Chapter 6

SERIES AND APPLICATIONS

Facts and Formulas

EXERCISE 1

Circle the correct answer:

1. The n^{th} term of an arithmetic sequence is given by:

(A) $T_n = a + (n - 1)d$ (B) $T_n = \frac{n}{2}[2a+(n-1)d]$

(C) $T_n = a + nd$ (D) $T_n = a(n - 1)d$

2. The sum of n terms of an arithmetic series, given a and d.

(A) $S_n = \frac{n}{2}[a+l]$ (B) $S_n = \frac{n}{2}[a+(n-1)d]$

(C) $S_n = \frac{n}{2}[2a+(n-1)d]$ (D) $S_n = 2n[2a+(n-1)d]$

3. The sum of n terms of an arithmetic series, given a and l.

(A) $S_n = \frac{n}{2}[a+2l]$ (B) $S_n = \frac{n}{2}[a+l]$

(C) $S_n = n[a+l]$ (D) $S_n = \frac{n}{2}[2a+l]$

4. The n^{th} term of a geometric progression is given by:

(A) $T_n = ar^n$ (B) $T_n = a + r^n$

(C) $T_n = ar^{n-1}$ (D) $T_n = a + r^{n-1}$

5. The sum of n terms of a geometric series is given by:

(A) $S_n = \frac{a(r^n-1)}{(r-1)}$ (B) $S_n = \frac{a(r-1)^n}{(r+1)}$

(C) $S_n = \frac{a^n(r-1)}{(r-1)}$ (D) $S_n = \frac{a(r^n-1)}{(1-r)}$

6. The limiting sum of a geometric series is given by:

(A) $S_\infty = \frac{a}{r-1}$ (B) $S_\infty = \frac{a}{r+1}$

(C) $S_\infty = \frac{a}{1-r}$ (D) $S_\infty = \frac{a-1}{r}$

7. The condition for an infinite geometric series to have a limiting sum is:

(A) $r > 1$ (B) $r < 1$

(C) $r > -1$ (D) $-1 < r < 1$

8. Formula for Compound Interest

(A) $A = P(1 - \frac{r}{100})^n$ (B) $A = P(1 + \frac{r}{100})^n$

EXERCISE 2

1. Which of the following are sequences and which are series?:

(a) 3, 5, 7, 9, 11, ...

(b) 2 + 4 + 8 + 16 + ...

(c) − 7 − 5 − 3 − 1 . . . .

(d) ½, ¼, ⅛, ...

2. Find the next two terms in each sequence:

(a) 4, 7, 10, 13, ...

(b) 2, 6, 18, 54, ...

(c) 1, 8, 27, 64, ...

(d) 0, 3, 8, 15, 24, ...

(e) 1, 1, 2, 3, 5, 8, ...

Arithmetic Progressions

EXERCISE 3

1. For each of the following, state the common difference:.

(a) 6, 9·4, 12·8, ...

(b) 8, −1, −10, −19, . . .

2. The n^{th} term of an arithmetic sequence is given by

$$T_n = \frac{5n+4}{2}$$

Evaluate the common difference d.

3. Find the value of x which makes the sequence 19, x, 81 arithmetic.

4. For an A.P. $a = 4$ and $d = 9$.

Find (a) T_{24}

(b) S_{24}

5.* For the A.P. : $\sqrt{5}$, $\sqrt{45}$, $\sqrt{125}$, $\sqrt{245}$, ...
Find d and T_{10}

6. Given the sequence 4, 7, 10, 13, ...
(a) Find T_{20}

(b) Find S_{20}

7. Find S_{15} for the A.P. 8, 12, 16, 20, . . .

8. Given the sequence 20, 14, 8, 2, ...

(a) Find T_{18}

(b) Find S_{18}

9. $x + 3$, $2x - 5$, $4x + 6$ are three consecutive terms of an A.P.

(a) Evaluate x.

(b) State the three terms.

(c) State the common difference.

10. $x - 8$, $3x + 4$, $6x - 10$ are three consecutive terms of an A.P.

(a) Evaluate x.

(b) State the three terms.

(c) State the common difference.

11. $y + 3$, $2y + 7$, $3y + 11$, $4y + 15$ are four consecutive terms of an A.P.

(a) If $y = 5$ evaluate the terms and state the common difference.

(b) If $y = -10$ evaluate the terms and state the common difference.

12. Which term of 2, 9, 16, 23, ... has a value of 254?

13. How many terms of 2, 9, 16, 23, ... are needed to give a sum of 16082?

14.* Which term of $-3, 1, 5, 9, 13, \ldots\ldots\ldots$ is the first term to have a value greater than 200?

15. How many terms of $8, 5, 2, -1, -4, \ldots\ldots$ are needed to give a sum of -4495?

16. Which term of $546, 543, 540, \ldots\ldots\ldots\ldots$ is the first term to be negative?

17. Evaluate $\sum_{n=1}^{20}(9n-4)$.

18. Evaluate $\sum_{n=1}^{18}(23-8n)$.

19. Evaluate $\sum_{n=1}^{12}(1-4n)$.

20. Evaluate $\sum_{n=5}^{30}(6n-10)$.

21. Evaluate k if $\sum_{n=1}^{k}(5n+7) = 1800$.

22.* Find the sum of all integers from 1 to 70 that are not divisible by 9.

23. The 50[th] term of an A.P. is 337. The 72[nd] term of this A.P. is 491. Find:

(a) d

(b) a

(c) T_{23}

(d) S_{23}

24.* A bell rings at 6:32 am and then every 3 minutes until it last rings at 10:14 am. Using arithmetic sequences calculate the number of times the bell rings.

25. An A.P. with $a = 13$ has $S_{30} = 3435$. Find the value of d.

26. An A.P. with $a = 24$ has $S_{30} = -3195$. Find the value of d.

27. Which term of an A.P. with $a = 2$ and $d = 3$ is the first term greater than 500?

28. For an A.P., $T_{20} = -61$ and $T_{48} = -173$. Evaluate a, d and S_{48}.

29. Michael wants to improve his fitness. He runs 1·5 km the first day, then increases the distance by 400 m every day. On which day does he run 7·5 km? How far has he run altogether from day one?

30. Michelle starts a new job and the first week she deposits \$45 into her savings account. She then increases the deposit by \$5 each week. How much does she deposit in the 17th week? How much has she saved altogether?

31. Find the sum of the first 200 positive integers. Hence, or otherwise, find the sum of :
1, 2, 4, 5, 7, 8, 10, 11, 13, ... 200.

32. The sum of n terms of a series is given by $S_n = \dfrac{n^2 + 5n}{2}$.

(a) Find S_{12}

(b) How many terms are required to give a sum of 525?

33. Evaluate $\sum_{n=5}^{30}(19 - 7n)$.

34. Find the sum of all consecutive even numbers between 56 and 193.

35. How many terms of the series $-18 - 15 - 12$ . . . . must be taken to give a sum of 1521?

36. The sum of the first 30 terms of an A.P. whose first term is 5, is 1890. Find the common difference.

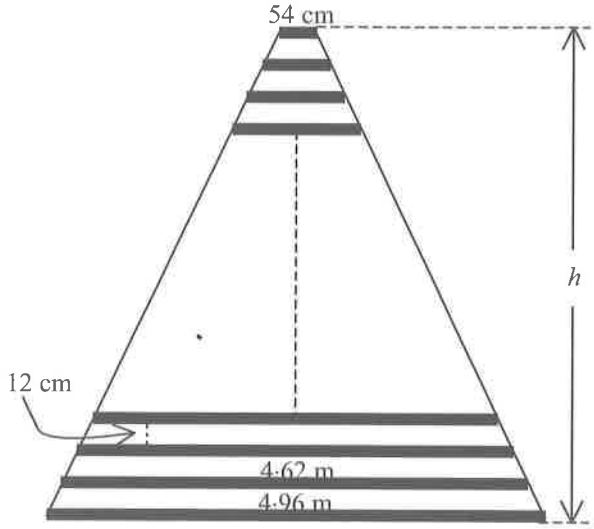
37. Tim opens a savings account with a small deposit. He increases his deposit weekly by a constant amount. After 8 weeks he has saved \$280, and after 40 weeks he has saved \$5240. What was his initial deposit and by how much did he increase his deposit each week?

38.* Peter throws a ball 12 m vertically into the air. Each successive throw is 14 cm less in height than the previous throw.

(a) How high does the ball reach on his 15th throw?

(b) What total distance does the ball travel in the air after the 15 throws?

39.* The horizontal cylindrical bars shown in the diagram are equally spaced and there is a common difference in the length of consecutive bars.



(a) How many bars are there altogether?

(b) What is the total length of all the horizontal bars?

(c) Each bar has a diameter of 5 cm, and the bars are spaced 12 cm apart. Calculate the height h.

Geometric Progressions

EXERCISE 4

1. For each of the following state the common ratio r:

(a) $2, 8, 32, \ldots\ldots$

(b) $27, 9, 3, \ldots\ldots$

(c) $4, -12, 36, \ldots$

(d) $12, -30, 75, \ldots$

(e) $b^2, b^6, b^{10}, \ldots$

2. The n^{th} term of a G.P. is given by :

$T_n = 5(2^{n-1})$

(a) Evaluate a.

(b) Evaluate r.

(c) Find T_{10}.

3. A positive integer x is inserted so that $48, x, 27, \ldots$ is a G.P: Find the value of x.

4. For a G.P. $a = 6$ and $r = 4$, find:

(a) T_7

(b) S_5

5. Find T_{12} for $x^3\ x^5, x^7, \ldots\ldots$

6. Find T_8 for $e^6, e^4, e^2, \ldots\ldots$

7. Evaluate x if x, 15, $9x \ldots\ldots$ form a G.P.

8. If $a = 4$ and $r = ½$ find:

(a) T_9.

(b) S_9.

9. Find T_{11} and S_{11} for 6, −18, 54, ...

10. For the sequence 20, 5, $1\frac{1}{4}$, ... find:

(a) T_8 (as a basic fraction)

(b) S_8 (to 1 dec. pl.)

11. Solve for n: $3^{n-1} = 19683$

12. What term of the sequence 3, 6, 12, ...is 12288?

13. How many terms of $\frac{1}{2}$, 1, 2, 4, ... are needed to give a sum of 4095·5?

14. For the sequence $2, \sqrt{8}, 4, \sqrt{32}, 8, \ldots\ldots$ find:

(a) T_{10} (as a surd)

(b) S_{10} (to 2 sig. fig.)

15. Which term of the sequence $\frac{1}{2}$, $1\frac{1}{2}$, $4\frac{1}{2}$, ... is the first to exceed 5000?

working continues next page

16.* Which term of the sequence 800, 700, 612·5, ... is the first term to be less than 80? (to 3 sig. fig.)

17. For the geometric sequence with $T_4 = 80$ and $T_9 = 2560$ find:

(a) r

(b) a

(c) T_7

(d) S_{12}

18. For a series $S_n = 2(3)^n - 2$, evaluate:

(a) S_6

(b) S_1

(c) S_2

(d) S_3

(e) a

(f) r

(g) T_7

19. (a) Evaluate $\sum_{n=1}^{9} 3(4)^{n-1}$

(b) Evaluate $\sum_{n=1}^{20} 12(\frac{1}{6})^{n-1}$

(c) Evaluate $\sum_{n=1}^{8} \frac{1}{4}(2)^{n-1}$

20. Evaluate $\sum_{n=1}^{10} 2^{2n-1}$

21. Evaluate $\sum_{n=5}^{12} (3)(-2)^{n}$

22. Evaluate
$1 + (\frac{1}{3}) + (\frac{1}{3})^2 + (\frac{1}{3})^3 + \ldots\ldots\ldots + (\frac{1}{3})^7$
(as an improper fraction)

23.* The sum of the 3rd and 6th terms of a G.P. is 18. The sum of the 6th and 9th terms of the same G.P. is 144. Evaluate a and r.

24. John receives a salary of $39 600 in the first year of his employment. His salary increases by 6% each year.

(a) How much will he earn in his 7th year? (nearest $)

(b) How much will he have earned altogether at the end of 7 years? (nearest $)

25.* Find an expression for the n^{th} term (T_n) of the sequence: 5, 10, 17, 28, 47, 82, ...
Hint: $T_1 = 2 + 3 = 5$
$T_2 = 4 + 6 = 10$
$T_3 = 8 + 9 = 17$ etc

Hence evaluate T_{15}.

Limiting Sum of an Infinite Geometric Series

EXERCISE 5

1. Determine which of the following G.P.'s have limiting sums and, if they exist, find their value:

(a) −10, 5, −2·5, ...

(b) 12, −18, 27, ...

(c) 1000, 900, 810, ...

(d) 9, 6, 4, ...

(e) 3, 3·3, 3·63, ...

(f) $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\ldots$

(g) $e^6, e^5, e^4, \ldots\ldots\ldots\ldots (e \approx 2.7)$

2. Determine the range of values of x for which the G.P. $x, \frac{3x^2}{2}, \frac{9x^3}{4}, \ldots\ldots$ will have a limiting sum.

3. Evaluate S_∞ for a G.P. given:

(a) $a = 10$ and $r = \frac{1}{5}$

(b) $a = 14$ and $r = -\frac{3}{4}$

4. A G.P. has a limiting sum of 40 and a common ratio of 0·4. Calculate:

(a) a

(b) T_5

5. John donates \$100 to his favourite charity. He decides to continue donating each year ¾ of the amount donated the previous year.

(a) How much will he have donated after 5 years?

(b) In theory, what is the total amount that the charity will receive?

6. Use the infinite geometric series formula to express each of the following as a basic fraction.

(a) $0\cdot\dot{4}1\dot{3}$

(b) $0\cdot 3\dot{8}$

(c) $2\cdot5\dot{1}\dot{6}$

7.* A ball is dropped from a height of 32 m. It rebounds to a height of 19·2 m and on each successive bounce it reaches 0·6 of its previous height. In theory, what is the total distance the ball will travel?

8. Evaluate $\sum_{n=1}^{\infty}(2)(\frac{1}{5})^{n-1}$.

9. Evaluate $\sum_{n=1}^{\infty}(6)(-3)^{n-1}$.

Compound Interest, Superannuation and Time Payment

EXERCISE 6

1. Martin invests \$7 000 at 5·25% p.a., compounded annually.

(a) What is the value of his investment after 10 years?

(b) How much interest did he receive?

(c) Find the equivalent simple interest rate (2 dec. pl.)

2. Janis invests \$8 000 at 6% p.a., compounded quarterly. What is the value of her investment after 5 years?

3. What amount of money needs to be invested at 5% p.a. (compounded annually) to grow to \$15 000 in 6 years? (nearest \$)

4. The purchase price of a car is \$36 000. It depreciates at a rate of 11% p.a. What is its value after 7 years?

5. Jasmine deposits \$600 into an account on the 1st December each year starting in 1995. She receives an interest rate of 6% p.a. compounded annually.

(a) How much will her total investment be worth by December 2008?

(b) How much interest has she received? (whole dollars)

6.* Suzanne invests \$2000 at the beginning of each year for 20 years. Interest is compounded 6 monthly at 5·6% p.a.

(a) Calculate the value of the investment at the end of this period.

(b) How much interest has been earned?

7.* Cathy borrows \$18 000 to buy a new car. She is charged interest at12% p.a. compounded monthly, on the balance owing. The loan is to be repaid in equal monthly instalments over 5 years. Let A_n be the amount owing after the n^{th} monthly repayment (M) has been made.

(a) Find expressions for A_1, A_2, A_3 and A_n.

(b) Calculate her monthly instalments.

(c) Calculate the equivalent simple interest rate.

8. Marena deposits \$20 000 into an investment account. Interest paid is 7% p.a. compounded annually. She withdraws \$P exactly 12 months after this deposit is made. She continues to withdraw \$P at the same time each year. After making 6 withdrawals her account balance is zero. Calculate the value of P.

9. Darren borrows \$70,000 and agrees to pay the loan plus interest in 40 equal quarterly instalments. Interest is charged at 4% per quarter and is calculated quarterly on the balance owing.
Let A_n = amount owing after n quarters
Let M = quarterly instalments

(a) Find an expression for A_1.

(b) Find an expression for A_2.

(c) Find an expression for A_3.

(d) Find an expression for A_n.

(e) Find the amount of each quarterly instalment.

(f) Find the amount owing after 3 years.

FORMULAS

For questions 1, 2, 3 and 4 complete each formula for Arithmetic Progressions:

1. $T_n =$

2. $S_n =$ (using a and d)

3. $S_n =$ (using a and l)

4. $T_n =$ (using S_n and S_{n-1})

For questions 5, 6 and 7 complete each formula for Geometric Progressions:

5. $T_n =$

6. $S_n =$

7. $S_\infty =$

8. Compound Interest Formula $A =$

9. In an arithmetic sequence there is a between successive terms.

10. In a geometric sequence there is a between successive terms.

11. The condition for the limiting sum of a G.P. to exist is .

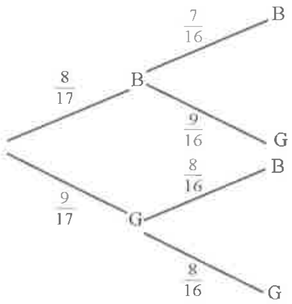
12. The symbol used for the summation of a series is .

Exercise 3 (page 95)

1. 279, 297, 729, 792, 927, 972

2. 11, 14, 16, 41, 44, 46, 61, 64, 66 **3.** *

4.(a)



(b) $\frac{7}{34}$ **(c)** $\frac{9}{17}$ **5.(a)** $\frac{1}{16}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{24}$ **(d)** 12

(e) $\frac{7}{48}$ **6.** * **7.(a)** $\frac{1}{320}$ **(b)** $\frac{1}{64}$ **(c)** $\frac{1}{40}$

8.(a) $\frac{1}{81}$ **(b)** $\frac{16}{81}$ **(c)** $\frac{65}{81}$ **(d)** $\frac{8}{81}$ **9.** $\frac{1}{138}$

10.(a) 16·25% **(b)** 48·75% **(c)** 7%

11.(a) 24 **(b)** ½ **12.(a)** $\frac{9}{100}$ **(b)** $\frac{26}{225}$

(c) $\frac{52}{225}$ **(d)** $\frac{51}{100}$

Exercise 4 (page 97)

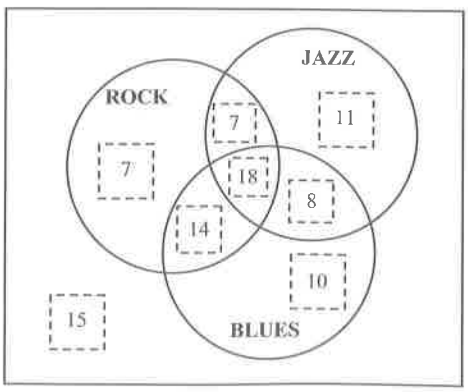
1. * **2.** * **3.(a)** $\frac{245}{1199}$ **(b)** $\frac{119}{801}$ **(c)** $\frac{101}{198}$

4.(a) ½ **(b)** $\frac{3}{10}$ **(c)** $\frac{13}{40}$ **5.** *

Exercise 5 (page 99)

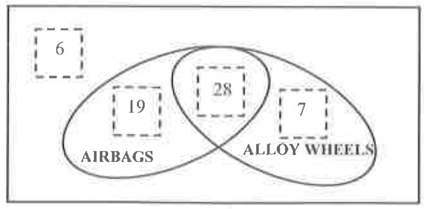
1. P(A) + P(B) – P(A and B) **2.** $\frac{31}{40}$ **3.** 8

4.(a)



(b) 15 **(c)** $\frac{7}{90}$ **(d)** $\frac{13}{18}$ **(e)** $\frac{5}{6}$

5.(a)



(b) $\frac{1}{10}$ **(c)** $\frac{7}{15}$ **(d)** $\frac{9}{10}$ **(e)** $\frac{19}{60}$ **6.** *

Summary (page 101)

1. outcomes, outcomes **2.** sample space **3.** 1

4. 0, 1 **5.** 0 **6.** 1 **7.** mutually exclusive

8. P(A) + P(B) **9.** P(A) + P(B) – P(A and B)

10. dependent **11.** independent **12.** not

13. complementary, 1 **14.** tree

15. probability tree **16.** c × d

17. product, P(A) × P(B) **18.** added

Series and Applications

Exercise 1 (page 103)

1. A **2.** C **3.** B **4.** C **5.** A **6.** C

7. D **8.** B

Exercise 2 (page 104)

1.(a) sequence **(b)** series **(c)** series **(d)** sequence

2.(a) 16, 19 **(b)** 162, 486 **(c)** 125, 216

(d) 35, 48 **(e)** 13, 21

Exercise 3 (page 104)

1.(a) 3·4 **(b)** –9 **2.** 2·5 **3.** 50

4.(a) 211 **(b)** 2580 **5.** *

6.(a) 61 **(b)** 650 **7.** 540

8.(a) –82 **(b)** –558 **9.(a)** –19

(b) –16, –43, –70 **(c)** –27 **10.(a)** 26

(b) 18, 82, 146 **(c)** 64 **11.(a)** 8, 17, 26, 35, $d = 9$

(b) –7, –13, –19, –25, $d = -6$ **12.** 37

13. 68 **14.** * **15.** 58 **16.** T_{184} **17.** 1810

18. –954 **19.** –300 **20.** 2470

21. 25 **22.** *

23.(a) 7 **(b)** –6 **(c)** 148 **(d)** 1633

24. * **25.** 7 **26.** –9 **27.** $T_{168} = 503$

28. 15, −4, −3792 **29.** Day 16, 72 km

30. \$125, \$1445 **31.** 13 467

32.(a) 102 **(b)** 30 **33.** −2691

34. 8500 **35.** 39 **36.** 4 **37.** \$14, \$6

38. * **39.** *

Exercise 4 (page 111)

1.(a) 4 **(b)** ⅓ **(c)** −3 **(d)** −2·5 **(e)** b^4

2.(a) 5 **(b)** 2 **(c)** 2560 **3.** 36 **4.(a)** 24576

(b) 2046 **5.** x^{25} **6.** e^{-8} **7.** ±5

8.(a) $\frac{1}{64}$ **(b)** $7\frac{63}{64}$ **9.** 354 294, 265 722

10.(a) $\frac{5}{4096}$ **(b)** 26·7 **11.** 10 **12.** T_{13}

13. 13 **14.(a)** $32\sqrt{2}$ **(b)** 150 **15.** $T_{10} = 9841{\cdot}5$

16. * **17.(a)** 2 **(b)** 10 **(c)** 640 **(d)** 40 950

18.(a) 1456 **(b)** 4 **(c)** 16 **(d)** 52

(e) 4 **(f)** 3 **(g)** 2916

19.(a) 262 143 **(b)** $14\frac{2}{5}$ **(c)** $63\frac{3}{4}$

20. 699 050 **21.** 8160 **22.** $\frac{3280}{2187}$

23. * **24.(a)** \$56 173 **(b)** \$332 396 **25.** *

Exercise 5 (page 115)

1.(a) $-6\frac{2}{3}$ **(b)** no **(c)** 10 000 **(d)** 27

(e) no **(f)** 0·5 **(g)** $\frac{e^7}{e-1}$ **2.** $-\frac{2}{3} < x < \frac{2}{3}$

3.(a) 12·5 **(b)** 8 **4.(a)** 24 **(b)** 0·6144

5.(a) \$305·08 **(b)** \$400 **6.(a)** $\frac{413}{999}$

(b) $\frac{7}{18}$ **(c)** $2\frac{511}{990}$ **7.** * **8.** 2·5 **9.** 1·5

Exercise 6 (page 117)

1.(a) \$11 676·67 **(b)** \$4676·67 **(c)** 6·68% p.a.

2. \$10 774·84 **3.** \$11 193 **4.** \$15 923·28

5.(a) \$12 009 **(b)** \$4209 **6.** * **7.** *

8. \$4195·92 **9(a)** 70 000(1·04) – M

(b) $70\,000(1{\cdot}04)^2 - M(1 + 1{\cdot}04)$

(c) $70\,000(1{\cdot}04)^3 - M(1 + 1{\cdot}04 + 1{\cdot}04^2)$

(d) $70\,000(1{\cdot}04)^n - 25M(1{\cdot}04^n - 1)$

(e) \$3536·64 **(f)** \$58 931·39

Formulas (page 121)

1. $a + (n-1)d$ **2.** $\frac{n}{2}\left[2a + (n-1)d\right]$ **3.** $\frac{n}{2}\left[a + l\right]$

4. $S_n - S_{n-1}$ **5.** ar^{n-1} **6.** $\dfrac{a(r^n - 1)}{r - 1}$

7. $\dfrac{a}{1-r}$ **8.** $P(1 + r\%)^n$

9. common difference **10.** common ratio

11. $-1 < r < 1$ **12.** $\sum$

Applications of Calculus to the Physical World

Exercise 1 (page 122)

1. B **2.** C **3.** B **4.** A **5.** C **6.** B

7. A **8.** C **9.** C **10.** A **11.** C **12.** B

Exercise 2 (page 123)

1.(a) 450 **(b)** 7 **2.** 351·8 **3.** *

4.(a) $\frac{2}{3}\pi rh$ **(b)** 36π **5.** *

Exercise 3 (page 124)

1.(a) 0 **(b)** 72 m **(c)** 3 s **(d)** 75 m

(e) 72 m **(f)** changed direction **(g)** 10 s

2.(a) 4, −2 **(b)** 9 s

3.(a) 1 m to the right **(b)** 16 m **(c)** 3 s

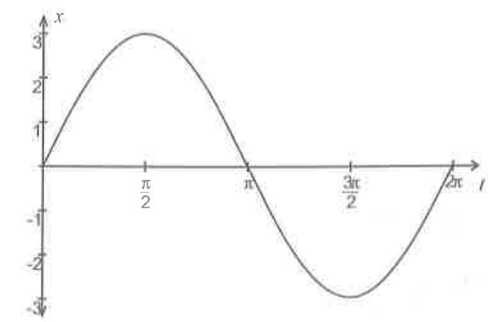
(d) 6 m to the left **(e)** 25 m

4.(a) after 7 s **(b)** 98 m **(c)** 130 m

5. 5 **6.(a)** 0 **(b)** after 4·5 s **(c)** 54, left

7. *

8.(a)



(b) 3 m **(c)** 1·5 m **(d)** π, 2π s **(e)** 12 m

(f) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ s **9.** 18·96 m **10.** *

Exercise 4 (page 128)

1. $v = 7$, a constant **2.** *

3.(a) $v > 0$ for all values of t

(b) 225 m/s **4.(a)** 104

Exercise 3

5 $A.P.\ \sqrt{5}, \sqrt{45}, \sqrt{125}, \sqrt{245}, \ldots$

$i.e.\ \sqrt{5}, \sqrt{9}\sqrt{5}, \sqrt{25}\sqrt{5}, \sqrt{49}\sqrt{5}, \ldots$

$\sqrt{5}, 3\sqrt{5}, 5\sqrt{5}, 7\sqrt{5}, \ldots$

$d = T_2 - T_1$

$= 3\sqrt{5} - \sqrt{5}$

$= 2\sqrt{5}$

$T_n = a + (n-1)d$

$T_{10} = \sqrt{5} + (10-1)2\sqrt{5}$

$= \sqrt{5} + (9)2\sqrt{5}$

$= \sqrt{5} + 18\sqrt{5}$

$= 19\sqrt{5}$

14 $A.P. -3, 1, 5, 9, 13 \ldots$

$a = -3 \quad d = T_2 - T_1 = 1 - -3 = 4$

$T_n = a + (n-1)d$

$T_n = (-3) + (n-1)(4)$

$= -3 + 4n - 4$

$= 4n - 7$

$Tn > 200$

$4n - 7 > 200$

$4n > 200 + 7$

$4n > 207$

$n > \frac{207}{4}$

$n > 51\frac{3}{4}$

$\therefore n = 52$

i.e. T_{52} is the first term > 200

22 Numbers divisible by 9

9, 18, 27, ... 63

A.P. a = 9 : d = 18 - 9 = 9

n = 7

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_7 = \frac{7}{2}\left(2(9) + (7-1)(9)\right)$

$= \frac{7}{2}\left(18 + (6)(9)\right)$

$= \frac{7}{2}\left(18 + 54\right)$

$= \frac{7}{2}\left(72\right)$

$= 7 \times 36$

$= 252$

Sum of all integers from 1 to 70

A.P. a = 1 : d = 1 : n = 70

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{70} = \frac{70}{2}\left(2(1) + (70-1)(1)\right)$

$= \frac{70}{2}\left(2 + (69)\right)$

$= 35(71)$

$= 2485$

Sum of integers from 1 to 70, NOT divisible by 9 = 2485 − 252

= 2233

24 From 6 : 32 am to 10 : 14 am

= 222 minutes

Working from 32 minutes past 6 to 254 minutes past 6 we get the sequence 32, 35, 38, ... 254

A.P. a = 32 : d = 3

$T_n = a + (n-1)d$

$254 = 32 + (n-1)(3)$

$254 = 32 + 3n - 3$

$254 = 29 + 3n$

$225 = 3n$

$3n = 225$

$n = 75$

38 $12, 11 \cdot 86, 11 \cdot 72 \ldots$

$A.P.\ a = 12 : d = -0 \cdot 14$

$T_n = a + (n-1)d$

$T_{15} = 12 + (15-1)(-0 \cdot 14)$

$= 12 + (14)(-0 \cdot 14)$

$= 12 - 1 \cdot 96$

$= 10 \cdot 04\text{ m}$

Distance travelled is both up and down $= 2 \times$ distance up

$S_n = \frac{n}{2}\left(a + l\right)$

$S_{15} = \frac{15}{2}\left(12 + 10 \cdot 04\right)$

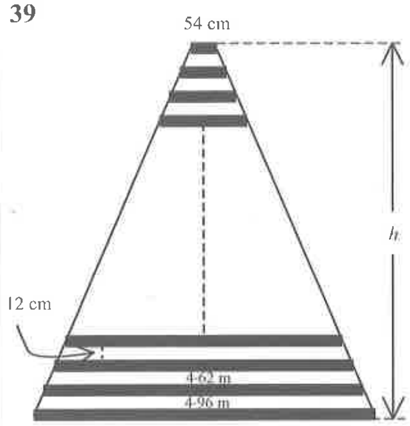
$= \frac{15}{2}\left(22 \cdot 04\right)$

$= 165 \cdot 3$

Distance travelled $= 2 \times 165 \cdot 3$

$= 330 \cdot 6\text{ m}$

39



(a) $A.P.\ 4 \cdot 96, 4 \cdot 62, \ldots 0 \cdot 54$

$a = 4 \cdot 96 : d = 4 \cdot 62 - 4 \cdot 96$

$= -0 \cdot 34$

$T_n = a + (n-1)d$

$0 \cdot 54 = 4 \cdot 96 + (n-1)(-0 \cdot 34)$

$0 \cdot 54 = 4 \cdot 96 + (-0 \cdot 34n + 0 \cdot 34)$

$0 \cdot 54 = 4 \cdot 96 - 0 \cdot 34n + 0 \cdot 34$

$0 \cdot 34n = 5 \cdot 3 - 0 \cdot 54$

$0 \cdot 34n = 4 \cdot 76$

$n = \frac{4 \cdot 76}{0 \cdot 34}$

$n = 14$

(b) $4 \cdot 96 + 4 \cdot 62, \ldots + 0 \cdot 54$

$S_n = \frac{n}{2}\left(a + l\right)$

$S_{14} = \frac{14}{2}\left(4 \cdot 96 + 0 \cdot 54\right)$

$= 7\left(5 \cdot 5\right)$

$= 38 \cdot 5\text{ m}$

(c) 14 bars 13 spaces

spaces $= 13 \times 12\text{ cm} = 13 \times 0 \cdot 12\text{ m}$

$= 1 \cdot 56\text{ m}$

bars $= 14 \times 5\text{ cm} = 1 \times 0 \cdot 05\text{ m}$

$= 0 \cdot 7\text{ m}$

height $= 1 \cdot 56\text{ m} + 0 \cdot 7\text{ m}$

$= 2 \cdot 26\text{ m}$

Exercise 4

16 800, 700, 612·5 . . . G.P.

$$a = 800 : \; r = \frac{T_2}{T_1} = \frac{700}{800} = \frac{7}{8}$$

$$T_n = ar^{n-1}$$

$$= 800\left(\frac{7}{8}\right)^{n-1}$$

But $T_n < 80$

$$800\left(\frac{7}{8}\right)^{n-1} < 80$$

$$\left(\frac{7}{8}\right)^{n-1} < \frac{80}{800}$$

$$\left(\frac{7}{8}\right)^{n-1} < \frac{1}{10}$$

$$\ln\left(\frac{7}{8}\right)^{n-1} < \ln\left(\frac{1}{10}\right)$$

$$(n-1)\ln\left(\frac{7}{8}\right) < \ln\left(\frac{1}{10}\right) \quad \left(\div \ \ln\left(\frac{7}{8}\right)\right)$$

$$(n-1)\frac{\ln\left(\frac{7}{8}\right)}{\ln\left(\frac{7}{8}\right)} > \frac{\ln\left(\frac{1}{10}\right)}{\ln\left(\frac{7}{8}\right)} \quad \text{(rev.sign} \div \text{by -)}$$

$$(n-1) > \frac{\ln\left(\frac{1}{10}\right)}{\ln\left(\frac{7}{8}\right)}$$

$$n - 1 > 17 \cdot 244$$

$$n > 17 \cdot 244 + 1$$

$$n > 18 \cdot 244$$

$$n = 19$$

23

$$ar^2 + ar^5 = 18$$

$$\Rightarrow ar^2(1 + r^3) = 18 \ . \ . \ . \ (1)$$

$$ar^5 + ar^8 = 144$$

$$\Rightarrow ar^5(1 + r^3) = 144 \ . \ . \ . \ (2)$$

$$(2) \div (1) \; : \; r^3 = 8$$

$$r = 2$$

When $r = 2$

$$a(2)^2 + a(2)^5 = 18$$

$$4a + 32a = 18$$

$$36a = 18$$

$$a = \frac{18}{36}$$

$$a = 0 \cdot 5$$

$$\therefore a = 0 \cdot 5, \; r = 2$$

25 5, 10, 17, 28, 47, 82, ……

Hint : $T_1 = 2 + 3 = 5$

$$T_2 = 4 + 6 = 10$$

$$T_3 = 8 + 9 = 17$$

First part of each term :

2, 4, 8, . . . $G.P.: a = 2, \; r = 2$

$$T_n = ar^{n-1}$$

$$T_n = (2)(2)^{n-1}$$

$$T_n = 2^n$$

Second part of each term :

3, 6, 9, . . . $A.P.: a = 3, \; d = 3$

$$T_n = a + (n-1)d$$

$$T_n = (3) + (n-1)(3)$$

$$T_n = 3 + 3n - 3$$

$$T_n = 3n$$

$\therefore$ For given sequence :

$$T_n = 2^n + 3n$$

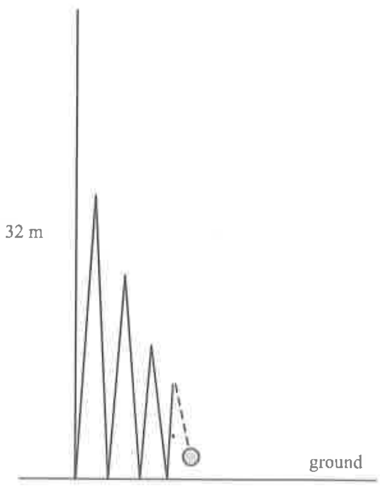
$$T_{15} = 2^{15} + 3(15)$$

$$= 32768 + 45$$

$$= 32813$$

Exercise 5

7



$$\text{Distance} = 32 + 2\underbrace{\left(19 \cdot 2 + 0 \cdot 6(19 \cdot 2) + 0 \cdot 6\{0 \cdot 6(19 \cdot 2)\} + \ldots\right)}_{\underbrace{G.P. \; a = 19 \cdot 2; \; r = 0 \cdot 6}}$$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{19 \cdot 2}{1 - 0 \cdot 6}$$

$$= \frac{19 \cdot 2}{0 \cdot 4}$$

$$= 48$$

$$\text{Distance} = 32 + 2(48)$$

$$= 32 + 96$$

$$= 128 \text{ m}$$

Exercise 6

6(a) $5 \cdot 6\%$ p.a. $= 0 \cdot 056$ p.a.

$= 0 \cdot 028$ per half year

$$A = 2000(1 \cdot 028)^{40} + 2000(1 \cdot 028)^{38} + 2000(1 \cdot 028)^{36} + ... + 2000(1 \cdot 028)^4 + 2000(1 \cdot 028)^2$$

$$= 2000\left[1 \cdot 028^{40} + 1 \cdot 028^{38} + 1 \cdot 028^{36} + \ ... \ + 1 \cdot 028^4 + (1 \cdot 028)^2\right]$$

$$= 2000\left[\underbrace{1 \cdot 028^2 + (1 \cdot 028)^4 + \ ... + (1 \cdot 028)^{40}}\right]$$

G.P. $a = 1 \cdot 028^2$; $r = 1 \cdot 028^2$; $n = 20$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{20} = \frac{1 \cdot 028^2[(1 \cdot 028^2)^{20} - 1]}{1 \cdot 028^2 - 1}$$

$$= \frac{1 \cdot 028^2(1 \cdot 028^{40} - 1)}{1 \cdot 028^2 - 1}$$

$$= 37 \cdot 5568718$$

$\therefore A = 2\,000 \times 37 \cdot 5568718$

$= \$75113 \cdot 74$

i.e. Amount is $\$75113 \cdot 74$

(b) Interest earned $= \$\$75113 \cdot 74 - (\$2000 \times 20)$

$= \$75113 \cdot 74 - \$40\,000$

$= \$35113 \cdot 74$

7 Loan $= \$18\,000$; Int $= 12\%$ p.a. $= 1\%$ per month; 5 years = 60 months

An = Amount owing after n^{th} payment; M = monthly payment

(a) $A_1 = 18000 + (0 \cdot 01 \times 18000) - M$

$= 18000(1 + 0 \cdot 01) - M$ (18000 is a common factor of the 1st two terms)

$= 18000(1 \cdot 01) - M$

$A_2 = A_1(1 \cdot 01) - M$

$A_2 = (18000(1 \cdot 01) - M)(1 \cdot 01) - M$

$A_2 = 18000(1 \cdot 01)^2 - M(1 \cdot 01) - M$ ($-M$ is a common factor of the last two terms)

$A_2 = 18000(1 \cdot 01)^2 - M\{(1 \cdot 01) + 1\}$

$A_2 = 18000(1 \cdot 01)^2 - M\{1 + (1 \cdot 01)\}$

$A_3 = A_2(1 \cdot 01) - M$

$A_3 = \left[18000(1 \cdot 01)^2 - M\{(1 \cdot 01) + 1\}\right](1 \cdot 01) - M$

$A_3 = 18000(1 \cdot 01)^3 - M(1 \cdot 01)^2 - M(1 \cdot 01) - M$ ($-M$ is a common factor of the last three terms)

$A_3 = 18000(1 \cdot 01)^3 - M\{(1 \cdot 01)^2 + (1 \cdot 01) + 1\}$

$A_3 = 18000(1 \cdot 01)^3 - M\{1 + (1 \cdot 01) + (1 \cdot 01)^2\}$

$A_n = 18000(1 \cdot 01)^n - M\{1 + (1 \cdot 01) + (1 \cdot 01)^2 + (1 \cdot 01)^3 + \ldots\ldots + (1 \cdot 01)^{n-1}\}$

$$= 18000(1 \cdot 01)^n - M\frac{(1 \cdot 01^n - 1)}{(1 \cdot 01 - 1)}$$

$$= 18000(1 \cdot 01)^n - 100M((1 \cdot 01^n - 1)$$

(b) When $n = 60$: $A_{60} = 0$ *i.e.* After 60^{th} payment there is zero owing

$$0 = 18000(1\cdot 01)^{60} - M\{\underbrace{1 + (1\cdot 01) + (1\cdot 01)^2 + (1\cdot 01)^3 + \ldots\ldots + (1\cdot 01)^{59}}_{G.P.\ a = 1;\ r = 1\cdot 01;\ n = 60}\}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{60} = \frac{1((1\cdot 01)^{60} - 1)}{1\cdot 01 - 1}$$

$$= \frac{(1\cdot 01)^{60} - 1}{0\cdot 01}$$

$$= 81\cdot 66966985$$

$$0 = 18000(1\cdot 01)^{60} - M(81\cdot 66966985)$$

$$M(81\cdot 66966985) = 18000(1\cdot 01)^{60}$$

$$M = \frac{18000(1\cdot 01)^{60}}{81\cdot 66966985}$$

$$M = 400\cdot 40$$

$\therefore$ Monthly instalment is $\$400\cdot 40$

(c) Total repayments = \$400.40 x 60
= \$24 024.00
Total interest = \$24 024 – \$18 000
= \$6024

Using Simple Interest formula $I = \dfrac{PRT}{100}$

$$\therefore R = \frac{100I}{PT}$$

$$= \frac{100 \times 6024}{18000 \times 5}$$

$$\therefore R = 6\cdot 69\%$$