VLSI CAD: LOGIC TO LAYOUT: kbdd Software Package

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kbdd is a BDD calculator done by Prof. Randy Bryant's[1] research group at Carnegie Mellon University (http://www.cs.cmu.edu/~bryant/). **kbdd** has all the operators you would want to use to manipulate Boolean functions, and a simple command line interface to type in functions, etc. We have installed kbdd in the Coursera cloud, and have created an interface to allow you to upload text files to solve useful Boolean computational problems. You upload your text file of commands; we run kbdd in the cloud; a textfile of kbdd outputs is presented back to you on your Coursera class page. You will use kbdd to do some computations that are too big to do by hand.

As a starting point, if you type the *help* command into *kbdd*, this is what will be printed, as a "quick reference" to what commands *kbdd* offers:

```
? [<command>]                   -- Print information about command
adder <n> <sum> <a> <b> <cin>   -- generate formulas for n-bit adder
alu181 <cout> <sum> <m> <s> <a> <b> <cin>
                                -- generate functions for 181 alu
bdd <f>                         -- print out representation for formula
boolean <v1>..<vn>              -- declare boolean variables
echo                            -- rest of line
evaluate <f> <exp>              -- create formula from expression
free <f1> ... <fn>              -- free formula(s)
garbage                         -- force bdd package to do garbage collection
implies <f1> <f2>               -- f1 imply f2 ?
ite <fd> <fi> <ft> <fe>         -- perform if fi then ft else fe
limit <n>                       -- set memory limit for bdds to be n bytes.
mux <n> <out> <sel> <in>        -- generate formulas for n to 2^n bit mux
quantify [<eu>] <fd> <fs> <v1>..<vn>
                                -- quantify formula over variables
quit                            -- exit program
replace <fd> <fs> <v1> <f1>..<vn> <fn>
                                -- replace variable vi with function fi
satisfy <f>                     -- print var assignments that satisfy formula
show [<command>]                -- List hidden commands/Show in menu.
size <f1>..<fn>                 -- print number of bdd nodes under formulas
sop <f>                         -- print sop representation of formula
source <file>                   -- Read commands from file
switch [<switch1>:<val1>..<switchn>:<valn>]
                                -- Set/check run time switches
totalsize                       -- print total number of nodes in bdd
verify <f1> <f2>                -- verify that two formulas are equal
```

[1] We gratefully acknowledge Prof. Randy Bryant of CMU for his permission to use his kbdd software package for our University of Illinois MOOC on VLSI CAD.

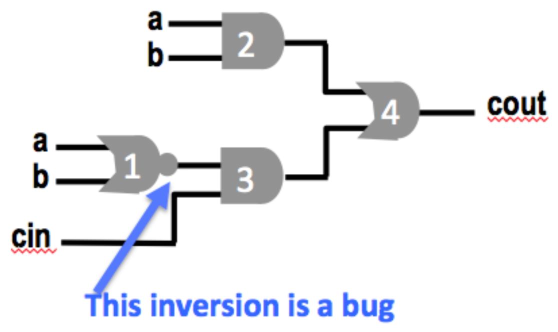
Here is this same information, but with a bit more explanation about what is happening with each command available in **kbdd**:

kbdd Quick Reference Information

boolean var ...	Declare variables and variable ordering
Extended naming	
var[*m* .. *n*]	Numeric range (ascending or descending)
{*s1,s2,...*}	Enumeration
evaluate *dest expr*	*dest* := bdd for boolean expression *expr* Can also type **eval** for short here
Operations	(decreasing precedence)
(*expr***)**	Parentheses work as usual in any expression
!	Complement
^	Exclusive-Or
&	And
+	Or
bdd *funct*	Print BDD DAG as lisp-like representation
sop *funct*	Print sum-of-products representation of *funct*
satisfy *funct*	Print all satisfying variable assignments of *funct*
verify *f1 f2*	Verify that two functions *f1 f2* are equivalent
size *funct* ...	Compute total BDD nodes for set of functions
replace *dest funct var replace*	dest := *funct* with variable *var* replaced by *replace* function output
quantify [u|e] <u>*dest funct var*</u> ...	*dest* := Quantification of function *funct* over variables *var*
e	Existential quantification is done
u	Universal quantification is done
adder *n Cout Sums As Bs Cin*	Compute functions for n -bit adder
n	Word size
Cout	Carry output or (*Sum.n)*
Sums	Destinations for sum outputs: *Sum.n-1 ... Sum.0*
As	A inputs: *A.n-1 ... A.2 A.1 A.0*
Bs	B inputs: *B.n-1 ... B.2 B.1 B.0*
Cin	Carry input
mux *n Out Sels Ins*	Compute functions for 2^n-bit multiplexor
n	Word size
Out	Destination for output function
Sels	Control inputs: *Sel.n-1 ... Sel.1 Sel.0*
Ins	Data inputs: $In.2^{n-1}$ ... *In.1 In.0*
help	print a quick reference of kbdd commands
# *anything*	This line is a comment for readability
quit	Exit KBDD

It is helpful to show a concrete example of a BDD computation that kbdd can do. Let us consider another version of the network repair problem we have already described in lecture.

Consider the logic network below. In this example, a simple 1-bit adder circuit for the carry-out *cout* has a NOR gate incorrectly where there should be an OR gate, like this:



We can employ the repair steps, via quantification, etc., as in the lecture video and notes on Computational Boolean Algebra. The basic recipe is:

1. Build a correct BDD for the function we want, *Cout*.
2. Build a BDD for the incorrect logic, but replace the suspect gate – the input NOR – with a 4:1 multiplexor (MUX), with new inputs *d0 d1 d2 d3* as the MUX data inputs.
3. Exclusive NOR (EXNOR) the correct and to-be-repaired functions. This new function *Z* can be satisfied only if the *d* inputs are set correctly to let the MUX mimic the correct gate.
4. Universally quantify away the real logic input (*a,b,cin*) here, so that the *Z* function depends only on the MUX d inputs.
5. Check is there is a satisfying assignment to the d inputs; if so, we have found a viable gate repair.

Pleasantly enough, this is all quite easy in **kbdd.**

The following shows an example of a session with **kbdd**. Inputs are in normal font (these are what *you* would type into a plain textfile, and upload to our Coursera cloud-based version of **kbdd**). **kbdd** outputs are **blue**, **kbdd's** prompts for input shown in bold as **KBDD**:

Kbdd Example Session for Adder Carry-out Repair

```
KBDD: # input variables
KBDD: boolean a b cin d0 d1 d2 d3
KBDD: #
KBDD: # define the correct equation for the adder’s carry out
KBDD: eval cout a&b + (a+b)&cin
cout: a&b + (a+b)&cin
KBDD: #
KBDD: # define the incorrect version of this equation (just for fun)
KBDD: eval wrong a&b + (!(a+b))&cin
wrong: a&b + (!(a+b))&cin
KBDD: #
KBDD: # define the to-be-repaired version with the MUX
KBDD: eval repair a&b + (d0&!a&!b + d1&!a&b + d2&a&!b + d3&a&b)&cin
repair: a&b + (d0&!a&!b + d1&!a&b + d2&a&!b + d3&a&b)&cin
KBDD: #
KBDD: # make the Z function that compares the right version of
KBDD: # the network and the version with the MUX replacing the
KBDD: # suspect gate  (this is EXNOR of cout and repair functions)
KBDD: eval Z repair&cout + !repair&!cout
Z: repair&cout + !repair&!cout
KBDD: # universally quantify away the non-mux vars: a b cin
KBDD: quantify u ForallZ  Z a b cin
KBDD: #
KBDD: # let’s ask kbdd to show an equation for this quantified function
KBDD: sop ForallZ
  !d0 & d1 & d2
KBDD: #
KBDD: # what values of the d’s make this function == 1?
KBDD: satisfy ForallZ
Variables: d0 d1 d2
011
KBDD: #
KBDD: # that’s it!
KBDD: quit
%
```

As always, it is important to use your brain to analyze what the software tool is telling you. Observer that kbdd says that a satisfying assignment of the MUX inputs is this:

```
Variables: d0 d1 d2
011
```

This means d3's value does not matter. So, in fact, there are two solutions: d0 d1 d2 d3 = 0111, and 0110. These specify and OR and an EXOR gate, respectively, as feasible repairs of the network.

Usage Notes for kbdd: Adders

Using the built-in functions like adders and the extended range notation

kbdd has basic n-bit adders built in, so this is very convenient. But, there is a bug in the online "help" output for this version of kbdd, for the syntax for the adder command. The example shown here clears up exactly how to use this:

```
KBDD: #declare inputs to a 4 bit adder
KBDD: boolean a[3..0] b[3..0] c0
KBDD: # now, build all the outputs of the 4b adder
KBDD: adder 4 c4 s[3..0] a[3..0] b[3..0] c0
KBDD: # now DRAW the BDD itself in text form
KBDD: # here is the low order sum bit s0
KBDD: bdd s0
(a0:1753429896
   (b0:1753429864
      (c0:1753429784)
      ![c0:1753429784])
   ![b0:1753429864])
KBDD: # now ask how BIG this s0 BDD is
KBDD: size s0
size [ s0 ]
3
```

Usage Notes for kbdd: Graph Structure

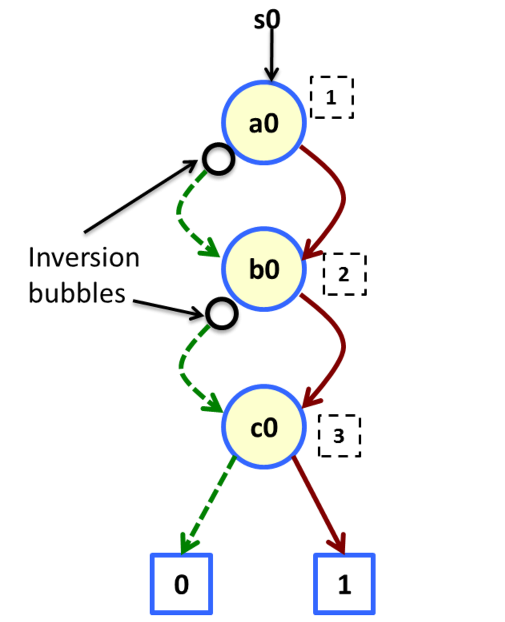
It is important to note that **kbdd** is using an additional "trick" that we did not discuss in lecture. This trick is something called **negation arcs**. In digital design, suppose we have a function **F**, and we want to build logic for the complement **F'**. We might optimize **F'** directly as gates. Or, we might just build **F** itself, and then send its output through a simple *inverter* gate. We would like to choose the option that gives us the fewest logic gates. One can apply a similar idea to BDDs. Sometimes, it is easiest to build the BDD for **F'** directly. But sometimes it is easier to just build **F**, and then indicate in the data structure that we have "inverted it". This is the idea of the negation arc: it is exactly like a simple inverter gate. We put an inversion bubble on the edge leaving a BDD node, and the bubble means "interpret the BDD to which this edge points as being inverted". It turns out that a simple set of Shannon factor tricks, and some basic DeMorgan complement laws, can be used to build the rules for how this can work. Nicely enough, one again creates *canonical* structures: a function **F** makes one and only one BDD, and always the *same* BDD. The complement bubbles just arrange themselves in the right places. The big advantage is that one can save, on average, about half the nodes in the BDD. The big disadvantage (and this is rather minor) is that BDDs become rather hard to "read", visually, as graphs.

For our BDD example, the printout with parentheses and big numbers, has this interpretation:

- **Letters**: these are the variable name
- **Numbers**: these are the actual BDD node addresses in memory
- **"!":** this is an inversion bubble on a negation arc
- **Indentation**: each indent means "we go down one level in the BDD graph"; children of a particular node are listed on lines with the same indent under their parents.
- **Ordering**: We first list the high-child (variable=1) on the *first* indented line under a BDD node. We list the low-child (variable=0) on the *last* indented line with the *same indent*, under a BDD node. If you see an indent anywhere, it means "this is the child of the thing above, one level less indented).
- **Constants**: kbdd will print **"[0]"** or **"[1]"** when an internal node has a child that one of the two constants. However, for nodes which have the "standard" children at the very bottom of the tree – that is a variable "x" whose high-child is **[1]** and low-child is **[0]** – **kbdd** omits printing these child nodes.

So, if we return to the BDD printout from our adder, this is the actual graph:

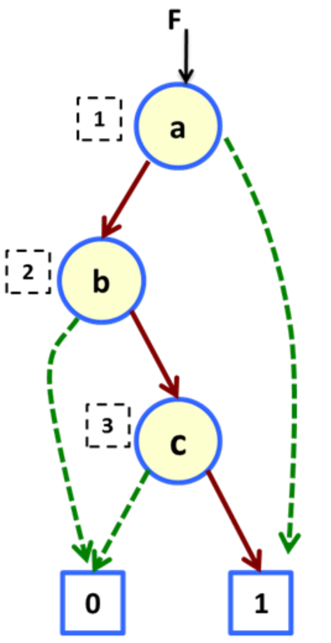
```
(a0:1753429896
   (b0:1753429864
      (c0:1753429784)
      ![c0:1753429784])
   ![b0:1753429864])
```



: for describing the diagram in "alternate text" for visually impaired course participants

We can also show another example to illustrate that we don't always need negation arcs. This BDD has a more familiar structure:

```
KBDD: boolean a b c
KBDD: eval F !a + b&c
F: !a + b&c
KBDD: bdd F
(a:1812523160
   (b:1812523176
      (c:1812523096)
      [0])
   [1])
```



: for describing the diagram in "alternate text" for visually impaired course participants