SUGGESTED MARKING ONLY.

2017 YEAR 12 MATHEMATICS

ASSESSMENT # 2 - SAMPLE SOLUTIONS.

SECTION I

1. $\sqrt{\frac{\pi+1}{5}} = 0.910 \; (3SF)$ B

2. $2x - 4y + 3 = 0$

$-4y = -2x - 3$

$y = \frac{-2x}{-4} + \frac{-3}{-4}$

$\therefore y = \frac{x}{2} + \frac{3}{4}$

ie Gradient, $m = \frac{1}{2}$ C

3. $2x - 3y = 8$

$-3y = -2x + 8$

$y = \frac{-2x}{-3} + \frac{8}{-3}$

$y = \frac{2x}{3} - \frac{8}{3}$.

$\therefore$ Gradient, $m_1 = \frac{2}{3}$.

ie $m_2 = \frac{-3}{2}$ $(2, 0)$.

$y - 0 = \frac{-3}{2}(x - 2)$

$2y = -3x + 6$

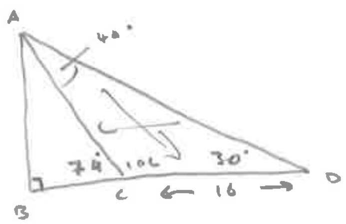
$3x + 2y - 6 = 0$

ie $3x + 2y = 6$ B.

4. $27x^3 - 8 = (3x)^3 - 2^3$

$= (3x - 2)(9x^2 + 6x + 4)$ B

5.



$\frac{AC}{\sin 30^\circ} = \frac{16}{\sin 44^\circ}$

$AC = \frac{16 \sin 30^\circ}{\sin 44^\circ}$

$\therefore \sin 74^\circ = \frac{AB}{AC}$

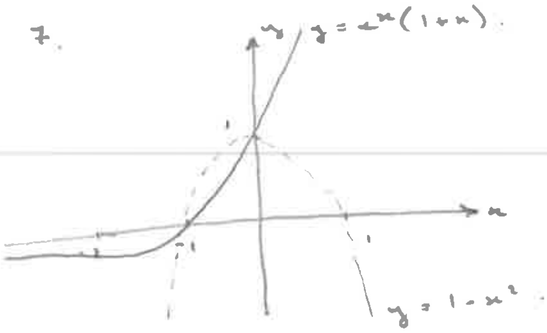
$AB = AC \sin 74^\circ$

$= \frac{16 \sin 30^\circ \sin 74^\circ}{\sin 44^\circ}$ B.

$\therefore AB = 11.07 \; (2DP)$

6. B

7.

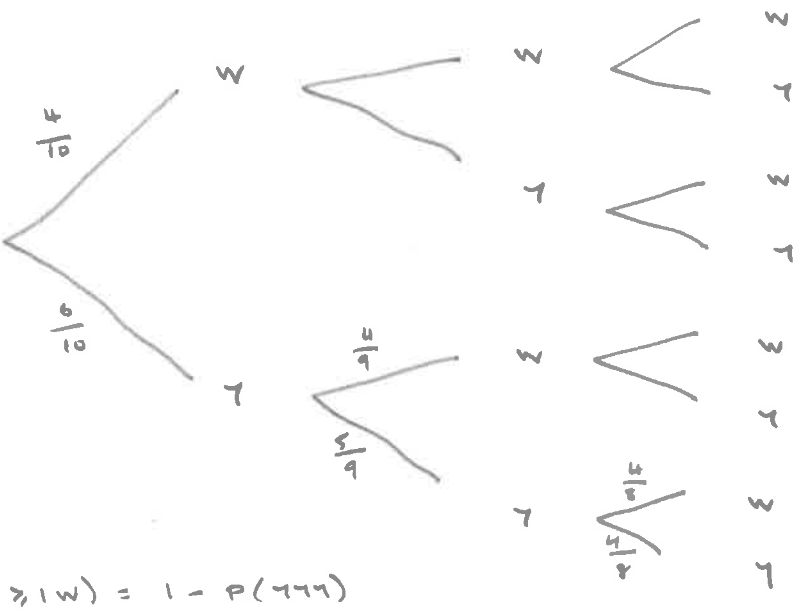


$y = 1 - x^2$

$y = (1-x)(1+x)$

C

8.



$P(\geq 1W) = 1 - P(YYY)$

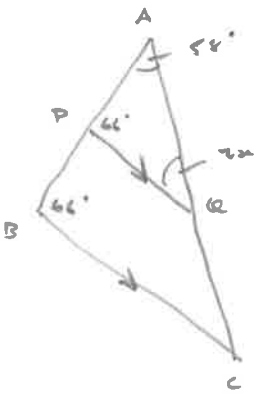
$= 1 - \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right)$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

C

9.



$2x + 58° + 66° = 180°$ (180° in Δ)

$2x + 124° = 180°$

$2x = 56°$

$x = 28°$

A.

10. C

SECTION II

Q11 (a) $y = 3x^2$, $x = 1$.

When $x = 1$
$y = 3$

$\frac{dy}{dx} = 6x$

When $x = 1$

$\frac{dy}{dx} = 6(1)$
$= 6$ (1)

$y - 3 = 6(x - 1)$
$y - 3 = 6x - 6$
$0 = 6x - y - 3$

$\therefore$ tangent is $6x - y - 3 = 0$. (1)

(b) $\sin\theta\cos\theta\csc^2\theta = \cot\theta$

LHS $= \sin\theta\cos\theta\csc^2\theta$

$= \cancel{\sin\theta} \cdot \cos\theta \cdot \frac{1}{\sin^2\theta}$ (1)

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$ (1)

$=$ RHS.

$\therefore \sin\theta\cos\theta\csc^2\theta = \cot\theta$

(c) $\lim_{x \to 1} \frac{x^4 - x^2}{x^2 - 1}$

$= \lim_{x \to 1} \frac{x^2(\cancel{x^2 - 1})}{(\cancel{x^2 - 1})}$

$= \lim_{x \to 1} x^2$ (1)

$= 1^2$

$= 1$

$\therefore \lim_{x \to 1} \frac{x^4 - x^2}{x^2 - 1} = 1$ (1)

(d) $\int e^{3x+1}\,dx = \frac{1}{3}e^{3x+1} + c$ (1) (1)

(e) $y = ax^3 + bx^2 + cx + d$

$\frac{dy}{dx} = 3ax^2 + 2bx + c$

$\frac{d^2y}{dx^2} = 6ax + 2b$ (1)

When $\frac{d^2y}{dx^2} = 0$

$6ax + 2b = 0$
$6ax = -2b$
$x = \frac{-2b}{6a}$
$\therefore x = \frac{-b}{3a}$ (1)

i.e. $p = \frac{-b}{3a}$.

(f) $y = x^2e^{2x}$

$\frac{dy}{dx} = x^2 \cdot e^{2x} \cdot 2 + e^{2x} \cdot 2x$ (1)

$= 2x^2e^{2x} + 2xe^{2x}$

$\therefore \frac{dy}{dx} = 2xe^{2x}(x + 1)$. (1)

(g) $\int_3^4 \frac{(4x-5)}{(2x^2-5x)}\,dx = \left[\log_e(2x^2 - 5x)\right]_3^4$ (1)

$= \left[\log_e(32 - 20) - \log_e(18 - 15)\right]$ (1)

$= \log_e 12 - \log_e 3$

$= \log_e\left(\frac{12}{3}\right)$

$\therefore \int_3^4 \frac{(4x-5)}{(2x^2-5x)}\,dx = \log_e 4$ (1)

Q12.

(a) $x^2 - 6x + 2 = 0$

(i) $\alpha + \beta = \frac{-(-6)}{1}$

$= 6$ (1)

(ii) $\alpha\beta = \frac{2}{1}$

$= 2$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{6}{2}$

$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = 3$ (1)

(b) $2^{2x+1} = 64$

$2^{2x+1} = 2^6$

$\therefore 2x + 1 = 6$ (1)

$2x = 5$

$\therefore x = \frac{5}{2}$ (1)

(c) $\frac{dy}{dx} = 6x - 2$ $(-1, 4)$

$y = \frac{6x^2}{2} - 2x + c$

$y = 3x^2 - 2x + c$ (1)

when $x = -1$
$y = 4$

$4 = 3(-1)^2 - 2(-1) + c$

$4 = 3 + 2 + c$

$4 = 5 + c$

$\therefore c = -1$

$\therefore y = 3x^2 - 2x - 1.$ (1)

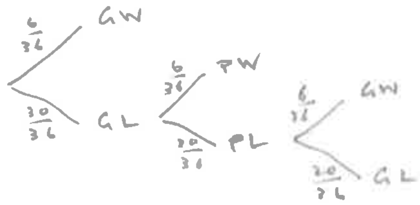
(d) NOT DEFECTIVE $= (1 - 0.04) \times 1200$ (1)

$= 0.96 \times 1200$

$= 1152.$

$\therefore$ 1152 would be expected to NOT be defective. (1)

(e)



(i) $P(GW) = \frac{6}{36}$

$= \frac{1}{6}$ (1)

(ii) P(GW on 1 or 2 THROW)

$= \frac{1}{6} + \left(\frac{30}{36} \times \frac{30}{36} \times \frac{6}{36}\right)$ (1)

$= \frac{1}{6} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{1}{6} + \frac{25}{216}$

$= \frac{61}{216}$ (1)

(f) Distance, $D \doteqdot \frac{5}{3}\left[173 + 168 + 4(81 + 195) + 2(127)\right]$ (1)

$= \frac{5}{3}\left[341 + 1104 + 254\right]$

$= \frac{5}{3}\left[1699\right]$

$= 2831.666\ldots$ (1)

$\therefore$ Distance $\doteqdot$ 2832 m (0DP). (1)

MARK ROUNDING HERE ONLY.

Q13 a (i) $F(x) = 9(2x+3)^5$

$F'(x) = 9 \cdot 5(2x+3)^4 \cdot 2$

$\therefore F'(x) = 90(2x+3)^4$ (1)

(ii) $y = \frac{5}{x\sqrt{x}}$

$= \frac{5}{x^1 \cdot x^{\frac{1}{2}}}$

$= \frac{5}{x^{\frac{3}{2}}}$

$= 5x^{-\frac{3}{2}}$ (1)

$\frac{dy}{dx} = 5 \cdot -\frac{3}{2} x^{-\frac{5}{2}}$

$= \frac{-15}{2} x^{-\frac{5}{2}}$ (1)

(iii) $y = (x-1)\log_e x$

$= (x-1) \cdot \frac{1}{x} + \log_e x \cdot 1$ (1)

$= \frac{(x-1)}{x} + \log_e x$ (1)

b. $f(x) = \frac{2x}{x-3}$, $x \neq 3$.

$f'(x) = \frac{(x-3) \cdot 2 - 2x(1)}{(x-3)^2}$

$= \frac{\cancel{2x} - 6 - \cancel{2x}}{(x-3)^2}$ (1)

$\therefore f'(x) = \frac{-6}{(x-3)^2}$ (1)

ie $f'(x) < 0$ for all x, except $x = 3$.

$\therefore f(x)$ is always a decreasing curve (1) $(x \neq 3)$.

c. $M = t^3 - 6t^2 + 9t$ $\quad 0 \le t \le 5$ (hours).

(i) $M = t^3 - 6t^2 + 9t$

$= t(t^2 - 6t + 9)$

$M = t(t-3)(t-3)$

when $M = 0$

$0 = t(t-3)(t-3)$

$\therefore t = 0$ or 3. (1)

ie Sagun has NO medicine (M) in his bloodstream when $t = 0$ or 3 hours.

(ii) $M = t^3 - 6t^2 + 9t$

$= t(t-3)(t-3)$

$\frac{dM}{dt} = 3t^2 - 12t + 9$

$= 3(t^2 - 4t + 3)$

$\therefore \frac{dM}{dt} = 3(t-3)(t-1)$

$\frac{d^2M}{dt^2} = 6t - 12$

$= 6(t-2)$

For ~~stationary~~ values, $\frac{dM}{dt} = 0$

$3(t-3)(t-1) = 0$

$\therefore t = 1$ or 3.

ie stationary points at $(1, 4)$ and $(3, 0)$ (1) (1)

when $x = 1$

$\frac{d^2M}{dt^2} = 6(1) - 12$

$= 6 - 12$

$= -6$

< 0 (1)

$\therefore$ at $(1, 4)$ concave down $\cap$, $\therefore$ MAXIMUM occurs.

When $x = 3$

$\frac{d^2M}{dt^2} = 6(3) - 12$

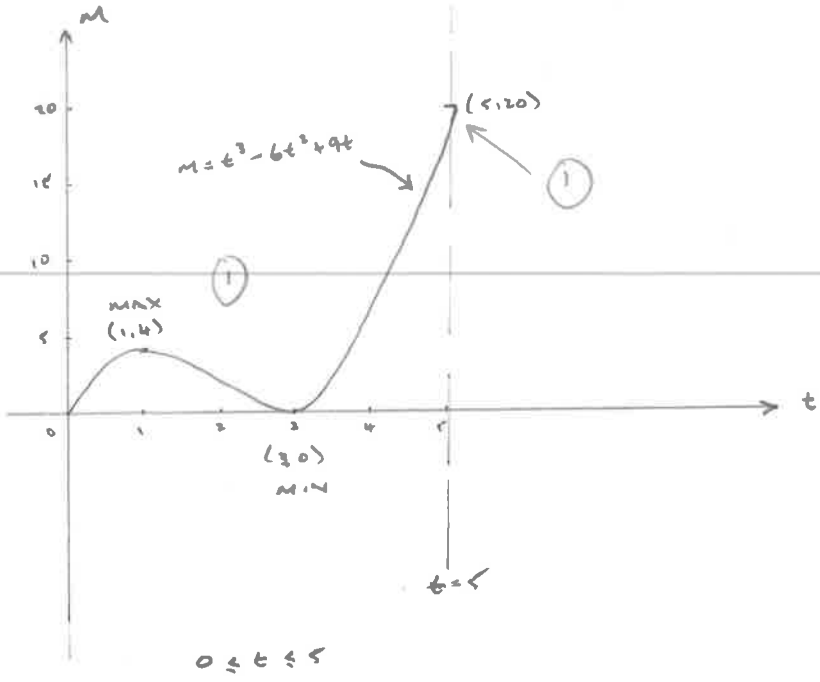
$= 18 - 12$

$= 6$

> 0 (1)

$\therefore$ at $(3, 0)$ concave up $\cup$, MINIMUM occurs.

(iii)



when $t = 5$

$M = 5^3 - 6(5)^2 + 9(5)$

$= 125 - 150 + 45$

$\therefore M = 20$

Q14 a (i) $ax^2 - 12x + 10 = 0 \quad a \neq 0$

$\Delta = (-12)^2 - 4(a)(10)$ 1

$\Delta = 144 - 40a$

(ii) When $ax^2 - 12x + 10 > 0$, $\Delta < 0$

$\therefore 144 - 40a < 0$ 1

$144 < 40a$

$\frac{144}{40} < a$

$\therefore a > 3\frac{3}{5}$ (or $\frac{18}{5}$) 1

b. $2\sin^2 x = \sin x$, $0° \leq x \leq 360°$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$ 1

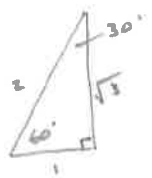
$\therefore \sin x = 0$

$\therefore x = 0°, 180°$ or $360°$ 1

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$



S | A
T | C

$\therefore x = 30°$ or $150°$ 1

$\therefore x = 0°, 30°, 150°, 180°$ or $360°$

c. (i) $y = 5x - x^2$ — (1)

$y = x^2 - 3x$ — (2)

Let (1) = (2)

$5x - x^2 = x^2 - 3x$

$5x - x^2 - x^2 = -3x$

$5x - 2x^2 = -3x$

$-2x^2 + 8x = 0$

$2x^2 - 8x = 0$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$\therefore x = 0$ or 4. 1

$\therefore$ upper limit $x = 4$

Lower limit $x = 0$

$A = \int_0^4 (5x - x^2) - (x^2 - 3x)\, dx$ 1

$= \int_0^4 5x - x^2 - x^2 + 3x\, dx$

$= \int_0^4 -2x^2 + 8x\, dx$

$\therefore A = \int_0^4 8x - 2x^2\, dx$ 1

PTO

Q14 c (ii)

$$A = \int_0^4 8x - 2x^2 \, dx$$

$$= 2\int_0^4 4x - x^2 \, dx$$

$$= 2\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 \quad (1)$$

$$= 2\left[\left(32 - \frac{64}{3}\right) - (0 - 0)\right]$$

$$= 2\left[\frac{32}{3}\right]$$

$$\therefore A = \frac{64}{3} \left(\text{or } 21\tfrac{1}{3}\right) u^2 \quad (1)$$

d. $y = e^x - e^{-x}$

$$y^2 = (e^x - e^{-x})(e^x - e^{-x})$$

$$= e^{2x} - e^0 - e^0 + e^{-2x}$$

$$\therefore y^2 = e^{2x} + e^{-2x} - 2 \quad (1)$$

$$\therefore \text{Volume, } V = \pi \int_0^{\frac{1}{2}} y^2 \, dx$$

$$= \pi \int_0^{\frac{1}{2}} (e^{2x} + e^{-2x} - 2) \, dx \quad (1)$$

$$= \pi\left[\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} - 2x\right]_0^{\frac{1}{2}} \quad (1)$$

$$= \pi\left[\left\{\frac{1}{2}e^{(2\times\frac{1}{2})} - \frac{1}{2}e^{-2(\frac{1}{2})} - 2\left(\tfrac{1}{2}\right)\right\} - \left\{\frac{1}{2}e^0 - \frac{1}{2}e^0 - 0\right\}\right]$$

$$= \pi\left[\left(\frac{e^1}{2} - \frac{e^{-1}}{2} - 1\right) - \left(\cancel{\tfrac{1}{2} - \tfrac{1}{2}}\right)\right]$$

$$= \pi\left[\frac{e}{2} - \frac{e^{-1}}{2} - 1\right] \qquad -\frac{e^{-1}}{2} = \frac{-\frac{1}{e}}{\frac{2}{1}}$$

$$= \pi\left[\frac{e}{2} - \frac{1}{2e} - 1\right] \qquad = \frac{-1}{2e}$$

$$\therefore \text{Vol, } V = \pi\left(\frac{e}{2} - \frac{1}{2e} - 1\right) u^3 \quad (1)$$