Chapter 2

INTEGRATION

Facts and Formulas

EXERCISE 1

Circle the correct answer.

1. $\int x^n dx$ is:

(A) $(n+1)x^{n+1}+c$ (B) $\frac{x^n}{n+1}+c$

(C) $\frac{x^{n+1}}{n+1}+c$ (D) $\frac{x^{n+1}}{n}+c$

2. $\int k\,dx$ is:

(A) $\frac{k^2}{2}+c$ (B) $\frac{kx^2}{2}+c$ (C) 0 (D) $kx+c$

3. $\int kx^n\,dx$ is:

(A) $\frac{kx^{n+1}}{n+1}+c$ (B) $\frac{(k+1)x^{n+1}}{n+1}+c$

(C) $\frac{kx^{n+1}}{n}$ (D) $\frac{kx^n}{n+1}$

4. $\int 1dx$ is:

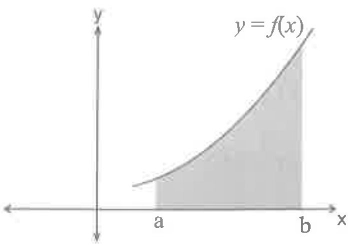
(A) $1+c$ (B) $x+c$ (C) $\frac{x^2}{2}+c$ (D) $0+c$

5. $\int (ax+b)^n\,dx$ is:

(A) $\frac{(ax+b)^{n+1}}{n+1}+c$ (B) $\frac{(ax+b)^n}{a(n+1)}$

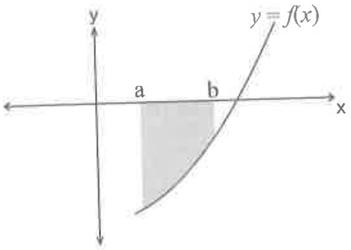
(C) $\frac{(ax+b)^{n+1}}{a(n+1)}+c$ (D) $\frac{a(ax+b)^{n+1}}{n+1}$

6. The area bounded by the curve $y=f(x)$, the x axis, and the lines $x=a$ and $x=b$ (as shown on the diagram) is:



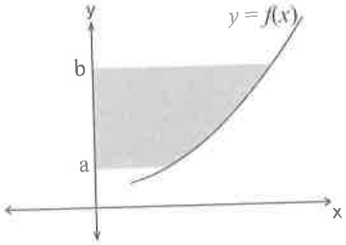
(A) $\int_b^a ydx$ (B) $\int_a^b ydx$ (C) $\int_b^a xdy$ (D) $\int_a^b xdy$

7. The area shown on the diagram is:



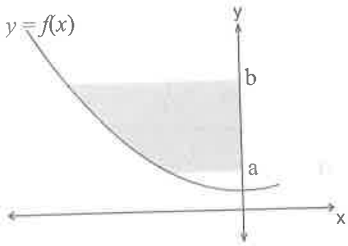
(A) $\int_a^b xdy$ (B) $\left|\int_a^b ydx\right|$ (C) $\int_a^b ydx$ (D) $\left|\int_a^b xdy\right|$

8. The area bounded by the curve $y = f(x)$, the y axis, and the lines $y = a$ and $y = b$ (as shown on the diagram) is:



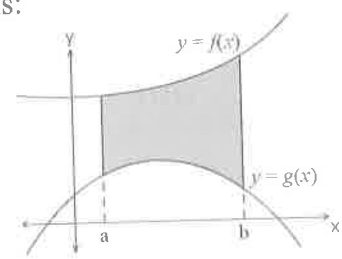
(A) $\int_b^a ydx$ (B) $\int_a^b ydx$ (C) $\int_b^a xdy$ (D) $\int_a^b xdy$

9. The area as shown in the diagram is:



(A) $\int_a^b xdy$ (B) $\int_b^a xdy$ (C) $\left|\int_a^b xdy\right|$ (D) $\left|\int_a^b ydx\right|$

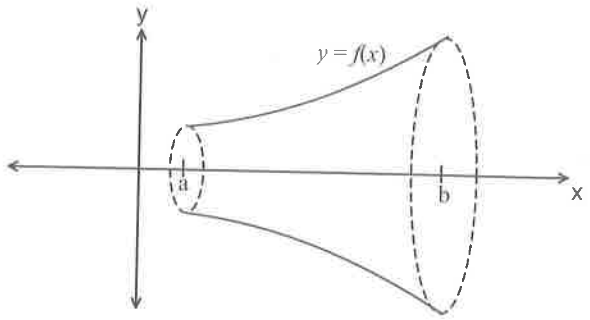
10. The area shown in the diagram is:



(A) $\int_a^b [f(x)+g(x)]dx$ (B) $\int_a^b [g(x)-f(x)]dx$

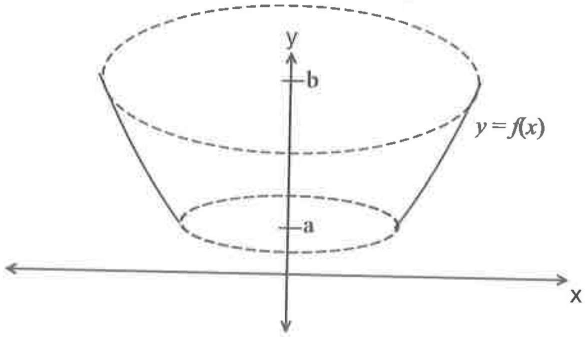
(C) $\int_a^b [f(x)-g(x)]dx$ (D) $\int_b^a [f(x)-g(x)]dx$

11. The volume of the solid obtained by rotating $y = f(x)$ about the x axis, as shown in the diagram is:



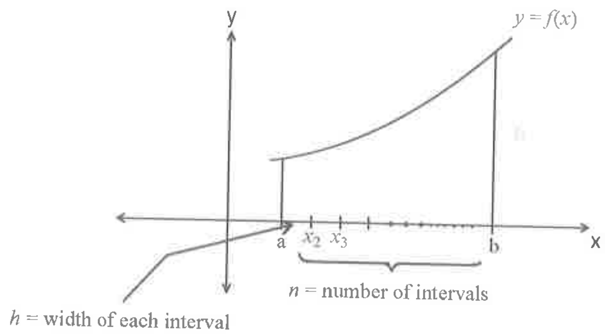
(A) $\pi \int_a^b x^2 dy$ (B) $\pi \int_a^b y dx$ (C) $\pi \int_a^b y^2 dx$ (D) $\int_a^b y^2 dx$

12. The volume of the solid obtained by rotating $y = f(x)$ about the y axis, as shown in the diagram is:



(A) $\pi \int_a^b x dy$ (B) $\pi \int_a^b y^2 dx$ (C) $\int_a^b x^2 dy$ (D) $\pi \int_a^b x^2 dy$

This diagram applies to questions **13** *and* **14**.



13. The extended Trapezoidal Rule for the given diagram states that:

(A) $A \approx \frac{h}{3}\left[f(a) + 2\left[f(x_2) + f(x_3) + \ldots\ldots\right] + f(b)\right]$ where $h = \frac{b-a}{n}$

(B) $A \approx \frac{h}{2}\left[f(a) + 2\left[f(x_2) + f(x_3) + \ldots\ldots\right] - f(b)\right]$ where $h = \frac{b-a}{n}$

(C) $A \approx \frac{h}{2}\left[f(a) + 2\left[f(x_2) + f(x_3) + \ldots\ldots\right] + f(b)\right]$ where $h = \frac{b-a}{n}$

(D) $A \approx \frac{n}{2}\left[f(a) + 2\left[f(x_2) + f(x_3) + \ldots\ldots\right] + f(b)\right]$

14. Simpson's Rule (extended) for the given diagram states that:

(A) $A \approx \frac{h}{3}\left[f(a)+4\left[f(x_2)+f(x_4)+\ldots\ldots\right]+2\left[f(x_3)+f(x_5)+\ldots\ldots\right]+f(b)\right]$ where $h=\frac{b-a}{n}$

(B) $A \approx \frac{h}{2}\left[f(a)+4\left[f(x_2)+f(x_4)+\ldots\ldots\right]+2\left[f(x_3)+f(x_5)+\ldots\ldots\right]+f(b)\right]$ where $h=\frac{b-a}{n}$

(C) $A \approx \frac{h}{3}\left[f(a)+4\left[f(x_3)+f(x_5)+\ldots\ldots\right]+2\left[f(x_2)+f(x_4)+\ldots\ldots\right]+f(b)\right]$ where $h=\frac{b-a}{n}$

(D) $A \approx \frac{h}{3}\left[f(a)+\left[f(x_3)+f(x_5)+\ldots\ldots\right]+2\left[f(x_2)+f(x_4)+\ldots\ldots\right]+f(b)\right]$ where $h=\frac{b-a}{n}$

Indefinite Integrals

EXERCISE 2

1. Find:

(a) $\int x^5\,dx$

(b) $\int p^7\,dp$

(c) $\int 5x^4\,dx$

(d) $\int 16x^3\,dx$

(e) $\int -3x^3\,dx$

(f) $\int 5x^9\,dx$

(g) $\int \frac{x^2}{4}\,dx$

(h) $\int \frac{3x^2}{5}\,dx$

2. Find:

(a) $\frac{2}{3}\int 5x^2\,dx$

(b) $\pi\int (3x^4)^2\,dx$

(c) $\int 3\,dx$

(d) $\int \pi\,dx$

(e) $\int \pi x\,dx$

(f) $\int -2\,dm$

(g) $\int \frac{dx}{3}$

EXERCISE 3

1. Find (answers in fraction form):

(a) $\int x^{-4}\,dx$

(b) $\int 6x^{-3}\,dx$

(c) $\int 2x^{-5}\,dx$

(d) $\int \frac{x^{-2}}{3} dx$

(e) $\int \frac{1}{x^6} dx$

(f) $\int \frac{3}{x^7} dx$

(g)* $\int \frac{1}{2x^9} dx$

2. Simplify:

(a) $\frac{x^{\frac{7}{5}}}{\frac{7}{5}}$

(b) $\frac{x^{1 \cdot 2}}{1 \cdot 2}$

(c) $\frac{14x^{\frac{7}{4}}}{\frac{7}{4}}$

EXERCISE 4

1. Find (answers in surd form):

(a) $\int x^{\frac{1}{2}} dx$

(b)* $\int x^{\frac{3}{4}} dx$

(c) $\int 3\sqrt{x}\, dx$

(d) $\int \sqrt[3]{x}\, dx$

(e) $\int 5\sqrt{x^3}\, dx$

(f) $\int 10\sqrt[4]{x}\,dx$

(g) $\int \frac{10}{\sqrt{x}}\,dx$

(h) $\int (3x^2 + 4x - 9)\,dx$

(i) $\int \left(8x + \frac{2}{x^2}\right)dx$

(j) $\int (x^5 + 6\sqrt{x} - \frac{2}{\sqrt{x}})\,dx$

(k)* $\int (10x + \frac{5}{x^6} + \sqrt[3]{x})\,dx$

(l) $\int (8 - \sqrt[3]{x^2})\,dx$

2. Find:

(a) $\int \left(\frac{6x^3 + 4x^2 - 3x}{x}\right)dx$

(b) $\int \left(\frac{5x-2}{x^3}\right)dx$

(c)* $\int \frac{6x - 2\sqrt{x}}{\sqrt{x}}\, dx$

(d) $\int \sqrt{x}(x - 8\sqrt{x})\, dx$

(e) $\int (2x+1)(3x+5)\, dx$

(f) $\int (9-x)(4+2x)\, dx$

(g)* $\int 2t^2(t^3 + 6t)\, dt$

(h) $\int 6p(p^2 - p + 5)\, dp$

EXERCISE 5

Find the following indefinite integrals:

1. $\int (2x+3)^5\, dx$

2. $\int (3-4x)^6\, dx$

3.* $\int (6x-1)^3\, dx$

4. $\int \left(\frac{x+5}{2}\right)^4 dx$

5. $\int \sqrt{x+6}\,dx$

6. $\int \sqrt{3x-1}\,dx$

7.* $\int \frac{1}{\sqrt{x+6}}\,dx$

8. $\int \frac{2}{\sqrt{3x-1}}\,dx$

9. $\int \frac{1}{(x+8)^2}\,dx$

10. $\int \frac{2}{(5x-2)^3}\,dx$

11. $\int \sqrt[3]{6x+1}\,dx$

Finding the Primitive Function

EXERCISE 6

1. If $\frac{dy}{dx} = 6x - 9$ then $y =$

2. Find the primitive function of $f'(x) = 3x^2 - 16x + 5$.

3. Find the primitive function of $f'(x) = 2x + 8$ given that $f(1) = 16$.

4.* The gradient function of a curve is $11 - 2x$. The curve passes through the point (2, 9). Find the equation of the curve.

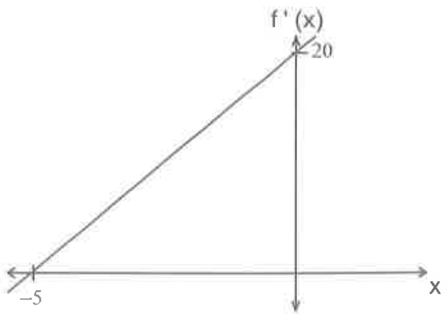
5. $f'(x) = 4x^3 - 6x$ and $f(0) = -1$.

(a) Find $f(x)$.

(b) Evaluate $f(3)$.

6. Find an expression for $f(x)$ if $f''(x) = 48x - 14$, $f'(-1) = 40$, $f(-2) = -100$

7.* The gradient function of a curve is shown below. The y intercept of the curve is 1. Find the equation of the curve.



8. If $\frac{d^2y}{dx^2} = -8$, find y in terms of x if $\frac{dy}{dx} = -27$ and $y = 6$ when $x = 4$.

Definite Integrals

EXERCISE 7

Simplify:

1. (a) $\int_0^a (3x^2 + 2)dx$

(b) $\int_1^b (8x^3 - 6x)dx$

2.* Find k, if $k>5$: $\int_5^k (2x+5)\ dx = 54$

3. Evaluate these definite integrals:

(a) $\int_2^4 (3x^2 + 5)dx$

(b) $\int_{-1}^3 (4x+9)dx$

(c) $\int_{-3}^{-2} (8x - 9x^2)dx$

(d) $\int_0^2 (3+2x)^3 dx$

(e) $\int_{-1}^2 (x^3+x^2+2)dx$

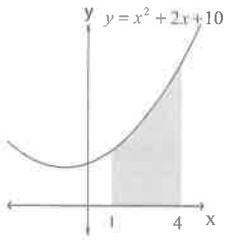
(f)* $\int_4^9 \frac{1}{\sqrt{x}} dx$

(g) $\int_{-1}^2 (2x^4+\frac{3}{\sqrt{x^2}})dx$

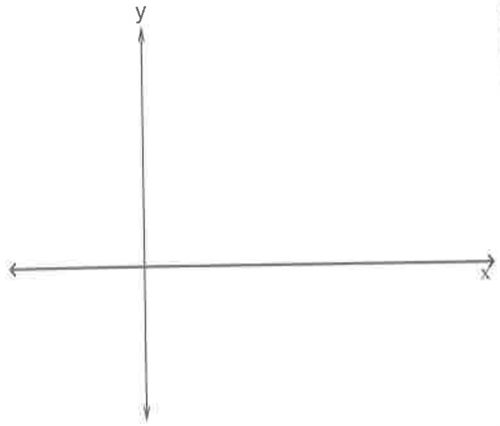
Area Under a Curve

EXERCISE 8

1. Find the area bounded by the curve $y = x^2 + 2x + 10$, the x axis and the ordinates $x = 1$ and $x = 4$.

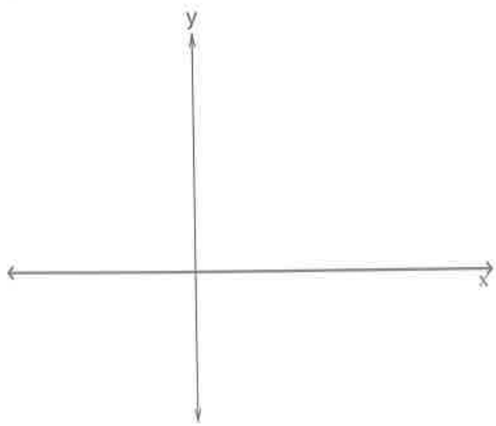


2. (a) Sketch the curve $y = 30x - 6x^2$.



(b) Find the area between the curve and the x axis.

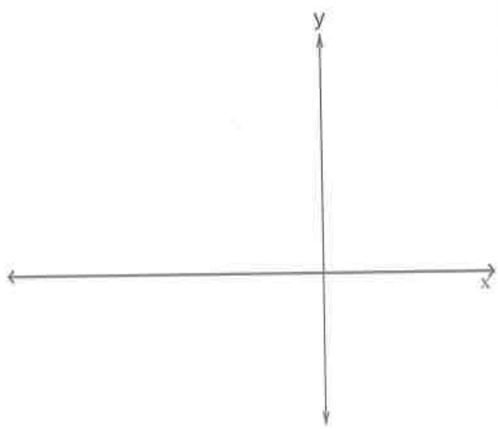
3. (a) Sketch the curve $y = x^2 - 3x - 10$



(b) Find the area between the curve and the x axis.

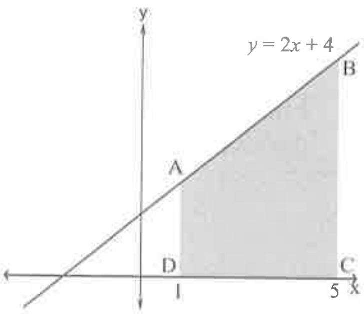
4. Explain why the area bounded by the curve $y = x^2 - 3x$, the x axis, the line $x = 2$ and the line $x = 6$ is not equal to $\int_2^6 (x^2 - 3x)dx$.

5.* (a) Sketch the curve $y = x^2 + 8x + 7$



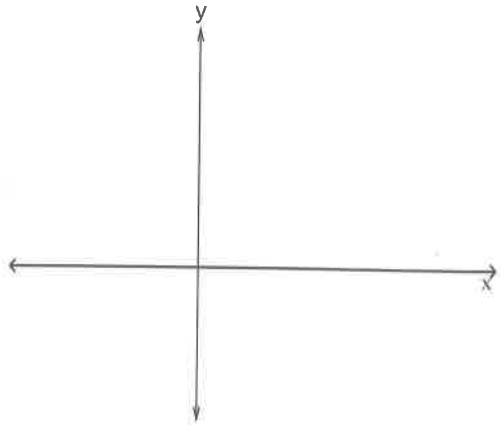
(b) Find the area enclosed by the curve, the x axis and the ordinates $x = -4$ and $x = 1$.

6. (a) Calculate the area of the trapezium ABCD, using the formula $A = \frac{1}{2}(a + b) \times h$.



(b) Use calculus to verify your answer in part (a).

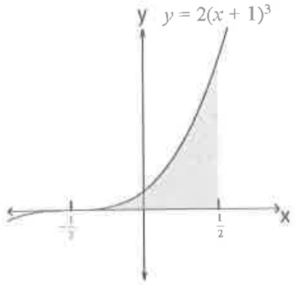
7. (a) Sketch the curve $y = 3(x-1)^2$.



(b) Evaluate the area bounded by the curve, the x axis, and the lines $x = -1$ and $x = 3$.

8. Evaluate the area bounded by the curve $y = \frac{2}{x^2}$, the x axis and the lines $x = 1$ and $x = 5$.

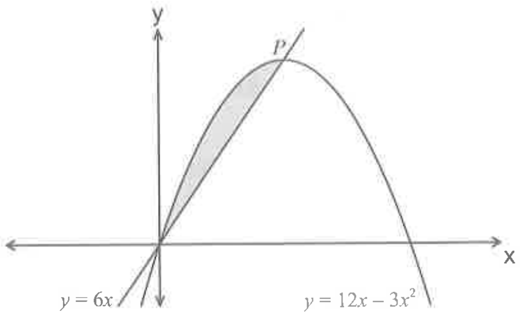
9. A section of the curve $y=(2x+1)^3$ is drawn below. Find the shaded area.



Areas between Curves

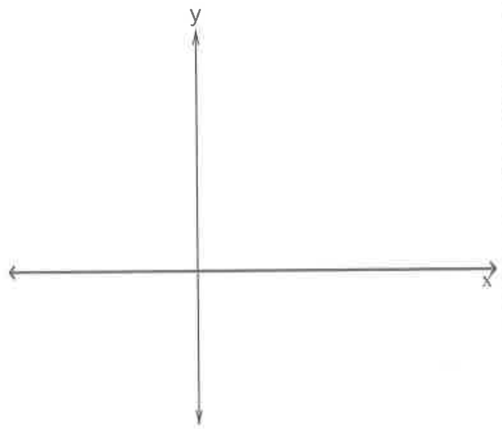
EXERCISE 9

1. (a) Find the coordinates of P as shown on the given diagram.



(b) Find the shaded area enclosed by the graphs of $y=6x$ and $y=12x-3x^2$.

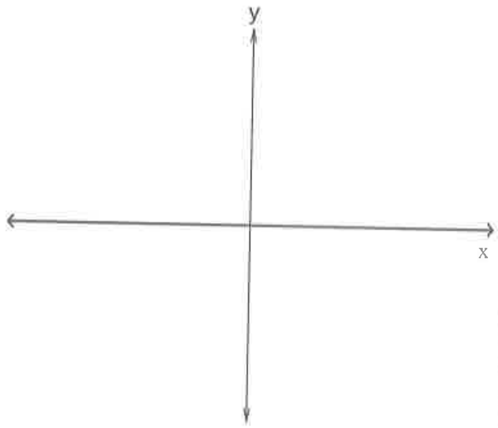
2. (a) On the axes below, sketch the functions $y=10x-x^2$ and $y=3x$.



(b) Find the points of intersection of the two graphs.

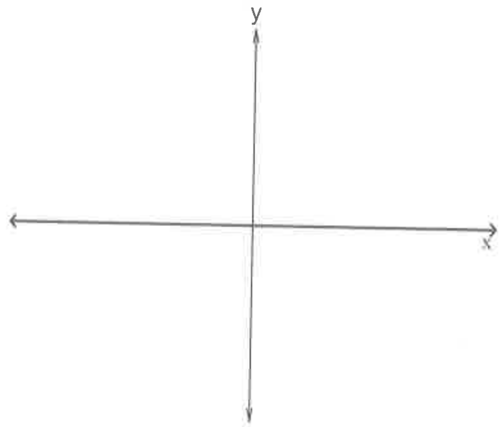
(c) Find the area enclosed by the graphs of $y=3x$ and $y=10x-x^2$.

3. (a) On the given axes, sketch the functions $y = x^2 - 3x - 10$ and $y = -x^2 + 3x + 10$.



(b) Find the area between the two curves.

4.* (a) On the given axes, sketch the functions $f(x) = 4x^3$ and $g(x) = 40 - 4x$.

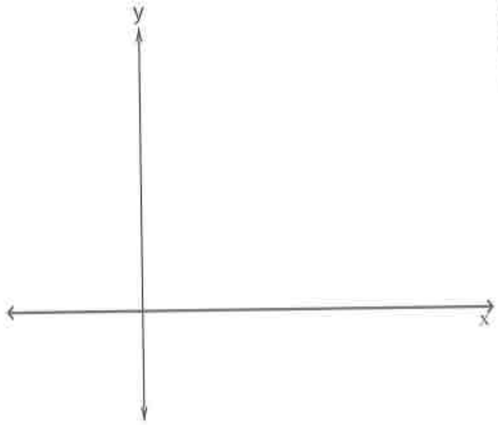


(b) Evaluate $f(2)$.

(c) Evaluate $g(2)$.

(d) Find the area bounded by $f(x)$, $g(x)$ and the x axis.

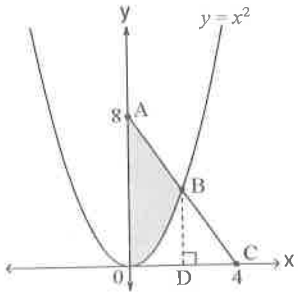
5. (a) On the given axes, sketch the functions $y=(x-2)^2$ and $y=(x-6)^2$.



(b) Find the point of intersection of the two curves.

(c) Find the area enclosed by the two curves and the x axis.

6.*

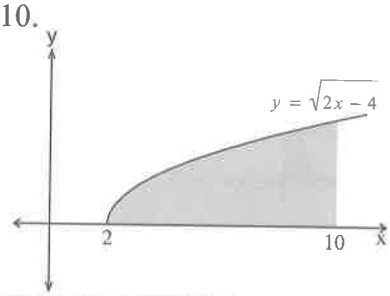


(a) Find the coordinates of B.

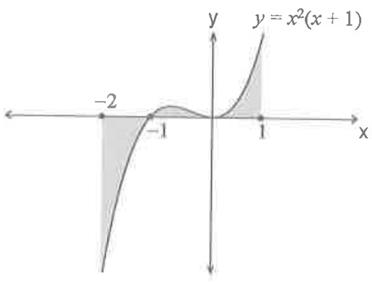
(b) Find the shaded area OAB.

EXERCISE 10

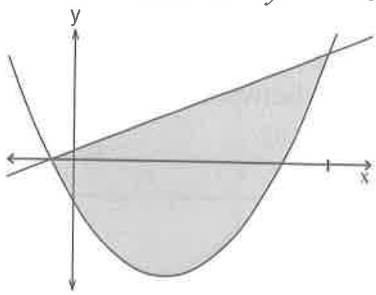
1. Find the area enclosed by the curve $y = \sqrt{2x - 4}$, the x axis and the line $x = 10$.



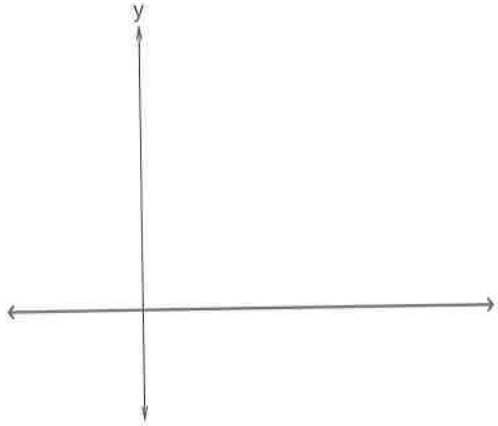
2. Find the area bounded by the curve $y = x^2(x + 1)$, the x axis and the lines $x = -2$ and $x = 1$.



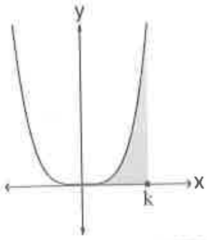
3. Find the area enclosed by the curves $y = x^2 - 8x - 9$ and $2x - y + 2 = 0$



4.* A(0, 0) and B(2, 24) are connected by a straight line section. A and B also lie on the parabola $y = 6x^2$. Sketch this information on the given axes and then find the area between the parabola and the straight line.

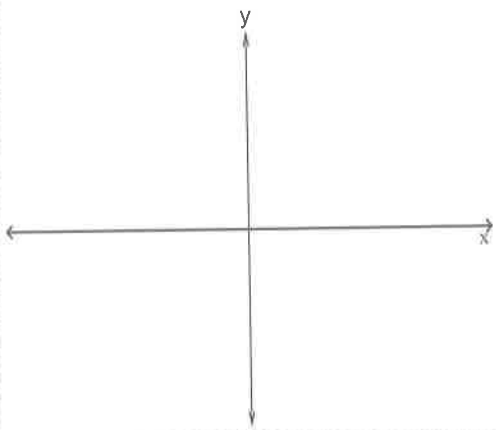


5. The area between the curve $y = 10x^4$, the x axis and the line $x = k$, (where $k > 0$) is 486 units^2. Evaluate k.



6.* (a) Sketch the piecemeal function defined by:

$$f(x) = \begin{cases} -x^2 & \text{for } x \leq 3 \\ 3x - 18 & \text{for } x > 3 \end{cases}$$

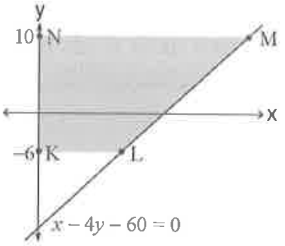


(b) Find the area bounded by the graph of $y = f(x)$ and the x axis.

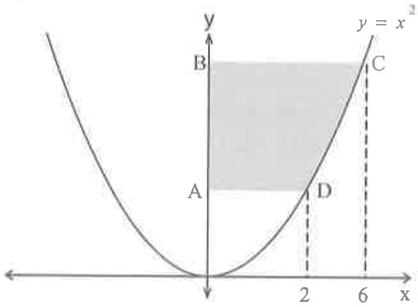
Area Between a Curve and the y axis

EXERCISE 11

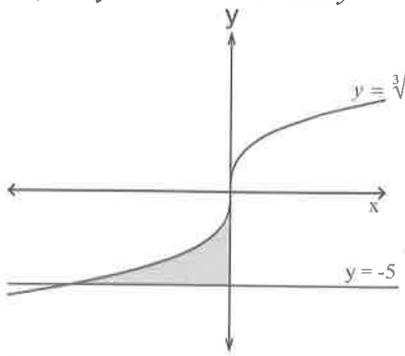
1. Use integration to find the area of KLMN.



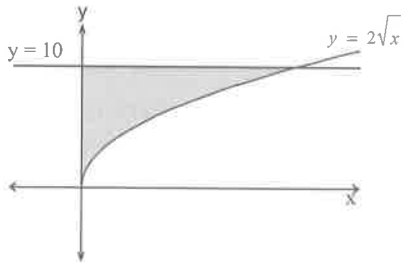
2. Evaluate the shaded area ABCD on the diagram below:



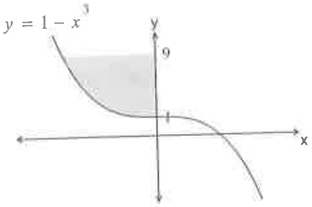
3. Find the area enclosed by the curve $y = \sqrt[3]{x}$, the y axis and the line $y = -5$.



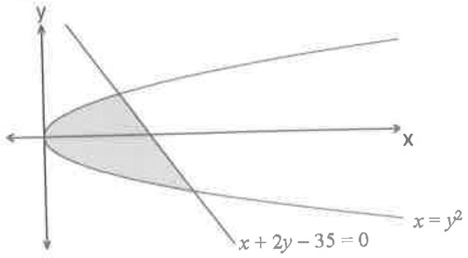
4. Find the area enclosed by the curve $y = 2\sqrt{x}$, the y axis and the line $y = 10$.



5.* Find the area enclosed by the curve $y = 1 - x^3$, the y axis and the line y = 9.



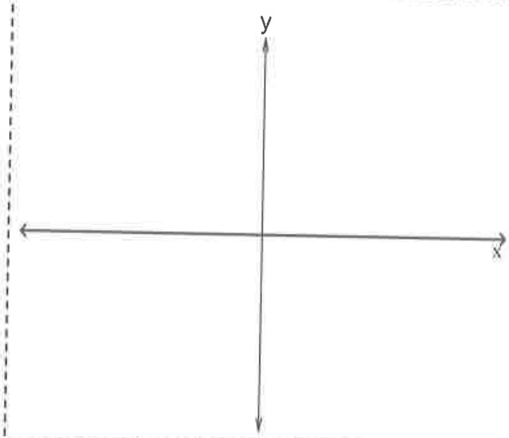
6. Find the area between $x = y^2$ and $x + 2y - 35 = 0$, as shown in the diagram below.



Volumes about the x axis

EXERCISE 12

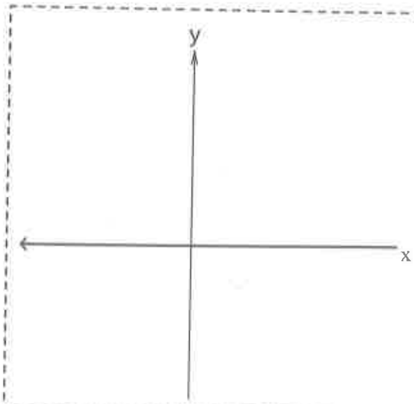
1. (a) Sketch the graph $y = -4x$, and the solid formed by rotating this line about the x axis from $x = 0$ to $x = 2$.



(b) Find the volume of the solid of revolution formed by rotating $y = -4x$ about the x axis from x = 0 to x = 2. (nearest integer)

2.* The section of the line $2y - x + 4 = 0$, from $x = 8$ to $x = 10$ is rotated about the x axis. Find the volume of the solid formed. (2 dec. pl.)

3. (a) Sketch $3x - y - 6 = 0$ on the given axes, and the solid formed by rotating this line about the x axis from $x = 2$ to $x = 4$.

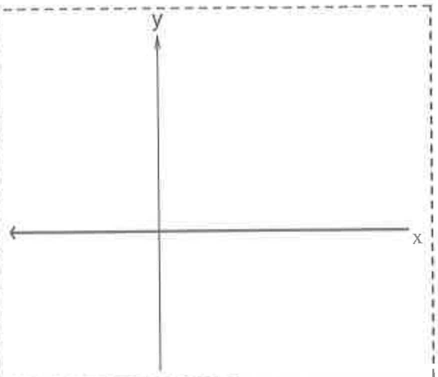


(b) Find the volume of the solid of revolution formed by rotating the section of $3x - y - 6 = 0$ from $x = 2$ to $x = 4$ around the x axis. (Express your answer as an exact value in terms of π.)

4. (a) Complete the table of values for $y = \sqrt{x+1}$.

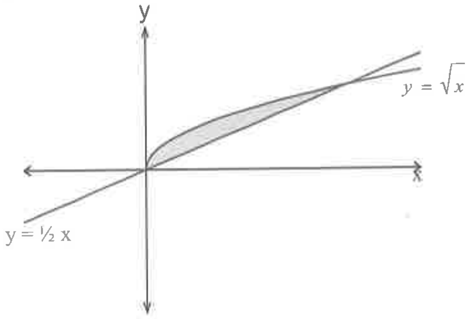
x	-1	0	3	8	15
y					

(b) Sketch $y = \sqrt{x+1}$ on the given axes, and the solid formed by rotating this curve about the x axis from $x = 0$ to $x = 8$.



(c) Find the volume of the solid in (b). (Express your answer as an exact value in terms of π)

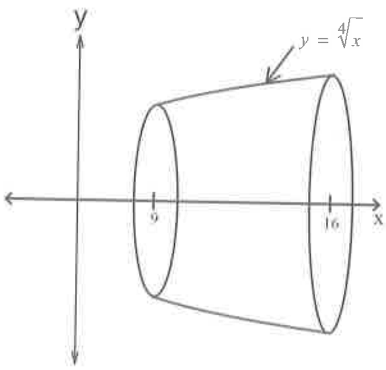
5. Find the volume of the solid formed by rotating the shaded area, shown on the diagram, around the x axis.



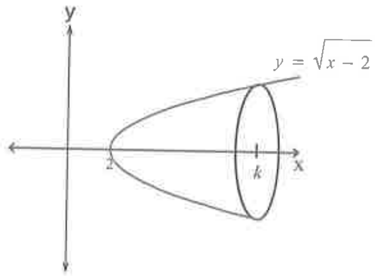
6. The line $y = 3$ is rotated around the x axis from $x = -2$ to $x = 2$. Find the volume of the solid formed. (1 dec. pl.)

7. $x = \sqrt{y}$ is rotated around the x axis from x = 1 to x = 3. Find the volume of the solid formed. (2 dec. pl.)

8. Find the volume of the solid of revolution shown in the given diagram.(1 dec. pl.)



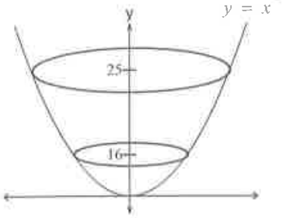
9.* Find k, (where k > 0), if the volume of the solid of revolution shown is 8π units^3.



Volumes about the *y* axis

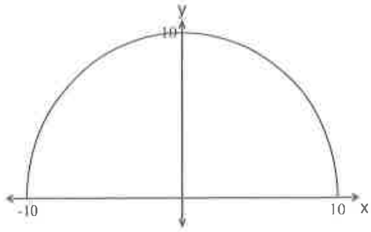
EXERCISE 13

1. The curve $y = x^2$ is rotated about the y axis from y = 16 to y = 25. Find the volume of the resulting solid. (1 dec. pl.)



2. $y = 6 - x$ is rotated about the y axis from $y = 2$ to $y = 5$. Find the volume of the solid of revolution in terms of π.

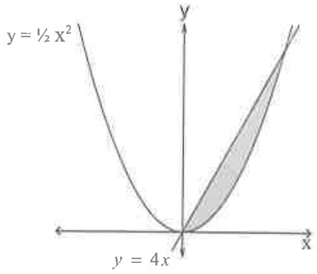
3.* The semi-circle, as shown below, is rotated about the y axis. Find the volume of the solid formed. (1 dec. pl.)



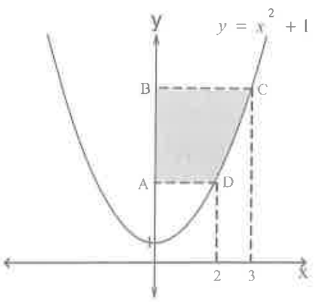
(a) by integration.

(b) by applying the formula for the volume of a sphere.

4. The shaded area, as shown in the diagram, is rotated about the y axis. Find the volume of the solid formed. (1 dec. pl.)



5.* The shaded area, as shown below, is rotated about the y axis. Find the volume of the solid formed (in terms of π).



Trapezoidal Rule

EXERCISE 14

1. (a) Complete the table of values for the function $y = \dfrac{8}{x+2}$.

x	2	3	4	5	6
y					

(b) Use the trapezoidal rule with five function values to approximate the area under the curve $y = \dfrac{8}{x+2}$, from $x = 2$ to $x = 6$. (3 sig. fig.)

2. (a) Complete the table of values for the function $y = \dfrac{1}{x^2+1}$.

x	0	1	2	3	4
y					

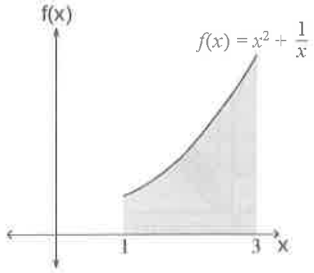
(b) Use the trapezoidal rule with four intervals to approximate the area under the curve $y = \dfrac{1}{x^2+1}$, from $x = 0$ to $x = 4$. (2 dec. pl.)

3.* The table of values for a section of $y = f(x)$ is given below:

x	4·0	4·2	4·4	4·6	4·8	5·0
y	4·00	3·50	3·14	2·88	2·67	2·50

Use the trapezoidal rule with five intervals to approximate $\int_4^5 f(x)\,dx$. (2 dec. pl.)

4.* Use the trapezoidal rule with 5 function values to approximate the shaded area in the given diagram (1 dec.pl.):

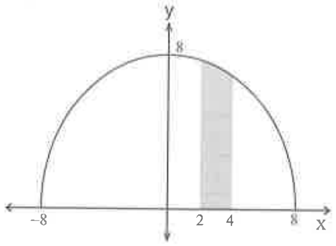


5. (a) Find the exact area bounded by the curve $y = x^3$, the x axis and the lines $x = 6$ and $x = 8$.

(b) Use the trapezoidal rule with 3 function values to approximate the area in part (a).

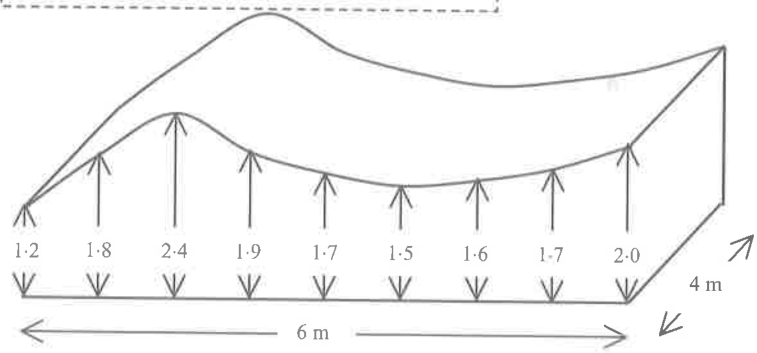
(c) What percentage error does the trapezoidal rule result in compared with the exact area?

6. Use the trapezoidal rule with 5 function values to approximate the shaded area on the semi-circle below. (2 dec. pl.)



x					
y					

7.



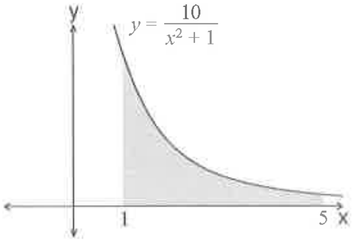
(a) Use the Trapezoidal rule to find an approximate value for the cross-sectional area in the diagram above. (All heights are in metres and these measurements are uniformly spaced.)

(b) What is the volume of the land mass?

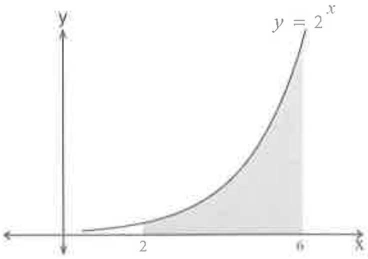
Simpson's Rule

EXERCISE 15

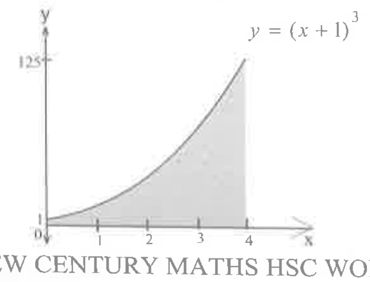
1. Use Simpson's rule with 3 function values to approximate the shaded area in the given diagram. (2 dec. pl.)



2. Use Simpson's rule with 3 function values to approximate the shaded area in the given diagram.



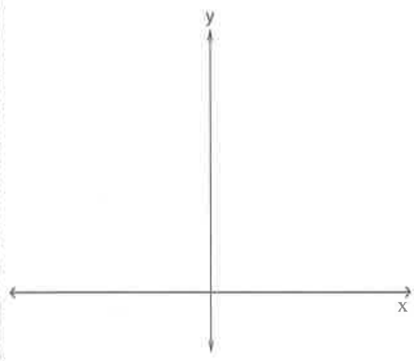
3. Use Simpson's rule with 5 function values to approximate the shaded area in the given diagram.



4. Apply Simpson's rule to the table of values given to evaluate $\int_3^6 f(x)dx$. (1 dec. pl.)

x	3	3·5	4	4·5	5	5·5	6
$f(x)$	3·16	3·64	4·12	4·6	5·1	5·59	6·08

5. (a) On the given set of axes, sketch the semi-circle $y = \sqrt{49 - x^2}$



(b) Find an approximate value for $\int_1^7 \sqrt{49-x^2}\,dx$ using Simpson's rule with 6 intervals. (1 dec. pl.)

x							
y							

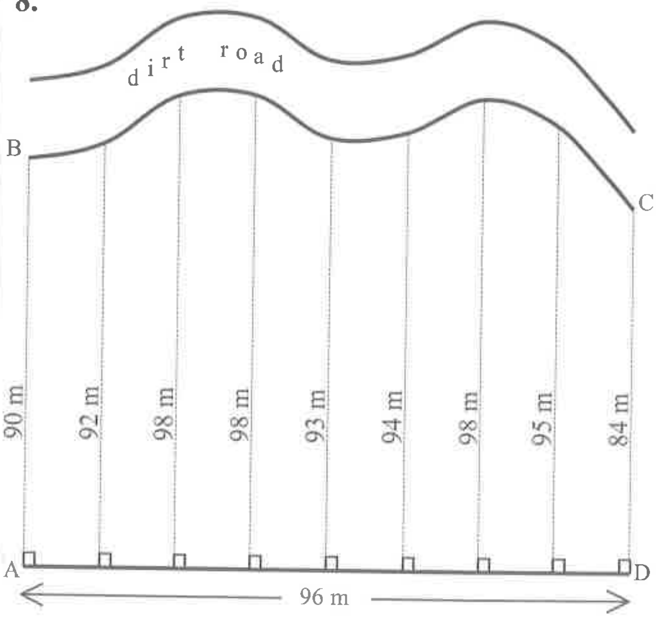
(c) Find an approximate value for $\int_1^7 \sqrt{49-x^2}\,dx$ using Trapezoidal Rule with 6 intervals. (1 dec. pl.)

6. Use Simpson's rule with 4 intervals to evaluate $\int_3^4 (x^2+2)dx$. (1 dec.pl.)

7. Apply Simpson's rule with 6 intervals to evaluate $\int_1^{10} f(x)dx$, using the table below. (1 dec. pl.)

x	1	2·5	4	5·5	7	8·5	10
y	1·4	2·32	3·84	6·36	10·54	17·46	28·93

8.



ABCD is a block of land.
Fence AD is marked off into 8 equal lengths, and measurements were taken to the edge of the dirt road, as shown.
Use Simpson's rule to approximate the area of the block.

9. Evaluate $\int_0^4 (4x^3 + 2)dx$ using:

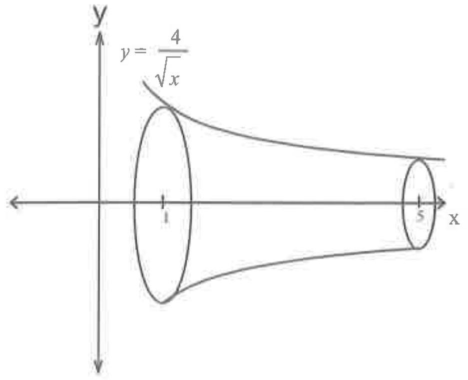
(a) Trapezoidal rule with four intervals.

x					
y					

(b) Simpson's rule with four intervals.

(c) Integration (i.e. exact value).

10.*



The section of the curve $y = \frac{4}{\sqrt{x}}$, from $x = 1$ to $x = 5$ is rotated about the x axis. Use Simpson's rule with 4 sub-intervals to find the volume of the solid of revolution formed. (4 sig. fig.)

x					
y					

FORMULAS

1. $\int x^n \, dx =$

2. $\int k \, dx =$

3. $\int (ax+b)^n \, dx =$

4. Area bounded by $y = f(x)$, the x axis and the ordinates $x = a$ and $x = b$ is:

5. Area bounded by $y = f(x)$, the y axis and the lines $y = a$ and $y = b$ is:

6. Volume of solid of revolution formed by rotating $y = f(x)$ around the x axis, from $x = a$ to $x = b$ is:

7. Volume of solid of revolution formed by rotating $y = f(x)$ around the y axis, from $y = a$ to $y = b$ is:

8. The basic trapezoidal rule, for 2 function values, is $A \approx$.

9. The extended trapezoidal rule is

   where $h =$ —— .

10. The basic Simpson's rule, for 3 function values is

    where $h =$ —— .

11. The extended Simpson's rule is

    where $h =$ ——

SUMMARY

1. The process of finding a function, given its derivative, is called finding the $\boxed{\quad\quad}$ function.
2. The primitive function is found by $\boxed{\quad\quad}$ the derived function.
3. An integral, without limits, $\int$ is called an $\boxed{\quad\quad}$ integral.
4. An integral, with limits, $\int_a^b$ is called a $\boxed{\quad\quad}$ integral.
5. If $F(x) = \int f(x)dx$ then $\int_a^b f(x)dx = \boxed{\quad\quad}$.
6. Two applications of integration are:

   (a) finding $\boxed{\quad\quad}$ under a curve.

   (b) finding $\boxed{\quad\quad}$ of a solid of revolution.

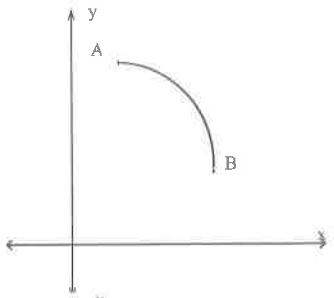
7. Two approximate methods of integration are:

   (a) $\boxed{\quad\quad\quad\quad}$

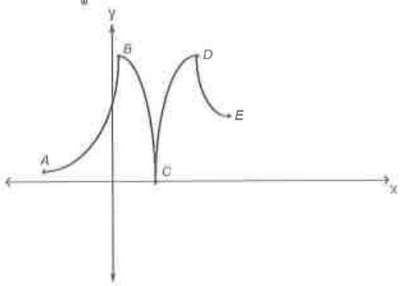
   (b) $\boxed{\quad\quad\quad\quad}$

8. When using the Trapezoidal rule or Simpson's rule, the number of function values is $\boxed{\quad\quad}$ the number of intervals.
9. To find the exact area under a curve, $\boxed{\quad\quad}$ the function.
10. When finding the area under a curve, always sketch the curve first to see whether the curve crosses the x axis between the limits of integration. If this happens, the integration must be separated into $\boxed{\quad\quad}$.
11. When finding the area between a curve and the x axis, if the area is $\boxed{\quad\quad}$ the x axis we find $\left|\int_a^b y\,dx\right|$.
12. When finding the area between a curve and the y axis, if the area is $\boxed{\quad\quad}$ the y axis we find $\left|\int_a^b x\,dy\right|$.
13. The exact area between 2 functions $f(x)$ and $g(x)$ is given by $\boxed{\quad\quad}$.

(d)



5.

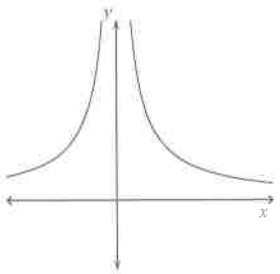


Exercise 8 (page 12)

1.(a) $f'(x) \neq 0,\ f''(x) \neq 0$ **(b)** even

(c) $0, 0, \infty$ **(d)** 0

(e)



2. *

Exercise 9 (page 14)

1. C **2.** D **3.** G **4.** A **5.** F **6.** B **7.** E

Exercise 10 (page 15)

1.(a) $A = 9x - 2x^2$ **(b)** $A = 10x^2 - x^3$

(c) $A = 2x + \frac{600}{x}$ **(d)** $A = \pm 3x\sqrt{25 - x^2}$

2.(a) $60 = 4y + 3x$ **(b)** $y = 15 - \frac{3}{4}x$

(d) $x = 10,\ y = 7\cdot5$

3. *

4.(a) base length and height = $5\frac{1}{3}$ cm **(b)** $151\cdot7$ cm^3

5. $r = 7\cdot2557$ cm, $h = 14\cdot5112$ cm

6.(a) $r = 7\cdot07$ cm, $h = 14\cdot15$ cm **(b)** $2222\cdot01$ cm^3

7. * **8.** * **9.** *

10. 30 **11.** *

Summary (page 19)

1. > **2.** < **3.** Max, Min **4.** 0, parallel

5. 0 **6.** <, =, > **7.** >, =, <

8. (a) 0 **(b)** sign, 2nd derivative **(c)** 0

9. Min **10.** Max **11.** > **12.** <

13. concavity

14. =, there is a change in the sign of $\frac{d^2y}{dx^2}$ on either side of the point.

15. =, =, there is a change in the sign of $\frac{d^2y}{dx^2}$ on either side of the point.

Integration

Exercise 1 (page 21)

1. C **2.** D **3.** A **4.** B **5.** C **6.** B

7. B **8.** D **9.** C **10.** C **11.** C **12.** D

13. C **14.** A

Exercise 2 (page 24)

1.(a) $\frac{x^6}{6} + c$ **(b)** $\frac{p^8}{8} + c$ **(c)** $x^5 + c$

(d) $4x^4 + c$ **(e)** $\frac{-3x^4}{4} + c$ **(f)** $\frac{x^{10}}{2} + c$

(g) $\frac{x^3}{12} + c$ **(h)** $\frac{x^3}{5} + c$

2.(a) $\frac{10x^3}{9} + c$ **(b)** $\pi x^9 + c$ **(c)** $3x + c$

(d) $\pi x + c$ **(e)** $\frac{\pi x^2}{2} + c$ **(f)** $-2m + c$

(g) $\frac{x}{3} + c$

Exercise 3 (page 24)

1.(a) $-\frac{1}{3x^3} + c$ **(b)** $-\frac{3}{x^2} + c$ **(c)** $-\frac{1}{2x^4} + c$

(d) $-\frac{1}{3x} + c$ **(e)** $-\frac{1}{5x^5} + c$ **(f)** $-\frac{1}{2x^6} + c$

(g) *

2.(a) $\frac{5}{7}x^{\frac{7}{5}}$ **(b)** $\frac{5}{6}x^{1\cdot2}$ **(c)** $8x^{\frac{7}{4}}$

Exercise 4 (page 25)

1.(a) $\frac{2}{3}\sqrt{x^3}+c$ **(b)** * **(c)** $2\sqrt{x^3}+c$

(d) $\frac{3}{4}\sqrt[3]{x^4}+c$ **(e)** $2\sqrt{x^5}+c$ **(f)** $8\sqrt[4]{x^5}+c$

(g) $20\sqrt{x}+c$ **(h)** x^3+2x^2-9x+c

(i) $4x^2-\dfrac{2}{x}+c$ **(j)** $\dfrac{x^6}{6}+4\sqrt{x^3}-4\sqrt{x}+c$

(k) * **(l)** $8x-\frac{3}{5}\sqrt[3]{x^5}+c$

2.(a) $2x^3+2x^2-3x+c$ **(b)** $\dfrac{1}{x^2}-\dfrac{5}{x}+c$ **(c)** *

(d) $\frac{2}{5}\sqrt{x^5}-4x^2+c$ **(e)** $2x^3+\dfrac{13x^2}{2}+5x+c$

(f) $36x+7x^2-\dfrac{2x^3}{3}+c$ **(g)** *

(h) $\dfrac{3p^4}{2}-2p^3+15p^2+c$

Exercise 5 (page 27)

1. $\dfrac{(2x+3)^6}{12}+c$ **2.** $-\dfrac{(3-4x)^7}{28}+c$ **3.** *

4. $\dfrac{(x+5)^5}{80}+c$ **5.** $\dfrac{2}{3}\sqrt{(x+6)^3}+c$

6. $\dfrac{2}{9}\sqrt{(3x-1)^3}+c$ **7.** * **8.** $\dfrac{4}{3}\sqrt{(3x-1)}+c$

9. $\dfrac{-1}{x+8}+c$ **10.** $\dfrac{-1}{5(5x-2)^2}+c$

11. $\dfrac{\sqrt[3]{(6x+1)^4}}{8}+c$

Exercise 6 (page 28)

1. $3x^2-9x+c$ **2.** $f(x)=x^3-8x^2+5x+c$

3. $f(x)=x^2+8x+7$ **4.** *

5.(a) $f(x)=x^4-3x^2-1$ **(b)** 53

6. $f(x)=8x^3-7x^2+2x-4$ **7.** *

8. $y=-4x^2+5x+50$

Exercise 7 (page 30)

1.(a) a^3+2a **(b)** $2b^4-3b^2+1$ **2.** *

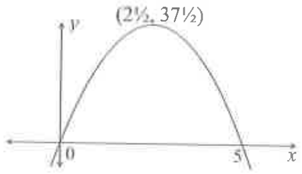
3.(a) 66 **(b)** 52 **(c)** −77

(d) 290 **(e)** 12¾ **(f)** * **(g)** $8\frac{7}{10}$

Exercise 8 (page 31)

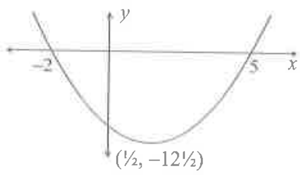
1. 66 units^2

2.(a)



(b) 125 units^2

3.(a)

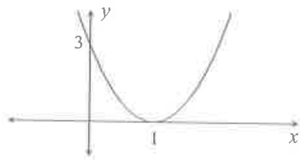


(b) $57\frac{1}{6}$ units^2

4. The curve crosses the x axis between the limits of integration **5.** *

6.(a) 40 units^2 **(b)** 40 units^2

7.(a)

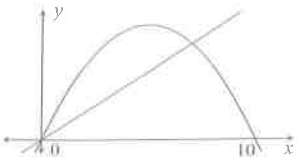


(b) 16 units^2 **8.** $1\frac{3}{5}$ units^2 **9.** 2 units^2

Exercise 9 (page 34)

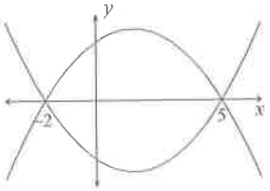
1.(a) P(2, 12) **(b)** 4 units^2

2.(a)



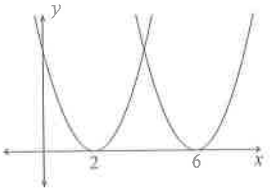
(b) (0, 0) and (7, 21) **(c)** $57\frac{1}{6}$ units^2

3.(a)



(b) $114\frac{1}{3}$ units^2 **4.** *

5.(a)



(b) (4, 4) **(c)** $5\frac{1}{3}$ units^2 **6.** *

Exercise 10 (page 37)

1. $21\frac{1}{3}$ units^2 **2.** $2\frac{1}{12}$ units^2 **3.** 288 units^2 **4.** *

5. 3 **6.** *

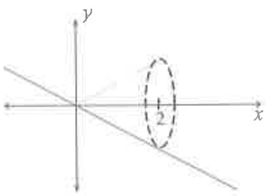
Exercise 11 (page 39)

1. 1088 units^2 **2.** $138\frac{2}{3}$ units^2 **3.** 156·25 units^2

4. $83\frac{1}{3}$ units^2 **5.** * **6.** 288 units^2

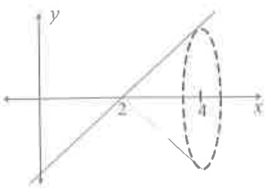
Exercise 12 (page 41)

1.(a)



(b) 134 units^3 **2.** *

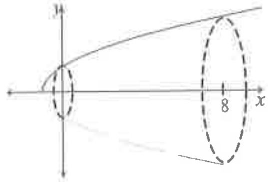
3.(a)



(b) 24π units^3

4.(a) 0 1 2 3 4

(b)



(c) 40π units^3 **5.** $\frac{8\pi}{3}$ units^3 **6.** 113·1 units^3

7. 152·05 units^3 **8.** 77·5 units^3 **9.** *

Exercise 13 (page 44)

1. 579·6 units^3 **2.** 21π units^3 **3.** *

4. 1072.3 units^3 **5.** *

Exercise 14 (page 45)

1.(a) 2 $1\frac{3}{5}$ $1\frac{1}{3}$ $1\frac{1}{7}$ 1 **(b)** 5·58 units^2

2.(a) 1 $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{17}$ **(b)** 1·33 units^2

3. * **4.** *

5.(a) 700 units^2 **(b)** 707 units^2 **(c)** 1%

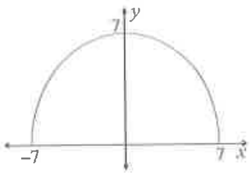
6. 14·77 units^2 **7.(a)** 10·65 m^2 **(b)** 42·6 m^3

Exercise 15 (page 48)

1. 6·26 units^2 **2.** 88 units^2

3. 156 units^2 **4.** 13·8

5.(a)



(b) 31·2 **(c)** 30·8

6. 14·3 **7.** 81·8 **8.** 9072 m^2

9.(a) 280 **(b)** 264 **(c)** 264

10. *

Formulas (page 51)

1. $\dfrac{x^{n+1}}{n+1}+c$ **2.** $kx+c$ **3.** $\dfrac{(ax+b)^{n+1}}{a(n+1)}+c$

4. $A=\int_a^b y\,dx$ **5.** $A=\int_a^b x\,dy$

6. $V=\pi\int_a^b y^2\,dx$ **7.** $V=\pi\int_a^b x^2\,dy$

8. $\frac{h}{2}\left[f(a)+f(b)\right]$

9. $A \cong \frac{h}{2}\left[f(a)+f(b)+2\left(f(x_2)+f(x_3).....\right)\right]$, $\frac{b-a}{n}$,

10. $\frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$, $\frac{b-a}{2}$

11. $A \cong \frac{h}{3}\left[f(a)+f(b)+4\left(f(x_2)+f(x_4)...\right)\right.$

$\left.+2\left(f(x_3)+f(x_5)...\right)\right]$, $\frac{b-a}{n}$

Summary (page 52)

1. primitive **2.** integrating **3.** indefinite

4. definite **5.** F(b) –F(a) **6.(a)** area

(b) volume **7.(a)** Trapezoidal Rule

(b) Simpson's Rule **8.** 1 more than **9.** integrate

10. sections **11.** below **12.** to the left of

13. $\int_a^b [f(x)-g(x)]\,dx$

Exponential and Logarithmic Functions

Exercise 1 (page 53)

1. B **2.** B **3.** C **4.** C **5.** B **6.** D

7. A **8.** B **9.** C **10.** D **11.** C **12.** A

13. D **14.** C **15.** A **16.** C **17.** A **18.** B

19. B **20.** C **21.** B

Exercise 2 (page 55)

1.(a) * **(b)** 0·2219 **(c)** 1·398

(d) 1·4313 **(e)** 0·5229 **(f)** 0·3495

(g) 2·398

2. $\log_x\left(\frac{a^3b^4}{3}\right)$ **3.** $18\ln 2$ **4.** $\frac{25\ln 3}{4}$

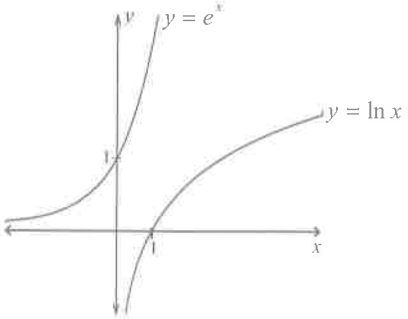
5. * **6.** *

Exercise 3 (page 56)

1.(a) 2·7 **(b)** 1 **(c)** 0 **(d)** e^y **(e)** $y = x$

(f) exponential

2.



(b) all real x **(c)** $y > 0$ **(d)** $x > 0$

(e) all real y

3.(a) $x > 0$ **(b)** $x > 2$ **(c)** $x > -3$

4. logarithm **5.(a)** E **(b)** B

(c) D **(d)** A **(e)** C

Exercise 4 (page 57)

1. 36·6 **2.** 1·9 **3.(a)** –1

(b) 0, –4 **(c)** 5 **(d)** *

(e) ln2 **(f)** 1·4 **(g)** 1·61

(h) 1·10 **(i)** 0·43 **(j)** 2·3

(k) 3 **(l)** 1100 **(m)** 264·9

(n) –1 **4.(a)** * **(b)** $e^x + 2$

(c) e^{8x} **(d)** 7 **(e)** $e^{2x} + e^{-2x} + 2$

5.(a) $125 = 5^y$ **(b)** $y = 4^3$ **(c)** $2 = e^x$

(d) $81 = x^2$ **(e)** $t = 2^m$ **(f)** $m = e^6$

6.(a) 16 **(b)** ¼ **(c)** 1·5 **(d)** $\frac{1}{5}$

Exercise 5 (page 58)

1. $y - y_1 = f'(x_1)(x - x_1)$

2. $y - y_1 = \frac{-1}{f'(x_1)}(x - x_1)$

3. $y' = vu' + uv'$ **4.** $y' = \frac{vu' - uv'}{v^2}$

5. $y' = n\left[f(x)\right]^{n-1} \times f'(x)$ **6.** gradient

Exercise 6 (page59)

1.(a) $5e^{5x}$ **(b)** $\frac{2}{3}e^{\frac{2x}{3}}$ **(c)** $\frac{1}{4}e^{\frac{x}{4}}$

(d) $-e^{-x}$ **(e)** $-\frac{1}{5}e^{\frac{-x}{5}}$ **(f)** $2e^{2x+5}$

(g) $-4e^{5-4x}$ **(h)** $6xe^{3x^2}$ **(i)** $6e^{2x}$

(j) $2x + 5e^x$ **(k)** $-2e^{-x}$ **(l)** $\frac{-1}{2e^x}$

(m) $\frac{-6}{e^{2x}}$ **(n)** $2e^{10x}$ **2.** 8·02

3.(a) $e^{2x}(2x+1)$ **(b)** $e^{4x}(12x+11)$

Exercise 3

1(g) $\int \frac{1}{2x^9} dx = \int \frac{1}{2} x^{-9} dx$

$= \frac{x^{-8}}{2(-8)} + c$

$= \frac{x^{-8}}{-16} + c$

$= \frac{-1}{16x^8} + c$

Exercise 4

1(b) $\int x^{\frac{3}{4}} dx = \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + c$

$= \frac{4}{7} x^{\frac{7}{4}} + c$

$= \frac{4}{7} \sqrt[4]{x^7} + c$

1(k) $\int (10x + \frac{5}{x^6} + \sqrt[3]{x})\, dx$

$= \int (10x + 5x^{-6} + x^{\frac{1}{3}})\, dx$

$= \frac{10x^2}{2} + \frac{5x^{-5}}{-5} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$= 5x^2 - x^{-5} + \frac{3}{4} x^{\frac{4}{3}} + c$

$= 5x^2 - \frac{1}{x^5} + \frac{3}{4}\sqrt[3]{x^4} + c$

2(c) $\int \left(\frac{6x - 2\sqrt{x}}{\sqrt{x}} \right) dx$

$= \int \left(\frac{6x}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}} \right) dx$

$= \int (6x^{\frac{1}{2}} - 2) dx$

$= \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + c$

$= \frac{2}{3}(6x^{\frac{3}{2}}) - 2x + c$

$= 4x^{\frac{3}{2}} - 2x + c$

$= 4\sqrt{x^3} - 2x + c$

2(g) $\int 2t^2 (t^3 + 6t)\, dt$

$= \int (2t^5 + 12t^3)\, dt$

$= \frac{2t^6}{6} + \frac{12t^4}{4} + c$

$= \frac{t^6}{3} + 3t^4 + c$

Exercise 5

3 $\int (6x-1)^3 dx = \frac{(6x-1)^4}{4 \times 6} + c$

$= \frac{(6x-1)^4}{24} + c$

7 $\int \frac{1}{\sqrt{x+6}} dx = \int (x+6)^{\frac{-1}{2}} dx$

$= \frac{(x+6)^{\frac{1}{2}}}{1 \times \frac{1}{2}} + c$

$= \frac{(x+6)^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2(x+6)^{\frac{1}{2}} + c$

$= 2\sqrt{(x+6)} + c$

Exercise 6

4 $\frac{dy}{dx} = 11 - 2x$

$y = 11x - x^2 + c$

Passes through $(2, 9)$

$\therefore 9 = 11(2) - (2)^2 + c$

$9 = 22 - 4 + c$

$c = -9$

$\therefore y = 11x - x^2 - 9$

7 From the graph, and using

$y = mx + b : m = \frac{20}{5} = 4,\ b = 20$

$\therefore f'(x) = 4x + 20$

thus $f(x) = 2x^2 + 20x + c$

now y intercept is 1

i.e. curve passes through $(0, 1)$

$\therefore 1 = 2(0)^2 + 20(0) + c$

$c = 1$

Curve is $f(x) = 2x^2 + 20x + 1$

Exercise 7

2 $\int_5^k (2x+5)dx = 54$

$\left[\frac{2x^2}{2} + 5x \right]_5^k = 54$

$\left[x^2 + 5x \right]_5^k = 54$

$\left[(k)^2 + 5(k) \right] - \left[(5)^2 + 5(5) \right] = 54$

$k^2 + 5k - 50 = 54$

$k^2 + 5k - 104 = 0$

$(k + 13)(k - 8) = 0$

$k = -13$ or $k = 8$ but $k > 5$

$\therefore k = 8$

3(f) $\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-\frac{1}{2}} dx$

$= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9$

$= \left[2x^{\frac{1}{2}} \right]_4^9$

$= \left[2\sqrt{x} \right]$

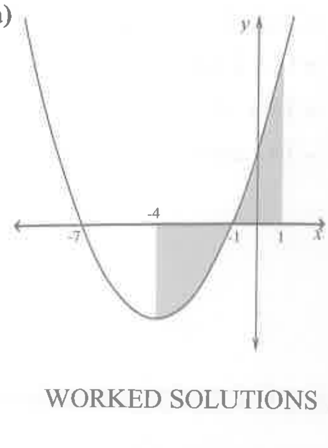
$= (2\sqrt{9}) - (2\sqrt{4})$

$= (6) - (4)$

$= 2$

Exercise 8

5(a)



(b) Using $A=\int_a^b y\,dx$

$$A=\left|\int_{-4}^{-1}(x^2+8x+7)\right|+\int_{-1}^{1}(x^2+8x+7)\,dx$$

$$=\left|\left[\frac{x^3}{3}+4x^2+7x\right]_{-4}^{-1}\right|+\left[\frac{x^3}{3}+4x^2+7x\right]_{-1}^{1}$$

$$=\left|\left(\frac{(-1)^3}{3}+4(-1)^2+7(-1)\right)-\left(\frac{(-4)^3}{3}+4(-4)^2+7(-4)\right)\right|$$

$$+\left[\left(\frac{(1)^3}{3}+4(1)^2+7(1)\right)-\left(\frac{(-1)^3}{3}+4(-1)^2+7(-1)\right)\right]$$

$$=\left|\left(-\tfrac{1}{3}+4-7\right)-\left(-21\tfrac{1}{3}+64-28\right)\right|+\left[\left(\tfrac{1}{3}+4+7\right)-\left(-\tfrac{1}{3}+4-7\right)\right]$$

$$=\left|\left(-3\tfrac{1}{3}\right)-\left(14\tfrac{2}{3}\right)\right|+\left[\left(11\tfrac{1}{3}\right)-\left(-3\tfrac{1}{3}\right)\right]$$

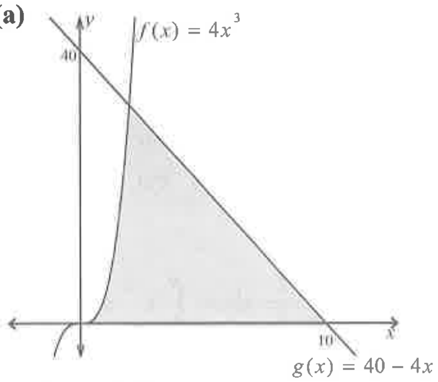
$$=\left|-18\right|+\left[14\tfrac{2}{3}\right]$$

$$=18+14\tfrac{2}{3}$$

$$=32\tfrac{2}{3}\text{ units}^2$$

Exercise 9

4(a)



(b) $f(x)=4x^3$

$$f(2)=4(2)^3$$
$$=32$$

(c) $g(x)=40-4x$

$$g(2)=40-4(2)$$
$$=32$$

(d) $A=\int_0^2 4x^3dx+\int_2^{10}(40-4x)\,dx$

$$=\left[x^4\right]_0^2+\left[40x-2x^2\right]_2^{10}$$

$$=\left[2^4-0^4\right]+\left[\left(40(10)-2(10)^2\right)-\left(40(2)-2(2)^2\right)\right]$$

$$=\left[16\right]+\left[(400-200)-(80-8)\right]$$

$$=16+\left[200-72\right]$$

$$=16+128$$

$$=144\text{ units}^2$$

6(a) Line AC has equation $y=mx+c$

$m=\dfrac{-8}{4}=-2,\ \therefore y=-2x+8$

Find B by solving simultaneously $y=x^2$ and $y=-2x+8$

$\therefore x^2=-2x+8 \Rightarrow\ x^2+2x-8=0$

$$(x+4)(x-2)=0$$
$$x=-4,2$$

As B is in the first quadrant, $x=2 \rightarrow y=4$

B is the point $(2,4)$

(b) Area OAB = Area ΔACO

− (Area under $y=x^2$ + Area ΔBCD)

Area OAB $=(\tfrac{1}{2}\times4\times8)-\left[\int_0^2 x^2dx+\tfrac{1}{2}\times2\times4\right]$

$$=16-\left(\left[\frac{x^3}{3}\right]_0^2+4\right)$$

$$=16-\left(\left[\frac{2^3}{3}-\frac{0^3}{3}\right]+4\right)$$

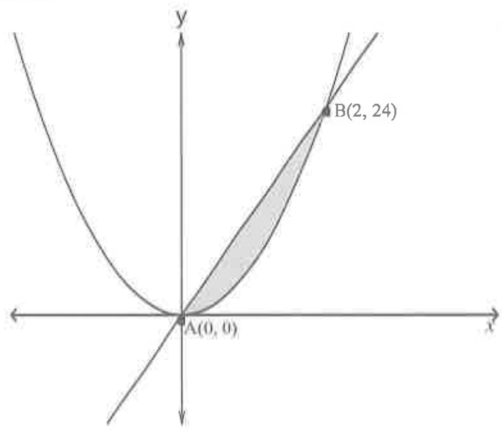
$$=16-\left(\frac{8}{3}+4\right)$$

$$=16-6\tfrac{2}{3}$$

$$=9\tfrac{1}{3}\text{ units}^2$$

Exercise 10

4

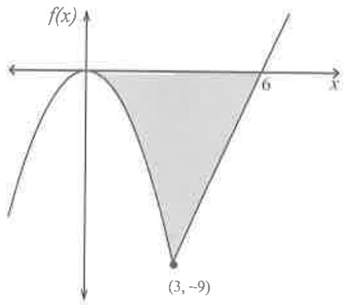


Line:

$$\begin{cases} y-y_1=m(x-x_1) \\ y-0=\frac{24}{2}(x-0) \\ y=12x \end{cases}$$

$$Area = \int_a^b f(x) - g(x)\,dx$$
$$= \int_0^2 12x - 6x^2\,dx$$
$$= \left[\frac{12x^2}{2} - \frac{6x^3}{3}\right]_0^2$$
$$= \left[6x^2 - 2x^3\right]_0^2$$
$$= (6(2)^2 - 2(2)^3) - (6(0)^2 - 2(0)^3)$$
$$= (6(4) - 2(8)) - 0$$
$$= (24 - 16)$$
$$= 8 \text{ units}^2$$

6(a) $f(x) = \begin{cases} -x^2 & \text{for } x \le 3 \\ 3x - 18 & \text{for } x > 3 \end{cases}$



(b) Shaded Area $= \left|\int_0^3 (-x^2)\,dx\right| + \frac{1}{2} \times 3 \times 9$
$$= \left|\left[\left(\frac{-x^3}{3}\right)\right]_0^3\right| + 13 \cdot 5$$
$$= \left|\left(\frac{-(3)^3}{3}\right) - \left(\frac{-(0)^3}{3}\right)\right| + 13 \cdot 5$$
$$= |-9 - 0| + 13 \cdot 5$$
$$= 9 + 13 \cdot 5$$
$$= 22 \cdot 5 \text{ units}^2$$

Exercise 11

5 $A = \left|\int_a^b x\,dy\right|$

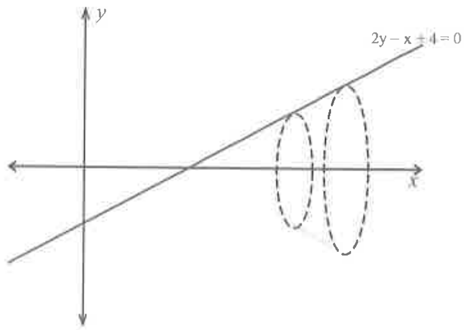
$y = 1 - x^3 \rightarrow x^3 = 1 - y \rightarrow x = \sqrt[3]{1 - y}$

i.e. $x = (1 - y)^{\frac{1}{3}}$

$$A = \left|\int_1^9 (1 - y)^{\frac{1}{3}}\,dy\right|$$
$$= \left|\left[\frac{(1-y)^{\frac{4}{3}}}{(-1) \times \frac{4}{3}}\right]_1^9\right|$$
$$= \left|\left[-\frac{3}{4}(1 - 9)^{\frac{4}{3}}\right] - \left[-\frac{3}{4}(1 - 1)^{\frac{4}{3}}\right]\right|$$
$$= \left|-\frac{3}{4}(-8)^{\frac{4}{3}} - 0\right|$$
$$= |-12|$$
$$= 12 \text{ units}^2$$

Exercise 12

2

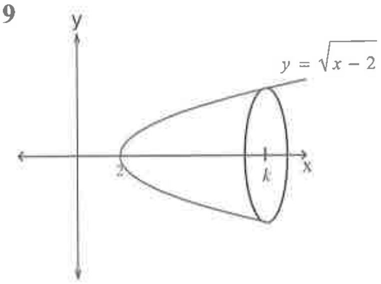


$V = \pi \int_a^b y^2\,dx$ now $2y - x + 4 = 0$

$$2y = x - 4$$
$$y = \frac{x - 4}{2}$$
$$y^2 = \left(\frac{x - 4}{2}\right)^2$$

$$\therefore V = \pi \int_8^{10} \frac{(x - 4)^2}{4}\,dx$$
$$= \frac{\pi}{4} \int_8^{10} (x - 4)^2\,dx$$
$$= \frac{\pi}{4}\left[\frac{(x - 4)^3}{3}\right]_8^{10}$$
$$= \frac{\pi}{4}\left[\frac{(10 - 4)^3}{3} - \frac{(8 - 4)^3}{3}\right]$$
$$= \frac{\pi}{4}\left(\frac{6^3}{3} - \frac{4^3}{3}\right)$$
$$= 39 \cdot 79 \text{ units}^3 \text{ (to 2 dec. pl.)}$$

9



$y = \sqrt{x-2} \rightarrow y^2 = x - 2$

$$V = \pi \int_a^b y^2\, dx$$
$$= \pi \int_2^k (x-2)\, dx$$
$$= \pi \left[\frac{x^2}{2} - 2x \right]_2^k$$
$$= \pi \left[\left(\frac{(k)^2}{2} - 2(k) \right) - \left(\frac{(2)^2}{2} - 2(2) \right) \right]$$
$$= \pi \left[\left(\frac{k^2}{2} - 2k \right) - (2-4) \right]$$
$$= \pi \left(\frac{k^2}{2} - 2k + 2 \right)$$

But $V = 8\pi \therefore \pi \left(\frac{k^2}{2} - 2k + 2 \right) = 8\pi$

$$\frac{k^2}{2} - 2k + 2 = 8$$
$$\frac{k^2}{2} - 2k - 6 = 0$$
$$k^2 - 4k - 12 = 0$$
$$(k-6)(k+2) = 0$$
$$k = 6 \text{ or } k = -2$$

But k lies on the positive side of x axis $\therefore k = 6$

Exercise 13

3 Graph is the semi – circle $y = \sqrt{100 - x^2}$

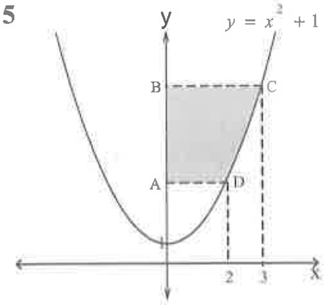
i.e. $y^2 = 100 - x^2 \rightarrow x^2 = 100 - y^2$

$$V = \pi \int_a^b x^2\, dy$$
$$= \pi \int_0^{10} 100 - y^2\, dy$$
$$= \pi \left[100y - \frac{y^3}{3} \right]_0^{10}$$
$$= \pi \left[\left(100(10) - \frac{(10)^3}{3} \right) - \left(100(0) - \frac{(0)^3}{3} \right) \right]$$
$$= \pi \left[\left(1000 - \frac{1000}{3} \right) - 0 \right]$$
$$= 2094 \cdot 4 \text{ units}^3$$

(b) $V = \frac{1}{2} \times \frac{4\pi}{3} (10^3)$

$= 2094.4 \text{ units}^3$

5



$y = x^2 + 1$: when $x = 2 \rightarrow y = 5$

when $x = 3 \rightarrow y = 10$

$y = x^2 + 1 \rightarrow x^2 = y - 1$

$$V = \pi \int_a^b x^2\, dy$$
$$= \pi \int_5^{10} (y-1)\, dy$$
$$= \pi \left[\frac{y^2}{2} - y \right]_5^{10}$$
$$= \pi \left[\left(\frac{(10)^2}{2} - (10) \right) - \left(\frac{(5)^2}{2} - (5) \right) \right]$$
$$= \pi \left[(50 - 10) - \left(12\tfrac{1}{2} - 5 \right) \right]$$
$$= \pi \left[40 - 7\tfrac{1}{2} \right]$$
$$= \pi \left[32\tfrac{1}{2} \right]$$
$$= \frac{65\pi}{2} \text{ units}^3$$

Exercise 14

3

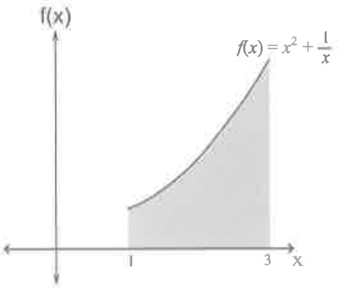
x	4·0	4·2	4·4	4·6	4·8	5·0
y	4·00	3·50	3·14	2·88	2·67	2·50

$$\int_a^b f(x)\, dx \approx \frac{h}{2} \left[f(a) + f(b) + 2\left[f(x_1) + f(x_2) + \ldots\ldots \right] \right]$$

$$\therefore \int_4^5 f(x)\, dx \approx \frac{0 \cdot 2}{2} \left[4 + 2 \cdot 5 + 2\left[3 \cdot 5 + 3 \cdot 14 + 2 \cdot 88 + 2 \cdot 67 \right] \right]$$

$$\approx 3 \cdot 09$$

4



5 function values $=4$ strips

$$h=\frac{b-a}{n}=\frac{3-1}{4}=0\cdot5$$

$f(x)=x^2+\frac{1}{x}$

$f(1)=(1)^2+\frac{1}{1}=2$

$f(1\cdot5)=(1\cdot5)^2+\frac{1}{1\cdot5}=2\cdot92$

$f(2)=(2)^2+\frac{1}{2}=4\cdot5$

$f(2\cdot5)=(2\cdot5)^2+\frac{1}{2\cdot5}=6\cdot65$

$f(3)=(3)^2+\frac{1}{3}=9\cdot33$

$A\approx\frac{h}{2}[f(a)+f(b)+2[f(x_1)+f(x_2)+\ldots\ldots]]$

$\approx\frac{0\cdot5}{2}[2+9\cdot33+2[2\cdot92+4\cdot5+6\cdot65]]$

$\approx9\cdot9\,\text{units}^2$

Exercise 15

10 $y=\frac{4}{\sqrt{x}}\rightarrow y^2=\frac{16}{x}$

$V=\pi\int_a^b y^2dx$

$=\pi\int_1^5\frac{16}{x}\,dx$

Trapezoidal rule is used to approximate this integral

Using $f(x)=\frac{16}{x}$ $\qquad \frac{b-a}{n}=\frac{5-1}{4}=1$

$f(1)=\frac{16}{1}=16$

$f(2)=\frac{16}{2}=8$

$f(3)=\frac{16}{3}=5\frac{1}{3}$

$f(4)=\frac{16}{4}=4$

$f(5)=\frac{16}{5}=3\frac{1}{5}$

$\therefore V\approx\pi\left\{\frac{1}{3}\left[16+3\frac{1}{5}+4(8+4)+2(5\frac{1}{3})\right]\right\}$

$\approx\pi\{25\cdot95\}$

$\approx81\cdot54\,\text{u}^3$