FEniCS for PDEs

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About FeniCS

- Created in 2003 as a collaboration between researchers from various universities and research institutions.
- It is an open-source computing platform for automated solution of PDEs using finite elements.
- The FEniCS software can be found at the FEniCS Project website, http://fenicsproject.org/.
- FEniCS includes a number of powerful features, examples are:
  - automated solution of variational problems
  - automated error control and adaptivity
  - extensive library of finite elements
  - provides an interface to linear algebra solvers and data-structures, such as PETSc
  - visualization via a simple interactive plotting function
  - can be used with Python and C++
  -extensive documentation: tutorial, handbook, demos etc.

Why FEniCS for PDEs?

- Said to be the only framework where the code stays compact, very close to the mathematical formulation, even when complexity of math and algorithm increases and when using a high-performance compute server (cluster).

Getting started with FEniCS

- Who is the audience?
- Windows: a bit tedious, everything else more straightforward explanation
- Install Docker, or do it via Linux for Windows
- Install editor: my preference Visual Studio Code
- Tutorials (handbook and the internet in general)

Example from tutorial

- Poisson equation: most fundamental task to solve in FEMs for PDEs:

Poisson equation is an example of a boundary value problem:

$$-\nabla^2 u(\boldsymbol{x}) = f(\boldsymbol{x}), \qquad \boldsymbol{x} \text{ in } \Omega, \tag{2.1}$$

$$u(\boldsymbol{x}) = u_{\mathrm{D}}(\boldsymbol{x}), \qquad \boldsymbol{x} \text{ on } \partial\Omega\,. \tag{2.2}$$

- In 2D:

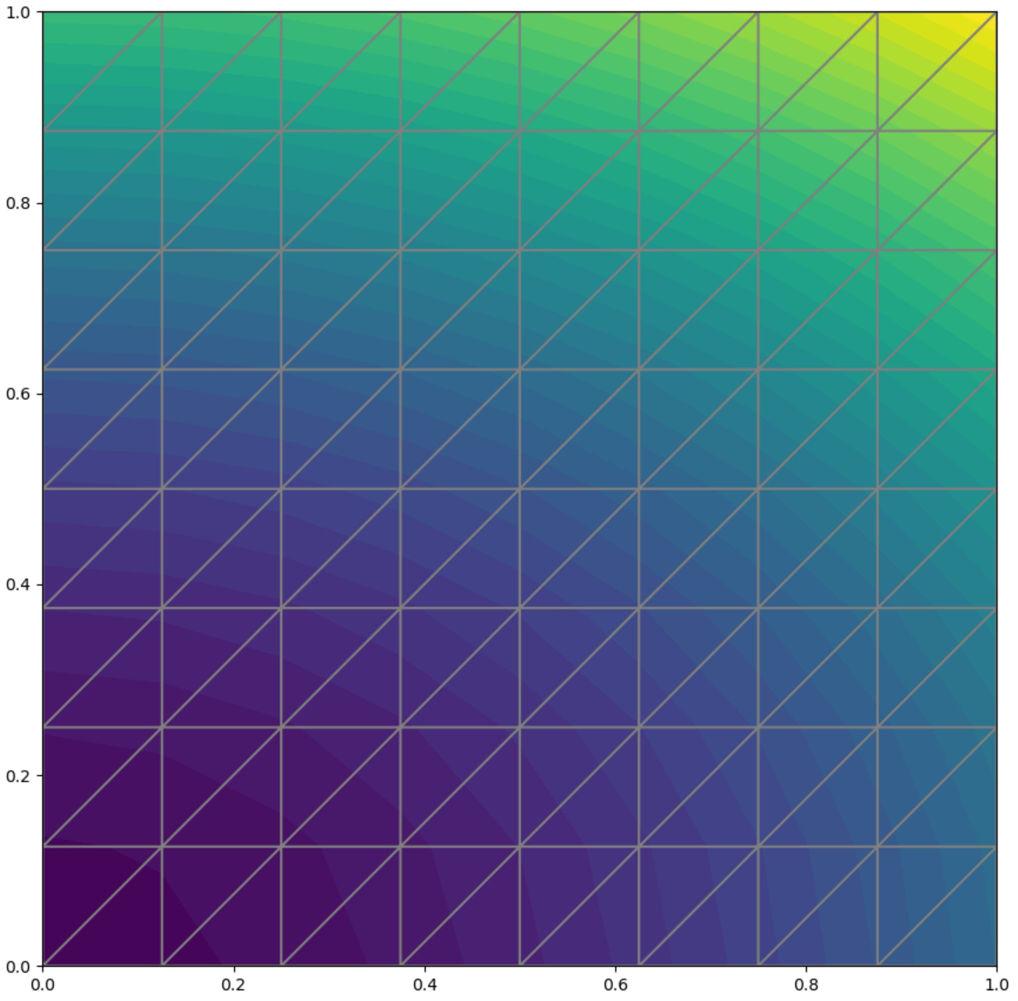
$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = f(x, y) \tag{2.3}$$

Example from tutorial: Poisson equation

- Applications: physics like heat conduction, electrostatics, twisting of elastic rods etc.
- Steps to solve Poisson equation in FEniCS:

1. Identify the computational domain Ω, the PDE, its boundary conditions and source terms
2. Reformulate the PDE as a finite element variational problem
3. Write Python program defining Ω, the variational problem, boundary conditions, source terms using FEniCS
4. Call FEniCS to solve the boundary-value problem

Example from tutorial: Poisson equation



Arbitrary quadratic function in 2D as the exact solution:

$$u_{\mathrm{e}}(x, y) = 1 + x^2 + 2y^2$$

Substitute into the Poisson equation gives that it is a solution if: f(x,y)= -6 and $u_D(x, y) = u_E$(x,y).

The domain is a unit square: Ω=[0,1]x[0,1]

FEniCS will compare the approximate solution u with the exact solution u_E(x,y).

The error in the L^2 and maximum norms are:

error_L2 = 0.008235098073354864
error_max = 1.3322676295501878e-15

How to contribute to the FEniCS community?

- Contribute to the documentation: the example discussed uses interactive() to plot but this is deprecated.
- Contribute by making tutorials: e.g. variations of existing ones.
- Performance comparison: FEniCS and FEniCSx (the updated version of FEniCS).

References

- Langtangen, H. P., & Logg, A. (2016). Solving PDEs in Minutes – The FEniCS Tutorial Volume I. Springer.
- Langtangen, H. P., & Logg, A. (2022). DOLFINx Tutorial. https://jsdokken.com/dolfinx-tutorial/chapter1/fundamentals.html
- Dokken, J. S. (2022). Adapted to FEniCSx by Jørgen S. Dokken. DOLFINx Tutorial. https://jsdokken.com/dolfinx-tutorial/chapter1/fundamentals.html