**ST PIUS X COLLEGE
CHATSWOOD**

2015 Stage 6 – Year 12

**ASSESSMENT TASK #2
MID-COURSE EXAMINATION**

30% of School Based Assessment

MATHEMATICS EXTENSION 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Draw diagrams using pencil
- Board-approved calculators may be used
- In Section II marks maybe deducted for careless or poorly arranged work
- Show all relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of all pages
- A table of standard integrals is included for reference

Total Marks – 70

| Section I | Multiple Choice |

10 marks

- Attempt Questions 1 – 10
- Enter solutions on Multiple Choice Answer Sheet

| Section II | Extended Response |

60 marks

- Attempt Questions 11 – 14
- Show all necessary working
- **Start each question in a SEPARATE booklet**

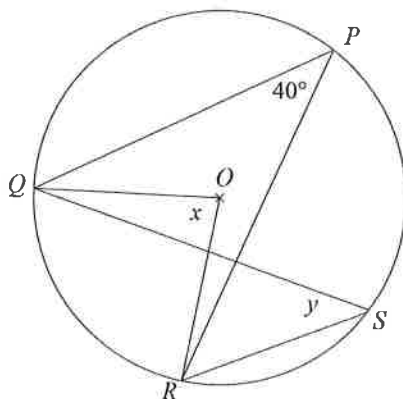**SECTION I – MULTIPLE CHOICE**   10 marks – each question is of equal value

Enter solutions on the Multiple Choice Answer Sheet provided

1   What is the solution to the equation $|2x-5| = x+2$?

(A) $x = 1$

(B) $x = 7$
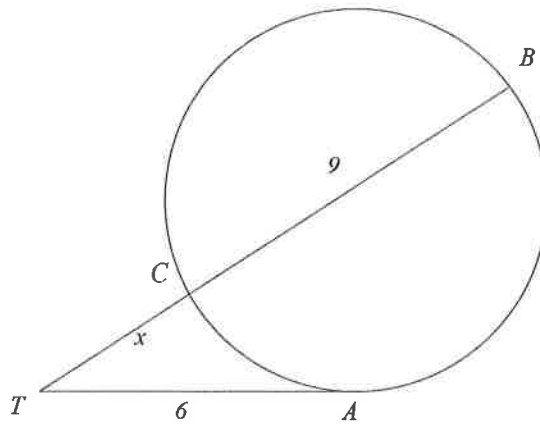
(C) $x = 1$ or $x = 7$

(D) $x = 1$ and $x = 7$

2 P, Q, R and S are points on a circle with centre O. $\angle QPR = 40°$.

What are the values of x and y?

(A) $x = 40°$ and $y = 20°$

(B) $x = 40°$ and $y = 40°$

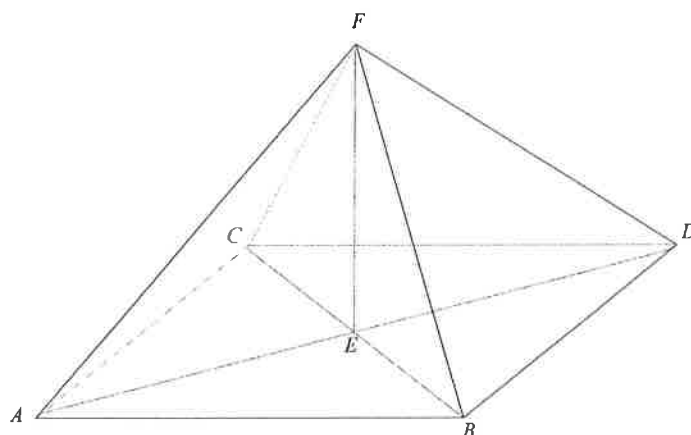(C) $x = 80°$ and $y = 20°$

(D) $x = 80°$ and $y = 40°$

3   Line *TA* is a tangent to the circle at *A* and *TB* is a secant meeting the circle at *B* and *C*.

Given that $TA = 6$, $CB = 9$ and $TC = x$, what is the value of x?

(A)   −12

(B)   2

(C)   3

(D)   4

4   What is the minimum value of $\cos\theta + 2\cos(\theta + 240°)$?

(A) $-2\sqrt{3}$

(B) $-\sqrt{3}$

(C) $-\dfrac{\sqrt{3}}{2}$

(D) $-\dfrac{\sqrt{3}}{4}$

5 A rectangular pyramid shown below has $AB = 16$ cm, $BD = 12$ cm and angle $FBE = 50°$.

What is the perpendicular height of the pyramid?

(A) 12 cm

(B) 14 cm

(C) 19 cm

(D) 24 cm

6 What is the acute angle between the lines $2x - y - 7 = 0$ and $3x - 5y - 2 = 0$?

(A) 4°24ʼ

(B) 32° 28ʼ

(C) 57°32ʼ

(D) 85°36ʼ

7 What are the coordinates of the point P that divides externally the interval joining the points $A(-5, 6)$ and $B(1, 0)$ in the ratio 3:1?

(A) $(2, -3)$

(B) $(2, -1\frac{1}{2})$

(C) $(4, -3)$

(D) $(4, -1\frac{1}{2})$

8 A curve has parametric equations $x = t+1$ and $y = 2t^2$.

What is Cartesian equation of this curve?

(A) $y = 2\sqrt{(x-1)}$

(B) $y = 2\overline{)(x+1)}$

(C) $y = 2(x-1)^2$

(D) $y = 2(x+1)^2$

9 How many arrangements of all of the letters of the word ADDITION are possible?

(A) 720

(B) 10 080

(C) 20 160

(D) 40 320

10 Let α, β and γ be the roots of $2x^3 + x^2 - 4x + 9 = 0$.

What is the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma}$?

(A) $-\dfrac{1}{2}$

(B) $-\dfrac{1}{9}$

(C) $\dfrac{1}{9}$

(D) $\dfrac{1}{2}$

END OF SECTION I

QUESTION 11   15 marks – allocation of marks as shown

Use a SEPARATE writing booklet   **Marks**

a.   Consider the function $= \frac{1}{x} + lnx$.

  (i)   Find the first derivative.   1

  (ii)   Find the second derivative.   1

  (iii) State the domain.   1

  (iv)   Show that $(1,1)$ is a minimum turning point.   2

  (v)   What happened to y as x approaches infinity.   1

  (vi)   Show that a point of inflexion occurs at $x = 2$.   2

  (vii) Draw on the same number plane, sketches of the graphs of

$$y = \frac{1}{x}, \quad y = lnx \text{ and } \quad y = \frac{1}{x} + lnx .$$ 2

b.   Differentiate $e^{-x}ln(2x)$.   2

c.   If $\int_1^5 \frac{dx}{2x-1} = ln(Z)$, find the value of Z.   3

QUESTION 12   15 marks – allocation of marks as shown

Use a SEPARATE writing booklet   **Marks**

a.   (i)   How many 10 letter "words" can be made by arranging the letters   **1**
          of the word SIMPLIFIES?

     (ii) In how many of these will the word MISS appear?   **2**

b.   Five women and their husbands belong to a Bridge Club.
     A committee of three is to be formed and it is decided that no man should be on
     the committee if his wife is also on it.
     In how many ways can the committee be formed?   **3**

c.   Prove by Mathematical induction for integers $n \geq 1$,

     $$1^2 + 3^2 + \cdots + (2n-1)^2 = \tfrac{1}{3}n(2n-1)(2n+1) \,.$$ **4**

d.   (i)   Sketch the function $= |1 - 2x|, -1 \leq x \leq 3$.   **1**

     (ii) Evaluate $\int_{-1}^{3}|1 - 2x|\, dx$.   **2**

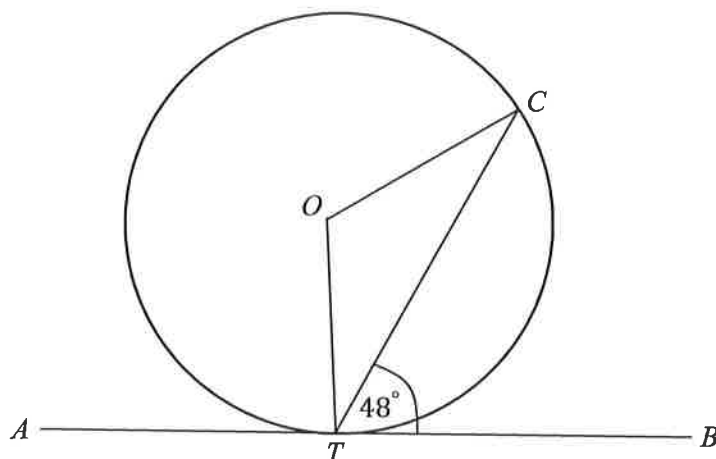e.   Express $\dfrac{1-x^{-1}}{x^{-1}-x^{-2}}$ in its simplest form.   **2**

QUESTION 13   15 marks – allocation of marks as shown
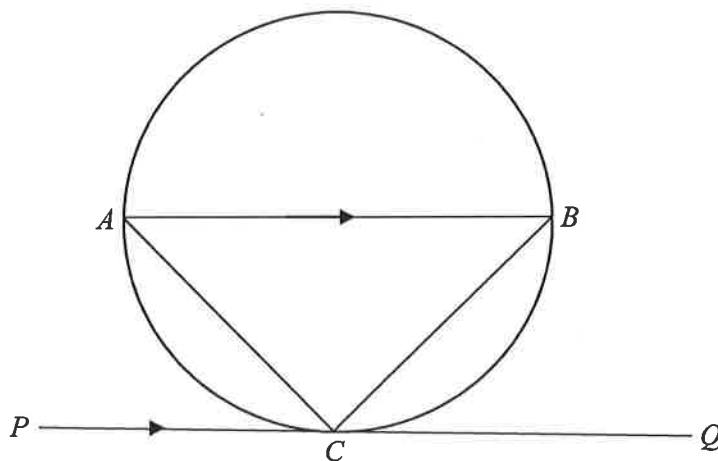
Use a SEPARATE writing booklet   **Marks**

a.   The diagram shows a circle with centre O and a tangent AB at T.

If the angle CTB is $48°$, find the value of the angle OCT.   **2**

b.   The diagram shows PQ, a tangent to the circle touching it at C. AB is a chord of the circle, parallel to PQ. Prove that the triangle ABC is isosceles.   **3**

c.   Find the indefinite integrals, using the indicated change of variable.

(i) $\int x^2\left(\sqrt{1+3x^3}\right) dx$ $[t = 1 + 3x^3]$ **3**

(ii) $\int \frac{e^{2x}}{e^x+1} dx$ $[t = e^x + 1]$ **4**

d.   If $p\cos x = 1 + \sin x$, prove that $\frac{p-1}{p+1} = t$, where $t = \tan\frac{x}{2}$.   **3**

QUESTION 14   15 marks – allocation of marks as shown

Start this question in a SEPARATE booklet   **Marks**

a.   Mickey is in Year Twelve at Disney High School and wants to buy a car
     when he finishes school.

     He has saved $P but wishes to save a total of $8000. Mickey sets up a car fund on
     1ˢᵗ January 2015.

     The fund is paying 6% per annum interest which is compounding monthly.

   (i)   If Mickey makes no further deposits it would take $9\frac{1}{2}$ years for his investment
         to accumulate to $8000. Show that $P = \$4531$ to the nearest dollar.   **2**

   (ii) To reach his target by the end of 2015, Mickey decides that after his initial
         deposit of $4531 he will make regular deposits of $Q at the beginning of each
         month starting in February 2015.

         Mickey plans to collect his savings of $8000 at the close of business on
         31ˢᵗ December 2015 just after the interest is paid.

         Let A_n represent the amount Mickey has saved after n months.

       (α)   Show that by the 30ᵗʰ April 2015, Mickey will have saved:

                    $$A_4 = 4531(1.005)^4 + Q(1.005 + 1.005^2 + 1.005^3)$$ **3**

       (β)   Find the value of his monthly deposits, $Q, correct to the nearest dollar.   **3**

b.   Consider the series $1 + \frac{1}{(1-x)} + \frac{1}{(1-x)^2} + \frac{1}{(1-x)^3} + \cdots$

     Find the set of x values for which this series has a sum to infinity.   **3**

c.   In a certain series the Sum is given by $S_n = 5n - n^2$.

   (i)   Find the expression for the general term T_n .   **2**

   (ii) Show that this Sum is representative of an arithmetic sequence.   **2**

END OF SECTION II

END OF ASSESSMENT