Optimal Seat Pricing With Multiple Classes & Why Economy Seats Feel so Small

1. Research Problem

While the optimal dynamic pricing of inventories over finite horizons in a monopoly setting has been extensively studied (see, for example, Bitran and Caldentey 2003 for an overview of the field), not much is known for the optimal pricing menu when managing inventories of various qualities. However, such settings are prevalent in several industries, such as the airline and the hotel industry, in which the simultaneous presence of inventories of multiple qualities (e.g., Economy and Business class seats, Standard and Deluxe rooms) can have important implications on both pricing strategies, capacity management, and consumer welfare.

In this paper, we focus on the context of airline seat pricing (although most of our results hold for more general settings) and provide a twofold contribution. First, we show that the optimal dynamic pricing menu for flights with seats of multiple qualities (classes) can be constructed in a simple way using the optimal dynamic strategy for flights in which all seats have the same quality. This result implies that what seems to be an intractable, multi-dimensional stochastic optimization problem (pricing seats of different qualities), is actually a sum of simpler, single-dimensional problems (pricing seats of identical quality).

Second, we use our characterization of the optimal seat pricing menu to provide a rigorous answer to a claim that has been repeatedly made in the popular press, namely that airline companies squeeze their Economy customers to be able to extract higher premiums from their Business travelers.[1] Using the optimal pricing menus as input, we compare the way a monopolist airline allocates space between Economy and Business seats with the objective of maximizing revenues, with the way a social planer would perform the same allocation with the objective of maximizing total welfare. We show that indeed, the monopolist airline chooses to allocate less space to Economy seats compared to the social planner, thus "squeezing" Economy customers.

[1] See, for example, "Feeling cramped? How to battle the shrinking airline seat", by D. Carrington on November 11, 2013 on CNN (link: http://www.cnn.com/2013/11/07/travel/feeling-cramped-battle-airline-seat/)

2. Assumptions & Methodology

We make the classic economic assumptions used in the revenue management literature; transactions take place in a monopolistic environment and customers are infinitely impatient (they see they posted price, and either purchase a ticket or exit the market).[2] Furthermore, we assume that while consumers can differ in their willingness to pay for each unit of quality, quality is an objectively measurable aspect of a seat. This assumption is aligned with how quality differences are treated in Economics (see Chapter 2 in Tirole 1988). We do not assume any functional form for the customer arrival process in the characterization of the optimal dynamic pricing menu with multiple qualities. For comparing the space allocation decisions of the monopolist airline and the social planner, we utilize the model of Section 5.5.1 of Gallego and Van Ryzin (1994), in which arrivals follow a Poisson process and consumer willingness to pay is distributed among consumers according to an exponential distribution.

Our results are derived using economic and combinatorial arguments. While explicit computation of optimal pricing strategies would require recursively solving non-trivial systems of differential equations, we are able to abstract away from these complexities through the use of strategy-stealing arguments[3]. That is, by appropriately coupling consumer behavior across flights we can perform comparative statics for the optimal revenues and prices, based only on knowledge of their existence (even without knowing them explicitly). This allows us to obtain structural results of the optimal prices and revenues using minimal assumptions, while keeping the proofs easy to follow and interpret. We use strategy stealing arguments to prove our results on the structure of optimal prices both when the objective is revenue optimization, and when the objective is welfare maximization.

Finally, when we compare the space allocation decisions of a revenue-maximizing monopoly airline and a social planner, we do not impose any specific functional form on the function that translates space (equivalently, legroom) to quality. The only requirement we make is that this function is increasing, i.e., more legroom is always more preferable than less legroom.

3. Main Results

We show that the optimal price of a specific seat in a flight with seats of multiple qualities can be decomposed to a quality-weighted sum of prices of a set of counterfactual flights, each of which contains seats of identical qualities. We also show that optimal revenues are subject to an analogous decomposition. As a result, knowledge of the solution to the optimal one-dimensional quality seat pricing problem is equivalent to knowledge of the solution to the multi-dimensional optimal

[2] There is recent work that relaxes the monopolistic assumption and the infinite impatience assumption. See, for example, Levin et al. (2009) and Gallego and Hu (2014)

[3] An argument of that flavor can be found in the proof of Theorem 1 in Zhao and Zheng (2000)

seat pricing problem. We prove that this decomposition holds both when the objective is revenue optimization and when the objective is welfare maximization.

Furthermore, we use our derived results on optimal pricing menus to provide a rigorous argument for the fact that a revenue-maximizing monopoly airline has an incentive to squeeze Economy customers in order to extract higher premiums from Business customers. More specifically, we consider how a revenue-maximizer and a welfare-maximizer would decide on the allocation of space among seats of two classes, Economy and Business, in an aircraft of fixed length. We prove that while both the revenue-maximizer and the welfare-maximizer price discriminate in order to match higher quality seats with customers of higher willingness to pay, the revenue maximizer does so more aggressively, in the sense that it designs less spacious Economy seats in order to be able to collect more from customers with a high willingness to pay.

As part of our work, using the flexibility of strategy-stealing arguments, we also prove that many traditional results on the behavior of optimal revenues and the prices that achieve them (such as Theorem 1 in Gallego and Van Ryzin 1994), also hold for optimal welfares and the prices a social planner uses to achieve them.

References

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