# adjusted $R^2$

- calculation
- uses

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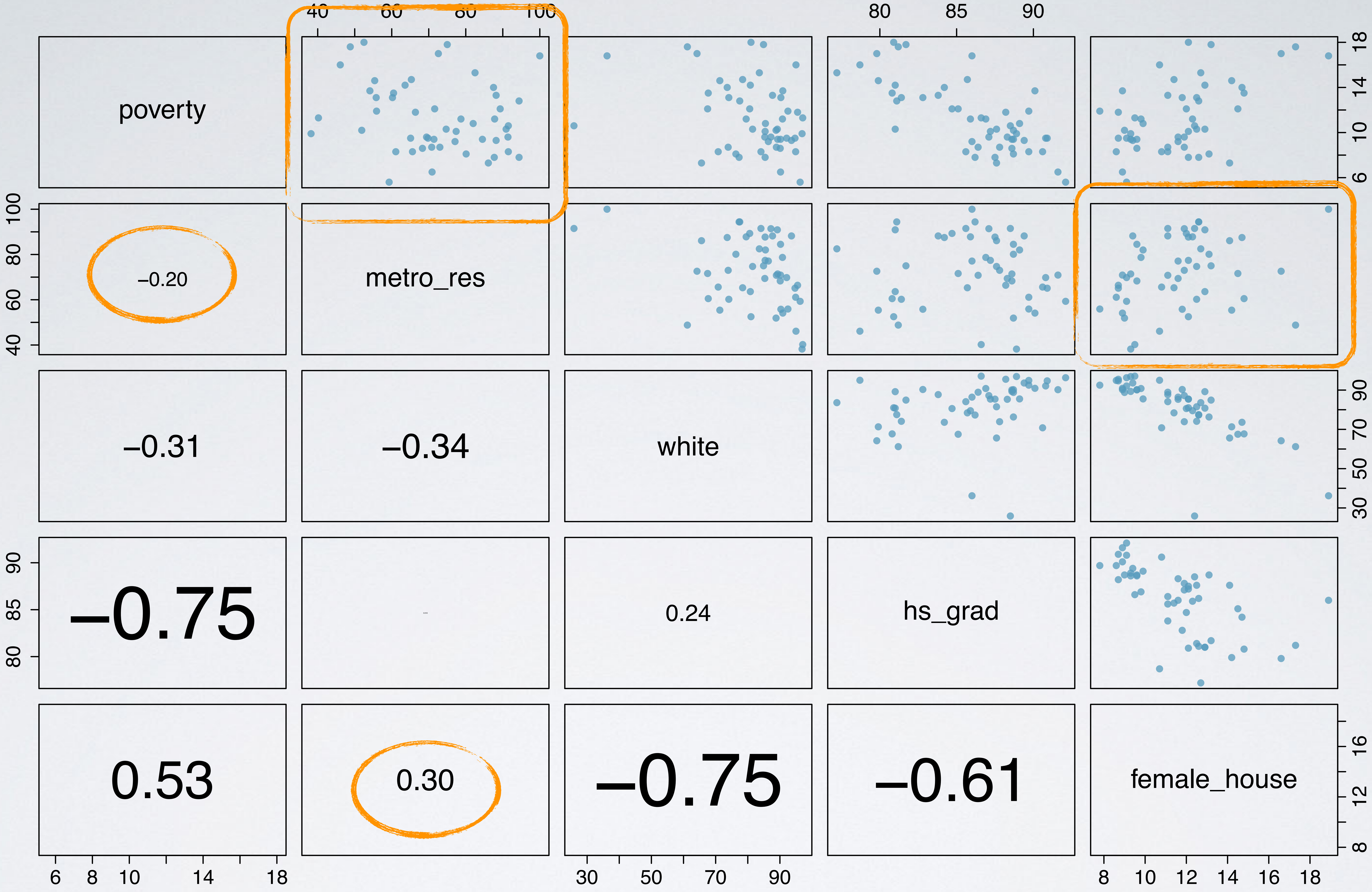


poverty
metro_res
white
hs_grad
female_house
−0.20
−0.31
−0.34
−0.75
0.24
0.53
0.30
−0.75
−0.61

R

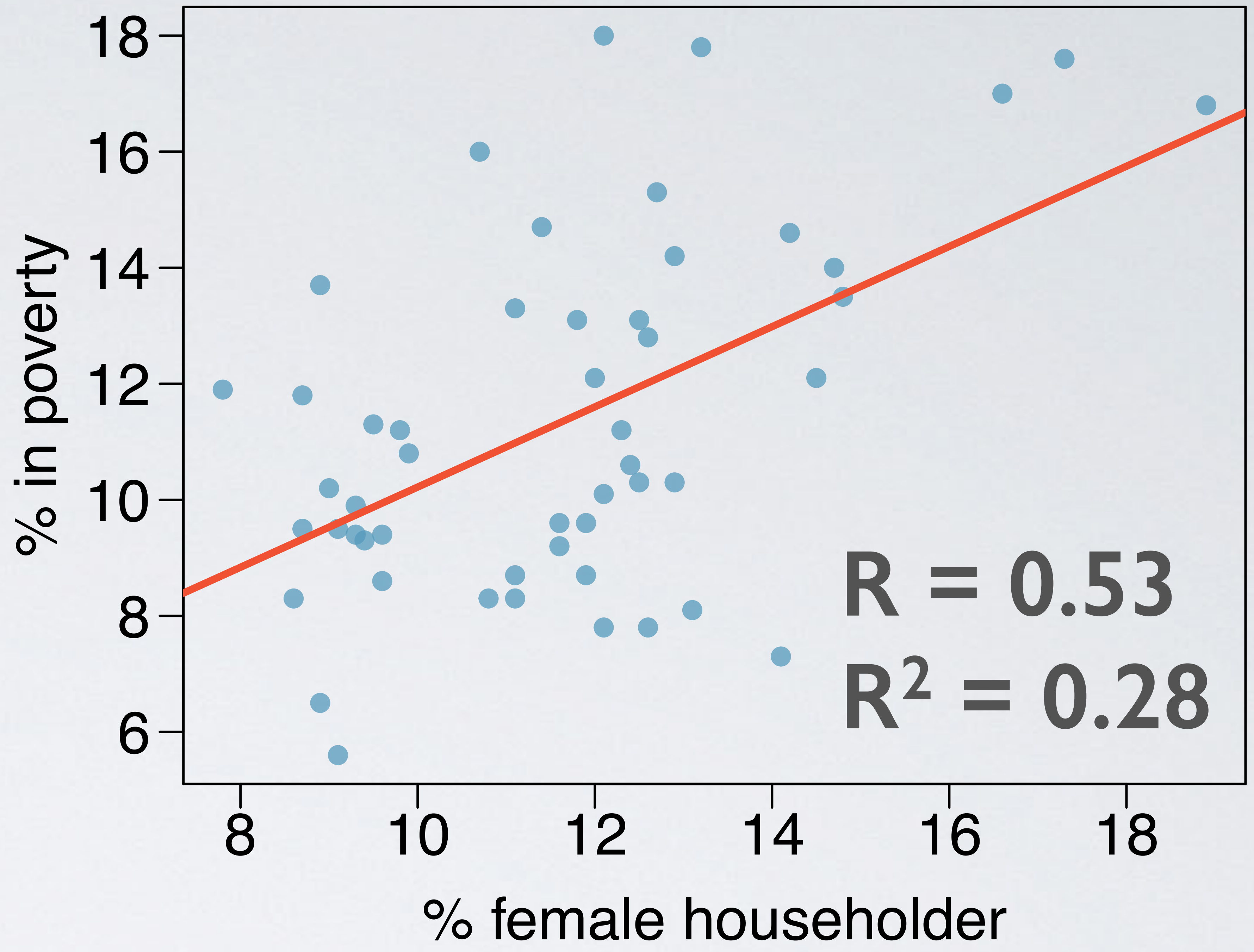
```
# load data
> states = read.csv("http://bit.ly/dasi_states")

# fit model
> pov_slr = lm(poverty ~ female_house, data = states)
> summary(pov_slr)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    3.3094     1.8970   1.745   0.0873 .
female_house   0.6911     0.1599   4.322 7.53e-05 ***

Residual standard error: 2.664 on 49 degrees of freedom
Multiple R-squared:  0.276,  Adjusted R-squared:  0.2613
F-statistic: 18.68 on 1 and 49 DF,  p-value: 7.534e-05
```

# predicting poverty from % female householder




| Linear model: | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 3.31 | 1.90 | 1.74 | 0.09 |
| female_house | 0.69 | 0.16 | 4.32 | 0.00 |

# another look at $R^2$

| **ANOVA:** | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| female_house | 1 | 132.57 | 132.57 | 18.68 | 0.00 |
| Residuals | 49 | 347.68 | 7.10 | | |
| Total | 50 | 480.25 | | | |

$$R^2 = \frac{\text{explained variability}}{\text{total variability}} = \frac{132.57}{480.25} = 0.28$$

# predicting poverty from % female householder + % white

R

```
> pov_mlr = lm(poverty ~ female_house + white, data = states)
> summary(pov_mlr)
```

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | -2.58 | 5.78 | -0.45 | 0.66 |
| female_house | 0.89 | 0.24 | 3.67 | 0.00 |
| white | 0.04 | 0.04 | 1.08 | 0.29 |

R

```
> anova(pov_mlr)
```

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| female_house | 1 | 132.57 | 132.57 | 18.74 | 0.00 |
| white | 1 | 8.21 | 8.21 | 1.16 | 0.29 |
| Residuals | 48 | 339.47 | 7.07 | | |
| Total | 50 | 480.25 | | | |

$$R^2 = \frac{132.57 + 8.21}{480.25} = 0.29$$

# adjusted $R^2$

**adjusted $R^2$:** $$R^2_{adj} = 1 - \left( \frac{SSE}{SST} \times \frac{n-1}{n-k-1} \right)$$

$k$ : number of predictors

practice

Calculate adjusted $R^2$ for the multiple linear regression model predicting % living in poverty from % female householders and % white.
Remember n = 51 (50 states + DC).

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| female_house | 1 | 132.57 | 132.57 | 18.74 | 0.00 |
| white | 1 | 8.21 | 8.21 | 1.16 | 0.29 |
| Residuals | 48 | 339.47 | 7.07 | | |
| Total | 50 | 480.25 | | | |

$$R^2_{adj} = 1 - \left( \frac{SSE}{SST} \times \frac{n-1}{n-k-1} \right)$$

$$= 1 - \left( \frac{339.47}{480.25} \times \frac{51-1}{51-2-1} \right) = 0.26$$

# $R^2$ vs. adjusted $R^2$

| | $R^2$ | adjusted $R^2$ |
|---|---|---|
| **Model 1 (poverty vs. female_house)** | 0.28 | 0.26 |
| **Model 2 (poverty vs. female_house + white)** | 0.29 | 0.26 |

- When **any** variable is added to the model $R^2$ increases.
- But if the added variable doesn't really provide any new information, or is completely unrelated, adjusted $R^2$ does not increase.

# properties of adjusted $R^2$

$$R^2_{adj} = 1 - \left( \frac{SSE}{SST} \times \frac{n-1}{n-k-1} \right)$$

- k is never negative → adjusted $R^2 < R^2$
- adjusted $R^2$ applies a penalty for the number of predictors included in the model
- we choose models with higher adjusted $R^2$ over others