Catastrophic Cancellation

Tuesday, September 10, 2024 3:07 PM

Let's work in a decimal (↗ vs. binary) floating point (≈ scientific notation; ↳ vs. fixed point) system

mantissa ↓ 3 digits

Float 3.1×10^{0} ↑ use "budget" for exponent but worth it!

Fixed 3.14

Ex: 7 digits (6 mantissa, 1 exponent)

$x = 3.14159 \times\times\times\times \cdot 10^{0}$

trustworthy | untrustworthy

(in reality, numbers stored in binary, and its conversion to decimal where we see these untrustworthy junk digits)

$y = 3.14150 \times\times\times\times \cdot 10^{0}$

$x+y = 6.28309 \times\times\times\times \cdot 10^{0}$ ↖ possibly corrupted

} $x+y$ is (about) the same as x, y themselves

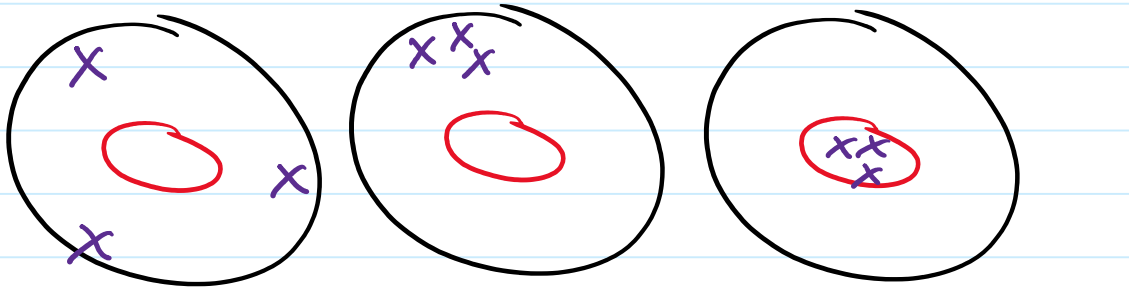
$x-y = 0.00009 \times\times\times\times \cdot 10^{0}$

$= 9.\times\times\times\times\times\times\times\times \cdot 10^{-5}$

} $x-y$ is catastrophic cancellation

these appear to be trustworthy but they aren't

Floating point numbers always appear precise but may not be accurate



Imprecise | Precise but inaccurate | Precise and Accurate