{dis,i,t}}}{u_{Y_{dis,i,t}}}\right) * 0.1 \quad \gamma_{\max} = 0.8$

B. Firms

Each firm is characterized by a level of firm capital, which determines their productive capacity. This is normally distributed (bounded at 0 and 100) with mean u_K and variance σ_K^2:

$$K_j \sim N\left(u_{K,t=0}, \sigma^2_{K.t=0}\right) \quad (2.1)$$

The productivity of each worker at a firm is given by the number of goods $Q_{i,t}$ they produce, as determined by the following production function:

$$Q_{i,t} = 5 * \frac{\mathrm{H_{i,t}}^{\varphi} \mathrm{K_{j,t}}^{1-\varphi}}{u_{K,t}} \quad (2.2)$$

Firm-wide productivity $\hat{Q}_{j,t}$ is the sum of all workers' productivities such that:

$$\hat{Q}_{j,t} = \sum_{1}^{m_j} Q_{i,t} \tag{2.3}$$

The wage $W_{i,t}$ of a worker is a certain proportion of their productivity such that:

$$W_{i,t} = (\frac{H_i\, K_j}{u_H})\, \theta^{1-u} \tag{2.4}$$

The costs firms face are based on the level of technology it is at, as well as the amount of goods its workers produce $\hat{Q}_{j,t}$ and the wages it pays out:

$$C_{j,t} = \hat{Q}_{j,t} * K_{j,t}^{\frac{K_{j,t}}{2u_K}} * \left[\pi \sin\left(\rho\, \hat{Q}_{j,t}\right) + \tau\right] + \sum_{i=1}^{m_j} W_{i,t} \tag{2.5}$$

The price a firm sets is a mark-up $\lambda_{j,t}$ over average costs:

$$p_{j,t} = \left(\frac{C_{j,t}}{\hat{Q}_{j,t}}\right)(1 + \lambda_{j,t}) \tag{2.6}$$

Through a sales process detailed below, a certain quantity of goods is sold $Q_{j,s,t}$. Based on this, the mark-up can be adjusted such that:

If $\frac{Q_{j,s,t}}{Q_{j,t}} = 1$ (i.e. all goods are sold)

$$\lambda_{j,t} = (1 + \psi) * \lambda_{j,t-1} \tag{2.7}$$

If $\frac{Q_{j,s,t}}{Q_{j,t}} < 1$

$$\lambda_{j,t} = \lambda_{j,t-1}(\frac{Q_{j,s,t}}{Q_{j,t}}) \tag{2.8}$$

The revenue a firm earns ($Q_{j,s,t} * p_{j,t}$) is used to maximize its objective function (2.9) between profits (or paying back debt) and investment in R&D. If the firm is in debt, all profits are used towards paying back debt and no technological progression occurs.

$$U_{j,f,t} = \left(D_{j,t}\right)^{\alpha_f} \left(S_{j,t}\right)^{1-\alpha_f} \tag{2.9}$$

Based on the amount invested in research and development, a firm's quality of capital progresses:

$$K_{j,t+1} = K_{j,t} + \gamma \frac{D_{j,t}}{K_{j,t}} \tag{2.10}$$

C. Processes

a) Sales

Assuming

- Imperfect Competition
- Limited Search Frictions
- Market dominance influences quantity sold and ability to charge different prices

After wages $W_{i,t}$ are paid out to workers, and after living costs for the household are paid, using their disposable income, each worker maximizes their utility. This yields a wished expenditure $\mathrm{x_{i,t}}$ for all consumers. Summing this together over all consumers we obtain the aggregated demand in a particular period:

$$\mathrm{AD_t} = \sum_1^{\mathrm{i}} \mathrm{x_{i,t}} \tag{3.1}$$

This is the amount of money that will be spent on goods. If there is a shortage of supply (i.e. the value of the goods is less that of aggregate demand) all goods are sold and consumers receive back an amount proportional to their original "wished" expenditure. In other words, they get back the money that was unable to buy goods. Aggregate supply can be seen as the total value of goods in the economy:

$$\mathrm{AQ_t} = \sum_1^{\mathrm{j}} \mathrm{p_{j,t}} * \mathrm{Q_{j,t}} \tag{3.2}$$

Next, firms are ordered in increasing terms of the prices they have set

$$p_{j\,1,t} < p_{j\,2,t} < \cdots < p_{jj\,-1,t} < p_{jj\,,t} \tag{3.3}$$

For each of the firms, market dominance is calculated as

$$\xi_{j\,,t} = \frac{p_{j\,,t} * Q_{j\,,t}}{AQ_t} \tag{3.4}$$

Now, starting from the firm with the lowest price, each firm sells a percentage of their stock equal to their market dominance $\xi_{j\,,t}$. This process is then repeated *iteratively* until either all goods are sold or the value of Aggregate Demand has been consumed. Goods are perishable and extra goods disappear at the end of each period.

b) Hiring and Firing Decisions

Hiring

Using a simple rule of thumb, firms decided whether or not to hire an additional worker. Firstly, they assess whether the entirety of their stock was sold last period, i.e. $\frac{Q_{j,s,t-1}}{Q_{j,t-1}} = 1$. If this holds, they are eligible to hire. Looking at the pool of unemployed, a firm checks the marginal benefit provided by each potential new worker, i.e. whether

$$Q_{i,t}(new\ worker) * \mathrm{p_{j,t-1}} > C_{j,t}(new\ worker) \tag{3.5}$$

Essentially, the firm hires workers which bring a marginal benefit to the firm. It offers wages to all potential hires within a limit of 10% of its own size (rounded upwards to minimum 1).

Firing

If a firm is in debt, and the entirety of its stock was not sold the last period, i.e. $\frac{Q_{j,s,t-1}}{Q_{j,t-1}} < 1$, it fires the workers with the least human capital such that:

$$sum(fired\ worker\ wages) > \beta *\ value\ of\ debt$$

c) The Bank Cycle and Entrepreneurship

Bank Cycle

1. Consumers withdraw last period savings for current period utility optimization. Bank pays these back multiplied by the savings rate.

$$S_{i,t} = S_{i,t-1} * \mathrm{r} \tag{3.6}$$

2. Firms take credit to finance current period production costs. Bank finances this which adds up to an amount $\overline{\mathrm{D}}_{\mathrm{f,t}}$ over all firms j

$$\overline{D}_{f,t} = \sum_1^{\mathrm{j}} C_{j,t} \tag{3.7}$$

3. After wages are paid out, consumers optimize their respective utilities and deposit savings into the bank. This totals to an amount $\overline{\mathrm{S}}_{\mathrm{c,t}}$ over all workers i

$$\bar{S}_{\mathrm{c},t} = \sum_1^{\mathrm{i}} S_{i,t} \tag{3.8}$$

4. Firms pay back their debt to the bank multiplied by the lending rate. Firms deposit additional savings. This totals to an amount

$$\bar{S}_{\mathrm{f},t} = \sum_{1}^{\mathrm{j}} D_{j,t} * \mathrm{r}_{\mathrm{l}} + \sum_{1}^{\mathrm{j}} \mathrm{P}_{j,t} \tag{3.9}$$

5. Bank uses an amount of money I_{t} to invest in entrepreneurship. This is based on a leverage ratio ? that is set as a parameter

$$\mathrm{I}_{\mathrm{t}} = \Xi(\bar{S}_{\mathrm{f},t} + \bar{S}_{\mathrm{c},t}) \tag{3.10}$$

6. The remaining money is assumed to be invested in government bonds (or other safe return assets) which yield a return r_{B} to bank savings:

$$S_{B,t+1} = (1 - \Xi)\left(\bar{S}_{\mathrm{f},t} + \bar{S}_{\mathrm{c},t}\right) * \mathrm{r}_{\mathrm{B}} \tag{3.11}$$

Entrepreneurship

New firms are established based on the human capital of unemployed workers v, i.e. entrepreneurs. The bank prioritizes creation of firms based on the unemployed workers v with highest human capital. The target capital level ? of the firm to be established is based on the unemployed worker's specific human capital, such that:

$$\widehat{K}_j = [H(v_i)]^{\frac{u_K}{u_H}} \tag{3.12}$$

Next, the costs of establishing the firm are

$$E_{j,t} = \frac{\widehat{K}^Z}{u_k} \tag{3.13}$$

Once the costs of establishment and target levels of new firms are established for multiple potential entrepreneurs, the bank maximizes the amount of capital $\widehat{K}$ created subject to the condition that the costs of establishment $E_{j,t}$ do not go over their planned investment I_{t}.

$$\max \sum_{1}^{\mathrm{j}} \widehat{K}_j \quad s.t. \ \sum_{1}^{\mathrm{j}} E_{j,t} < \mathrm{I}_{\mathrm{t}} \tag{3.14}$$

D. Model Timeline

1. Workers advance one period of life
2. Marriage rate and fertility rate checked and upheld
3. Savings accrue at the savings rate
4. Workers get hired or fired
5. Firms take credit to finance production costs
6. Production occurs and wages are paid out
7. Households use their income to pay living costs
8. Increase in human capital occurs based on household income
9. Earners receive back proportional remaining household income to maximize their utility between consumption and savings
10. Sales process occurs and goods are bough/sold
11. Firms use the revenue earned to pay back debt. Excess money is split between profits (savings for next period) and investment in R&D
12. Firms advance Technologically
13. The bank leverages part of their money for planned investment.
14. New firms are created.

TABLE 4 – Initial Conditions

Initial Condition	Amount
Number of workers	1,000
Firm Sizes	10% of workers in firm size N(30, 5)
	25% of workers in firm size N(15, 3)
	55% of workers in firm size N(5, 1)
Unemployment Rate	0.1
Marriage Rate	50% of adults married (= 50% of households consist of minimum two people)
Mean of Human Capital	50
Variance of Human Capital	5
Mean of Firm Capital	50
Variance of Firm Capital	5
Interest Rate	0.02
Lending Rate	0.03
Bond Rate	0.05
Granularity of Goods	5

Next, a summary of all the model's parameters can be seen in the following table:

TABLE 5 – Model Parameters

Parameter(s)	Symbol(s)	Description	Value
Fertility Rate Adjustment	Φ_G	Determines a base level of children per woman	1.2
Living Costs Adjustment	Ω, Φ	Determines base costs and how they scale with age	0.2, 30
Human Capital Acquisition	χ	Determines rate at which human capital progresses	0.1
Return to Human Capital	θ	Determines wage of worker	2
Production Function Weight	φ	Determines relative importance of H, K in production	0.4
Firm's Costs to Scale	π, ρ, τ		2, 0.04, 5
Price Mark-up adjustment	ψ	Determines how much mark-up increases when a firm sells the entirety of its stock	0.2
Firm's propensity to fire	β	Determines how many workers a firm fires in attempt to cover its debt	0.4
Firm's Propensity to invest in R&D	α_f	Determines amount invested in R&D	0.1
Firm Capital Acquisition	γ	Determines rate at which firm capital progresses	0.2
Costs of Entrepreneurship	Z	Determines cost of establishing a new firm	2
Bank leverage ratio	Ξ	Determines the amount of investment that occurs	0.01

Lastly, it is perhaps worth explicitly making clear some assumptions that arise out of the model formulation:

1. Fertility Rate depends on the standards of living of the workers. Higher human capital reflects higher standards of living
2. Workers with higher than average human capital pursue more education than average
3. The progress of a child's human capital is based on familial income
4. Agents with a (much) higher than average income consume proportionally less of their income
5. Firms scale such that there is first an increase in costs, then economies of scale are reached, and lastly diseconomies of scale
6. Firms only increase prices if they manage to sell the entirety of their stock
7. Firms only invest in capital if they are not indebted
8. Market dominance of a firm influences both the price it can set as well as the quantity sold
9. A firm can only hire additional workers if it is not indebted

4. Result Analysis

4.1 Model Output

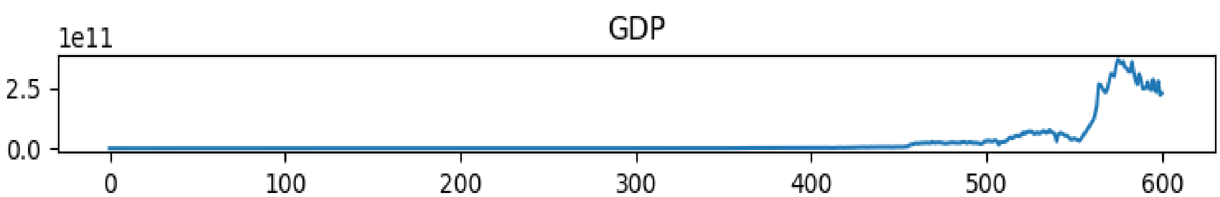
As a note, due to technical reasons, I was only able to successfully simulate a 600 year time period. In order to perform a Monte Carlo simulation, as well as prolong the length of the simulation, I would need access to a significantly better computer.

I begin the analysis by providing the model output across a spectrum of quantities. I analyze this first, clarifying quantities where needed and explaining what is happening. After this, in the next section of the Result Analysis, I provide a moment comparison exercise where the model's data is checked against empirically observed stylized facts.

Before beginning the analysis, it is important to mention that the first 200 years of the program are used to initialize the economy and balance various quantities, e.g. number of firms, number of workers, Aggregate Demand/ Aggregate Supply imbalances, etc. This is an approach used in most ACE models.

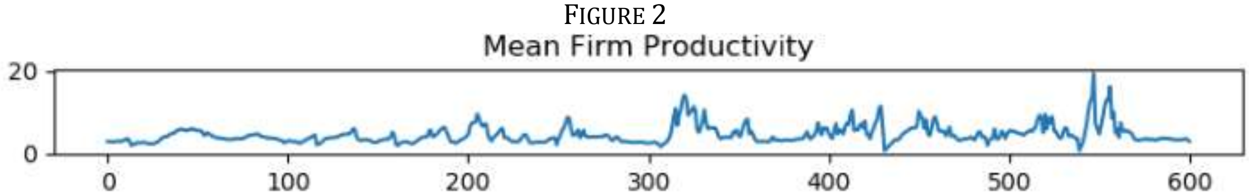
Starting with perhaps the most important quantity, in the graph below, one can see the simulated GDP of the economy over a period of 600 years. Perhaps the most striking fact is that (just like the real world), it showcases a "hockey-stick" shape of growth, whereby due to exponentiality, the majority of the growth is concentrated in the last 100 years.

FIGURE 1



Further, a cyclical movement can (slightly) be discerned as a boom until around year 520, followed by a crash and another boom until the year 580, after which another crash follows. This pattern can more clearly be seen in the mean of firm productivity (graph below). Firm productivity is indicative of how much is produced in the economy, GDP being the result of that production at the current price level.

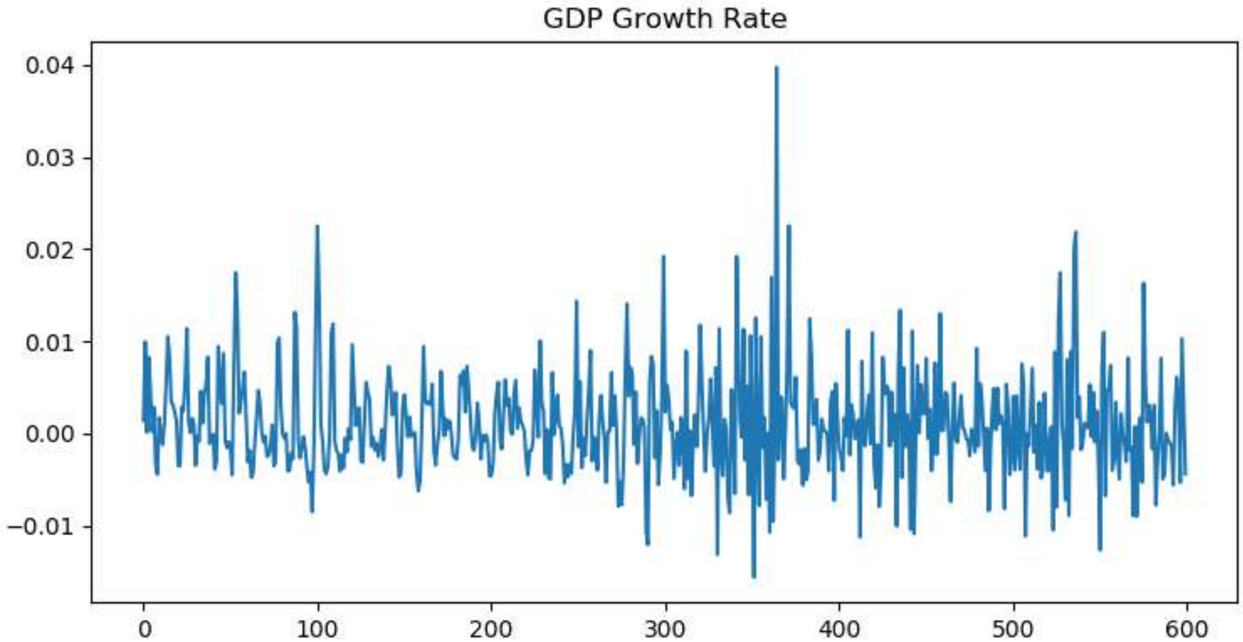
FIGURE 2



Cyclical variation in the figure above occurs with a period of 40-100 years, however it seems rather irregular. Nevertheless, variation in firm productivity indicates the existence of cycles in GDP (which are however difficult to see due to the scale of exponential growth).

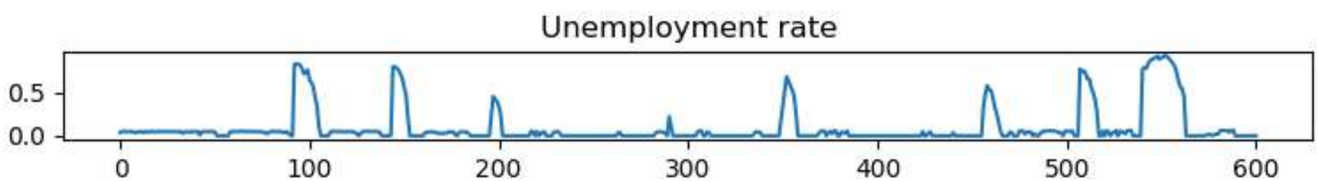
With reference to the growth rate (figure below), no clear 50+ year K-wave pattern can be detected, but the cyclical movement is obvious, if rather too erratic.

FIGURE 3



Next, looking at the unemployment rate, several things come out. Firstly, for most of the time, the economy is close to employment capacity signaling that growth is occurring. However, this is interrupted by periods of mass unemployment reaching 50% or more. Undoubtedly, the roughness of these transitions is the result of a poorly written model (at least in this regard), something that should be fixed in future work. Nevertheless, these periods of unemployment also occur at roughly 50 year intervals with striking regularity.

FIGURE 4



Below, we many note the number of new firms being created in the economy. Since this is a closed system, it is perhaps less volatile than the real world. What may be seen from the graph is that there are a relatively larger number of firms being created in the first 200 years than the rest of the time. This could be due to the monopolistic nature of capitalism whereby a few firms are able to gain (and retain) a competitive advantage by virtue of their size. As such, as the economy evolves, most of the employment is concentrated in these fewer larger firms. Due to computation issues, I was unable to simulate past period 600 reliably so it is difficult to say how this quantity will continue to progress over time.

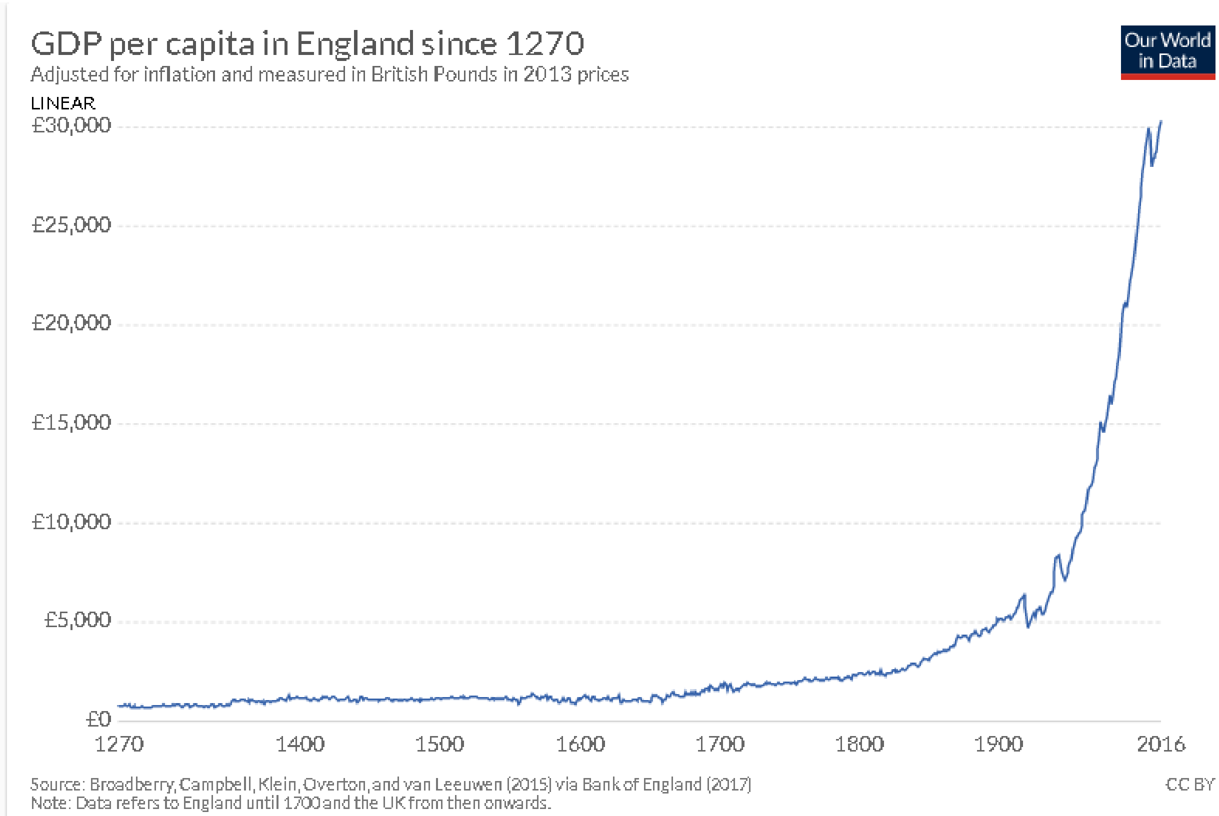
FIGURE 5

4.2 Moment Comparison Exercise

a) GDP Growth

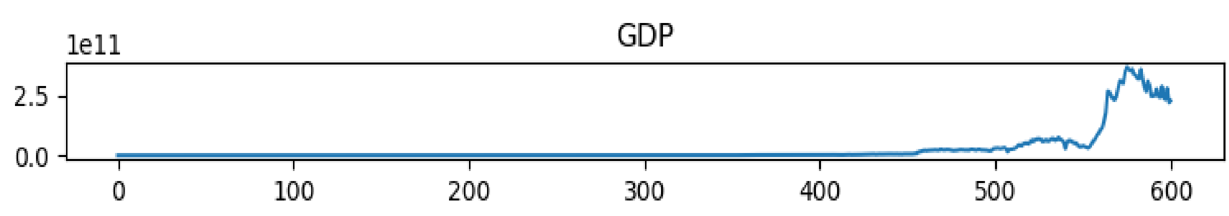
Below, we can see the GDP per capita in England over a period of 700 years. One can immediately note the hockey stick shape in growth due to exponentiality.

FIGURE 6



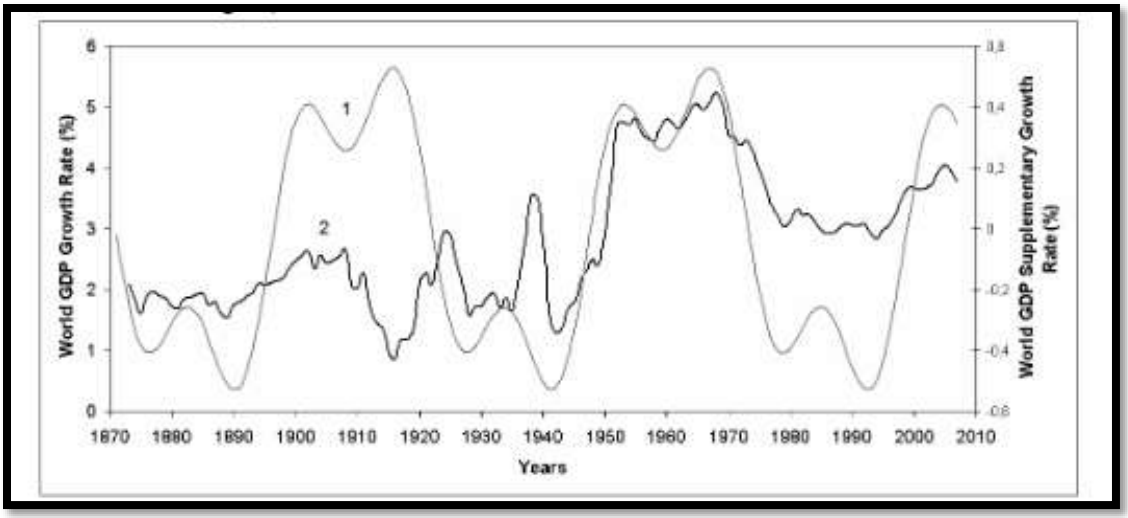
Similarly, the same hockey-stick shape can be noted in the simulated economy. However, a clear difference is the degree to which GDP varies. Whereas in the British economy, booms and busts are relatively insignificant to the principal growth path, in the simulated economy the crisis appear to be more severe and so GDP varies more erratically.

FIGURE 7



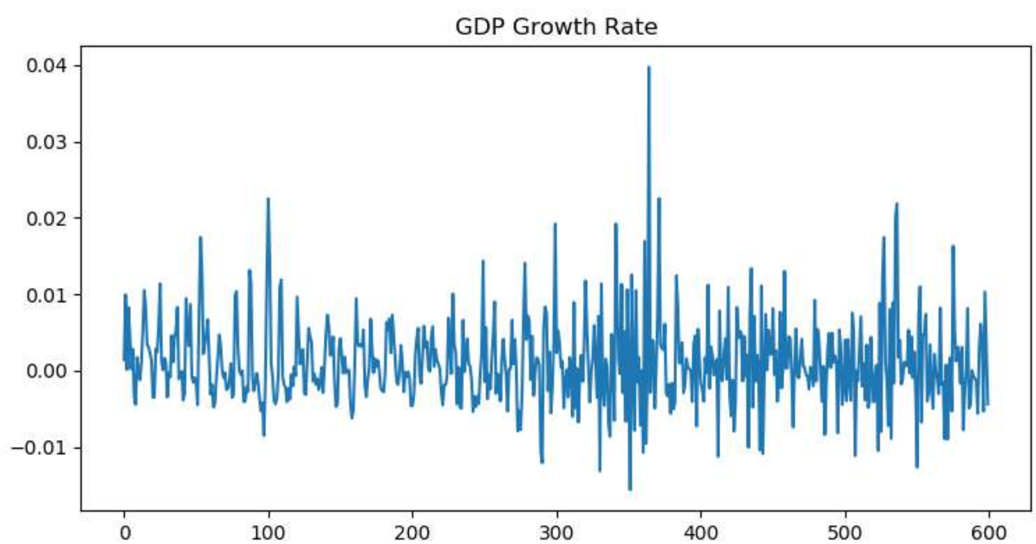
b) GDP Growth Rate

FIGURE 8 - Constructed K-wave (curve1) against 11-year average world GDP Growth Rate



Firstly, above we can see the world GDP growth rate together with a fitted K-wave performed by . Of note is the fact that both the fitted K-wave and the world GDP growth rate have amplitude of around 4% (ranging from around 1% to around 5%). Comparing this with the model's output, it can be noted that in terms of amplitude it recreates the moment relatively well. However, in terms of actual magnitude, the simulated economy often has periods of negative growth; this is because the "world" GDP always increases, whereas an isolated system can also have periods of negative growth

FIGURE 9



In all other aspects of the targeted moments, the model performs too poorly to be worth analyzing. This was in part to be expected by the rough formulation of such a complex system that is the economy.

5. Conclusions

5.1 Conclusions

Overall, to summarize simply, the model performs well in some regards (particularly in simulating cyclical behavior), but extremely poorly in other regards. With reference to the poor performance, the model is unable to even closely replicate most of the targeted moments. This is a result of multiple factors, most notably, a far from ideal formulation of the complexity of an economic system.

Nevertheless, a list of conclusions may be drawn from the observed behavior of the simulated economy:

1. Growth (or increased productivity) occurs due to an increase in both Human and Firm Capital
2. The cyclicality of growth (specifically, long term technological growth) arises out of an imbalance between Human Capital and Firm Capital and their attempt to catch up to each other.
3. Overall, growth seems as it will continue regardless of the crises that accompany it, so the aim of policy should not be to eliminate crises, but rather mitigate the effects of them, especially on the most vulnerable.

5.2 Policy Recommendations

The main policy prescription that may come across is cemented in the understanding of how technological cycles function: variation in the relative quality of human capital to firm capital creates a disparity in productivity engendering cyclicality. To mitigate the magnitude of this, and thereby decrease the severity of crises, governments should take note of the state of the mismatch. For example, at this moment, the most developed economies (e.g. US or UK) are plagued by a mismatch such that the majority of their populations do not have the skills required for the current job market. This can be seen by the disparity in incomes and the pronounced effects of poverty on the less fortunate which directly affects employment and productivity (despite how governments may wish to disguise their figures behind part-time or zero-hour contracts).

5.3 Plans for Future Work

The main scope of future work is twofold. Firstly, refine the model formulation to be able to simulate and replicate more moments. The performance at the moment is starkly disappointing. Secondly, get access to more computing power in order to be able to run a full-fledged Monte Carlo simulation.

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7. Annex

Section One - Moments and Stylized Facts

Growth Path of the Economy:

Table 1. Long Waves and Their Phases Identified by Kondratieff

Long wave number	*Long wave phase*	*Dates of the beginning*	*Dates of the end*
One	A: upswing	"The end of the 1780s or beginning of the 1790s"	1810–1817
	B: downswing	1810–1817	1844–1851
Two	A: upswing	1844–1851	1870–1875
	B: downswing	1870–1875	1890–1896
Three	A: upswing	1890–1896	1914–1920
	B: downswing	1914–1920	

The subsequent students of Kondratieff cycles identified additionally the following long-waves in the post-World War 1 period (see Table 2):

Table 2. "Post-Kondratieff" Long Waves and Their Phases

Long wave number	*Long wave phase*	*Dates of the beginning*	*Dates of the end*
Three	A: upswing	1890–1896	1914–1920
	B: downswing	From 1914 to 1928/29	1939–1950
Four	A: upswing	1939–1950	1968–1974
	B: downswing	1968–1974	1984–1991
Five	A: upswing	1984–1991	2008–2010?
	B: downswing	2008–2010?	?

Sources: Mandel 1980; Dickson 1983; Van Duijn 1983: 155; Wallerstein 1984; Goldstein 1988: 67; Modelski, Thompson 1996; Bobrovnikov 2004: 47; Pantin, Lapkin 2006: 283–285, 315; Ayres 2006; Linstone 2006: Fig. 1; Tausch 2006: 101–104; Thompson 2007: Table 5. Jourdon 2008: 1040–1043. The last date is suggested by the authors of the present paper. It was also suggested earlier by Lynch 2004; Pantin, Lapkin 2006: 315; see also Akaev 2009.

FIGURE 1 – Tables of Kondratieff Cycles[27]

[27] Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*

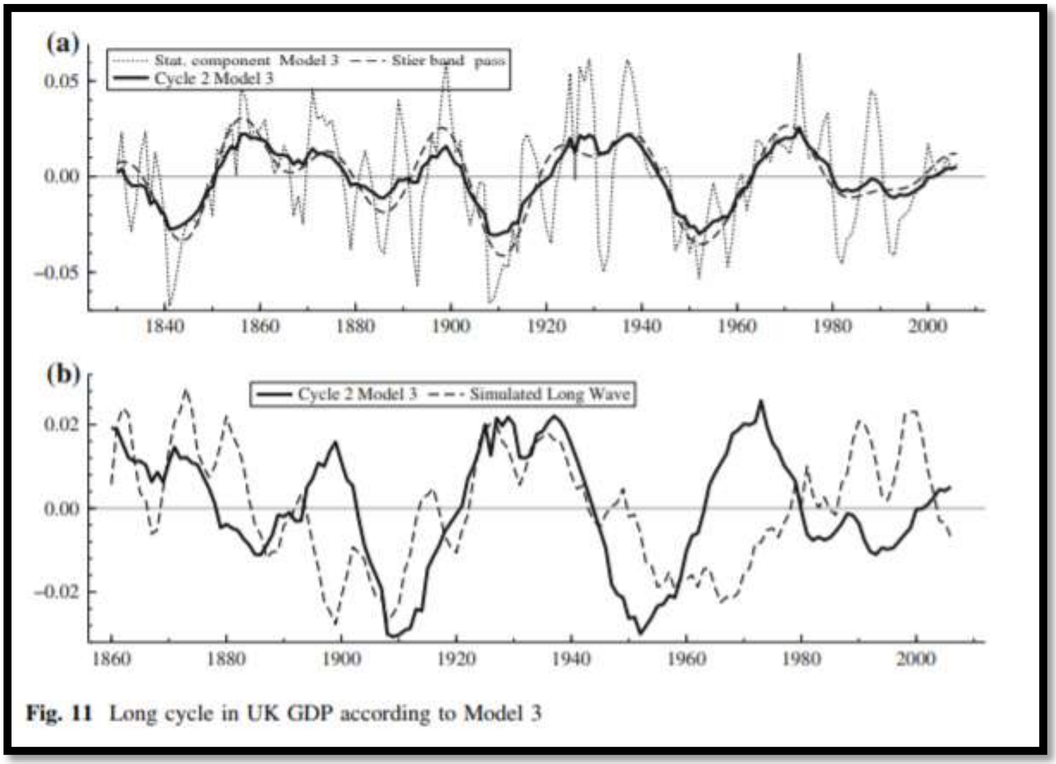
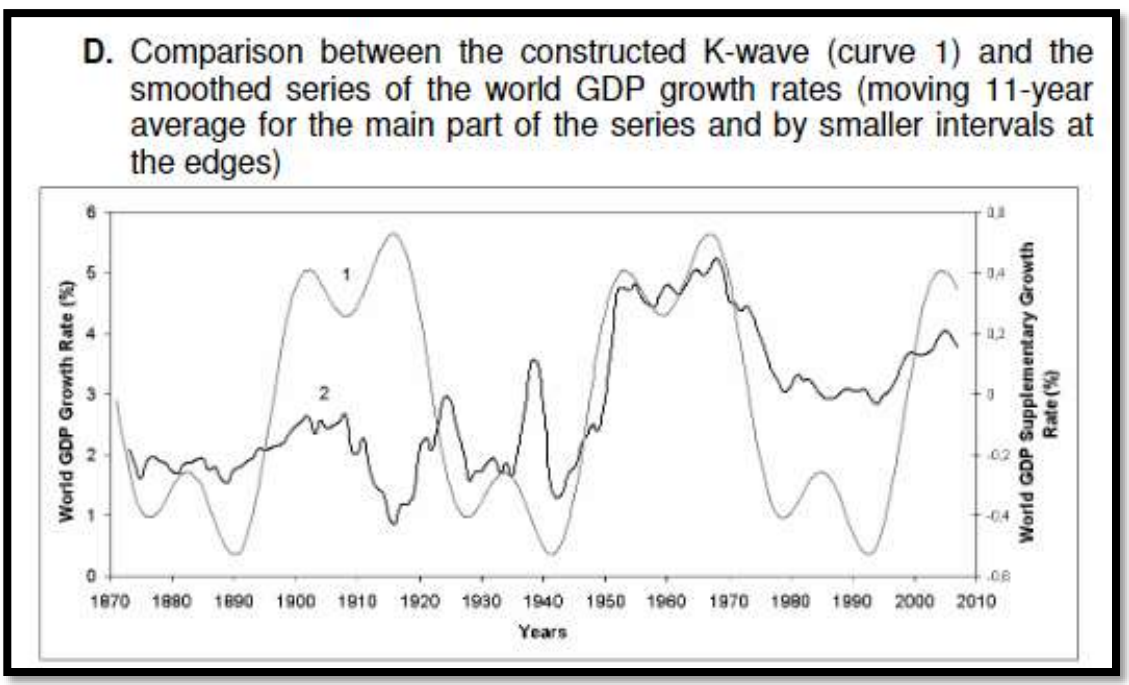


FIGURE 2 – Long Cycle in UK GDP[28]



29

FIGURE 3 – Constructed K-wave against 11-year average world GDP Growth Rate

28 Metz (2010), *Do Kondratieff Waves Exist?*

29 Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*

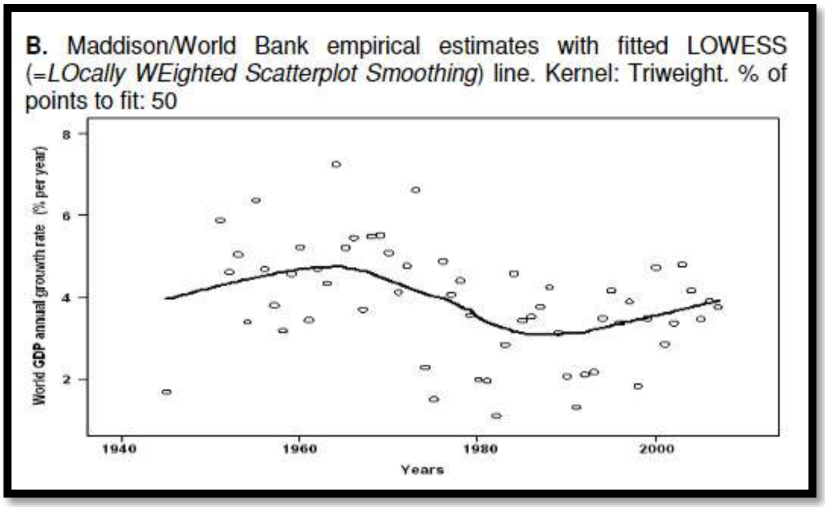


FIGURE 4 – K-Wave Fitted on World Bank Estimates of Global Growth Rate[30]

Multiple papers corroborate the following historical progression of long waves:

- 1789-1849 – First cycle caused by the Industrial Revolution and social changes
- 1849-1896 – Second cycle caused by the widespread use of steam engines
- 1896 – 1945 – Third cycle caused by the widespread use of electricity
- 1945 – 1995 – Fourth cycle caused by manufacturing becoming widespread
- 1995 – Onwards – Fifth cycle currently occurring due to information technology

As Kondratieff described, something important to note is that a new cycle begins not when the discovery of a major new innovation occurs, but when this innovation becomes of widespread use. This is simulated in the model by the fact that while new highly technologically advanced firms are created during a recession, it is only when the whole economy moves up to their level of technology that a new cycle begins.

$$AQ_t = \sum_1^j p_{j,t} * Q_{j,t}$$

(1.1)

$$g_t = \frac{AQ_t - AQ_{t-1}}{AQ_{t-1}}$$

(1.2)

$$\overline{g_t} = \frac{\sum_1^t g_t}{t}$$

30 Korotayev, Tsirel (2010), *A Spectral Analysis of World GDP Dynamics*

(1.3)

$$g_{\square,\square} = \frac{\sum_1^t (g_t - \bar{g}_t)^2}{t}$$

(1.4)

Equation (1.1) identifies aggregate productivity (or GDP) in the economy. This should develop cyclically out of the model formulation, following the dynamics of a Kondratieff cycle. Equation (1.2) identifies the growth rate; this should vary cyclically from positive to negative. Overall, the average growth rate, as given by equation (1.3) should be positive. The variance of the growth rate, equation (1.4), describes the amplitude of cycles, i.e. the height and depth of peaks or troughs. The period of the cycle is determined jointly by the acquisition parameters of firm capital and human capital (these are further discussed below).

Rate of Change of Human Capital: Acemoglu (2013)[31] discusses how technology affects labour markets in the following ways

- increase in the supply of *skilled* workers results in Skill-Biased Technical Change (SBTC)
- increase in the supply of *unskilled* workers results in skill replacing technologies

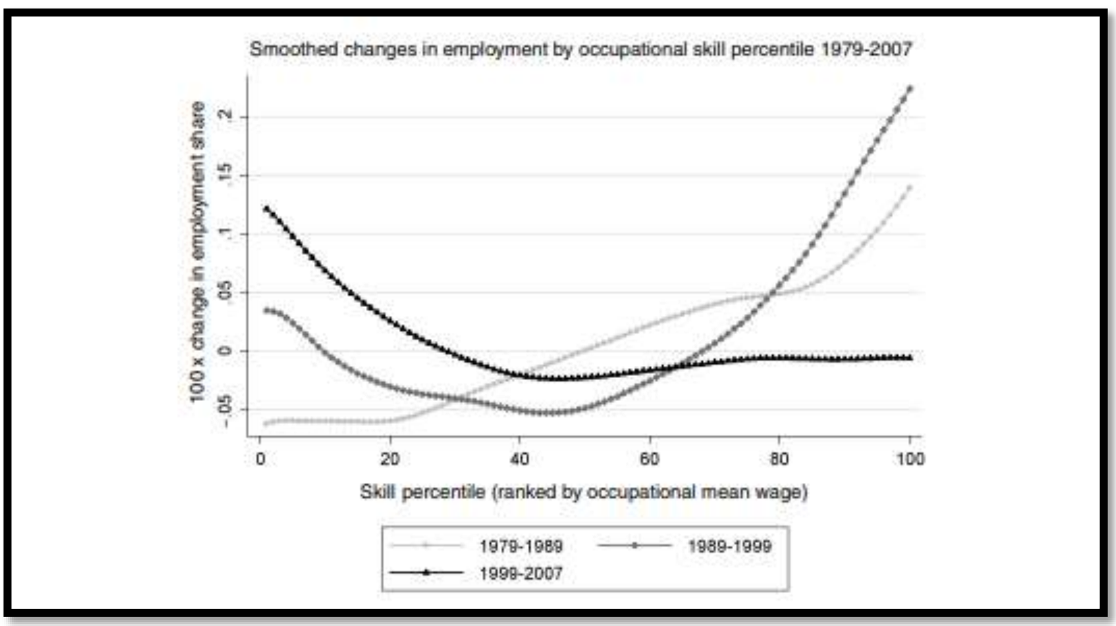


FIGURE 5 – Changes in Employment by Skill Level[32]

[31] Acemoglu (2003), *Human Capital and the Nature of Technological Progress*

[32] Acemoglu (2010), *Skills, Tasks and Technologies: Implications for Employment and Earnings*

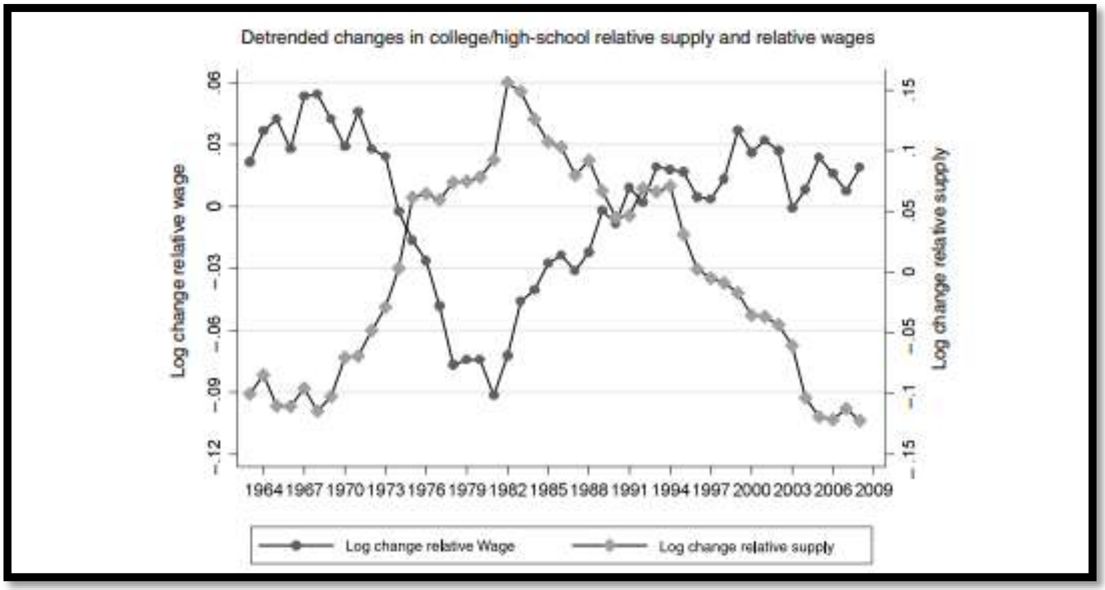


FIGURE 6 – Relative Changes in Supply and Wages (Highschool vs. College)[33]

In terms of the model, we can see this with the following equations:

$$AH_t = \sum_1^i H_{i,t} \tag{2.1}$$

$$g_{H,t} = \frac{AH_t - AH_{t-1}}{AH_{t-1}} \tag{2.2}$$

$$\overline{g_{H,t}} = \frac{\sum_1^t g_{H,t}}{t} \tag{2.3}$$

Equation (2.1) identifies aggregate human capital in the economy. Equation (2.2) finds the growth rate of the current period and equation (2.3) the average growth rate until that period. According to Acemoglu, when the current period's growth rate (2.2) is larger than the average growth rate (2.3), we have an increase in the supply of skilled workers and thus should see SBTC (i.e. technology – skill complementarity and a decrease in the mismatch). This is in fact part of my hypothesis that human capital advances quicker at a time when firm capital does so slower. Conversely, when the growth rate (2.2) is smaller than the average growth rate (2.3) we have an increase in unskilled worker and should thus see an increase in firm capital or technology as skill replacing technologies. This dynamic can be achieved through the way firms' production functions are formulated.

Rate of change of Firm Capital Quality (Technological Advancement)

[33] Idem

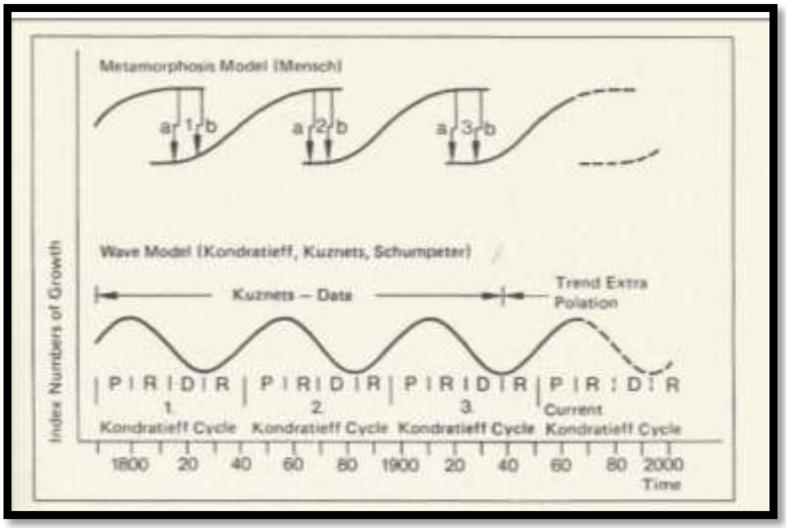


FIGURE 7 – Mensch's Technological Advancement Model over K-Waves

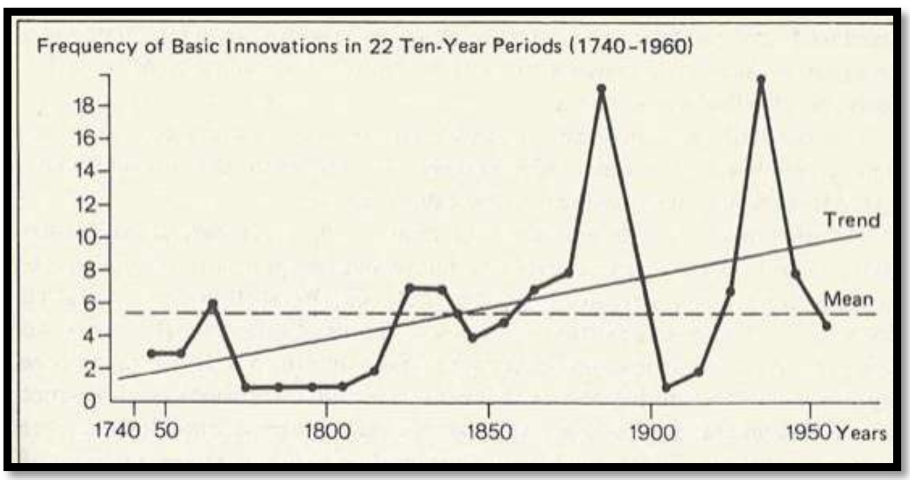


FIGURE 8 – Frequency of Basic Innovations

Tushman, Anderson (1990): Two types of technological progression: 1) an era of incremental growth, where small changes are made to a dominant design and 2) a technological discontinuity (i.e. a major innovation) implemented to change industry standards.

"Hypothesis 4: After each technological discontinuity, sales of all versions of the new technology will peak after the emergence of a dominant design, not during the era of ferment."

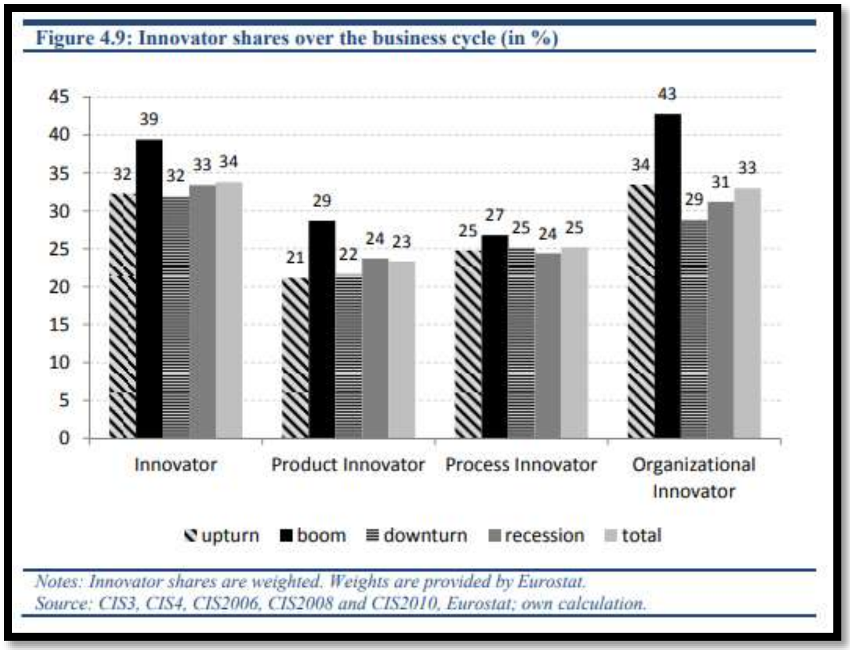


FIGURE 9 – Innovator Shares over the Business Cycle

Rate of change of Inflation via Unemployment: as the Philips curve shows, there is a negative relationship between inflation and unemployment. The model should replicate this and it may also shed more light on this dynamic through the following hypothesis: it is in fact the aggregate labour's share of income (which indeed changes due to unemployment, but also income inequality) which influences inflation. With this view in mind, more can be understood about why the Philips Curve itself shifts upwards or downwards.

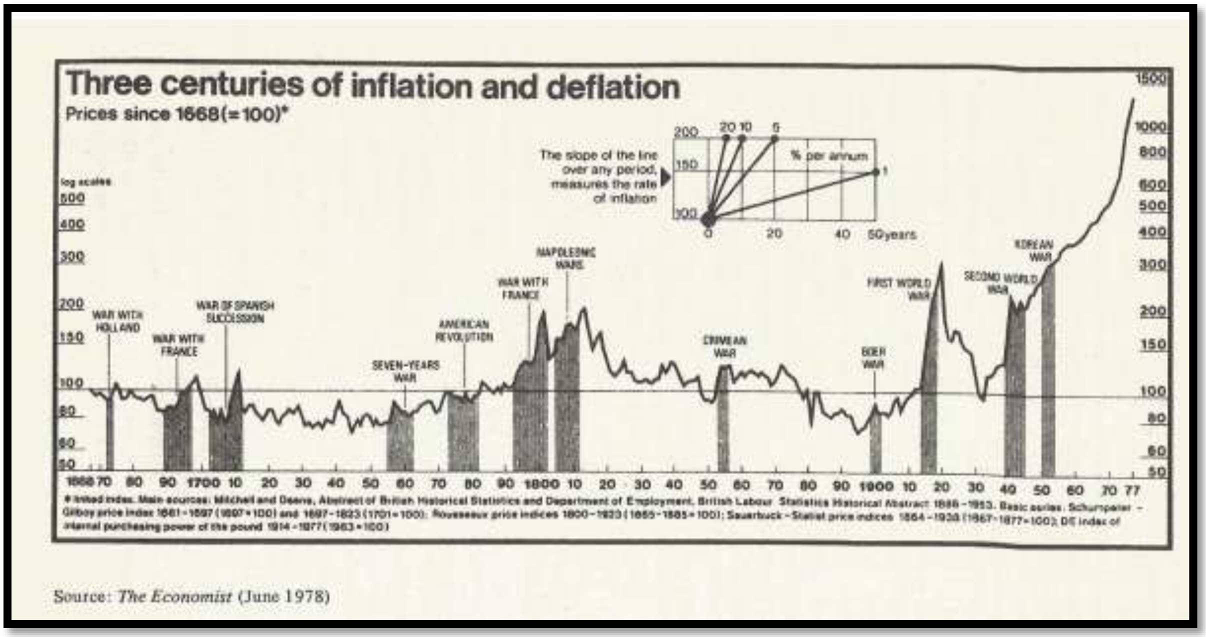


FIGURE 10 – British Prices

FIGURE 11 – U.S. Inflation and Unemployment Rate

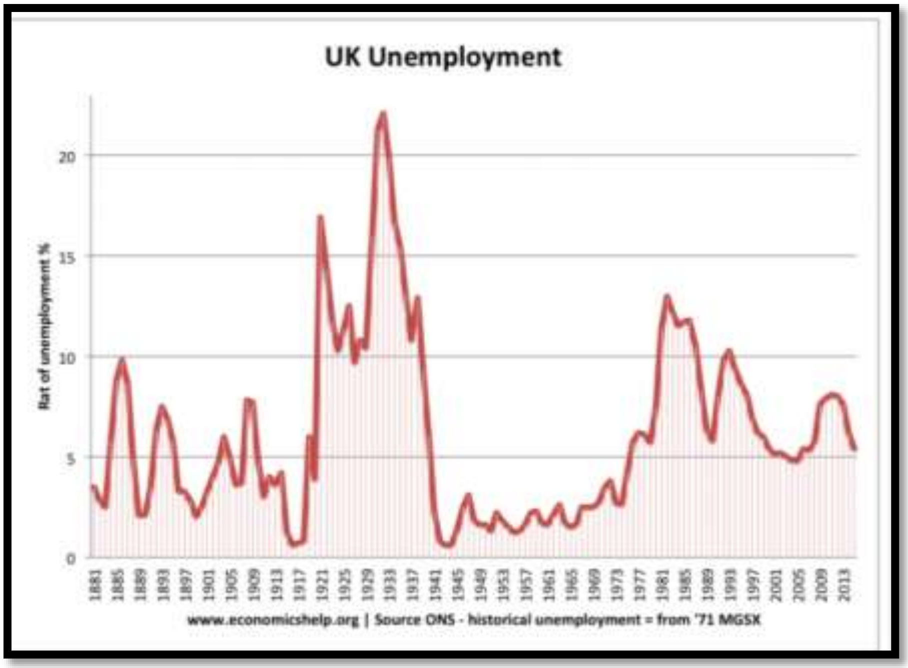


FIGURE 12 – UK Unemployment

$$\overline{p_t} = \frac{\sum_1^j p_{j,t}}{j} \tag{1}$$

$$i_t = \frac{\overline{p_t} - \overline{p_{t-1}}}{\overline{p_{t-1}}} \tag{2}$$

$$i_t = \beta_1 \theta_t + \beta_2 \theta_t^2 \tag{3}$$

$$AY_t = \sum_1^k Y_{HH,k,t} \tag{4}$$

$$AP_t = \sum_1^j P_{j,t} \tag{5}$$

Equation (3.1a) identifies the average price in the economy in period t. Equation (3.1b) expresses inflation in terms of changes in these average prices. Equation (3.1c) is a regression that can be ran with respect to unemployment; this data will also be graphed on a scatter plot.

Further, equation (3.2) identifies aggregate labour's income from k households. Equation (3.3) identifies aggregate profits from j firms. If the above hypothesis is correct, there should be a relation between the ratio of the two aggregate incomes and the inflation rate.

Positive Relationship between price mark-ups and growth rate:

FIGURE 13 – US Price Mark-ups

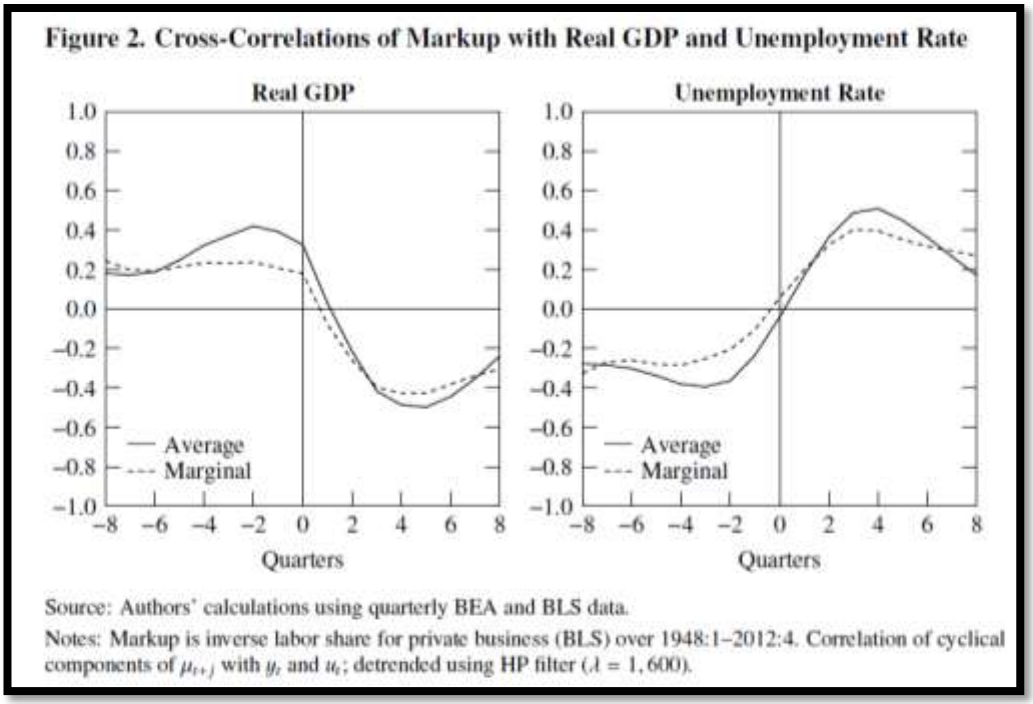


FIGURE 14 – Cross-Correlations of Mark-up w/ GDP and Unemployment

In terms of the model:

$$\overline{\lambda_t} = \frac{\sum_1^j \lambda_{j,t}}{j} \tag{6}$$

$$\Delta_{\lambda,t} = \frac{\overline{\lambda_t} - \overline{\lambda_{t-1}}}{\overline{\lambda_{t-1}}} \tag{7}$$

Equation (4.1) identifies the average mark-up across all firms in the economy. Equation (4.2) identifies the change in average mark-up across periods. Most papers find the mark-up to be countercyclical; however, these are all based on the same methods. New papers that approach the situation differently find that the mark-up may be cyclical conditional on a technology shock. As such, there should be a positive relationship between the change in mark-up (4.2) and the growth rate previously discussed (1.2).

Fluctuations in Firm Characteristics:

$$j_t \tag{5.1}$$

$$\delta_t = \frac{\sum_1^j \frac{\square_{\square,\square} * \square_{\square,\square}}{\square\square_{\square}}}{j_t} \tag{5.2}$$

Equation (5.1) refers to the number of firms in the economy; this will be plotted against time in order to observe how it progresses over time. Equation (5.2) measures the average market share of each firm, which is a way to observe how firm size varies over the cycles (while accounting for differences in costs that arise due to the level of technology a specific firm is at). These two quantities should vary according to empirically observed data. Potentially more characteristics than these can be tracked.

SECTION TWO – CODE DOCUMENTATION

Packages Used

```
import numpy as np

import matplotlib.pyplot as plt

from scipy import optimize

from time import time

import pdb
```

Initial Conditions and Parameters

- Initial Conditions
    - Worker population = 600
    - Marriage Rate = 0.5
    - Death age = 70
    - Fertility rate = 1.5
    - Initial number of firms = 100
    - Seniority age = 60
- Parameters
    - cphi = 30.0 # capital phi
    - comega = 0.2 # capital omega
    - chi = 0.5 # adjusts human capital growth

- lphi = 0.4 # lowercase phi, relative importance of H,K in production
- psi = 0.2 # capital psi, price mark-up adjustment
- alpha_f = 0.1 # Investment in R&D (in the document it appears as 1-alpha_f)
- theta = 0.9 #0.8 # theta, proportion of worker productivity for wage
- lomega = 0.8 # lowercase omega, scaling with productivity
- s = 1.2 # scales with number of workers
- o = 0.2 # scales with number of workers
- lgamma = 0.5 # lowercase gamma, rate of capital progress
- lRate = 1.06 # costs of debts
- iRate = 1.05 # return on savings
- bRate = 1.1 # government bonds rate
- eParam = 0.05 # Entrepreneurship adjustment parameter
- z = 1.5 # cost of entrepreneurship

Empty Lists and Computed values

Household list, human list, firm list

Mean income, mean firm capital, mean human capital, aggregated supply, aggregated demand

Classes and their attributes

- Class HUMAN
  - INITIALIZED VARIABLES:
    - Human Capital
    - Age
    - Married (T/F)[34]
    - Household (I.D.)[35]
    - Adult (T/F)

[34] Boolean variable
[35] Variable is the I.D. of a specific household. This indicates the worker is part of the other entity

        - Senior (T/F)
        - Living Costs
        - Wage
        - Savings
        - Income
        - Expenditure on old goods
        - Expenditure on new goods
        - Eligible for Employment (T/F)
        - Employer (I.D.)
        - Productivity
    - CLASS FUNCTIONS:
        - Check Retirement
        - Check Death
        - Age by one year
        - Compute Living Costs
        - Check eligibility for employment

- Class HOUSEHOLD
    - INITIALIZED VARIABLES:
        - Number of members
        - Members (I.D.)
        - Household Savings
        - Total income
        - Disposable incomes
    - CLASS FUNCTIONS:
        - Remove household

    - Increase Human Capital
    - Pay Living Costs
    - Optimize Utility

- Class FIRM
  - INITIALIZED VARIABLES:
    - Employees (I.D.)
    - Costs
    - Debt
    - Capital Level
    - Bankrupt (T/F)
    - Firm Productivity
    - Wages
    - Price
    - Mark-up
    - Expenditure on R&D
    - Goods Type
    - Number of goods produced
    - Number of goods sold
    - Market Dominance
    - Profit
    - Savings
  - CLASS FUNCTIONS
    - Check Good Type
    - Compute firm productivity

        - Compute firm costs
        - Set Mark-up and price
        - Take loan
        - Set initial price in period 1
        - Optimize objective function
        - Advance Firm Capital
        - Check market dominance
        - Check Bankruptcy
        - Go bankrupt
        - Check worker for hire
        - Hire worker
        - Check if worker must be fired
        - Fire worker

- Class BANK
    - INITIALIZED VARIABLES:
        - Funds
        - Consumer Savings
        - Firm Savings
        - Value of debt
        - Balance
        - Investment
    - CLASS FUNCTIONS:
        - Allow withdrawal of savings
        - Check balance
        - Check which new firms to invest in

        - Invest in new firms

2. Functions operating on classes

- Uphold marriage rate
- Establish new household (with a single member)
- Establish new household (with two+ members)
- UPDATE:
    - Mean Human Capital
    - Mean Firm Capital
    - Mean Income
- Sales Process (for both old and new goods)
    - Aggregate demand
    - Sell goods until demand satisfied or supply consumed
- Identify unemployed workers
- Uphold fertility rate
-

3. Initial organization of Agents

- Create initial age structure of population
- Create initial Households until marriage rate
- Distribute children to households until natality rate
- Distribute workers to firms until employment rate