life_expectancy

July 3, 2023

[313]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import sklearn.metrics
from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels.api as sm
from statsmodels.tools.tools import add_constant

from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.impute import KNNImputer
from sklearn.impute import SimpleImputer
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import StandardScaler
from sklearn.neighbors import KNeighborsRegressor
```

This first mini-project is focused on supervised machine learning. For this mini-project I have selected the Life Expectancy dataset that is publicly available on Kaggle.com. From this dataset, I am most interested in understanding the correlation between the factors chosen and longevity, specifically, I am interested in the relative importance of each of these factors in determining longevity. Given my interest is not primarily prediction, but more the predictive power of each factor on longevity, it is important that the resulting model remain interpretable, even at the expense of predictive accuracy.

The life expectancy dataset is a time series over 15 years, with country data from 178 unique countries. contains health and socioeconomic information on over 178 countries. The features of the dataset include: - country (qualitative) - the name of the country the record is relevant for - year (qualitative) - year that the record was taken - status (qualitative) - economic status of the country (Developing/ Developed) - Hepatitis B (quantitative) - percentage of the population that was immunized against Hep B - Measeles (quantitative) - cases per 1000 people of Measles - Polio (quantitative) - percentage of the population immunized against Polio - HIV/AIDs (quantitative) - Deaths caused by HIV/AIDs - Infant Deaths (quantitative) - number of infant deaths per 1000 people - under-five deaths (quantitative) - number of deaths of people under 5 years old per 1000 people - total expenditure (quantitative) - The ratio of government medical-health expenses to total government expenses - GDP (quantitative) - Gross Domestic Product - BMI (quantitative) - The average body mass index of the entire population of the country - thinness (quantitative) - Prevalence of thinness among people 19 years old in percentage - Alcohol (quantitative) - Liters of alcohol consumption among people over 15 years old - Schooling (quantitative) - The number of years that people study - life expectancy (quantitative) - Country life expectancy

There are 2848 total records in the file

[314]:
```python
df = pd.read_csv('life_expectancy.csv')
df
```

[314]:
```
          Country  Year      Status  Population  Hepatitis B  Measles  Polio
0     Afghanistan  2015  Developing  33736494.0         65.0     1154    6.0  \
1     Afghanistan  2014  Developing    327582.0         62.0      492   58.0
2     Afghanistan  2013  Developing  31731688.0         64.0      430   62.0
3     Afghanistan  2012  Developing   3696958.0         67.0     2787   67.0
4     Afghanistan  2011  Developing   2978599.0         68.0     3013   68.0
…             …   …         …         …          …      …   …
2843     Zimbabwe  2004  Developing  12777511.0         68.0       31   67.0
2844     Zimbabwe  2003  Developing  12633897.0          7.0      998    7.0
2845     Zimbabwe  2002  Developing    125525.0         73.0      304   73.0
2846     Zimbabwe  2001  Developing  12366165.0         76.0      529   76.0
2847     Zimbabwe  2000  Developing  12222251.0         79.0     1483   78.0

      Diphtheria  HIV/AIDS  infant deaths  under-five deaths
0           65.0       0.1             62                 83  \
1           62.0       0.1             64                 86
2           64.0       0.1             66                 89
3           67.0       0.1             69                 93
4           68.0       0.1             71                 97
…          …       …            …                 …
2843        65.0      33.6             27                 42
2844        68.0      36.7             26                 41
2845        71.0      39.8             25                 40
2846        75.0      42.1             25                 39
2847        78.0      43.5             24                 39

      Total expenditure         GDP   BMI  thinness  1-19 years  Alcohol
0                  8.16  584.259210  19.1                  17.2     0.01  \
1                  8.18  612.696514  18.6                  17.5     0.01
2                  8.13  631.744976  18.1                  17.7     0.01
3                  8.52  669.959000  17.6                  17.9     0.01
4                  7.87   63.537231  17.2                  18.2     0.01
…                …         …   …                     …       …
2843               7.13  454.366654  27.1                   9.4     4.36
2844               6.52  453.351155  26.7                   9.8     4.06
2845               6.53   57.348340  26.3                   1.2     4.43
2846               6.16  548.587312  25.9                   1.6     1.72
2847               7.10  547.358878  25.5                  11.0     1.68

      Schooling  Life expectancy
0          10.1             65.0
```

```
1          10.0             59.9
2           9.9             59.9
3           9.8             59.5
4           9.5             59.2
…          …               …
2843        9.2             44.3
2844        9.5             44.5
2845       10.0             44.8
2846        9.8             45.3
2847        9.8             46.0

[2848 rows x 18 columns]
```

[315]:
```
df.describe()
```

[315]:
```
              Year    Population  Hepatitis B        Measles        Polio  
count  2848.000000  2.204000e+03  2306.000000    2848.000000  2829.000000  \
mean   2007.500000  1.283457e+07    81.076756    2083.082163    82.682220   
std       4.610582  6.196094e+07    25.019068   10249.107207    23.434954   
min    2000.000000  3.400000e+01     1.000000       0.000000     3.000000   
25%    2003.750000  1.967585e+05    77.000000       0.000000    78.000000   
50%    2007.500000  1.391756e+06    92.000000      16.000000    93.000000   
75%    2011.250000  7.438947e+06    97.000000     336.750000    97.000000   
max    2015.000000  1.293859e+09    99.000000  212183.000000    99.000000   

        Diphtheria     HIV/AIDS  infant deaths  under-five deaths  
count  2829.000000  2848.000000    2848.000000        2848.000000  \
mean     82.451396     1.756461      28.359902          39.500000   
std      23.693936     5.148935     117.188032         159.800866   
min       2.000000     0.100000       0.000000           0.000000   
25%      78.000000     0.100000       0.000000           0.000000   
50%      93.000000     0.100000       3.000000           4.000000   
75%      97.000000     0.700000      20.000000          25.000000   
max      99.000000    50.600000    1800.000000        2500.000000   

       Total expenditure            GDP          BMI  thinness  1-19 years  
count        2627.000000    2406.000000  2816.000000             2816.000000  \
mean            5.935577    7664.398813    38.503374                4.847230   
std             2.504439   14466.241793    19.955485                4.443695   
min             0.370000       1.681350     1.000000                0.100000   
25%             4.240000     477.541713    19.500000                1.600000   
50%             5.760000    1841.086830    43.900000                3.300000   
75%             7.530000    6265.658907    56.200000                7.125000   
max            17.600000  119172.741800    77.600000               27.700000   

           Alcohol    Schooling  Life expectancy  
count  2660.000000  2688.000000      2848.000000  
```

```
mean      4.638932   12.060156        69.347402
std       4.064721    3.320160         9.528332
min       0.010000    0.000000        36.300000
25%       0.930000   10.200000        63.500000
50%       3.785000   12.400000        72.200000
75%       7.810000   14.300000        75.800000
max      17.870000   20.700000        89.000000
```

0.1 Exploratory Data Analysis

In the following, I will go through each feature and perform some initial testing to identify at a high level any relevant insights for each feature.

First, we need to understand how and where there are missing values. Because this is a time series data set, we have the ability to interpolate missing values using an estimator from the existing values. However, before we interpolate, we should consider the distribution of data within each feature, whether it is normalized, and also how many missing values there are and whether those missing values are clustered or normally distributed in the data. There may be some columns from which there is too much data missing for us to realistically impute or interpolate missing values, and for those we will likely populate with the median value from that feature.

Becuase we are drawing inferences from the data, and our primary goal is to determine which factors contribute to Life Expectancy, with a secondary goal of prediction accuracy, we are going to choose a highly interpretable model. Because the response variable "Life expectancy" is quantitative data, we are going to use linear regression as a model to predict Life expectancy based on the feature set. Linear Regression is highly intepretable, and we will be able to look at the coefficient values in the model summary, as well as the p-values to determine the relative strength and impact of the feature on the model.

0.2 First we are going to look at our missing values

[316]:
```python
#Missing data, first we want to see what data we are missing.
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2848 entries, 0 to 2847
Data columns (total 18 columns):
 #   Column                 Non-Null Count  Dtype
---  ------                 --------------  -----
 0   Country                2848 non-null   object
 1   Year                   2848 non-null   int64
 2   Status                 2848 non-null   object
 3   Population             2204 non-null   float64
 4   Hepatitis B            2306 non-null   float64
 5   Measles                2848 non-null   int64
 6   Polio                  2829 non-null   float64
 7   Diphtheria             2829 non-null   float64
 8   HIV/AIDS               2848 non-null   float64
 9   infant deaths          2848 non-null   int64
```

```
 10  under-five deaths     2848 non-null   int64
 11  Total expenditure     2627 non-null   float64
 12  GDP                   2406 non-null   float64
 13  BMI                   2816 non-null   float64
 14  thinness  1-19 years  2816 non-null   float64
 15  Alcohol               2660 non-null   float64
 16  Schooling             2688 non-null   float64
 17  Life expectancy       2848 non-null   float64
dtypes: float64(12), int64(4), object(2)
memory usage: 400.6+ KB
```

Above we have all the features and the number of values missing in each feature. It looks like there are many missing values, specifically in Population where we are missing almost 23% of the data. We also know that Population is not normally distributed, and thus likely to be skewed. For this reason, we will likely drop Population data, especially as the impact on Life Expectancy seems questionable.

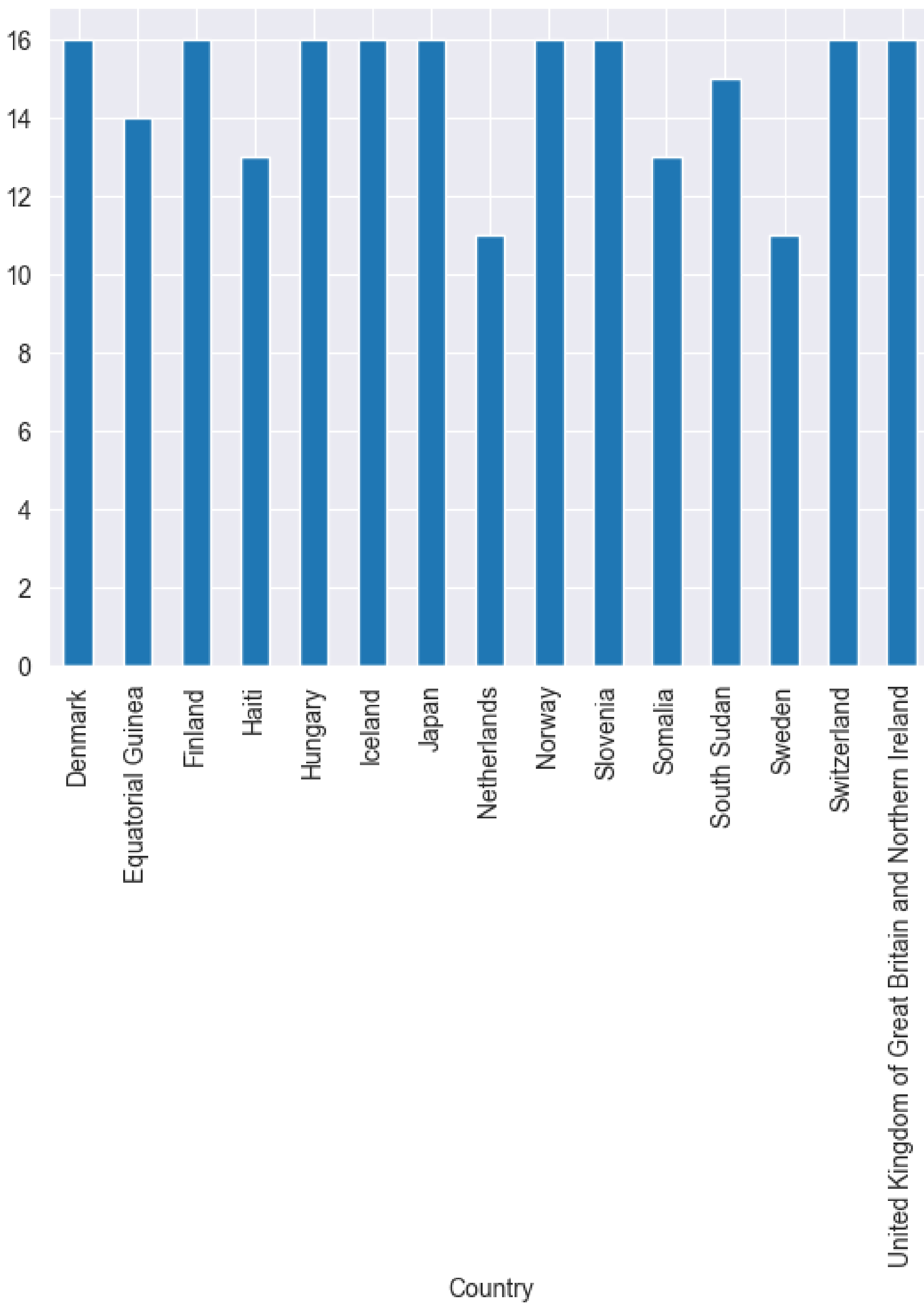
[317]:
```
df_missing = df.set_index(['Country','Year'])
df_missing = df_missing[df_missing.isna().any(axis=1)]
print(f'{len(df_missing)/len(df)} % of rows are missing data')
```

```
0.4434691011235955 % of rows are missing data
```

It looks like we're missing quite a lot of data, and that almost half of our records are missing one or more values. Let's take a given feature and see how many countries are missing all of their feature data, vs just some of the feature data.

[318]:
```
df_missing_hep = df_missing['Hepatitis B']
df_missing_hep_na = df_missing_hep.isna().groupby(level=0).sum()
df_missing_hep_na[df_missing_hep_na > 10].plot(kind='bar')
```

[318]:
```
<Axes: xlabel='Country'>
```



As we can see above, only a handful of countries are missing all of their ‘Hepatitis B’ data, which means our interpolation strategy of inferring missing feature values based on previous feature values in the time series is promising.

[319]:
```
df_pivot = df.pivot(columns='Year',index='Country')
df_pivot.head()
```

[319]:
```
                         Status
Year                       2000       2001       2002       2003
Country
Afghanistan          Developing Developing Developing Developing  \
Albania              Developing Developing Developing Developing
Algeria              Developing Developing Developing Developing
Angola               Developing Developing Developing Developing
Antigua and Barbuda  Developing Developing Developing Developing


Year                       2004       2005       2006       2007
Country
Afghanistan          Developing Developing Developing Developing  \
Albania              Developing Developing Developing Developing
Algeria              Developing Developing Developing Developing
Angola               Developing Developing Developing Developing
Antigua and Barbuda  Developing Developing Developing Developing


                                            … Life expectancy
Year                       2008       2009  …            2006 2007 2008
Country                                     …
Afghanistan          Developing Developing  …            57.3 57.5 58.1  \
Albania              Developing Developing  …            74.2 75.9 75.3
Algeria              Developing Developing  …            73.4 73.8 74.1
Angola               Developing Developing  …            47.7 48.2 48.7
Antigua and Barbuda  Developing Developing  …            74.8 75.0 75.2


Year                 2009 2010 2011 2012 2013 2014 2015
Country
Afghanistan          58.6 58.8 59.2 59.5 59.9 59.9 65.0
Albania              76.1 76.2 76.6 76.9 77.2 77.5 77.8
Algeria              74.4 74.7 74.9 75.1 75.3 75.4 75.6
Angola               49.1 49.6 51.0 56.0 51.1 51.7 52.4
Antigua and Barbuda  75.4 75.6 75.7 75.9 76.1 76.2 76.4

[5 rows x 256 columns]
```

[320]:
```
#Proportion of countries for which there is any missing population data.
print(f' {len(df_pivot.loc[df_pivot.Population.isnull().any(axis=1)])/len(df.
 ↪Country.unique())* 100}% of countries are missing population data')
```

```
23.03370786516854% of countries are missing population data
```

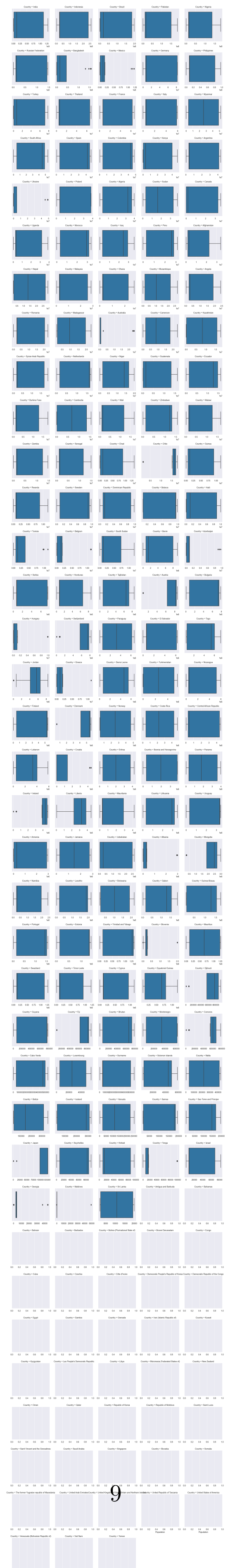
Below I want to visually represent all of the missing data for Population, just to confirm the sparseness of the data, before dropping it from the table.

[321]:
```
df_pivot = df.pivot(index='Year', columns='Country', values='Population')
std_devs = df_pivot.std().sort_values(ascending=False)

country_order = std_devs.index.tolist()
g = sns.FacetGrid(df, col="Country", col_order=country_order, col_wrap=5,␣
  ↪sharex=False)

# Apply a boxplot on each face
g.map(sns.boxplot, "Population")
plt.show()
```

```
/Users/alexcullen/Library/Python/3.10/lib/python/site-
packages/seaborn/axisgrid.py:712: UserWarning: Using the boxplot function
without specifying `order` is likely to produce an incorrect plot.
  warnings.warn(warning)
```



Not only are there many missing values, we can see from the box plots that there are several outliers in the dataset. The data collected for this feature is poor and may bias our dataset.

The only categorical variable in the dataset is economic Status, which is binary and has a value of either "Developed" or "Developing". We will need to convert this categorical variable into a single dummy variable column with value 1 if the country is "Developed" and 0 otherwise.

Also, looking at the distribution of Status, we can see that every country that was "Developed" at period 0 stayed "Developed" through the last period, and every country that was "Developing" at period 0, remained "Developing" through the last period. This is promising for our interpolation strategy, as the estimator we choose will likely be extremely accurate.

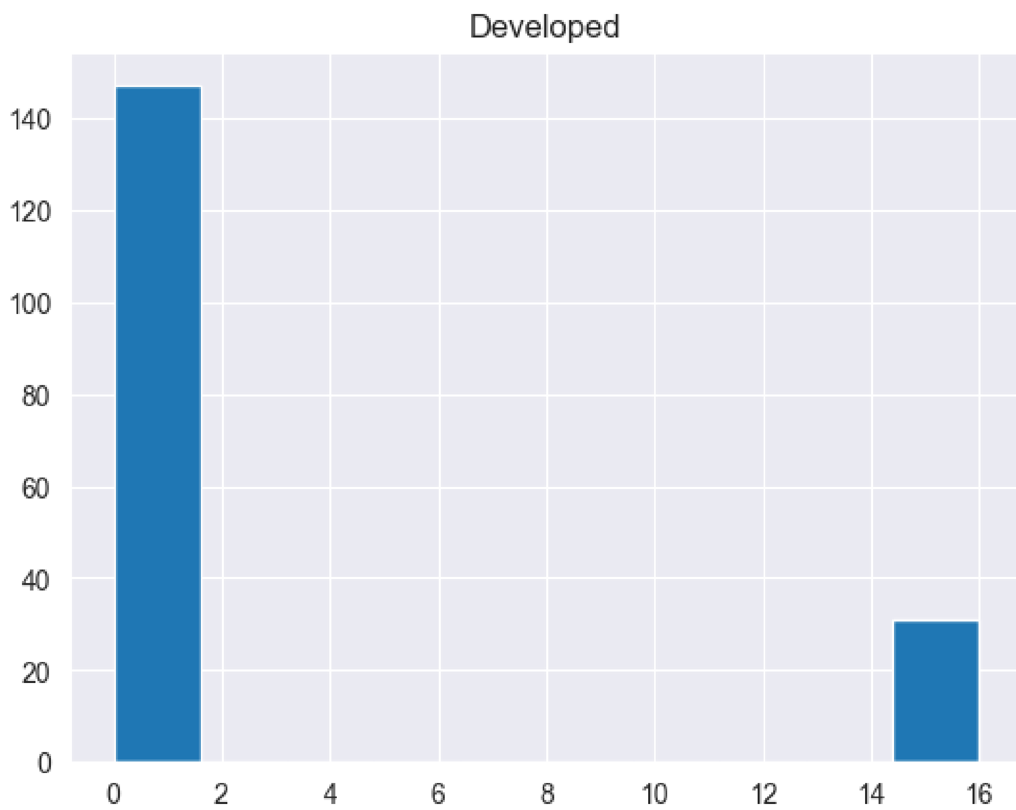
[322]:
```
#Economic Status (developing or developed) of each country

df_status = pd.concat([df.drop('Status',axis=1),pd.get_dummies(df.Status)],␣
 ↪axis=1)[['Country','Developed']]
df_status.head()
```

[322]:

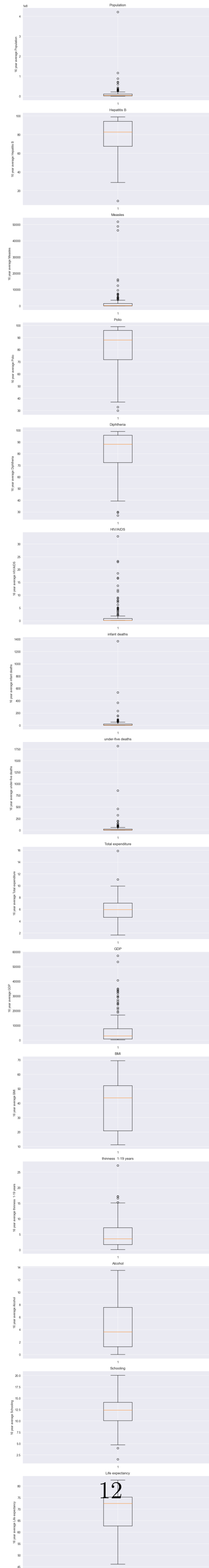
	Country	Developed
0	Afghanistan	False
1	Afghanistan	False
2	Afghanistan	False
3	Afghanistan	False
4	Afghanistan	False

[323]:
```
df_status.groupby('Country').sum().hist();
```



Next we are going to take the mean of each feature value by country, and plot those using a box plot to better understand the distribution of data and whether it is normally distributed.

[324]:
```
fig, axs = plt.subplots(len(df.drop(['Country', 'Year', 'Status'], axis=1).
 ↪columns), 1, figsize=(10, 5*len(df.drop(['Country', 'Year', 'Status'],␣
 ↪axis=1).columns)))
for idx, col in enumerate(df.drop(['Country', 'Year', 'Status'], axis=1).
 ↪columns):
    df_pivot = df.pivot(index='Year', columns='Country', values=col)
    means = df_pivot.mean().dropna()
    axs[idx].boxplot(means)
    axs[idx].set_title(col)
    axs[idx].set_ylabel(f'16 year average {col}')
plt.tight_layout()
plt.show()
```

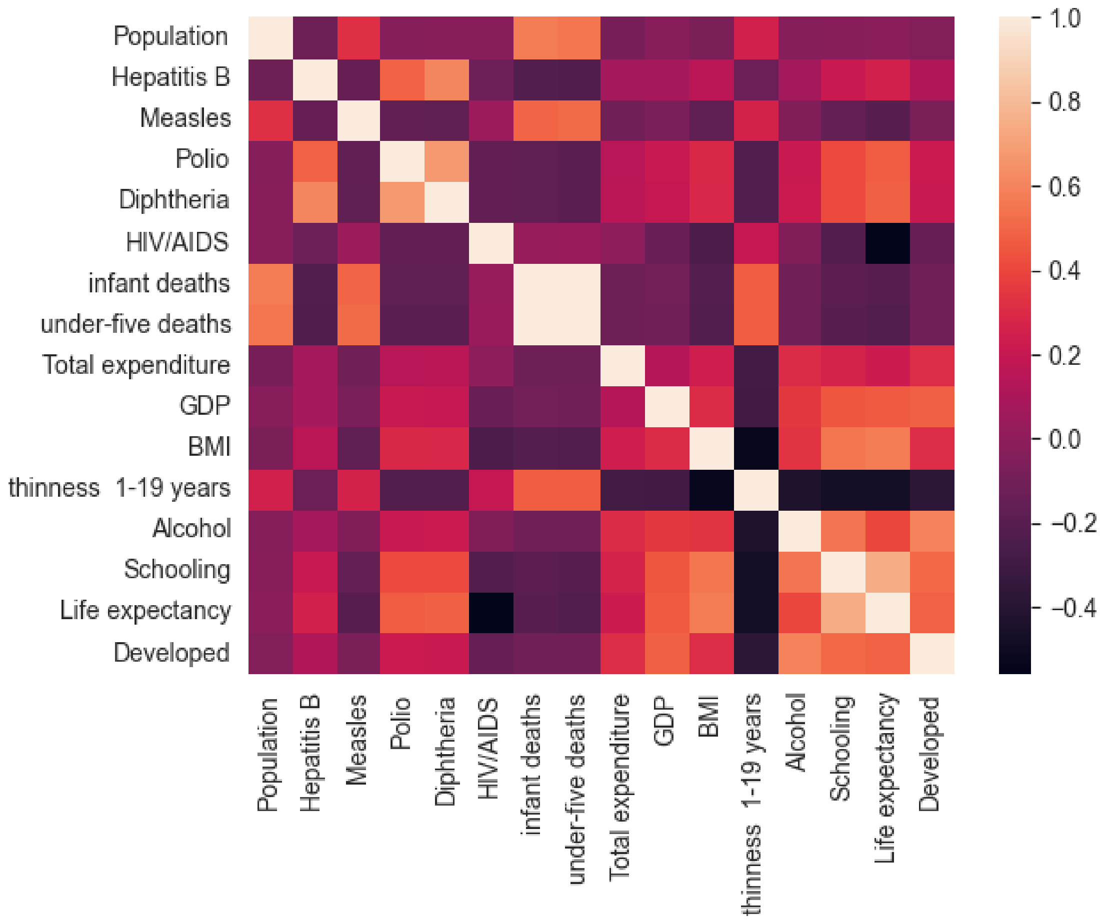

Population
Hepatitis B
Measles
Polio
Diphtheria
HIV/AIDS
infant deaths
under-five deaths
Total expenditure
GDP
BMI
thinness 1-19 years
Alcohol
Schooling
Life expectancy

We can see that many of the features are not normally distributed, like GDP, Total Expenditure, HIV/AIDS, and others. These values we will need to log transform so that they do not skew/bias our model's predictions.

Below we look at the correlation between features, and we can see that infant deaths and under-five deaths are almost 1:1 correlated. This will likely be an interaction that we need to account for in our model. Other features are correlated with each other, for example vaccination rates, but not to a material degree that they must be accomodated for.

[325]:
```
df_status = pd.concat([df.drop('Status',axis=1),pd.get_dummies(df.Status)],␣
↪axis=1).drop('Developing',axis=1)
sns.heatmap(df_status.set_index(['Year','Country']).corr())
```

[325]: <Axes: >



In the following section, we are going to perform the following transformations on the data based on what we learned from EDA above. 1. Convert Status as a categorical variable to a numeric

0/1 binary column 2. Log transform postive numeric columns that are not normally distributed 3. Interpolate missing values in features where possible 4. Impute all remaining missing values.

[326]:
```
df_melt = df_status.melt(id_vars=['Year', 'Country'],
                         value_vars=df_status.drop(['Year', 'Country'],axis=1).
 ↪columns,
                        var_name='Feature',
                        value_name='Value')

df_pivot = df_melt.pivot_table(index='Year',
                               columns=['Country', 'Feature'],
                               values='Value')
df_pivot
```

[326]:
```
Country Afghanistan
Feature     Alcohol   BMI Developed Diphtheria         GDP HIV/AIDS
Year
2000           0.01  12.2       0.0       24.0      114.56      0.1  \
2001           0.01  12.6       0.0       33.0   117.49698      0.1
2002           0.01  13.0       0.0       36.0   187.84595      0.1
2003           0.01  13.4       0.0       41.0  198.728544      0.1
2004           0.02  13.8       0.0        5.0  219.141353      0.1
2005           0.02  14.2       0.0       58.0    25.29413      0.1
2006           0.03  14.7       0.0       58.0   272.56377      0.1
2007           0.02  15.2       0.0       63.0  369.835796      0.1
2008           0.03  15.7       0.0       64.0  373.361116      0.1
2009           0.01  16.2       0.0       63.0  445.893298      0.1
2010           0.01  16.7       0.0       66.0   553.32894      0.1
2011           0.01  17.2       0.0       68.0   63.537231      0.1
2012           0.01  17.6       0.0       67.0     669.959      0.1
2013           0.01  18.1       0.0       64.0  631.744976      0.1
2014           0.01  18.6       0.0       62.0  612.696514      0.1
2015           0.01  19.1       0.0       65.0   584.25921      0.1

Country                                              …    Zimbabwe
Feature Hepatitis B Life expectancy Measles Polio  … Hepatitis B
Year                                                 …
2000           62.0            54.8  6532.0  24.0  …        79.0  \
2001           63.0            55.3  8762.0  35.0  …        76.0
2002           64.0            56.2  2486.0  36.0  …        73.0
2003           65.0            56.7   798.0  41.0  …         7.0
2004           67.0            57.0   466.0   5.0  …        68.0
2005           66.0            57.3  1296.0  58.0  …        65.0
2006           64.0            57.3  1990.0  58.0  …        68.0
2007           63.0            57.5  1141.0  63.0  …        72.0
2008           64.0            58.1  1599.0  64.0  …        75.0
2009           63.0            58.6  2861.0  63.0  …        73.0
```

```
2010          66.0            58.8  1989.0  66.0  …          9.0
2011          68.0            59.2  3013.0  68.0  …         94.0
2012          67.0            59.5  2787.0  67.0  …         97.0
2013          64.0            59.9   430.0  62.0  …         95.0
2014          62.0            59.9   492.0  58.0  …         91.0
2015          65.0            65.0  1154.0   6.0  …         87.0

Country
Feature Life expectancy Measles Polio  Population Schooling Total expenditure
Year
2000                46.0  1483.0  78.0  12222251.0       9.8               7.1  \
2001                45.3   529.0  76.0  12366165.0       9.8              6.16
2002                44.8   304.0  73.0    125525.0      10.0              6.53
2003                44.5   998.0   7.0  12633897.0       9.5              6.52
2004                44.3    31.0  67.0  12777511.0       9.2              7.13
2005                44.6   420.0  69.0    129432.0       9.3              6.44
2006                45.4   212.0  71.0  13124267.0       9.5              5.12
2007                46.6   242.0  73.0   1332999.0       9.6              4.47
2008                48.2     0.0  75.0  13558469.0       9.7              4.96
2009                50.0   853.0  69.0   1381599.0       9.9              6.26
2010                52.4  9696.0  89.0   1486317.0      10.0              5.37
2011                54.9     0.0  93.0  14386649.0      10.1              6.31
2012                56.6     0.0  95.0   1471826.0       9.8              6.69
2013                58.0     0.0  95.0    155456.0      10.4              6.88
2014                59.2     0.0  92.0  15411675.0      10.3              6.44
2015                67.0     0.0  88.0  15777451.0      10.3               NaN

Country
Feature infant deaths thinness  1-19 years under-five deaths
Year
2000             24.0                  11.0               39.0
2001             25.0                   1.6               39.0
2002             25.0                   1.2               40.0
2003             26.0                   9.8               41.0
2004             27.0                   9.4               42.0
2005             28.0                   9.0               43.0
2006             28.0                   8.6               45.0
2007             29.0                   8.2               46.0
2008             30.0                   7.8               46.0
2009             30.0                   7.5               45.0
2010             29.0                   7.1               44.0
2011             28.0                   6.8               42.0
2012             26.0                   6.5               39.0
2013             25.0                   6.2               36.0
2014             23.0                   5.9               34.0
2015             22.0                   5.6               32.0
```

```
[16 rows x 2757 columns]
```

In this section we are removing all outlier values by country by feature. For example, we look at a given country's rates of vaccination for Hep B, and we remove any extreme (3 std deviations) outliers in Hep B based on the existing data for that country. There are many outlier values in the data set. We will interpolate or impute the missing values that we are removing from the dataset.

[327]:
```
threshold = 3

def calculate_z_scores(col):
    if col.std() == 0:
        return [np.nan] * len(col)
    else:
        return (col - col.mean()) / col.std()

z_scores = df_pivot.apply(calculate_z_scores, axis=0)

outliers = np.abs(z_scores) > threshold

df_pivot[outliers] = np.nan
```

[328]:
```
df_pre_impute = df_pivot.copy()
```

[329]:
```
df_stacked = df_pre_impute.stack(level=0)

df_reset = df_stacked.reset_index()
```

Number of missing values

[330]:
```
df_reset.isna().sum().sum()
```

[330]: 2612

We will use a complex KNN Imputer in order to impute the remaining missing values. This is preferred over a more simple approach because it is likely that there is a high degree of clustering in the dataset. The initial "Status" column is an indication that factors amongst the group of developing or developed nations are dissimilar from each other, but similar within their class.

[331]:
```
for col in df_reset.drop('Country',axis =1).columns:
    df_reset[col] = pd.to_numeric(df_reset[col])

numeric_columns = df_reset.select_dtypes(include=[np.number])

imputer = KNNImputer(n_neighbors=5)

df_imputed_numeric = imputer.fit_transform(numeric_columns)

df_imputed_numeric = pd.DataFrame(df_imputed_numeric, columns=numeric_columns.
 ↪columns)
```

```python
df_imputed = pd.concat([df_reset.drop(numeric_columns.columns, axis=1),
 ↪df_imputed_numeric], axis=1)
```

Here we are log transforming the non-normally distributed columns

[332]:
```python
df_transformed = df_imputed.copy()

log_transform_cols = ['GDP', 'Total expenditure', 'under-five deaths', 'infant
 ↪deaths', 'Measles', 'HIV/AIDS']

for col in log_transform_cols:
    df_transformed[col] = df_transformed[col].apply(lambda x: np.log(x+ 1))
```

[333]:
```python
df_transformed
```

[333]:
```
Feature                          Country    Year  Alcohol   BMI  Developed
0                            Afghanistan  2000.0    0.010  12.2        0.0  \
1                                Albania  2000.0    3.660  45.0        0.0
2                                Algeria  2000.0    0.250  44.4        0.0
3                                 Angola  2000.0    1.850  15.4        0.0
4                    Antigua and Barbuda  2000.0    7.270  38.2        0.0
…                                      …       …        …     …          …
2843     Venezuela (Bolivarian Republic of)  2015.0    5.784  62.1        0.0
2844                            Viet Nam  2015.0    1.248  17.5        0.0
2845                               Yemen  2015.0    2.444  41.3        0.0
2846                              Zambia  2015.0    3.048  23.4        0.0
2847                            Zimbabwe  2015.0    1.582  31.8        0.0

Feature  Diphtheria       GDP  HIV/AIDS  Hepatitis B  Life expectancy
0              24.0  4.749790  0.095310         62.0             54.8  \
1              97.0  7.070545  0.095310         96.0             72.6
2              86.0  7.472033  0.095310         86.0             71.3
3              28.0  6.321302  1.098612         69.0             45.3
4              95.0  9.197879  0.095310         96.4             73.6
…                 …         …         …            …                …
2843           87.0  7.947697  0.095310         87.0             74.1
2844           97.0  6.280968  0.095310         97.0             76.0
2845           69.0  8.116740  0.095310         69.0             65.7
2846            9.0  7.181508  1.629241          9.0             61.8
2847           87.0  4.784937  1.974081         87.0             67.0

Feature   Measles  Polio    Population  Schooling  Total expenditure
0        8.784622   24.0  2.937560e+05        5.5           2.219203  \
1        2.104134   97.0  3.892700e+04       10.7           1.982380
2        0.000000   86.0  3.118366e+06       10.7           1.501853
3        7.705262    3.0  1.644924e+06        4.6           1.332366
4        0.000000   96.0  1.058848e+07        0.0           1.635106
```

```
…          …     …          …          …                 …
2843    0.000000   87.0  2.258149e+07       14.3          1.926873
2844    5.549076   97.0  2.354233e+06       12.6          1.746762
2845    6.150603   63.0  1.281210e+07        9.0          1.781036
2846    2.302585    9.0  1.615870e+05       12.5          1.796083
2847    0.000000   88.0  1.577745e+07       10.3          1.907466

Feature  infant deaths  thinness  1-19 years  under-five deaths
0             4.488636                   2.3           4.812184
1             0.693147                   2.1           0.693147
2             3.091042                   6.5           3.258097
3             4.584967                   1.9           5.081404
4             0.000000                   3.7           0.000000
…                  …                   …                 …
2843          2.302585                   1.6           2.397895
2844          3.367296                  14.2           3.583519
2845          3.637586                  13.6           3.871201
2846          3.332205                   6.3           3.713572
2847          3.135494                   5.6           3.496508

[2848 rows x 18 columns]
```

Now that all of the data is cleaning and transformed, we will run a simple OLS linear regrssion using Statsmodel as our baseline to see how a normal linear regression model performs.

[334]:
```
df_final = df_transformed.set_index(['Year', 'Country']).
 ↪drop('Population',axis=1)
X = df_final.drop('Life expectancy', axis=1)
y = df_final['Life expectancy']

X = sm.add_constant(X)

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
 ↪random_state=42)

model = sm.OLS(y_train, X_train)
results = model.fit()

print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.847
Model:                            OLS   Adj. R-squared:                  0.846
Method:                 Least Squares   F-statistic:                     897.8
Date:                Mon, 03 Jul 2023   Prob (F-statistic):               0.00
Time:                        21:22:55   Log-Likelihood:                -6218.5
No. Observations:                2278   AIC:                         1.247e+04
```

```
Df Residuals:                    2263   BIC:                         1.255e+04
Df Model:                          14
Covariance Type:            nonrobust
================================================================================
========
                          coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
--------
const                  54.3579      0.773     70.338      0.000      52.842
55.873
Alcohol                -0.0366      0.028     -1.324      0.186      -0.091
0.018
BMI                     0.0207      0.006      3.716      0.000       0.010
0.032
Developed               2.2710      0.277      8.184      0.000       1.727
2.815
Diphtheria              0.0340      0.006      6.151      0.000       0.023
0.045
GDP                     0.4779      0.057      8.445      0.000       0.367
0.589
HIV/AIDS               -5.5321      0.130    -42.499      0.000      -5.787
-5.277
Hepatitis B            -0.0155      0.004     -3.543      0.000      -0.024
-0.007
Measles                 0.0257      0.033      0.767      0.443      -0.040
0.091
Polio                   0.0207      0.005      3.888      0.000       0.010
0.031
Schooling               0.8868      0.040     22.346      0.000       0.809
0.965
Total expenditure       0.4077      0.216      1.887      0.059      -0.016
0.831
infant deaths           3.1913      0.579      5.509      0.000       2.055
4.327
thinness  1-19 years   -0.0517      0.023     -2.203      0.028      -0.098
-0.006
under-five deaths      -3.6897      0.555     -6.651      0.000      -4.778
-2.602
==============================================================================
Omnibus:                       74.006   Durbin-Watson:                   2.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              159.576
Skew:                          -0.185   Prob(JB):                     2.23e-35
Kurtosis:                       4.243   Cond. No.                     1.71e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
```

```
specified.
[2] The condition number is large, 1.71e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

We can see from the summary table that Simple Linear Regression is actaully quite effective at modeling the data. The adj R-squared for our model is 84.6% which means that our model explains 84.6% of the variance in the errors.

Looking at each parameter and the assessment, we can see based on the coefficient and p-value of each the relevance and impact of each parameter on the model. Immediately we can see that Total Expenditure on healthcare is above our p-value threshold of .05, as well as # of cases of measles. We can see that infant deaths and under-five deaths are extremely correlated with each other, and so we will need to include that interaction in our model.

From first review, it seems that HIV/AIDS is the strongest predictor of life expectancy, followed by whether or not the country is "Developed".

[335]:
```python
y_pred = results.predict(X_test)

mean_squared_error(y_test, y_pred) #baseline score
```

[335]:
```
13.256831860025624
```

To better analyze multicollinearity, we will use VIF below. We note that infant deaths and under-five deaths are highly correlated.

[336]:
```python
vif_data = pd.DataFrame()
vif_data["feature"] = X.columns

vif_data["VIF"] = [variance_inflation_factor(X.values, i) for i in range(len(X.
 ↪columns))]

print(vif_data)
```

```
                feature         VIF
0                 const   99.328038
1               Alcohol    1.985335
2                   BMI    1.919887
3             Developed    1.822120
4            Diphtheria    2.472871
5                   GDP    1.701669
6              HIV/AIDS    1.585255
7           Hepatitis B    1.629814
8               Measles    1.824198
9                 Polio    2.145212
10            Schooling    2.752249
11    Total expenditure    1.141331
12        infant deaths  145.658011
13  thinness  1-19 years    1.775081
14     under-five deaths  153.457055
```

Here we are creating the interaction between infant deaths and under-five deaths

[337]:
```
df_forward_select = df_final.copy()

df_forward_select['infant death * under-five deaths'] =
 ↪df_forward_select['infant deaths'] * df_forward_select['under-five deaths']
```

We split the data to validate the accuracy of our predictions on unseen data.

[338]:
```
X = df_forward_select.drop('Life expectancy', axis=1)
y = df_forward_select['Life expectancy']

X = sm.add_constant(X)

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
 ↪random_state=42)
```

Below we are going to implement the hybrid approach of forward selecting and backwards selecting features in each iteration of the loop until our model settles and we are no longer adding features. Each forward step we are adding the feature which improves the mean_squared_error the most, each backward step we are removing any features with p-values > our threshold.

[339]:
```
def get_score(X_train, X_test, y_train, y_test):
    model = sm.OLS(y_train, add_constant(X_train))
    results = model.fit()
    y_pred = results.predict(add_constant(X_test))
    return mean_squared_error(y_test, y_pred)

remaining = set(X_train.columns)
selected = []
n = len(X_train)
best_new_score, current_score = float('inf'), float('inf')
while remaining:
    changed = False

    # Forward step
    scores_with_candidates = []
    for candidate in remaining:
        temp_selected = selected + [candidate]
        score = get_score(X_train[temp_selected], X_test[temp_selected],
 ↪y_train, y_test)
        scores_with_candidates.append((score, candidate))

    scores_with_candidates.sort()
    best_new_score, best_candidate = scores_with_candidates[0]

    if best_new_score < current_score:
        remaining.remove(best_candidate)
```

```python
            selected.append(best_candidate)
            current_score = best_new_score
            changed = True

    # Backward step
    model = sm.OLS(y_train, add_constant(X_train[selected]))
    pvalues = model.fit().pvalues[1:]  # exclude intercept
    worst_pval = pvalues.max()
    if worst_pval > 0.05:
        worst_feature = pvalues.idxmax()
        selected.remove(worst_feature)
        remaining.add(worst_feature)
        changed = True

    if not changed:
        break

best_model = sm.OLS(y_train, add_constant(X_train[selected]))
results = best_model.fit()
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.846
Model:                            OLS   Adj. R-squared:                  0.845
Method:                 Least Squares   F-statistic:                     1246.
Date:                Mon, 03 Jul 2023   Prob (F-statistic):               0.00
Time:                        21:22:56   Log-Likelihood:                -6228.6
No. Observations:                2278   AIC:                         1.248e+04
Df Residuals:                    2267   BIC:                         1.254e+04
Df Model:                          10
Covariance Type:            nonrobust
================================================================================
========
                           coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
--------
const                   55.5427      0.683     81.315      0.000      54.203
56.882
HIV/AIDS                -5.6245      0.126    -44.570      0.000      -5.872
-5.377
Schooling                0.9070      0.038     24.127      0.000       0.833
0.981
GDP                      0.4986      0.056      8.942      0.000       0.389
0.608
under-five deaths       -3.8362      0.555     -6.917      0.000      -4.924
-2.749
```

```
Polio                     0.0207      0.005      3.884      0.000       0.010
0.031
Developed                 2.1944      0.251      8.755      0.000       1.703
2.686
infant deaths             3.3615      0.580      5.795      0.000       2.224
4.499
Diphtheria                0.0347      0.006      6.261      0.000       0.024
0.046
Hepatitis B              -0.0154      0.004     -3.523      0.000      -0.024
-0.007
thinness  1-19 years     -0.0767      0.022     -3.550      0.000      -0.119
-0.034
==============================================================================
Omnibus:                       70.443   Durbin-Watson:                   1.998
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              142.079
Skew:                          -0.198   Prob(JB):                     1.41e-31
Kurtosis:                       4.157   Cond. No.                     1.58e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.58e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[340]:
```python
print(f'Final score = {mean_squared_error(y_test, results.
 ↪predict(add_constant(X_test[selected])))}')
print(f'Selected attributes ranked in order of adj. r-squared ')
[col for col in selected]
```

```
Final score = 13.154045372207346
Selected attributes ranked in order of adj. r-squared
```

[340]:
```
['HIV/AIDS',
 'Schooling',
 'GDP',
 'under-five deaths',
 'Polio',
 'Developed',
 'infant deaths',
 'Diphtheria',
 'Hepatitis B',
 'thinness  1-19 years']
```

1 Final Analysis:

Based on our modeling, it is clear that the most immediate predictor of a country's Life Expectancy is it's incidence of HIV/AIDS, which is a deadly and serious disease. The correlation is strongly negative which means that the higher the prevelance the lower the Life Expectancy of that country.

After that, we see that metrics related to wealth like Schooling, GDP, and Status are positively correlated with Life Expectancy, which is not unusual. However, what is unusual is that Total expenditure on health care does not at all have predictive power on life expectancy.

Linear Regression was a very effective model for the analysis that we were performing today. It is surprising to see how accurate our model was, with our best performing model able to predict a given country's life expectancy with a root mean squared error of 4 years, which is a highly accurate measurement given the range of Life expectancies seen in the data.

Also, with Linear Regression we did not compromise on interpretability, we were readily able to infer from the data which factors were most important in predicting Life Expectancy by country

[354]:
```
#As a final step, we will look at the relative performance of KNN which is more
↪difficult to interpret, but might be more accurate at predicting life
↪expectancy.
```

[341]:
```
model = KNeighborsRegressor(n_neighbors=3)
model.fit(X_train, y_train)

y_pred = model.predict(X_test)

mean_squared_error(y_test, y_pred)
```

[341]:
```
15.671372553606236
```

[356]:
```
X_train.loc[2000.0]
```

[356]:
```
              const  Alcohol   BMI  Developed  Diphtheria       GDP  HIV/AIDS
Country
Timor-Leste     1.0     0.50  11.9        0.0        93.6  6.048049  0.095310  \
Mauritius       1.0     4.60  25.3        0.0        88.0  8.259024  0.095310
Gambia          1.0     2.18  18.0        0.0        85.6  5.088505  1.098612
Hungary         1.0    12.22  56.1        1.0        99.0  8.439116  0.095310
Israel          1.0     2.53  58.3        0.0        93.0  7.674684  0.095310
…               …      …     …         …           …         …        …
Guinea          1.0     0.17  16.6        0.0        46.0  3.570009  1.386294
Tunisia         1.0     1.21  48.1        0.0        97.0  7.702969  0.095310
Comoros         1.0     0.09  17.3        0.0         7.0  5.931855  0.095310
Botswana        1.0     5.37  29.9        0.0        97.0  8.116921  3.683867
Saint Lucia     1.0    11.69  36.8        0.0         7.0  7.553143  0.336472

              Hepatitis B   Measles  Polio  Schooling  Total expenditure
Country
Timor-Leste          94.6  0.000000   90.8       7.68           1.449269  \
Mauritius            88.0  0.000000   88.0      12.10           1.564441
Gambia               91.0  5.519860   84.0       6.50           1.528228
Hungary              83.2  0.693147   99.0      13.90           2.151762
```

```
Israel                98.0  3.610918   93.0      15.20           2.095561
…                      …         …     …       …                  …
Guinea                36.2  9.067809   47.0       4.80           1.495149
Tunisia               94.0  3.871201   97.0      12.80           1.856298
Comoros               65.8  0.000000    7.0       8.20           1.517323
Botswana              86.0  0.336472   97.0      11.70           1.729884
Saint Lucia           42.6  0.000000    7.0      12.80           1.876407

             infant deaths  thinness  1-19 years  under-five deaths
Country
Timor-Leste       1.386294                  12.2           1.609438  \
Mauritius         0.000000                   8.1           0.000000
Gambia            1.386294                   1.2           1.945910
Hungary           0.693147                   2.3           0.693147
Israel            0.693147                   1.1           0.693147
…                      …                    …                 …
Guinea            3.637586                   1.3           4.110874
Tunisia           1.609438                   6.6           1.791759
Comoros           0.693147                   7.9           1.098612
Botswana          1.098612                  12.3           1.609438
Saint Lucia       0.000000                   4.5           0.000000

             infant death * under-five deaths
Country
Timor-Leste                          2.231155
Mauritius                            0.000000
Gambia                               2.697604
Hungary                              0.480453
Israel                               0.480453
…                                       …
Guinea                              14.953658
Tunisia                              2.883726
Comoros                              0.761500
Botswana                             1.768148
Saint Lucia                          0.000000

[146 rows x 16 columns]
```

[379]:
```
X_test.head()
```

[379]:
```
                    const  Alcohol   BMI  Developed  Diphtheria        GDP
Year   Country
2008.0 Switzerland    1.0    10.29  54.6        1.0        95.0  11.186095  \
2001.0 Spain          1.0     9.86  58.2        1.0        96.0   9.637215
2011.0 Ukraine        1.0     8.48  59.0        0.0         5.0   8.180533
2014.0 Brazil         1.0     7.32  55.3        0.0        93.0   7.112830
2012.0 Cyprus         1.0    10.55  58.7        1.0        99.0  10.273400
```

```
                        HIV/AIDS  Hepatitis B   Measles  Polio  Schooling
Year   Country
2008.0 Switzerland  0.095310         98.0  7.612337   96.0       15.3  \
2001.0 Spain        0.095310         83.0  0.000000   95.0       15.7
2011.0 Ukraine      0.182322         21.0  7.195937   54.0       14.9
2014.0 Brazil       0.095310         96.0  5.564520   96.0       15.2
2012.0 Cyprus       0.095310         96.0  0.693147   99.0       13.8

                    Total expenditure  infant deaths  thinness  1-19 years
Year   Country
2008.0 Switzerland           0.828552       0.000000                  0.5  \
2001.0 Spain                 2.109000       1.098612                  0.6
2011.0 Ukraine               2.076938       1.791759                  2.4
2014.0 Brazil                2.232163       3.806662                  2.7
2012.0 Cyprus                2.132982       0.000000                  0.9

                    under-five deaths  infant death * under-five deaths
Year   Country
2008.0 Switzerland           0.000000                          0.000000
2001.0 Spain                 1.098612                          1.206949
2011.0 Ukraine               1.791759                          3.210402
2014.0 Brazil                3.912023                         14.891751
2012.0 Cyprus                0.000000                          0.000000
```

[383]:
```python
distances, indices = model.kneighbors(X=pd.DataFrame(X_test.loc[2008.0].
    loc['Switzerland']).T)
```

[385]:
```python
print(distances)
X_test.iloc[indices[0]]
```

```
[[1.86587358 2.40112778 3.79775239]]
```

[385]:
```
                const  Alcohol   BMI  Developed  Diphtheria       GDP
Year   Country
2005.0 Ireland    1.0    13.31  55.1        1.0        96.0  8.680642  \
2003.0 Israel     1.0     2.32  59.6        0.0        93.0  9.849454
2004.0 Senegal    1.0     0.35  19.1        0.0        87.0  6.599301

                HIV/AIDS  Hepatitis B   Measles  Polio  Schooling
Year   Country
2005.0 Ireland  0.095310         98.4  4.564348   96.8       17.5  \
2003.0 Israel   0.095310         98.0  4.828314   93.0       16.0
2004.0 Senegal  0.530628         76.0  3.465736   87.0        6.2

                Total expenditure  infant deaths  thinness  1-19 years
Year   Country
2005.0 Ireland           2.112635       0.000000                  0.3  \
```

```
2003.0 Israel            2.132982       0.693147                      1.1
2004.0 Senegal           1.899118       3.218876                     11.6

                 under-five deaths  infant death * under-five deaths
Year   Country
2005.0 Ireland           0.000000                           0.000000
2003.0 Israel            0.693147                           0.480453
2004.0 Senegal           3.761200                          12.106836
```

[382]:
```
y_test.iloc[indices[0]]
```

[382]:
```
Year     Country
2005.0   Ireland    78.7
2003.0   Israel     79.7
2004.0   Senegal    59.7
Name: Life expectancy, dtype: float64
```

Above we performed KNN on the same transformed and cleaned dataset, and we achieved a lower accuracy score than even the baseline Linear Regression model. We can also see that when use KNN to return the nearest neighbors, we get some non-intuitive results upon further analysis. For example, 2008 Switzerland's nearest neighbors are Ireland and Israel which make sense, but then Senegal is the third closest, however, as we can see from the data, the Life Expectancy of the Senegalese is far below Switzerland's own.

[ ]:
```

```