1. (9 points) Convert the following unsigned base 2 numbers (binary) to base 16 numbers (hexadecimal):
   A. 0110 0001 1111
      - 0110=(0x2^0 + 1x2^1 +1x2^2 + 0x2^3) -> (6)_D -> (6)_H
      - 0001= (1x2^0 + 0x2^1 + 0x2^2+0x2^3) -> (1)_D -> (1)_H
      - 1111= (1x2^0 + 1x2^1 + 1x2^2+1x2^3) -> (15)_D (F)_H
      - => (61F)__H

   B. 1000 1111 1100
      - 1000= (0x2^0 + 1x2^1 +1x2^2 + 1x2^3) ->(8)_D ->(8)_H
      - 1111= (1x2^0 + 1x2^1 +1x2^2 + 1x2^3) ->(15)_D ->(F)_H
      - 1100= (0x2^0 + 0x2^1 +1x2^2 + 1x2^3) ->(12)_D ->(C)_H
      - => (8FC)_H

   C. 0001 0110 0100 0101
      - 0001= (1x2^0 + 0x2^1 +0x2^2 + 0x2^3) →(1)_D -> (1)_H
      - 0110= (0x2^0 + 1x2^1 +1x2^2 + 0x2^3) →(6)_D -> (6)_H
      - 0100= (0x2^0 + 0x2^1 +1x2^2 + 0x2^3) →(4)_D -> (4)_H
      - 0101= (1x2^0 + 0x2^1 +0x2^2 + 0x2^3) →(5)_D -> (5)_H
      - => (1645)_H

2. (27 points) Convert the following binary numbers to base 10 numbers (decimal). Each time if binary numbers are represented in:
a) Signed magnitude representation.
1) 1100 1010 =
- 0x2^0 + 1x2^1 + 0x2^2 + 1x2^3 + 0x2^4 + 0x2^5 + 1x2^6
- 2+8+64
- => (-74)_D

2) 1111 0010 =
- 0x2^0 + 1x2^1 + 0x2^2 + 0x2^3 + 1x2^4 + 1x2^5 + 1x2^6
- 2+16+32+64
- => (-114)_D

3) 1000 0111 =
- 1x2^0 + 1x2^1 + 1x2^2 + 0x2^3 + 0x2^4 + 0x2^5 + 0x2^6
- 1+2+4
- => (-7)_D

b) One’s complement representation.
1) 1100 1010 =
- -(1x2^0 + 0x2^1 + 1x2^2 + 0x2^3 + 1x2^4 + 1x2^5 + 1x2^6)
- => (-53)_D

2) 1111 0010 =
- -(1x2^0 + 0x2^1 + 1x2^2 + 1x2^3)
- => (-13)_D

3) 1000 0111 =

- -(1x2^3 + 1x2^4 + 1x2^5 + 1x2^6)
- => (-120)_D

c) Two's complement representation.

1) 1100 1010 =

- (1011 0110) – 2's complement
- -(1x2^1 + 1x2^2 + 1x2^4 + 1x2^5)
- => (-54)_D

2) 1111 0010 =

- (1000 1110) – 2's complement
- -(1x2^1 + 1x2^2 + 1x2^3)
- => (-14)_D

3) 1000 0111 =

- (1111 1001) – 2's complement
- -(1x2^0 + 1x2^3 + 1x2^4 + 1x2^5 + 1x2^6)
- (-121)_D

For example, question A, if 1100 1010 is a binary number represented in signed magnitude representation, what is the decimal value? Also do it again if 1100 1010 is a binary number in one's
complement representation and two's complement representation. There are 9 separate answers in
total.

3. (36 points) Convert the following base 10 (decimal) values to binary numbers (8-bits). Each binary
result represented in:

a) Signed magnitude representation.

1) -100_d =

- 2 – 100 → 0
- 2 – 50 → 0
- 2 – 25 → 1
- 2 – 12 → 0
- 2 – 6 → 0
- 2 – 3 → 1
- 2 – 1 → 1
- => 11100100

2) -16_d =

- 2 – 16 → 0
- 2 – 8 → 0
- 2 – 4 → 0
- 2 – 2 → 0
- 2 – 1 → 1
- => 10010000

3) -21_d =

- 2 – 21 → 1
- 2 – 10→ 0
- 2 – 5 → 1
- 2 – 2 → 0
- 2 – 1 → 1
- => 10010101

4) -0_d =

- => 10000000

b) One’s complement representation.

1) -100_d =

- 11100100
- => 10011011 – 1’s complement

2) -16_d =

- 10010000
- => 11101111 – 1’s complement

3) -21_d =

- 10010101
- => 11101010 – 1’s complement

4) -0_d =

- 10000000
- => 11111111 – 1’s complement

c) Two’s complement representation.

1) -100_d =

- 11100100
- => 10011100 – 2’s complement

2) -16_d =

- 10010000
- => 11110000 – 2’s complement

3) -21_d =

- 10010101
- => 11101011 – 2’s complement

4) -0_d =

- 10000000
- => 10000000 – 2’s complement

(There are 12 separate answers in total.)

4. (4 points) What is the range of:
A. An unsigned 7-bit number?

- 0 – (2^n -1), n=7
- 0 – (2^7 -1)
- 0 – (128 -1)
- => 0 to 127

B. A signed 7-bit number?

- -2^(n-1) – (2^(n-1) -1)
- -2^(7-1) – (2^(7-1) -1)
- -2^6 – (2^6 – 1)
- => -64 to 63

5. (12 points) Solve following bitwise operations (∧ = AND, ∨ = OR)
e.g. 0101 ∧ 0011 = 0001
1. 1000 ∧ 1110
    - =>1000 ∧ 1110 = 1000

2. 1000 ∨ 1110
    - => 1000 ∨ 1110 = 1110

3. (1000 ∧ 1110) ∨ (1001 ∧ 1110)
    - 1000 ∧ 1110 = 1000
    - 1001 ∧ 1110 = 1000
    - =>1000 ∨ 1000 = 1000

6. (9 points) Please demonstrate each step in the calculation of the arithmetic operation 25 - 65. (both
25 and 65 are signed decimal numbers)

- 25
- 2 – 25 → 1
- 2 – 12 → 0
- 2 – 6 → 0
- 2 – 3 → 1
- 00011001
- 00011001 – 2's complement
- (25) 00011001 + (-65) 10111111
- => (10101000) – 2's complement → (-40)_D

7. (3 points) Mathematically the answer in Q6 is -40_d. Please verify your answer in Q6 using a conversion of 2's and decimal numbers

- (-40)_D → (10101000) - 2's complement
- -(0x2^0 + 0x2^1 + 0x2^2 + 1x2^3 + 0x2^4 + 1x2^5 + 0x2^6)
- -(8 + 32)
- => (-40)_D