SAMPLE SOLUTIONS ONLY.

2017 YEAR 12 MATHEMATICS - MID COURSE EXAMINATION

SECTION I - MULTIPLE CHOICE.

1. $\sqrt{\frac{\pi+1}{5}} = 0.910 \ (3SF)$ B

2. $2x - 4y + 3 = 0$

$-4y = -2x - 3$

$4y = 2x + 3$

$y = \frac{2x}{4} + \frac{3}{4}$

$\therefore m = \frac{1}{2}$ C

3. $2x - 3y = 8$

$-3y = -2x + 8$

$3y = 2x - 8$

$y = \frac{2x}{3} - \frac{8}{3}$.

$\therefore m_1 = \frac{2}{3}$.

ie $\perp$ has $m_2 = \frac{-3}{2}$ $P(2,0)$

$y - 0 = \frac{-3}{2}(x-2)$

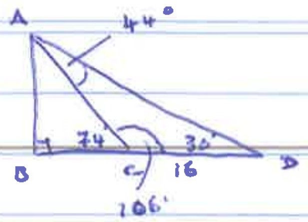
$2y = -3x + 6$

$3x + 2y - 6 = 0$ B.

4. $27x^3 - 8 = (3x)^3 - 2^3$

$= (3x-2)(9x^2 + 6x + 4)$ B

5.



$$\frac{16}{\sin 44^\circ} = \frac{AC}{\sin 30^\circ}$$

$$AC \sin 44^\circ = 16 \sin 30^\circ$$

$$AC = \frac{16 \sin 30^\circ}{\sin 44^\circ}$$

$$AC = 11.52$$

$$\therefore \sin 74^\circ = \frac{AB}{11.52}$$

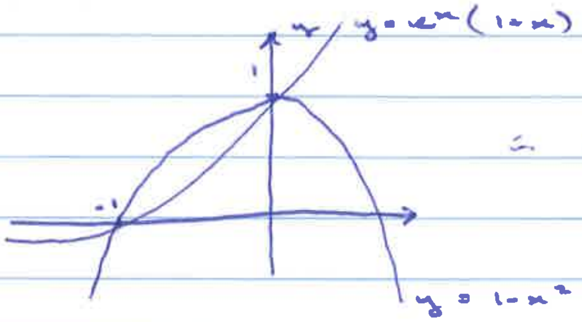
$$AB = 11.52 \sin 74^\circ$$

$$\therefore AB = 11.07$$ B.

6. $\sin\theta > 0$, $\cos\theta < 0$

$\therefore$ 2nd quadrant B

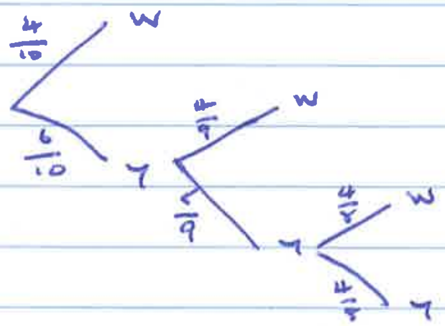
7.



$\therefore e^x(1+x) = 1-x^2$

has two solutions C

8. $P(\geq W) = 1 - P(YYY)$.



$$\therefore P(\geq 1W) = 1 - P(YYY)$$

$$= 1 - \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right)$$

$$= 1 - \frac{120}{720}$$

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$ C

9. $2x + 58^\circ + 66^\circ = 180^\circ$

$2x + 124^\circ = 180^\circ$

$2x = 56^\circ$

$\therefore x = 28^\circ$

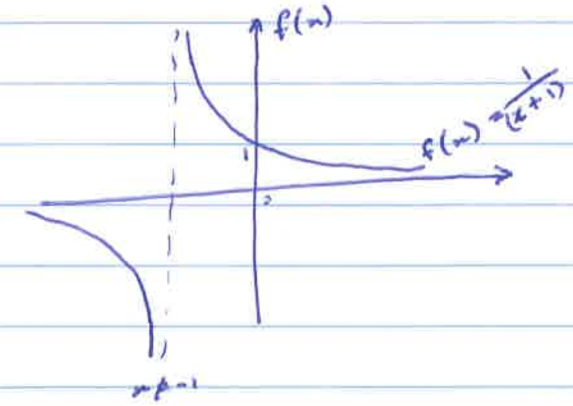
A.

10. $f(x) = \log_e(x+1)$

$f'(x) = \frac{1}{(x+1)} \cdot 1$

$= \frac{1}{x+1}$

A.



SECTION II

Q11. (a) $y = 3x^2$

when $x = 1$

$y = 3(1)^2$

$= 3$.

$\therefore P(1,3)$.

$\frac{dy}{dx} = 6x$

when $x = 1$

$\frac{dy}{dx} = 6(1)$

$= 6$.

$\therefore y - 3 = 6(x-1)$

$y - 3 = 6x - 6$

$0 = 6x - 6 - y + 3$

$6x - y - 3 = 0$.

ie tangent is $6x - y - 3 = 0$.

(b) $\sin^2\theta \cos\theta \csc^2\theta = \cot\theta$

$LHS = \sin^2\theta \cos\theta \csc^2\theta$

$= \cancel{\sin^2\theta} \cos\theta \times \frac{1}{\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= RHS$.

ie $\sin^2\theta \cos\theta \csc^2\theta = \cot\theta$

8 (c) $\lim_{x \to 1} \dfrac{x^4 - x^2}{x^2 - 1}$

$= \lim_{x \to 1} \dfrac{x^2 \cancel{(x^2 - 1)}}{\cancel{(x^2 - 1)}}$

$= \lim_{x \to 1} x^2$

$= 1^2$

$= 1$

$\therefore \lim_{x \to 1} \dfrac{x^4 - x^2}{x^2 - 1} = 1$

(d) $\int e^{3x+1}\, dx = \frac{1}{3} e^{3x+1} + c$

(e) $y = ax^3 + bx^2 + cx + d$

$\dfrac{dy}{dx} = 3ax^2 + 2bx + c$

$\dfrac{d^2y}{dx^2} = 6ax + 2b$

For point of inflexion $\dfrac{d^2y}{dx^2} = 0$

ie. $6ax + 2b = 0$

$6ax = -2b$

$x = \dfrac{-2b}{6a}$

i.e. $x = \dfrac{-b}{3a}$

ie $p = \dfrac{-b}{3a}$.

11 (f) $y = x^2 e^{2x}$

$$\frac{dy}{dx} = x^2 \cdot e^{2x} \cdot 2 + e^{2x} \cdot 2x$$

$$= 2x^2 e^{2x} + 2x e^{2x}$$

$$\therefore \frac{dy}{dx} = 2xe^{2x}(x+1)$$

(g) $$\int_3^4 \frac{4x-5}{2x^2-5x}\,dx = \left[\log_e(2x^2-5x)\right]_3^4$$

$$= \left[\log_e(32-20) - \log_e(18-15)\right]$$

$$= \left[\log_e(12) - \log_e(3)\right]$$

$$= \log_e\left(\frac{12}{3}\right)$$

$$\therefore \int_3^4 \frac{4x-5}{2x^2-5x} = \log_e 4$$

Q12 (a) (i) $x^2 - 6x + 2 = 0$

$$\alpha + \beta = \frac{-(-6)}{1}$$
$$= 6$$

(ii) $$\alpha\beta = \frac{2}{1}$$
$$= 2$$

(iii) $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$
$$= \frac{6}{2}$$
$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = 3$$

(b) $$2^{2x+1} = 64$$
$$= 2^6$$
$$\therefore 2x + 1 = 6$$
$$2x = 5$$
$$\therefore x = \frac{5}{2}$$

(c) $P(-1, 4)$ $\frac{dy}{dx} = 6x - 2$

$$y = \int 6x - 2 \, dx$$
$$y = \frac{6x^2}{2} - 2x + c$$
$$y = 3x^2 - 2x + c$$

when $x = -1$
$y = 4$

$$4 = 3(-1)^2 - 2(-1) + c$$
$$4 = 3 + 2 + c$$
$$4 = 5 + c$$
$$\therefore c = -1$$

Now $y = 3x^2 - 2x - 1$.

12 (d) P(defective) = 0.04

P(non-defective) = 1 - 0.04

= 0.96

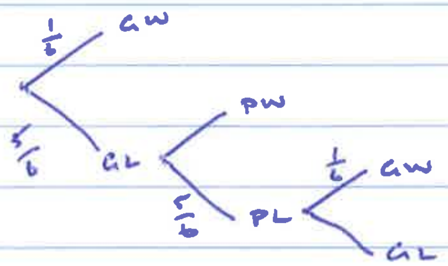
Expected non-defective = 0.96 × 1200

= 1152.

(e) (i) P(GW) = $\frac{6}{36}$

= $\frac{1}{6}$

(ii)



∴ P(GW on 1 or 2 throw) = $\frac{1}{6} + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{1}{6} + \frac{25}{216}$

$= \frac{61}{216}$ or 0.2824 (4DP).

(f) Distance, d ≑ $\frac{5}{3}\left[173 + 168 + 4(81 + 195) + 2(127)\right]$

$= \frac{5}{3}\left[341 + 1104 + 254\right]$

$= \frac{5}{3}\left[1699\right]$

= 2831.66...

∴ Distance, d = 2832 m (0DP).

Q 13 (a) (i) $F(x) = 9(2x+3)^5$

$F'(x) = 9.5(2x+3)^4 \cdot 2$

$= 45(2x+3)^4 . 2$

$\therefore F'(x) = 90(2x+3)^4$

(ii) $y = \dfrac{5}{x\sqrt{x}}$

$= \dfrac{5}{x^1 . x^{\frac{1}{2}}}$

$= \dfrac{5}{x^{\frac{3}{2}}}$

$\therefore y = 5x^{-\frac{3}{2}}$

$\dfrac{dy}{dx} = 5 . -\dfrac{3}{2} x^{\left(-\frac{3}{2} - \frac{2}{2}\right)}$

$\therefore \dfrac{dy}{dx} = -\dfrac{15}{2} x^{-\frac{5}{2}}$

(iii) $y = (x-1)\log_e x$

$\dfrac{dy}{dx} = (x-1) \cdot \dfrac{1}{x} . 1 + \log_e x \cdot 1$

$\therefore \dfrac{dy}{dx} = \dfrac{(x-1)}{x} + \log_e x$

(b) $f(x) = \dfrac{2x}{x-3}$, $x \neq 3$

$f'(x) = \dfrac{(x-3).2 - 2x.1}{(x-3)^2}$

$= \dfrac{\cancel{2x} - 6 - \cancel{2x}}{(x-3)^2}$

$\therefore f'(x) = \dfrac{-6}{(x-3)^2}$

For decreasing ~~curve~~ curves $f'(x) < 0$.

$f'(x) < 0$ for all x, except $x = 3$ ($x \neq 3$).

13 (c) (i) $M = t^3 - 6t^2 + 9t$, $0 \le t \le 5$ (t hours).

$= t(t^2 - 6t + 9)$

$M = t(t-3)(t-3)$

When $M = 0$

$t = 0$ or 3 hours.

ie Angus has no medicine in his blood at $t = 0$ ~~or~~ & 3 hours.

(ii) $M = t^3 - \cancel{6t} 6t^2 + 9t$

$= t(t-3)(t-3)$

$$\frac{dM}{dt} = 3t^2 - 12t + 9$$

$$= 3(t^2 - 4t + 3)$$

$$= 3(t-3)(t-1)$$

$$\frac{d^2M}{dt^2} = 6t - 12$$

$$= 6(t-2)$$

For stationary values $\frac{dM}{dt} = 0$

ie $3(t-3)(t-1) = 0$

$\therefore$ $t = 1$ or 3.

$\therefore$ stationary point at $(1, 4)$ and $(3, 0)$

For max or min

When $x = 1$

$$\frac{d^2M}{dt^2} = 6(1) - 12$$

$$= 6 - 12$$

$$= -6$$

$$< 0$$

$\therefore$ at $(1, 4)$ a MAX value (ie. concave down). $\cap$

When $x = 3$

$\frac{d^2M}{dt^2} = 6(3) - 12$

$= 18 - 12$

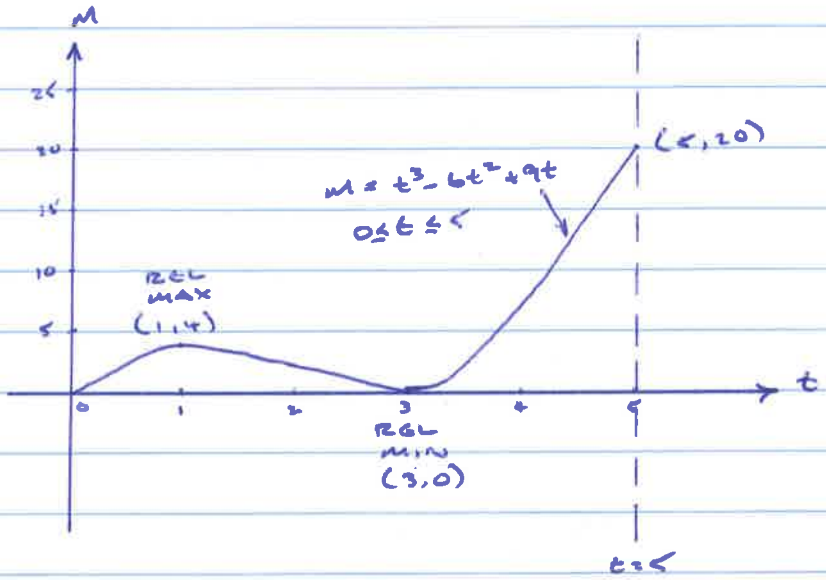
$= 6$

> 0.

$\therefore$ at $(3, 0)$ a ~~max~~ MIN value (i.e. concave up). ✓

(iii) When $t = 5$

$M = 20$



Q14 (a) (i) $ax^2 - 12x + 10 = 0$, $a \neq 0$

$\Delta = (-12)^2 - 4(a)(10)$

$\therefore \Delta = 144 - 40a$

(ii) When $ax^2 - 12x + 10 > 0$

Then $\Delta < 0$

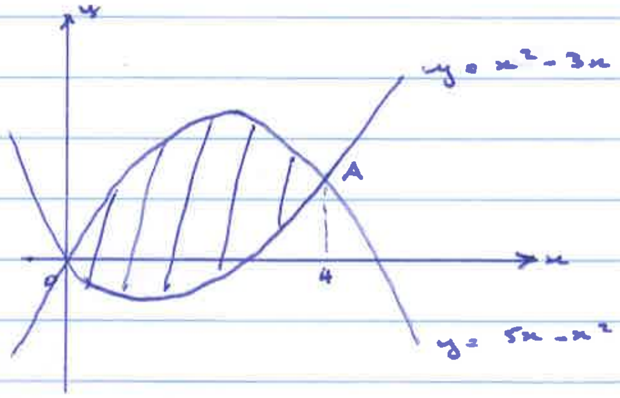
$\therefore$ $144 - 40a < 0$

$144 < 40a$

$\frac{144}{40} < a$

$\therefore a > \frac{18}{5}$ or (3.6).

b.



(i) $y = x^2 - 3x$ — ①

$y = 5x - x^2$ — ②

Let ① = ②

$x^2 - 3x = 5x - x^2$

$2x^2 - 8x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$\therefore x = 0$ or 4

$\therefore$ Upper limit 4

Lower limit 0

$$\text{Area, } A = \int_0^4 (5x - x^2) - (x^2 - 3x)\,dx$$

$$= \int_0^4 (5x - x^2 - x^2 + 3x)\,dx$$

$$\therefore A = \int_0^4 8x - 2x^2\,dx$$

(ii) $$A = \int_0^4 8x - 2x^2\,dx$$

$$= \left[\frac{8x^2}{2} - \frac{2x^3}{3}\right]_0^4$$

$$= \left[4x^2 - \frac{2x^3}{3}\right]_0^4$$

$$= \left[\left(64 - \frac{128}{3}\right) - (0 - 0)\right]$$

$$= \left(64 - \frac{128}{3}\right)$$

$$= \frac{64}{3}$$

$$\therefore A = 21\tfrac{1}{3}\ u^2. \quad (\underline{\text{or}}\ 21.\dot{3}\ u^2).$$

(d.) $$y = e^x - e^{-x} \qquad 0 \le x \le \tfrac{1}{2}$$

$$y^2 = (e^x - e^{-x})(e^x - e^{-x})$$

$$= e^{(x+x)} - e^{(x-x)} - e^{(-x+x)} + e^{(-x-x)}$$

$$= e^{2x} - e^0 - e^0 + e^{-2x}$$

$$\therefore y^2 = e^{2x} + e^{-2x} - 2$$

$$\therefore \text{Vol, } V = \pi \int_0^{\frac{1}{2}} e^{2x} + e^{-2x} - 2\,dx$$

$$= \pi\left[\tfrac{1}{2}e^{2x} - \tfrac{1}{2}e^{-2x} - 2x\right]_0^{\frac{1}{2}}$$

$$= \pi\left[\left(\tfrac{1}{2}e^1 - \tfrac{1}{2}e^{-1} - 1\right) - \left(\tfrac{1}{2}e^0 - \tfrac{1}{2}e^0 - 0\right)\right]$$

$\therefore$ Vol, $V = \pi\left[\left(\frac{e}{2} - \frac{1}{2e} - 1\right) - \left(\cancel{\frac{1}{2} - \frac{1}{2}} - 0\right)\right]$

i.e. $V = \pi\left[\frac{e}{2} - \frac{1}{2e} - 1\right]$