Community Analysis Proposal

Modelling of a Finite-Speed Thermal Wave in Solids using OpenFOAM

CSCI 5636: Numerical Solution of Partial Differential Equations

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Physics of the problem

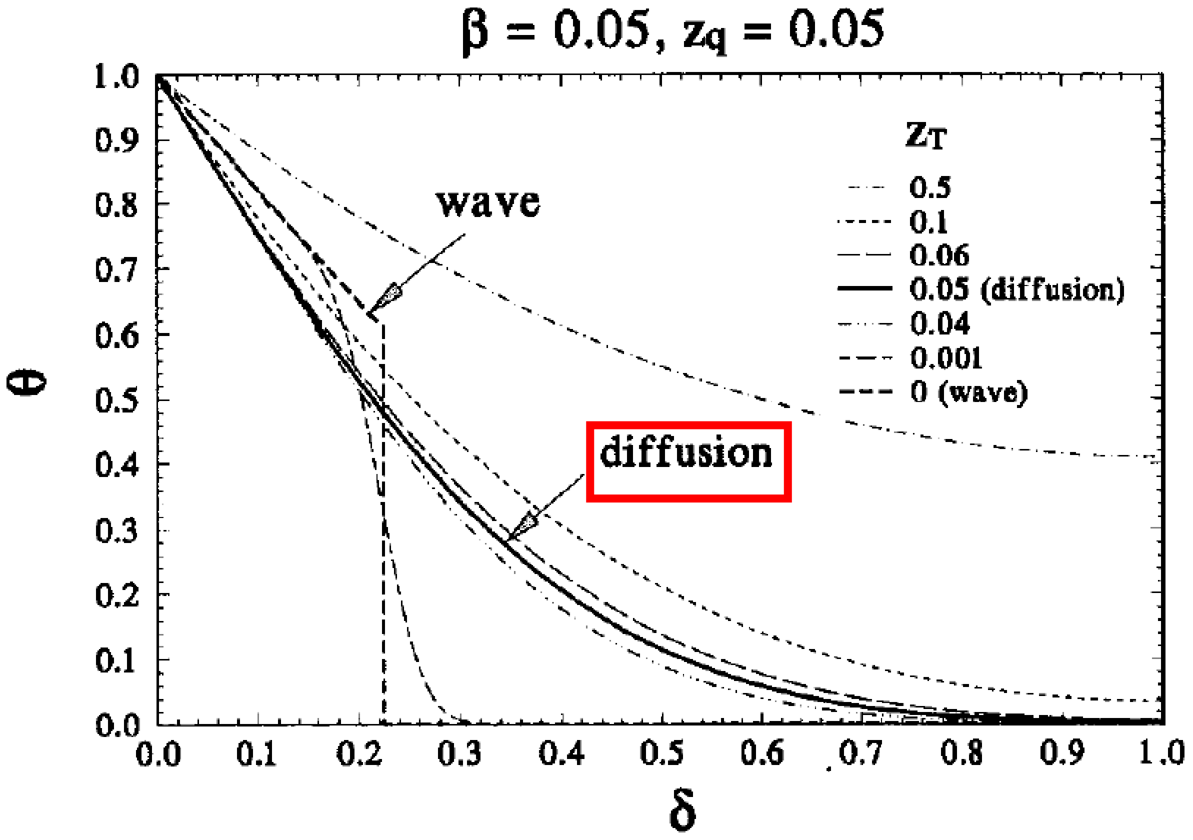
- Heat propagates through **phonons** – modes of vibrations of atoms in a solid.
- Usual approximation : assume an infinitely fast thermal wave i.e. Fourier's Law of Heat Conduction.

$$-k\nabla T(r) = q(r)$$

Constitutive Relation

$$\boxed{-\alpha\nabla^2 T = \frac{\partial T}{\partial t}}$$

Governing Equation



Effect of phase lag(s) on temperature gradient at non-dimensional time β=0.05. δ is the non-dimensional coordinate, z is the non-dimensional phase lag. *Source: Tzou et al., 1995.*

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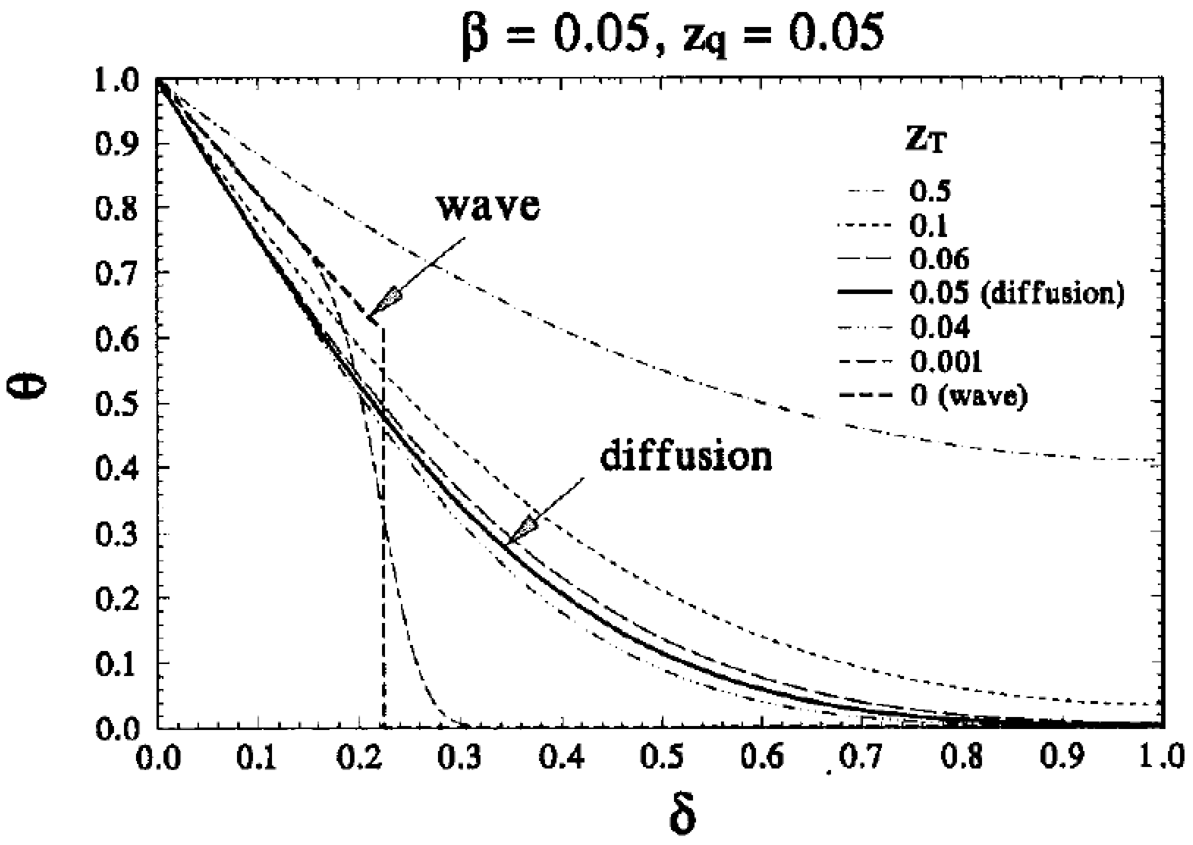
- In certain cases, this is not valid : biological systems are a prime example.
- We need to define a finite wave speed for such cases.



Physics of the problem

- Incorporating a time delay in constitutive relation can solve the problem.
- Time delay (or **phase lag**) accounts for microscopic effects like electron-phonon and phonon-phonon interactions in solids.

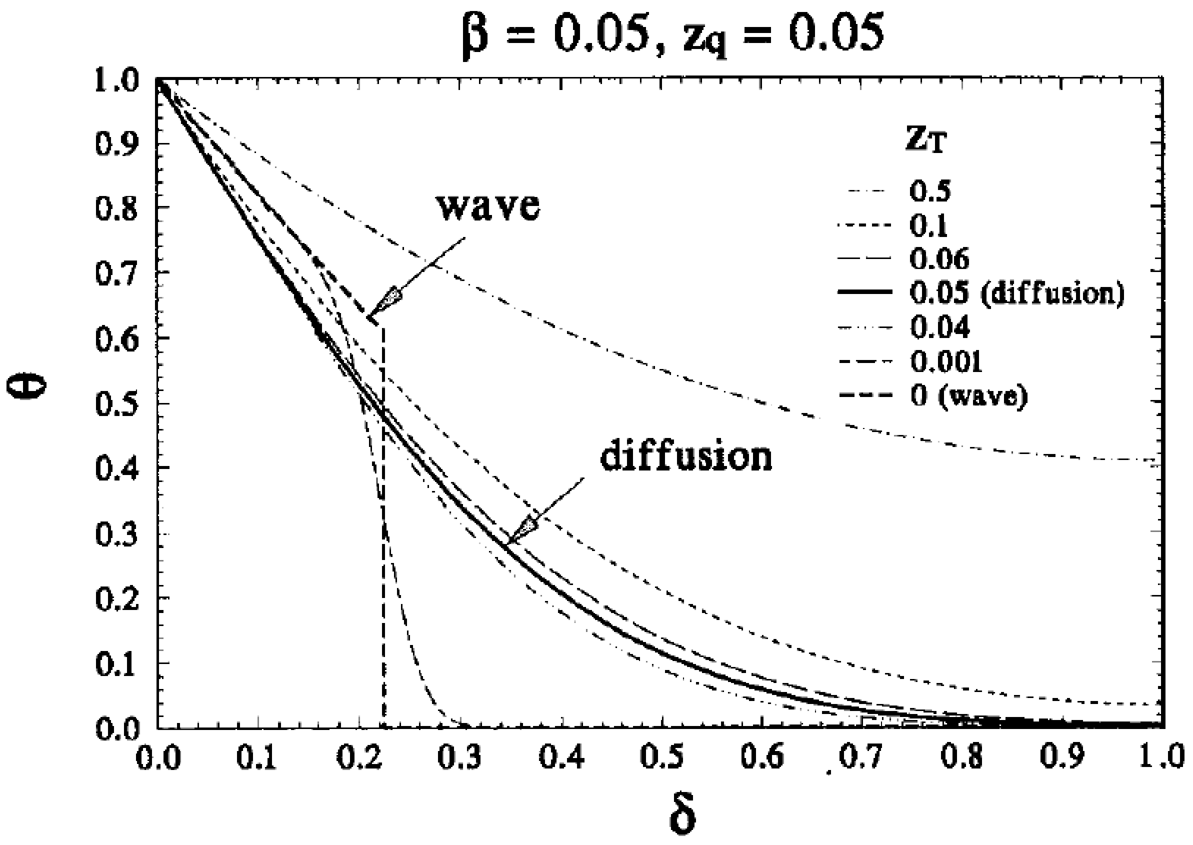
$$-k\nabla T(r, t + \tau_T) = q(r, t + \tau_q)$$

New Constitutive Relation

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}(\nabla^2 T) = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2}$$

New Governing Equation

- τ_T = phase lag in temperature
- τ_q = phase lag in heat flux



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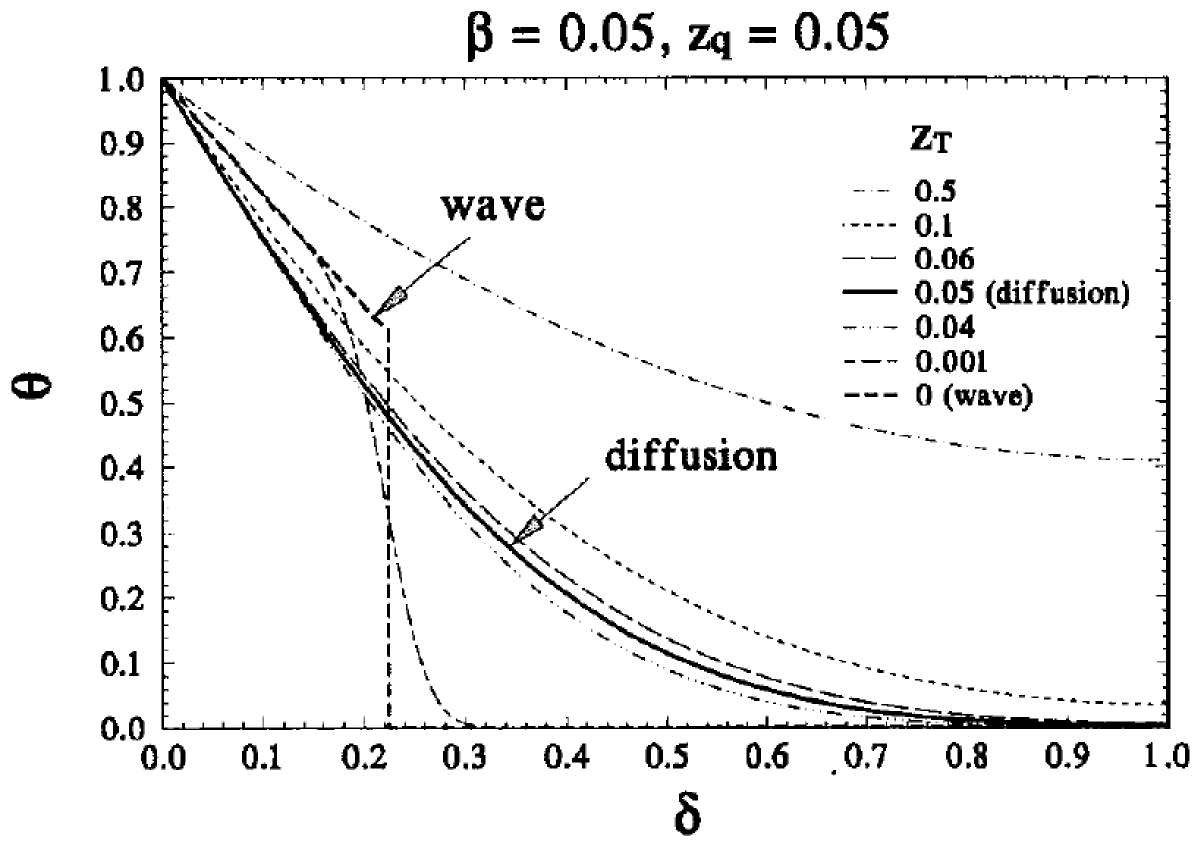

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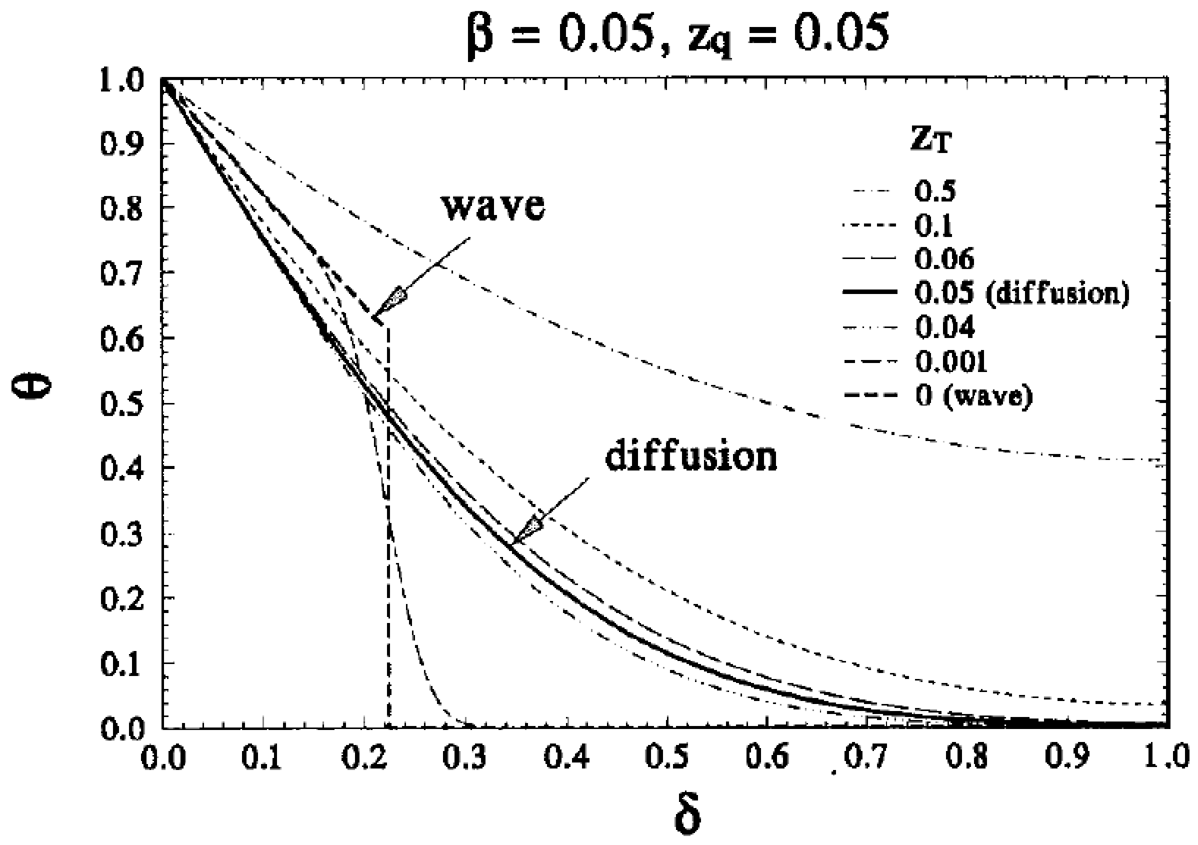

OpenFOAM Essentials

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  - Problems solved – fluid mechanics, heat transfer, solid mechanics (limited), turbomachinery, electromagnetics, and many more.

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- Why OpenFOAM?

  - Open-source: one can tweak source-code to suit their purposes.

  - Validated for numerous cases and is being validated for many others.

  - Ability to solve stiff problems using discretization schemes of your choice.

  - Structured! No need to break the entire package to solve a new problem.

OpenFOAM Statistics

- Year of founding : 2004
- Number of contributors : 8 (contributions are *heavily* regulated)
- Number of distinct affiliations : 6
- How does the project accept contributions : Signing the OpenFOAM **contributor agreement, Github commits**.
- Automated test suite? : in the dev version, yes.
- Continuous integration? : yes
- Legal/licencing steps? : yes.
- Language: C++

References

- Da Yu Tzou, The generalized lagging response in small-scale and high rate heating, International Journal of Heat and Mass Transfer 38(17) (1995) 3231.
- https://www.openfoam.com/documentation/guides/latest/doc/guide-fos-field-ddt.html
- https://www.openfoam.com