PINN Case study: gravity modeling

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Martin, J., Schaub, H. Physics-informed neural networks for gravity field modeling of small bodies. *Celest Mech Dyn Astron* **134**, 46 (2022).
https://doi.org/10.1007/s10569-022-10101-8

Gravity Modeling

What kinds of questions to ask?

- "Geopotential Models" for Earth
  Infer density ρ of Earth given observations of moon and satellites
- What is gravity (or acceleration due to gravity) around $\underbrace{\text{asteroids}}$, $\underbrace{\text{planets}}$, ...

Small, irregular shapes

more spherical but may have other issues (eg. atmospheres)

CU's aerospace dept. excels at "small-body" modeling
(Profs. Schaub, Scheeres, McMahon ...)

Background

$$F = \frac{G m_1 m_2}{r^2}$$ right?

force

For Earth, we can assume $\underbrace{\text{all mass is at center-of-mass}}$

Yes, in Physics 101. Exactly true for perfect "spheres" (w/ uniform ρ)

more generally,
use Superposition:
sum (or integral) over all "point" masses

$\vec{r}$ is 3D coordinate,
$r = \|\vec{r}\|_2$
$\hat{r} = \vec{r}/r$ unit vector

Gauss' Law, derivative form $\quad \vec{\nabla}\cdot\vec{g} = -4\pi G\rho$

density

$$\vec{g}(\vec{r}) := -\frac{GM}{r^2}\hat{r}$$

... so in a region with no mass, $\rho = 0$, so $\vec{\nabla}\cdot\vec{g} = 0$

Gravity modeling (p. 2: background)

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... background...

if $\vec{F}$ is the force due to gravity, then in a region w/ $\rho = 0$,

① $\vec{\nabla} \cdot \vec{F} = 0$ (no sources nor sinks, "solenoidal", "incompressible", "divergence free")

divergence

② Also, gravity is a conservative field (i.e. path-independent)

so it's irrotational, i.e. $\vec{\nabla} \times \vec{F} = 0$

curl

$\Rightarrow \vec{F} = \nabla U$ [Recall $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$, $\vec{\nabla} \times (\nabla U) = 0$]

gradient

i.e.

② $\vec{F} = \nabla U$

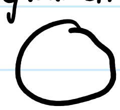
① $\vec{\nabla} \cdot (\nabla U) = 0$ in regions with zero density, aka $\Delta U = 0$ Laplace Eq

... plus boundary conditions ...

Classical methods: (goal: find U) → so all automatically satisfy ②

ⓐ treat object as a point source ... ok if spherically symmetric

$\vec{r}$



Not ok:

$\vec{r}$

ⓑ $\vec{F}(\vec{r}) = G \int_V \frac{1}{\|\vec{x} - \vec{r}\|} \cdot \rho(\vec{r}) \, dV(\vec{r})$ Impractical to use often, ρ and V often unknown

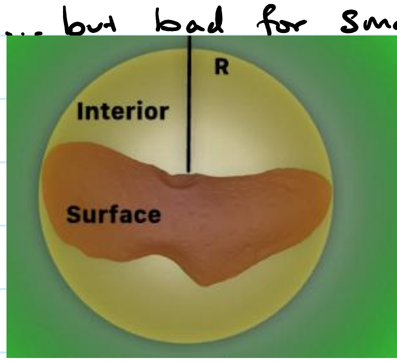
ⓒ Spherical harmonic expansion of U

If body is an ellipse, this is not bad

$$U_l(r) = \frac{\mu}{r} \sum_{l=0}^{l} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^l P_{l,m}[\sin(\phi)][C_{l,m}\cos(m\lambda) + S_{l,m}\sin(m\lambda)],$$

... but bad for small (irregular) bodies

Eros



R = "Brillouin" radius

Spherical harmonics "unstable" if $r < R$

(origin usually at center-of-mass)

Gravity modeling (p. 3)

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(d) polyhedral models (Werner + Scheeres '97)

Describe shape/volume V via (thousands) of edges and facets

PINN approach, ver 1

Goal: send spacecraft to asteroid, want to know

acceleration (of spacecraft) due to gravity [= force, since craft mass is constant, $F=ma$]

Setup 1: we have lots of training data of $\{\vec{x}_i, \vec{a}_i\}$

craft position

(noisy) acceleration readings

want gravity map

$\vec{x} \longmapsto \vec{a}$ i.e. via a neural net

PINN idea:

$f_\omega : \vec{x} \longmapsto U$, then $\vec{a} := \nabla U$ (or $\vec{a} = -\nabla U$ in some conventions)

train:

gradient via backprop

$$\min_\omega \frac{1}{N} \sum_{i=1}^{N} \| \vec{a}_i - (-\nabla f_\omega(x_i)) \|^2$$

Automatically enforces $\vec{\nabla} \times \vec{F} = 0$ (i.e. conservative field)

... but ignores Laplace Eq, $\vec{\nabla} \cdot \vec{F} = 0$, i.e. $\Delta U = 0$.

Does it work? (also do tricks w/ preprocessing / normalization / coordinate system)

3 training scenarios: all assume access to "a"!

"r" points drawn from surface out to $3 \cdot R$ Unrealistic

"r*" " " from $2R$ to $3R$ Idea: measurements from a craft orbiting at a safe distance

"F" hybrid: like r* but 10% closer to surface
(inspired by ejecta or gravity poppers!)

Gravity modeling (p. 4: results)

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Hyperparameters

Parameter Value Activation -- GeLU
Batch size -- 5,000
Optimizer -- ADAM
Epochs -- 7,500
Hidden layers -- 8
Nodes per layer -- 20
Weight initialization -- Glorot normal
Learning rate -- 0.002

$N = 5{,}000$

Add noise to $\vec{a}$ of 20%

Results: "r" works well
"r^*" doesn't extrapolate to low altitude orbits well
"$\bar{r}$" better than r^*

PINN Approach, ver 2

$\vec{a} = -\nabla U$ automatically enforced $\vec{\nabla} \times \vec{F} = 0$ (i.e. $U(\vec{x}) = \overbrace{f_w(\vec{x})}^{\text{neural net}}$)

... but $\vec{\nabla} \cdot \nabla U \; (= \Delta U) = 0$ was ignored

so try

$$\text{loss} = \frac{1}{N}\left(\sum_{i=1}^{N} \| \vec{a}_i - (-\nabla f_w(x_i)) \|^2 + \underbrace{\| \Delta f_w(x_i) \|^2}_{\text{new!}} \right)$$

via backprop

* they also add $\| \vec{\nabla} \times \nabla f_w(x_i) \|^2$
which doesn't make sense to me as it should automatically be 0

- Also try a transformer-inspired architecture w/ attention and skip-connections/residual
  Works well though more expressive and prone to overfitting

Experiment: online / "in situ" model creation

"NEAR-Shoemaker", year-long orbit of 433-Eros in 2000, (irregular shape but smooth surface) 

10-min data.

PINNs start to work well after day 250 ... need data! Especially close passes

also "OSIRIS-REx" to Bennu, 2 years (spherical but irregular surface)