Chapter 3

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Facts and Formulas

EXERCISE 1

Circle the correct answer:

1. Derivative of e^x.

(A) xe^{x-1} (B) e^x (C) xe^x (D) $e^x + c$

2. Derivative of e^{ax+b}.

(A) $\frac{1}{a}e^{ax+b}$ (B) ae^{ax+b} (C) e^{ax+b} (D) $(ax+b)e^{ax+b}$

3. Derivative of $e^{f(x)}$

(A) $e^{f(x)}$ (B) $\frac{e^{f(x)}}{f'(x)}$ (C) $f'(x)e^{f(x)}$ (D) $f(x)e^{f(x)}$

4. $\int e^x dx$.

(A) $\frac{e^{x+1}}{(x+1)} + c$ (B) $\frac{e^x}{x} + c$ (C) $e^x + c$ (D) $e^{x+1} + c$

5. $\int e^{ax+b} dx$.

(A) $ae^{ax+b} + c$ (B) $\frac{1}{a}e^{ax+b} + c$ (C) $e^{ax+b} + c$ (D) $e^{ax+b+1} + c$

6. $\int f'(x)e^{f(x)} dx$.

(A) $f'(x)e^{f(x)} + c$ (B) $\frac{e^{f(x)}}{f'(x)} + c$ (C) $e^x + c$ (D) $e^{f(x)} + c$

7. Derivative of $ln\,x$.

(A) $\frac{1}{x}$ (B) e^x (C) e^{-x} (D) $\frac{1}{x^2}$

8. Derivative of $\ln f(x)$.

(A) $\frac{f(x)}{f'(x)}$ (B) $\frac{f'(x)}{f(x)}$ (C) $\frac{f'(x)}{\ln f(x)}$ (D) $\frac{x}{f(x)}$

9. $\int \frac{1}{x} dx$.

(A) $-\frac{1}{x^2} + c$ (B) $e^x + c$ (C) $ln\,x + c$ (D) $-ln\,x + c$

10. $\int \frac{f'(x)}{f(x)} dx$.

(A) $ln\,f'(x) + c$ (B) $e^{f(x)} + c$ (C) $e^{f'(x)} + c$ (D) $ln\,f(x) + c$

11. $\log_m(ab) =$

(A) $\log_m a \times \log_m b$ (B) $\log_m a - \log_m b$

(C) $\log_m a + \log_m b$ (D) $\log_m a \div \log_m b$

12. $\log_m\left(\frac{a}{b}\right) =$

(A) $\log_m a - \log_m b$ (B) $\frac{\log_m a}{\log_m b}$

(C) $\log_m b - \log_m a$ (D) $\log_m a + \log_m b$

13. $\log_m a^b =$

(A) $a\log_m b$ (B) $\log_m ab$ (C) $\log_m b^a$ (D) $b\log_m a$

14. $\log_b a =$

(A) $\log_a b$ (B) $\frac{\log_m b}{\log_m a}$

(C) $\frac{\log_m a}{\log_m b}$ (D) $\log_m a - \log_m b$

15. If $y = a^x$ then

(A) $x = \log_a y$ (B) $y = \log_a x$ (C) $x = \log_y a$ (D) $a = \log_x y$

16. If $y = \ln x$ then

(A) $y = e^x$ (B) $y = x^e$ (C) $x = e^y$ (D) $x = y^e$

17. Derivative of a^x

(A) $a^x \ln a$ (B) a^x (C) $a^x \ln x$ (D) $\frac{a^x}{\ln a}$

18. If $y = e^x$ then

(A) $x = e^y$ (B) $x = \ln y$ (C) $y = \ln x$ (D) $x = y^e$

19. $e^{\ln x}$ is

(A) $\ln x$ (B) x (C) e^x (D) $\ln ex$

20. Derivative of $\log_a x$ is

(A) $\frac{1}{x}$ (B) $\frac{a}{x}$ (C) $\frac{1}{x\ln a}$ (D) $\frac{x}{a}$

21. $\ln e^x$ is

(A) $\ln x$ (B) x (C) e^x (D) xe^x

Laws for Logarithms

EXERCISE 2

1. Given that $\log 3 = 0.4771$, $\log 5 = 0.6990$ and $\log 2 = 0.3010$, evaluate:

(a)* $\log 15$

(b) $\log (1\frac{2}{3})$

(c) $\log 25$

(d) $\log 27$

(e) $\log (3\frac{1}{3})$

(f) $\log \sqrt{5}$

(g) $\log 250$

2. Simplify

$3\log_x a + 4\log_x b - \frac{1}{3}\log_x 27$

3. Write $ln8 + ln16 + ln32 + ln\ 64$ in the form $xln2$

4. Write $ln\sqrt{3} + ln9 - ln\sqrt[4]{3} + ln81$ in the form $xln3$.

5.* Evaluate (to 2 decimal places):

(a) $\log_3 8$

(b) $\log_7 15$

6.* Solve for x:

(a) $ln(x-2)+ln(x-5)=ln(x+3)$

(b) $ln(x+10)-ln2=ln(x+4)$

(c) $5ln2=ln x+ln8$

Basic Logarithmic and Exponential Functions

EXERCISE 3

1. Complete the following:

(a) The value of e to 1 decimal place is

(b) $ln e=$

(c) $ln 1=$

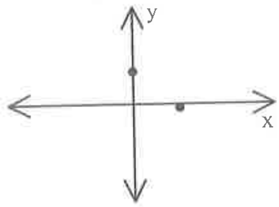
(d) If $y=ln x$ then $x=$

(e) The graphs of $y=e^x$ and $y=ln x$ are reflections in the line on the number plane.

(f) Some logarithmic equations are best solved by converting them to form.

eg. $\ln x=8$, $\log_3(6x-7)=2$

2. (a) Sketch $y=e^x$ and $y=ln x$ on the diagram below, indicating intercepts.



(b) The domain of $y=e^x$ is

(c) The range of $y=e^x$ is

(d) The domain of $y=ln x$ is

(e) The range of $y=ln x$ is

3. State the domain of each of the following:

(a) $y=ln x$

(b) $y=ln(x-2)$

(c) $y=ln(x+3)$

4. To solve an equation where the pronumeral is a power, e.g. $2^x=1000$, take the of both sides.

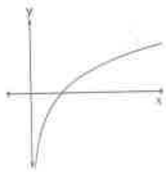
5. Label each curve below using the choices given:

(A) $y=e^x$ (B) $y=e^{-x}$

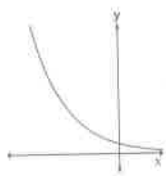
(C) $y=-e^{-x}$ (D) $y=-e^x$

(E) $y=ln x$

(a)



(b)



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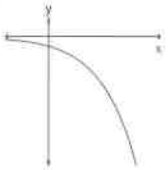
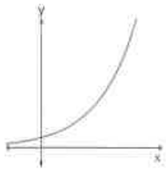
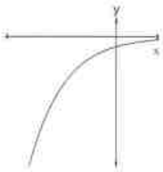


(d)



(e)



EXERCISE 4

1. Evaluate $e^{3.6}$ (1 dec. pl.)

2. Evaluate $ln\, 6.8$ (1 dec. pl.)

3. Solve for x:

(a) $e^x(x+1)=0$

(b) $3e^x(x^2+4x)=0$

(c) $(x-5)e^{-x}=0$

(d)* $e^x - e^{-x} = 0$

(e) $e^{2x} - e^x - 2 = 0$ (1 dec. pl.)

(f) $5^x = 9$ (1 dec. pl.)

(g) $e^x = 5$ (3 sig.fig)

(h) $e^{2x} = 9$ (2 dec. pl.)

(i) $4^{3x} = 6$ (2 dec. pl.)

(j) $2^{4x-5} = 20$ (1 dec. pl.)

(k) $ln\, e^{5x-3} = 12$

(l) $ln\, x = 7$ (2 sig.fig)

(m) $200 = e^{0.02x}$ (4 sig.fig)

(n) $\ln e^{5x+2} = -3$

4. (a)* Factorise $e^{2x} - 4$

(b) Simplify $\dfrac{e^{2x} - 4}{e^{x} - 2}$

(c) Simplify $\left(e^{4x}\right)^2$

(d) Evaluate $e^{\ln 7}$

(e) Expand and simplify $\left(e^{x} + e^{-x}\right)^2$

5. Write each of the following in exponential form:

(a) $\log_5 125 = y$

(b) $\log_4 y = 3$

(c) $\ln 2 = x$

(d) $\log_x 81 = 2$

(e) $\log_2 t = m$

(f) $6 = \ln m$

6. Write in exponential form and hence solve

(a) $\log_5(2x - 7) = 2.$

(b) $\log_9 \sqrt{3} = x$

(c) $\log_4 8 = x$

(d) $\log_{25} x = -\dfrac{1}{2}$

EXERCISE 5

Complete:

1. Equation of the tangent to a curve

2. Equation of the normal to a curve

3. Product rule

4. Quotient rule

5. Function of a function rule

6. The first derivative is called the ______ function.

Derivative and Integral of Exponential Functions

EXERCISE 6

1. Differentiate:

(a) e^{5x}

(b) $e^{\frac{2x}{3}}$

(c) $e^{\frac{x}{4}}$

(d) e^{-x}

(e) $e^{\frac{-x}{5}}$

(f) e^{2x+5}

(g) e^{5-4x}

(h) e^{3x^2}

(i) $3e^{2x}$

(j) $x^2 + 5e^x$

(k) $\dfrac{2}{e^x}$

(l) $\dfrac{1}{2e^x}$

(m) $\dfrac{3}{e^{2x}}$

(n) $\dfrac{e^{10x}}{5}$

2. If $f(x) = x^2 - e^{-x}$, evaluate $f'(4)$ (3 sig. fig.)

3. Differentiate:

(a) xe^{2x}

(b) $(3x+2)e^{4x}$

(c) $\dfrac{e^{6x}+1}{x^2}$

(d) $\left(e^{3x}+1\right)^5$

(e) $\dfrac{2}{e^{2x}-1}$

4. What is the gradient of the tangent to the curve $y = e^x$ at $x = 3$? (3 sig. fig)

5. What is the gradient of the normal to the curve $y = e^{2x} + 2x$ at $x = 1$? (1 sig. fig)

6. Find the equation of the tangent to the curve $y = e^{2x}$ at the point $(2, e^4)$.

7.* The tangent to the curve $y = e^{2x}$ at the point $(2, e^4)$ meets the x axis at S and the y axis at T. Find the area of Δ SOT.

8. Find the equation of the normal to the curve $y = e^x - 3x$ at the point $(0,1)$.

9. Find the y value of the point whose x value is 7 if the point lies on the curve $y = f(x)$ defined by $f'(x) = f(x)$.

10.* Find the value of x for which the gradient of the tangent to the curve $y = e^{-3x}$ equals the gradient of the tangent to the curve $y = \dfrac{3}{x-1}$.

11. If $y = e^{8-2x}$, evaluate c if $cy - y'' = 0$.

12.* For the curve $y = (x-5)e^x$:

(a) locate turning point(s) and determine whether maximum or minimum.

(b) locate any points of inflexion.

(c) determine x and y intercepts.

(d) consider what happens to y for extreme values of x.

(e) sketch the curve.

(f) State the domain for which the curve is monotonic increasing

13. The gradient function of a curve is $4e^{4x} - 2$. Find the equation of the curve if $(2, e^8 - 1)$ is a point on the curve.

14. Integrate each of the following:

(a) e^{4x}

(b) e^{-x}

(c) $e^{\frac{1}{2}x}$

(d) $e^{\frac{x}{3}}$

(e) $e^{\frac{4x}{5}}$

(f) e^{4-2x}

(g) $\frac{1}{3}e^{6x}$

(h) $\frac{e^{4x}}{2}$

(i) πe^{2x}

(j) $16e^{8x}$

(k) $\frac{e^{2x}+5}{e^{x}}$

(l) $e^{0\cdot75x}$

(m) $12xe^{6x^2}$

(n) $40xe^{10x^2}$

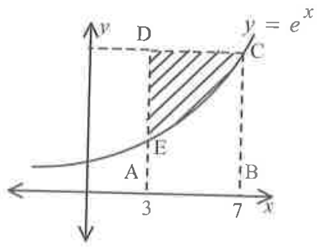
(o) $8xe^{8x^2}$

15. Evaluate $\int_{-1}^{2}(e^x + 4)dx$

16. Find the area bounded by the curve $y=e^{4x}$, the x axis and the lines $x = 0$ and $x = 2$.(1 dec.pl.)

17. Find the area enclosed by the curve $x=e^{y}$, the y axis and the lines $y = 1$ and $y = 5$. (1 dec.pl.)

18.

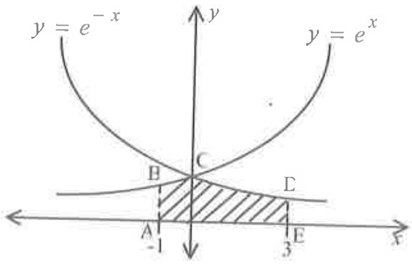


(a) Find the exact area of the rectangle ABCD.

(b) Hence find the exact shaded area CDE.
(Hint: Find Area of ABCE first.)

(b) Find the volume of the solid of revolution formed by rotating this region about the x axis. (Answer in terms of π.)

19.



(a) Find the exact area of ABCDE .

20. Find the exact area between the curve $y = -e^{-x}$, the x axis and the lines $x = 1$ and $x = 4$.

21.* If $f(x) = (e^x + e^{-x})^2$

(a) determine whether the function is even, odd or neither.

(b) Calculate the area bounded by the curve $f(x) = (e^x + e^{-x})^2$, the x axis and the lines $x = -3$ and $x = 3$. (to 1 dec.pl.)

22. Find $f(x)$ given that $f''(x) = 5 + e^{-x}$, and when $x = 0$, $f'(x) = 4$ and $f(x) = 9$.

Derivative and Integral of Logarithmic Functions

EXERCISE 7

1. Find the gradient of the tangent to the curve $y = \ln x$ when $x = 4$.

2. Find the gradient of the normal to the curve $y = \ln(x - 2)$ when $x = 5$.

3. Find the gradient of the tangent to the curve $y = \ln(3x + 6)$ when $x = -1$.

4. Find :

(a) the equation of the tangent to the curve $y = 3\ln(2x - 4)$ when $x = 3$.

(b) the equation of the normal to the curve $y = \frac{1}{2}\ln(3x)$ when $x = 2$.

5. The tangent to the curve $y = 2\ln x$ at the point where $x = 4$, meets the y axis at P. Find the coordinates of P.

EXERCISE 8

1. Differentiate:

(a) $y = \ln(6 - 2x)$

(b) $y = \dfrac{\ln 3x}{4}$

(c) $y = (\ln x)^2$

(d) $y = x\ln x$

(e) $y = (2x+5)\ln x$

(f) $y = \dfrac{5x}{\ln x}$

(g) $y = \ln e^x$

(h) $y = 2\ln e^{3x}$

(i) $y = \ln(2x+7)^5$

(j)* $y = [\ln(2x+7)]^5$

2. Differentiate: (Hint: use logarithm laws first)

(a) $y = \ln[(x+2)(x+3)]$

(b) $y = ln(2x+3)^4$

(c)* $y = ln\left(\dfrac{3x+2}{2x+1}\right)$

(d)* $y = ln\left(\dfrac{(2x+6)^7}{x}\right)$

3. Evaluate $f'(4)$ if $f(x) = ln(x^2 + 8)$

4. Find the point on the curve $y = 6 ln x$ at which the gradient is 2.

5. Find the point on the curve $y = ln(2x+4)$ at which the tangent is parallel to the line $x - y + 2 = 0$.

6. (a) Sketch the curve $y = ln(x^2 + 3)$.

x	-3	-2	-1	0	1	2	3
y							

(b) Use the trapezoidal rule with 4 strips to approximate the area bounded by the curve $y = ln(x^2 + 3)$, the x axis and the lines $x = -3$ and $x = 1$.

7. Integrate each of the following:

(a) $\int \frac{2}{x} dx$

(b) $\int \frac{1}{2x} dx$

(c) $\int \frac{2}{3x} dx$

(d) $\int 5x^{-1} dx$

(e) $\int_{-1}^{2} \frac{3}{(3x+5)} dx$

(f) $\int \frac{dx}{(x+6)}$

(g) $\int \frac{3}{(5-3x)} dx$

(h) $\int \frac{16}{(8x-3)} dx$

(i) $\int \frac{3}{(12x+5)} dx$

(j) $\int \frac{3}{(2x-6)} dx$

(k) $\int \frac{4x}{(x^2+8)} dx$

(l) $\int \frac{(16x+3)}{(8x^2+3x)}dx$

(m) $\int \frac{(7-20x)}{(10x^2-7x)}dx$

(n) $\int \frac{(6x-2)}{(6x^2-4x)}dx$

(o)* $\int \frac{(12x-8)}{(9x^2-12x)}dx$

(p) $\int \frac{2e^{2x}}{(e^{2x}+3)}dx$

(q) $\int \frac{(3x^5-4x^2+2x-5)}{x^2}dx$

(r) $\int \frac{(e^{2x}+6)}{(e^{2x}+12x)}dx$

8. Evaluate:

(a) $\int_1^3 \frac{2x}{x^2+3}dx$

(b) $\int_0^4 e^{2x}dx$

(c)* $\int_1^2 3xe^{x^2}dx$

(d) $\int_1^2 \left(3x^2 + \frac{2}{x}\right) dx$

(e) $\int_1^3 \frac{(5x^3 + 4)}{x} dx$

Area and Volume

EXERCISE 9

1. (a) Find the exact area bounded by the curve $y = \frac{2}{(2x-6)}$, the x axis and the lines $x = 5$, $x = 6$.

(b) Find the volume of the solid formed by rotating the curve in (a) around the x axis.

2. Find the volume of the solid of revolution formed by rotating the curve $y = \ln x$, from $y = -1$ to $y = 3$, about the y axis.

3.* Use Simpson's rule with 5 function values to approximate the area bounded by the curve $y = ln(x + 2)$, the x axis and the line $x = 3$. (3 dec. pl.)

4.* Use Simpson's rule with 5 function values to approximate the volume of the solid formed by rotating the curve $y = ln\,x$ from $x = 1$ to $x = 5$,around the x axis.(3 dec. pl)

5. Find the exact area bounded by the curve $y = -ln\,x$, the y axis and the lines $y = 0$ and $y = 1$.

6. Calculate the volume of the solid formed by rotating the curve $y = \frac{2}{\sqrt{(x-1)}}$ around the x axis, from $x = 2$ to $x = 5$. (correct to 4 sig. fig.)

FORMULAS

1. $\frac{d}{dx}\left(e^{x}\right)=$

2. $\frac{d}{dx}(e^{ax+b})=$

3. $\frac{d}{dx}(e^{f(x)})=$

4. $\int e^{x}dx=$

5. $\int e^{ax+b}\,dx=$

6. $\int f'(x)e^{f(x)}\,dx=$

7. $\frac{d}{dx}(ln\,x)=$

8. $\frac{d}{dx}[ln\,f(x)]=$

9. $\int\frac{1}{x}\,dx=$

10. $\int\frac{f'(x)}{f(x)}dx=$

11. $\log_{m}(ab)=$

12. $\log_{m}\left(\frac{a}{b}\right)=$

13. $\log_{m}a^{b}=$

14. $\log_{b}a=$ (change of base rule)

15. Exponential form of $y=\log_{a}x$

16. Exponential form of $y=ln\,x$

17. $\frac{d}{dx}\left(a^{x}\right)=$

18. Logarithmic equation for $y=e^{x}$

19. $e^{\ln x}=$

20. $\frac{d}{dx}(\log_{a}x)=$

21. $ln\,e^{x}=$

22. $ln\,1=$

23. $ln\,e=$

9. $A \cong \frac{h}{2}\left[f(a)+f(b)+2\left(f(x_2)+f(x_3).....\right)\right]$, $\frac{b-a}{n}$,

10. $\frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$, $\frac{b-a}{2}$

11. $A \cong \frac{h}{3}\left[f(a)+f(b)+4\left(f(x_2)+f(x_4)...\right)\right.$

$\left.+2\left(f(x_3)+f(x_5)...\right)\right]$, $\frac{b-a}{n}$

Summary (page 52)

1. primitive 2. integrating 3. indefinite
4. definite 5. F(b) –F(a) 6.(a) area
(b) volume 7.(a) Trapezoidal Rule
(b) Simpson's Rule 8. 1 more than 9. integrate
10. sections 11. below 12. to the left of

13. $\int_a^b [f(x)-g(x)]\,dx$

Exponential and Logarithmic Functions

Exercise 1 (page 53)

1. B 2. B 3. C 4. C 5. B 6. D
7. A 8. B 9. C 10. D 11. C 12. A
13. D 14. C 15. A 16. C 17. A 18. B
19. B 20. C 21. B

Exercise 2 (page 55)

1.(a) * (b) 0·2219 (c) 1·398
(d) 1·4313 (e) 0·5229 (f) 0·3495
(g) 2·398

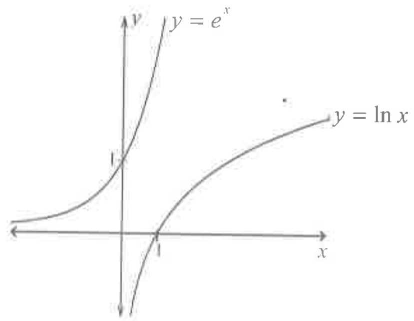
2. $\log_x\left(\frac{a^3b^4}{3}\right)$ 3. $18\ln 2$ 4. $\frac{25\ln 3}{4}$

5. * 6. *

Exercise 3 (page 56)

1.(a) 2·7 (b) 1 (c) 0 (d) e^y (e) $y = x$
(f) exponential

2.



(b) all real x (c) $y > 0$ (d) $x > 0$
(e) all real y
3.(a) $x > 0$ (b) $x > 2$ (c) $x > -3$
4. logarithm 5.(a) E (b) B
(c) D (d) A (e) C

Exercise 4 (page 57)

1. 36·6 2. 1·9 3.(a) –1
(b) 0, –4 (c) 5 (d) *
(e) ln2 (f) 1·4 (g) 1·61
(h) 1·10 (i) 0·43 (j) 2·3
(k) 3 (l) 1100 (m) 264·9
(n) –1 4.(a) * (b) $e^x + 2$
(c) e^{8x} (d) 7 (e) $e^{2x} + e^{-2x} + 2$
5.(a) $125 = 5^y$ (b) $y = 4^3$ (c) $2 = e^x$
(d) $81 = x^2$ (e) $t = 2^m$ (f) $m = e^6$
6.(a) 16 (b) ¼ (c) 1·5 (d) $\frac{1}{5}$

Exercise 5 (page 58)

1. $y - y_1 = f'(x_1)(x - x_1)$

2. $y - y_1 = \frac{-1}{f'(x_1)}(x - x_1)$

3. $y' = vu' + uv'$ 4. $y' = \frac{vu' - uv'}{v^2}$

5. $y' = n\left[f(x)\right]^{n-1} \times f'(x)$ 6. gradient

Exercise 6 (page59)

1.(a) $5e^{5x}$ (b) $\frac{2}{3}e^{\frac{2x}{3}}$ (c) $\frac{1}{4}e^{\frac{x}{4}}$

(d) $-e^{-x}$ (e) $-\frac{1}{5}e^{\frac{-x}{5}}$ (f) $2e^{2x+5}$

(g) $-4e^{5-4x}$ (h) $6xe^{3x^2}$ (i) $6e^{2x}$

(j) $2x + 5e^x$ (k) $-2e^{-x}$ (l) $\frac{-1}{2e^x}$

(m) $\frac{-6}{e^{2x}}$ (n) $2e^{10x}$ 2. 8·02

3.(a) $e^{2x}(2x+1)$ (b) $e^{4x}(12x+11)$

(c) $\dfrac{2(3xe^{6x} - e^{6x} - 1)}{x^3}$ (d) $15e^{3x}(e^{3x} + 1)^4$

(e) $\dfrac{-4e^{2x}}{(e^{2x} - 1)^2}$

4. 20·1 5. −0·06 6. $y = 2e^4 x - 3e^4$

7. * 8. $x - 2y + 2 = 0$ 9. e^7

10. * 11. 4 12. *

13. $y = e^{4x} - 2x + 3$ 14.(a) $\dfrac{e^{4x}}{4} + c$

(b) $-e^{-x} + c$ (c) $2e^{\frac{x}{2}} + c$ (d) $3e^{\frac{x}{3}} + c$

(e) $\dfrac{5}{4}e^{\frac{4x}{5}} + c$ (f) $-\dfrac{e^{4-2x}}{2} + c$ (g) $\dfrac{e^{6x}}{18} + c$

(h) $\dfrac{e^{4x}}{8} + c$ (i) $\dfrac{\pi e^{2x}}{2} + c$ (j) $2e^{8x} + c$

(k) $e^x - 5e^{-x} + c$ (l) $\dfrac{4e^{0\cdot75x}}{3} + c$

(m) $e^{6x^2} + c$ (n) $2e^{10x^2} + c$ (o) $\dfrac{e^{8x^2}}{2} + c$

15. $e^2 - e^{-1} + 12$ 16. 745·0 units²

17. 145·7 units² 18.(a) $4e^7$ (b) $(3e^7 + e^3)$ units²

19.(a) $\left(2 - \dfrac{1}{e} - \dfrac{1}{e^3}\right)$ units²

(b) $\pi\left(1 - \dfrac{1}{2e^2} - \dfrac{1}{2e^6}\right)$ units³

20. $\left(\dfrac{1}{e} - \dfrac{1}{e^4}\right)$ units² 21. *

22. $f(x) = \dfrac{5x^2}{2} + e^{-x} + 5x + 8$

Exercise 7 (page 64)

1. ¼ 2. −3 3. 1 4.(a) $y = 3x + 3\ln 2 - 9$

(b) $y = -4x + \dfrac{1}{2}\ln 6 + 8$ 5. $(0,\ 2\ln 4 - 2)$

Exercise 8 (page 65)

1.(a) $\dfrac{1}{x-3}$ (b) $\dfrac{1}{4x}$ (c) $\dfrac{2\ln x}{x}$

(d) $1 + \ln x$ (e) $2 + \dfrac{5}{x} + 2\ln x$ (f) $\dfrac{5\ln x - 5}{(\ln x)^2}$

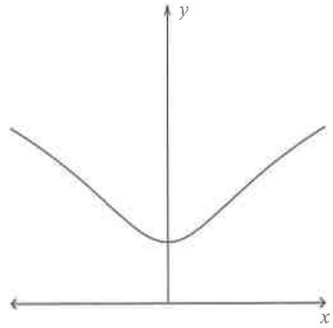
(g) 1 (h) 6 (i) $\dfrac{10}{2x+7}$ (j) *

2.(a) $\dfrac{2x+5}{(x+2)(x+3)}$ (b) $\dfrac{8}{2x+3}$

(c) * (d) * 3. ⅓ 4. $(3,\ 6\ln 3)$

5. $(-1,\ \ln 2)$

6.(a) $y = 2{\cdot}5,\ 1{\cdot}9,\ 1{\cdot}4,\ 1{\cdot}1,\ 1{\cdot}4,\ 1{\cdot}9,\ 2{\cdot}5$



(b) 6·37 units² 7.(a) $2\ln x + c$ (b) $\dfrac{1}{2}\ln x + c$

(c) $\dfrac{2}{3}\ln x + c$ (d) $5\ln x + c$ (e) $\ln 5\cdot5$

(f) $\ln(x+6) + c$ (g) $-\ln(5 - 3x) + c$

(h) $2\ln(8x - 3) + c$ (i) $\dfrac{1}{4}\ln(12x + 5) + c$

(j) $\dfrac{3}{2}\ln(2x - 6) + c$ (k) $2\ln(x^2 + 8) + c$

(l) $\ln(8x^2 + 3x) + c$ (m) $-\ln(10x^2 - 7x) + c$

(n) $\dfrac{1}{2}\ln(6x^2 - 4x) + c$ (o) *

(p) $\ln(e^{2x} + 3) + c$ (q) $\dfrac{3x^4}{4} - 4x + 2\ln x + \dfrac{5}{x} + c$

(r) $\dfrac{1}{2}\ln(e^{2x} + 12x) + c$ 8.(a) $\ln 3$ (b) $\dfrac{e^8 - 1}{2}$

(c) * (d) $7 + 2\ln 2$ (e) $43\tfrac{1}{3} + 4\ln 3$

Exercise 9 (page 69)

1.(a) $\ln 1\cdot5$ (b) $\frac{\pi}{6}$ units³ 2. $\frac{\pi}{2}(e^6 - e^{-2})$ units³

3. * 4. * 5. $\left(1 - \frac{1}{e}\right)$ units²

6. 17·42 units³

Formulas (page 71)

1. e^x **2.** ae^{ax+b} **3.** $f'(x)e^{f(x)}$ **4.** $e^x + c$

5. $\dfrac{e^{ax+b}}{a} + c$ **6.** $e^{f(x)} + c$ **7.** $\dfrac{1}{x}$ **8.** $\dfrac{f'(x)}{f(x)}$

9. $\ln x + c$ **10.** $\ln f(x) + c$ **11.** $\log_m a + \log_m b$

12. $\log_m a - \log_m b$ **13.** $b \log_m a$

14. $\dfrac{\log_m a}{\log_m b}$ **15.** $x = a^y$ **16.** $x = e^y$

17. $a^x . \ln a$ **18.** $x = \ln y$ **19.** x

20. $\dfrac{1}{x \ln a}$ **21.** x **22.** 0 **23.** 1

Trigonometric functions

Exercise 1 (page 72)

1. C **2.** D **3.** A **4.** A **5.** A **6.** B
7. A **8.** A **9.** B **10.** D **11.** A **12.** B
13. A **14.** A **15.** C **16.** B **17.** D

Exercise 2 (page 73)

1. 57 **2.** π **3.** $\frac{180}{\pi}$

4. $\frac{\pi}{180}$ **5.** small, radians **6.** a, $\frac{2\pi}{n}$

7. a, $\frac{2\pi}{n}$ **8.** $\frac{\pi}{n}$

Exercise 3 (page 74)

1.(a) $\frac{13\pi}{36}$ **(b)** $\frac{3\pi}{20}$ **(c)** $\frac{7\pi}{6}$

(d) $\frac{29\pi}{36}$ **(e)** $\frac{16\pi}{9}$ **2.(a)** 0·68

(b) 0·26 **(c)** 1·78 **(d)** 3·49

(e) 5·41 **3.(a)** 11°28′ **(b)** 91°40′

(c) 183°21′ **(d)** 179°55′ **(e)** 269°52′

4.(a) 36° **(b)** 22·5° **(c)** 20°

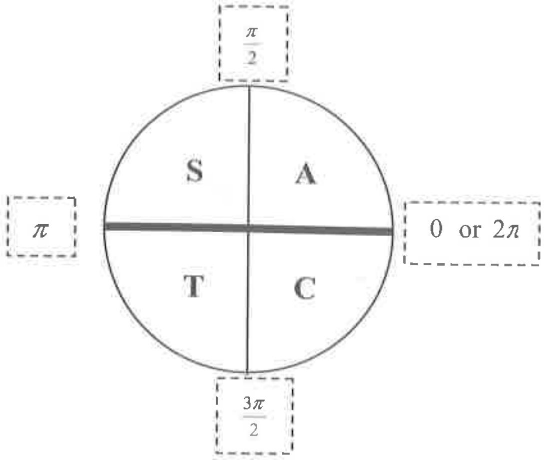
(d) 120° **(e)** 300°

5.(a) $\frac{\pi}{4}, \frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$ (base angle), $\frac{\pi}{6}$ **6.(a)** 1·5

(b) $\frac{\sqrt{6}}{2}$ **(c)** 0 **(d)** ⅔

(e) $\frac{\sqrt{2}}{4}$ **(f)** 5

7.



8. π, 2π, quadrant **9. (a)** * **(b)** $\frac{-\sqrt{2}}{2}$

(c) −1 **(d)** * **(e)** * **(f)** 1 **(g)** −0·5

(h) * **(i)** 0·5 **(j)** −1 **(k)** * **(l)** $\frac{\sqrt{2}}{2}$

10.(a) 1·56 **(b)** 0·75 **(c)** 0·88

(d) −1·02 **11.** $\sqrt{3}$ **12.(a)** 0·795, 5·488

(b) 3·467, 5·957 **(c)** 1·903, 5·044

Exercise 4 (page 77)

1.(a) $\frac{\pi}{3}, \frac{2\pi}{3}$ **(b)** $\frac{\pi}{4}, \frac{7\pi}{4}$ **(c)** $\frac{\pi}{6}, \frac{7\pi}{6}$

(d) $\frac{\pi}{6}, \frac{5\pi}{6}$ **(e)** $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{2}$ **2.(a)** 0·4115, 2·7301

(b) * **(c)** 2·2143, 4·0689

3(a) $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ **(b)** $-\pi, -\frac{3\pi}{4}, 0, \frac{\pi}{4}, \pi$

(c) 3·3657, 6·0591, 1·5708 **(d)** $\frac{3\pi}{4}, \frac{11\pi}{4}$

Exercise 5 (page 79)

1.(a) 5·2 cm, 10·4 cm^2, 2·69 cm^2

(b) 2π cm, 10π cm^2, 2·03 cm^2

(c) $\frac{7\pi}{3}$ cm, 14π cm^2, 2·68 cm^2

(d) $\frac{120}{\pi}$, 286·48 cm^2, 7·31 cm^2

(e) 2·5, 125 cm^2, 95·08 cm^2 **2.** 61·1 cm

3. 49 m **4.(a)** 18·8 cm **(b)** 22 cm^2

5. 12 cm **6.** 5 m **7.** *

8.(a) 40·9 cm **(b)** 45 cm **(c)** 143·5 cm

(d) 675 cm^2 **(e)** 226·1 cm^2 **(f)** 2601·3 cm^2

Exercise 2

1(a) $\log 15 = \log(3 \times 5)$
$= \log 3 + \log 5$
$= 0.4771 + 0.6990$
$= 1.1761$

5(a) Let $x = \log_3 8$

$= \dfrac{\ln 8}{\ln 3}$

$= 1 \cdot 89$

(b) Let $x = \log_7 15$

$= \dfrac{\ln 15}{\ln 7}$

$= 1 \cdot 39$

6(a)

$\ln(x-2)+\ln(x-5)=\ln(x+3)$
$\ln((x-2)(x-5))=\ln(x+3)$
$(x-2)(x-5)=x+3$
$x^2-7x+10=x+3$
$x^2-8x+7=0$
$(x-7)(x-1)=0$
$x=7,1$ But $x=1 \rightarrow \ln(\text{negative})$
$\therefore x=7$

(b) $\ln(x+10)-\ln 2=\ln(x+4)$
$\ln((x+10)\div 2)=\ln(x+4)$
$\ln\dfrac{x+10}{2}=\ln(x+4)$
$\dfrac{x+10}{2}=x+4$
$x+10=2x+8$
$2=x$
$x=2$

(c) $\ln 2^5=\ln 8x$
$2^5=8x$
$8x=32$
$x=4$

Exercise 4

3(d)

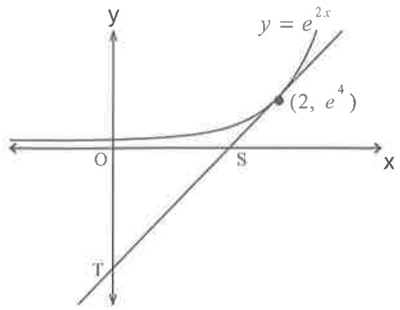
$e^x-e^{-x}=0$
$e^{-x}(e^{2x}-1)=0$ but $e^{-x}\neq 0$
$\therefore\ e^{2x}-1=0$
$e^{2x}=1 \Rightarrow 2x=0$
$x=0$

4(a)

$e^{2x}-4=0$
$(e^x)^2-2^2=0$
$(e^x-2)(e^x+2)=0$

Exercise 6

7



$y=e^{2x}$

$\dfrac{dy}{dx}=2e^{2x}$

at $(2,e^4)$ $\dfrac{dy}{dx}=2e^{2(2)}=2e^4$

Eq. of tangent : $(2,e^4)$, $m=2e^4$
$y-y_1=m(x-x_1)$
$y-e^4=2e^4(x-2)$
$y-e^4=2e^4x-4e^4$
$y=2e^4x-3e^4$
When $y=0: 0=2e^4x-3e^4$
$-2e^4x=-3e^4$
$x=\frac{3}{2}\ \therefore\ S\left(\frac{3}{2},0\right)$
When $x=0: y=2e^4(0)-3e^4$
$y=-3e^4\ \therefore\ T(0,-3e^4)$
Area $=\frac{1}{2}\times\text{base}\times\text{height}$
$=\frac{1}{2}\times OS\times OT$
$=\frac{1}{2}\times\frac{3}{2}\times 3e^4$ (Dist. $OT=\left|-3e^4\right|$)
$=\frac{9}{4}e^4\,\text{units}^2$

10

$y=e^{-3x}$

$\dfrac{dy}{dx}=-3e^{-3x}$

$=\dfrac{-3}{e^{3x}}$

$y=\dfrac{3}{x-1}=3(x-1)^{-1}$

$\dfrac{dy}{dx}=-3(x-1)^{-2}$

$=\dfrac{-3}{(x-1)^2}$

Gradients equal

$\therefore\ \dfrac{-3}{e^{3x}}=\dfrac{-3}{(x-1)^2}\quad(\div -3)$

$\dfrac{1}{e^{3x}}=\dfrac{1}{(x-1)^2}$

$e^{3x}=(x-1)^2$

By inspection $x=0$

12(a) $y=(x-5)e^x$
$y'=vu'+uv'$
$y'=e^x(1)+(x-5)e^x$
$y'=e^x(1+x-5)$
$y'=e^x(x-4)$
$y''=(x-4)e^x+e^x(1)$
$y''=e^x(x-4+1)$
$y''=e^x(x-3)$
St. Pts. when $y'=0$
$\therefore e^x(x-4)=0\quad but\ e^x\neq 0$
$x-4=0$
$x=4\Rightarrow y=(4-5)e^4=-e^4$
St. Pt. $(4,-e^4)$
Check nature
When $x=4: y''=e^4(4-3)$
$=e^4>0$
$\therefore$ Min Turn. Pt. at $(4,-e^4)$

(b) Possible pts. of inflex. $y'' = 0$

i.e. $e^x(x-3)=0$: but $e^x \neq 0$

$\therefore (x-3)=0$

$x = 3 \Rightarrow y = (3-5)e^3 = -2e^3$

i.e. $(3, -2e^3)$

Check concavity either side of $x = 3$

when $x = 4$: $y'' = e^4(4-3)$

$= e^4 > 0$

when $x = 2$: $y'' = e^2(2-3)$

$= -e^4 < 0$

Concavity changes

$\therefore$ Pt. of inflex. at $(3, -2e^3)$

(c) x intercepts when $y = 0$

$0 = (x-5)e^x \quad but\ e^x \neq 0$

$\therefore (x-5) \Rightarrow x = 5$

x intercept $(5, 0)$

y intercept when $x = 0$

$y = (0-5)e^0$

$\therefore y = (-5)(1) \Rightarrow y = -5$

y intercept $(0, -5)$

(d) As $x \to \infty$:

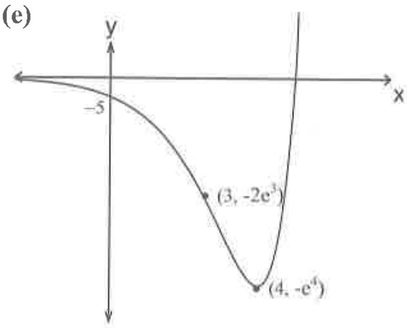
Both (x - 5) $\to \infty$

and $e^x \to \infty$

So as $x \to \infty$, $y \to \infty$

As $x \to -\infty$, $y \to 0$

(e)



(f) Monotonic increasing x > 4

21(a) $f(x) = (e^x + e^{-x})^2$

$f(a) = (e^a + e^{-a})^2$

$f(-a) = (e^{-a} + e^{-(-a)})^2$

$= (e^{-a} + e^a)^2$

$= (e^a + e^{-a})^2$

$f(a) = f(-a)$ $\therefore$ Even Function

(b) 415.4 units^2

Exercise 8

1(j) $y = [\ln(2x+7)]^5$

$y' = 5[\ln(2x+7)]^4 \cdot \left(\frac{2}{2x+7}\right)$

$= \frac{10[\ln(2x+7)]^4}{2x+7}$

2(c) $y = \ln\left(\frac{3x+2}{2x+1}\right)$

i.e. $y = \ln(3x+2) - \ln(2x+1)$

$y' = \frac{3}{3x+2} - \frac{2}{2x+1}$

or $y' = \frac{3(2x+1) - 2(3x+2)}{(3x+2)(2x+1)}$

$= \frac{6x+3-6x-4}{6x^2+7x+2}$

$= \frac{-1}{6x^2+7x+2}$

(d) $y = \ln\left(\frac{(2x+6)^7}{x}\right)$

i.e. $y = \ln(2x+6)^7 - \ln x$

$\therefore y = 7\ln(2x+6) - \ln x$

$y' = 7 \times \frac{2}{2x+6} - \frac{1}{x}$

$y' = 7 \times \frac{2}{2(x+3)} - \frac{1}{x}$

$y' = \frac{7}{x+3} - \frac{1}{x}$

or $y' = \frac{7x - x - 3}{(x+3)x}$

$y' = \frac{6x-3}{x^2+3x}$

7(o) $\int \frac{12x-8}{9x^2-12x}\,dx$

$= \int \frac{4(3x-2)}{3(3x^2-4x)}\,dx$

$= \frac{4}{3}\int \frac{3x-2}{3x^2-4x}\,dx$

[Now, $\frac{dy}{dx}$ of denominator $= 6x-4$

Need numerator to be this format]

$= \frac{2}{3}\int \frac{6x-4}{3x^2-4x}\,dx$

$= \frac{2}{3}\ln(3x^2-4x) + c$

8(c)

$\int_1^2 3xe^{x^2}\,dx \quad$ Note $\frac{d(e^{x^2})}{dx} = 2xe^{x^2}$

$\int_1^2 3xe^{x^2}\,dx = \int_1^2 \frac{3}{2} \times 2xe^{x^2}\,dx$

$= \frac{3}{2}\int_1^2 2xe^{x^2}\,dx$

$= \left[\frac{3}{2}e^{x^2}\right]_1^2$

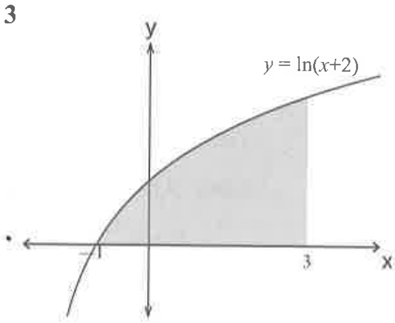
$= \frac{3}{2}e^{(2)^2} - \frac{3}{2}e^{(1)^2}$

$= \frac{3}{2}e^4 - \frac{3}{2}e$

$= \frac{3}{2}e(e^3 - 1)$

Exercise 9

3

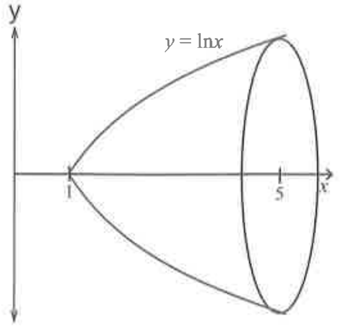


x	-1	0	1	2	3
y	ln1	ln2	ln3	ln4	ln5

$A \cong \frac{1}{3}\{0 + \ln 5 + 4[\ln 2 + \ln 4] + 2\ln 3\}$

$\cong 4 \cdot 041$ units^2

4



$$V = \pi \int_a^b y^2\, dx$$

$y = \ln x$

$y^2 = (\ln x)^2$

$$V = \pi \int_1^5 (\ln x)^2\, dx$$

Simpson's Rule is used to find an approximation for $\int_1^5 (\ln x)^2\, dx$

$f(x) = (\ln x)^2$

x	1	2	3	4	5
$f(x)$	0	$(\ln 2)^2$	$(\ln 3)^2$	$(\ln 4)^2$	$(\ln 5)^2$

$$A \cong \frac{1}{3}\left\{0 + (\ln 5)^2 + 4\left[(\ln 2)^2 + (\ln 4)^2\right] + 2\left[(\ln 3)^2\right]\right\}$$

$$\cong \frac{1}{3}(14 \cdot 61324859)$$

$$\cong 4 \cdot 871082865$$

$$\therefore V \cong \pi \times 4 \cdot 871082865$$

$$= 15 \cdot 303 \text{ units}^3 \text{ (to 3 dec. pl.)}$$