VAE (ch. 19 Bishop)

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ch. 18 (Bishop) Normalizing Flows

Also uses latent space, $x = f_w(z)$ and $z = g_w(x)$ — Require to be invertible? so typically $\dim(z) = \dim(x)$

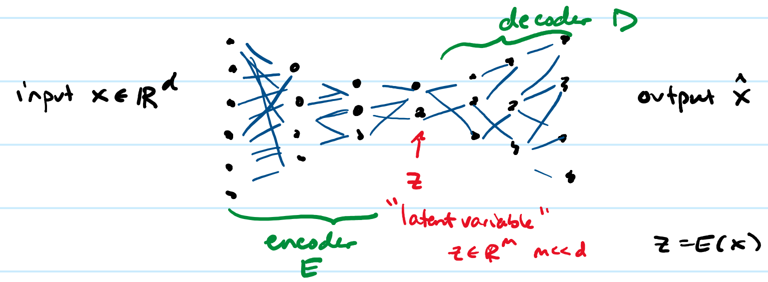
and use $p_x(x) = p_z(g(x)) \cdot |\det J(x)|$ — Jacobian, $\frac{\partial g}{\partial x}$

popularized in ML by Rezende + Mohamed 2015

Relation to neural ODEs

ch. 19 Autoencoders and Variational Autoencoders (VAE)

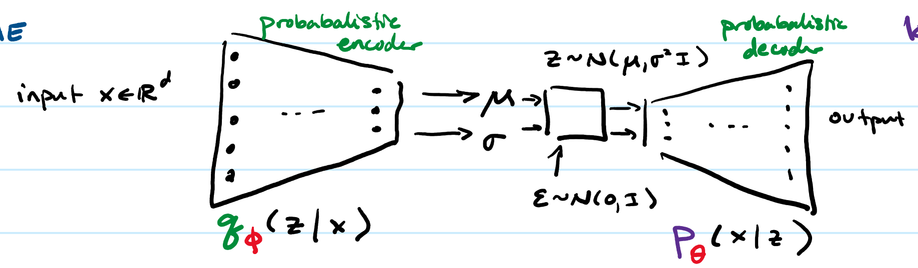
AE



$z = E(x)$

self-supervised: loss is $\| x - D(E(x)) \|^2$

VAE — Kingma + Welling '13, '19



Objective Function: maximum likelihood, $p(x)$ or $\log(p(x))$

$$\log(p(x)) = \int \underbrace{\log(p(x))}_{\text{constant w.r.t. } z} q(z|x)\,dz \underbrace{+ \int \log\left(\frac{p(z|x)}{q(z|x)}\right) q(z|x)\,dz + \int \log\left(\frac{q(z|x)}{p(z|x)}\right) q(z|x)\,dz}_{=0}$$

flip numerator/denominator

$$= \underbrace{\int \log\left(p(x)\frac{p(z|x)}{q(z|x)}\right) q(z|x)\,dz}_{\text{ELBO}} + \underbrace{KL(q(z|x) \,\|\, p(z|x))}_{\geq 0}$$

≥ 0 intractable so ignore

can rewrite

$$\text{ELBO}(x; \phi, \theta) = \underbrace{\mathbb{E}_{q_\phi} \log(p_\theta(x|z))}_{\text{Reconstruction}} - \underbrace{KL(q_\phi(z|x) \,\|\, p(z))}_{\text{Regularization}}$$

Fixed target

we want $q(z|x)$ to look like $p(z)$ so that we can sample a new $z \sim p(z)$ later and it "could have come" from a datapoint x... hence $p(x|z)$ is good

VAE (p. 2)

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3 key ideas to make VAE work:

key trick in "variational inference"

1. approximate likelihood function with ELBO
2. amortized inference: encoder $q_\phi(z|x)$ approximates posterior distribution of z
3. reparameterization trick to make it implementable