Student Number

ST PIUS X COLLEGE
CHATSWOOD

2015 Stage 6 – Year 12

ASSESSMENT TASK #2
MID-COURSE EXAMINATION

25% of School Based Assessment

MATHEMATICS

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- Board-approved calculators may be used
- In Section II marks maybe deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of all pages
- A table of standard integrals is included for reference

Total Marks – 80

Section I Multiple Choice

10 marks

- Attempt Questions 1 – 10
- Enter solutions on Multiple Choice Answer Sheet

Section II Extended Response

70 marks

- Attempt Questions 11 – 15
- Show all necessary working
- ***Start each question in a SEPARATE booklet***

SECTION I – MULTIPLE CHOICE 10 marks – each question is of equal value

Enter solutions on the MULTIPLE CHOICE ANSWER SHEET provided

1 Which equation would you use in first principle differentiation?

(A) $\lim\limits_{h\to 0}\frac{f(x^2)-f(x)}{h}$

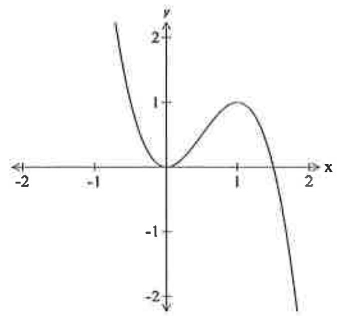
(B) $\lim\limits_{h\to 0}\frac{f(h)-f(x)}{h}$

(C) $\lim\limits_{h\to 0}\frac{f(y)-f(x)}{h}$

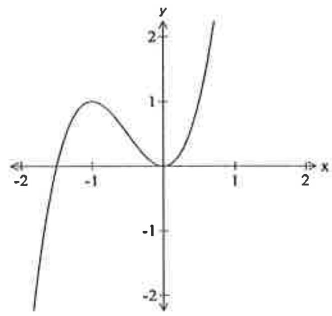
(D) $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

2 Which of the following is the graph of $(x) = 2x^3 - 3x^2$?

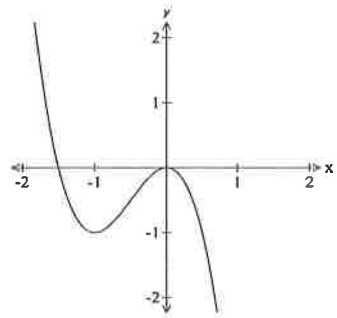
(A)



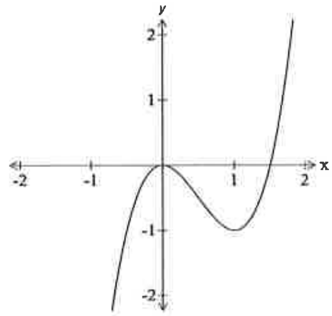
(B)



(C)



(D)



3 What is the equation of the tangent to the curve, $f(x) = x^2 + 2x$ at the point $(1,3)$?

(A) $y = 1 - 4x$

(B) $y = x + 1$

(C) $y = 4x - 1$

(D) $y = 2x - 1$

4 The general integral of $\int \frac{1}{x^2}\, dx$ is

(A) $\frac{1}{x} + c$

(B) $-\frac{1}{x} + c$

(C) $\frac{1}{x^3} + c$

(D) $-\frac{1}{x^3} + c$

5 The vertex of the parabola, $f(x) = x^2 - 4x + 8$, is

(A) (2,3)
(B) (1,4)
(C) (2,4)
(D) $(-1,-4)$

6 What is the solution to the equation, $sin^2x - 2sinx + 1 = 0, for\ 0 \leq x \leq 90°$

(A) $x = 60°$
(B) $x = 45°$
(C) $x = 30°$
(D) $x = 90°$

7 The equation, $f(x) = x^2 + 4x + 6$, has no real solutions because?

(A) $b^2 - 4ac < 0$
(B) $b^2 - 4ac \leq 0$
(C) $b^2 - 4ac \neq 0$
(D) $b^2 - 4ac = 0$

8 $\cot(180 - \theta) =$

(A) $cot\theta$
(B) $tan\theta$
(C) $\frac{-1}{tan\theta}$
(D) $\tan(180 - \theta)$

9 A fair coin is tossed three consecutive times. The outcomes are heads (h) and tails (t) on each occasion. The probability of obtaining at least one head is?

(A) $1 - t^3$

(B) $1 + t^3$

(C) $h^2 t$

(D) $h^2 t + h t^2$

10 The function, $f(x) = x^3(4 - x)$, has a turning point at $x = 3$. The conditions for this turning point to be a local maximum is?

(A) $f'(3) = 0$ and $f''(3) > 0$
(B) $f'(3) = 0$ and $f''(3) < 0$
(C) $f'(3) = 0$ and $f''(3) = 0$
(D) $f'(3) < 0$ and $f''(3) > 0$

END OF SECTION I

SECTION II

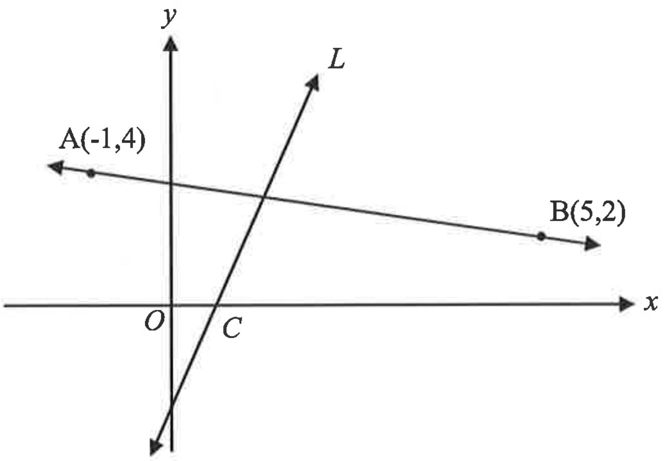
QUESTION 11 15 marks – allocation of marks as shown

Start this question in a SEPARATE booklet	**Marks**
a. Fully factorise $2x^2 - 18$.	**1**
b. Solve $x^4 = 5x^2 + 6$.	**3**
c. If $\frac{\sqrt{2}-1}{\sqrt{2}+1} = a - 2\sqrt{b}$, find values for a and b.	**2**
d. A packet of sweets contains 5 red and 14 green sweets. Two sweets are selected at random without replacement.	
i. Draw a tree diagram to show possible outcomes, include probabilities on each branch.	**1**
ii. What is the probability that the two sweets are different colours?	**2**
e. Solve $2^{2x+1} = 32$	**2**
f. Solve $\sqrt{3} tan\theta = -1$, for $0^\circ \leq \theta \leq 360^\circ$	**2**
g. Prove $tan\theta sin\theta + cos\theta = sec\theta$.	**2**

QUESTION 12 10 marks – allocation of marks as shown

Start this question in a SEPARATE booklet **Marks**

a. The diagram below shows the points $A(-1, 4)$ and $B(5, 2)$. The line L has Equation $3x - y - 4 = 0$ and cuts the x-axis at C.



i. Show that the length of AB is $2\sqrt{10}$ units. **1**

ii. Find the coordinates of M, the midpoint of AB. **1**

iii. Find the gradient of AB. **1**

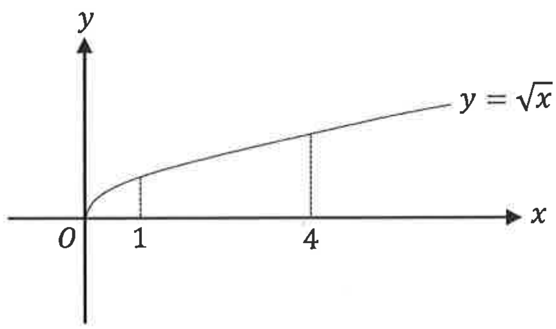
iv. Show that the equation of AB is $x + 3y - 11 = 0$. **1**

v. Prove that L is the perpendicular bisector of AB. **2**

vi. Find the coordinated of C. **1**

vii. Write down the equation of the circle with AB as the diameter. **1**

b.



The graph of $y = \sqrt{x}$ is shown in the diagram above. The arc of the curve between $x = 1$ and $x = 4$ is rotated about the x-axis.

Calculate the volume thus formed. **2**

QUESTION 13 15 marks – allocation of marks as shown

Start this question in a SEPARATE booklet **Marks**

a. Differentiate the following with respect to x.

i. $(1-3x)^4$ **2**

ii. $2x(x^2-4)$ **2**

iii. $\frac{x}{1+x^2}$ **2**

b. Find the indefinite integral of:

i. $\int \left(x^3+2x^2-\frac{3}{x^3}\right)\ dx$ **2**

ii. $\int (2x+4)^3\ dx$ **2**

c. Evaluate $\int_{-2}^{-1} \left(x-\frac{1}{x}\right)^2 \text{dx}$ **2**

d. i. Complete the table of values for the function $y=|x^2-1|$. **1**

x	-2	-1	0	1	2
y					

ii. Hence draw the graph of $y=|x^2-1|$ for $-2\leq x\leq 2$. **2**

QUESTION 14 15 marks – allocation of marks as shown

Start this question in a SEPARATE booklet **Marks**

a. Consider the function $y = x^3 - 6x^2 + 2$ for $-2 \leq x \leq 5$.

i. Find any stationary points and determine their nature. **4**

ii. Locate any points of inflexion. **2**

iii. Sketch this function showing its critical points. **3**

iv. Determine the values for which the function is increasing in the given domain. **1**

v. Determine the set of values for which the function is concave up. **1**

vi. What is the maximum value of this function? **1**

b. Using Simpson's rule, with 5 function values, find an approximation for $\int_1^5 \sqrt{x-1}\, dx$. Leave your answer in exact form. **3**

QUESTION 15 15 marks – allocation of marks as shown

Start this question in a SEPARATE booklet **Marks**

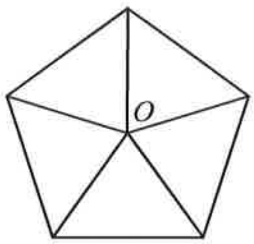
a. Consider the functions $y = 2x - x^2$ and $y = 2 - x$.

i. Find the x co-ordinates of their points of intersection. 1

ii. On the same set of axes sketch the curves. 2

iii. Find the area enclosed by the curves. 2

b.



The above figure shows a regular pentagon. Each internal angle is equal to $72°$ and each arm is 1 unit in length. O is the centre such that the five triangles are congruent.

i. Show that the area of the above pentagon is $\frac{5}{2}\sin 72°$. 2

ii. Show that the perimeter of the above pentagon is $10\sin 36°$. 2

c. A piece of wire 14cm long is cut into two portions.
One piece is bent to form a circle and the other piece to form a square.

i. Show that $r = \frac{7-2x}{\pi}$ where r is the radius of the circle and x is the length of the square. 1

ii. Write an exact expression for the sum of the areas of the circle and square in terms of x. 1

iii. Find the exact circumference of the circle if the sum of the areas of the circle and the square is to be a minimum. Justify your answer. 4

END OF SECTION II

END OF ASSESSMENT

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Multiple Choice Answer Sheet

Student Number

Instructions for use:

- Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think that you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

- Attempt all multiple choice questions.

Question				
1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○