**St Pius X College Chatswood**

MATHEMATICS DEPARTMENT

**2018 Stage 6 Year 11 Accelerated Mathematics Extension 1 HSC Course**



**2018 – Stage 6 Year 11 Accelerated**

**Mathematics Extension 1 HSC Course**

**Scope and Sequence**

  
Text references:

Jones and Couchman, ***3 Unit Mathematics Book 1***, Longman, Sydney, 1997.  
Fitzpatrick, **New Senior Mathematics 3 Unit Course, Second Edition**, Pearson, Melbourne 2013.

**Outcomes and Objectives**

|  |  |
| --- | --- |
| **Objectives** | **HSC Mathematics** |
| **Outcomes** |
| Students will develop: | A student: |
| appreciation of the scope, usefulness, beauty and elegance of mathematics | H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts |
| the ability to reason in a broad range of mathematical contexts | H2 - constructs arguments to prove and justify results |
| skills in applying mathematical techniques to the solution of practical problems | H3 - manipulates algebraic expressions involving logarithmic and exponential functions  H4 - expresses practical problems in mathematical terms based on simple given models  H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems |
| understanding of the key concepts of calculus and the ability to differentiate and integrate a range of functions | H6 - uses the derivative to determine the features of the graph of a function  H7 - uses the features of a graph to deduce information about the derivative  H8 - uses techniques of integration to calculate areas and volumes |
| the ability to interpret and communicate mathematics in a variety of forms | H9 - communicates using mathematical language, notation, diagrams and graphs |

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| **Objectives** | **HSC Mathematics Extension 1** |
| **Outcomes** |
| Students will develop: | A student: |
| appreciation of the scope, usefulness, beauty and elegance of mathematics | HE1 - appreciates interrelationships between ideas drawn from different areas of mathematics |
| the ability to reason in a broad range of mathematical contexts | HE2 - uses inductive reasoning in the construction of proofs |
| skills in applying mathematical techniques to the solution of practical problems | HE3 - uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay |
| understanding of the key concepts of calculus and the ability to differentiate and integrate a range of functions | HE4 - uses the relationship between functions, inverse functions and their derivatives  HE5 - applies the chain rule to problems including those involving velocity and acceleration as functions of displacement  HE6 - determines integrals by reduction to a standard form through a given substitution |
| the ability to interpret and communicate mathematics in a variety of forms | HE7 - evaluates mathematical solutions to problems and communicates them in an appropriate forms |

**2018 – Stage 6 Year 11 Accelerated**

**Mathematics Extension 1 HSC Course**

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# Stage 6 Year 11 Accelerated Mathematics Extension 1

# HSC Course

# Syllabus Reference Board of Studies NSW, *Mathematics 2/3 Unit Year 11-12 Syllabus*, Board of Studies NSW, 1982 (Revised for web May 2008).

# <http://www.boardofstudies.nsw.edu.au/syllabus_hsc/mathematics-advanced.html>

# Assessment Reference

# Board of Studies NSW, *Assessment and Reporting in Mathematics, Mathematics Extension 1 and Mathematics Extension 2 Stage 6*, Board of Studies NSW, September 2011.

# <http://www.boardofstudies.nsw.edu.au/syllabus_hsc/pdf_doc/maths-ext-1-and-2-a-and-r-from2012.pdf>

# Textbooks Jones and Couchman, *3 Unit Mathematics Book 1*, Longman, Sydney, 1997.

# Fitzpatrick, New Senior Mathematics 3 Unit Course, Pearson, Melbourne 1983.

# Introduction

The Board recognises that the aims and objectives of the syllabus may be achieved in a variety of ways and by the application of many different techniques. Success in the achievement of these aims and objectives is the concern of the Board which does not, however, either stipulate or evaluate specific teaching methods.

# The Mathematics Extension 1 Course

The content of this course, *which includes the whole of the Mathematics course*, and its depth of treatment as specified in Part A and Part B indicate that it is intended for students who have demonstrated a mastery of the skills included in the School Certificate mathematics course and who are interested in the study of further skills and ideas in mathematics.

The Extension 1 course is intended to give these students a *thorough* understanding of, and competence in, aspects of mathematics including many which are applicable to the real world. The course has general educational merit and is also useful for concurrent studies of science, industrial arts and commerce. It is a *recommended minimum* basis for further studies in mathematics as a *major* discipline at a tertiary level, *and for the study of* *mathematics in support of the physical and engineering sciences*. *Although the Extension 1* *course is sufficient for these purposes, it is recommended that students of outstanding* *mathematical ability should consider undertaking the Extension 2 course*.

**Mathematics Extension 1 Course** **Objectives**

Specific objectives of the course are:

(a) to give an understanding of important mathematical ideas such as variable, function, limit, etc, and to introduce students to mathematical techniques which are relevant to the real world;

(b) to understand the need to prove results, *to appreciate* the role of deductive reasoning in establishing such proofs, and to develop the ability to construct these proofs;

(c) to enhance those mathematical skills required for further studies in *mathematics, the physical sciences and the technological sciences*. For achievement of these aims, the following points are important:

(i) Understanding of the basic ideas and precise use of language must be emphasised;

(ii) A clear distinction must be made between results which are proved, and results which are merely stated or made plausible;

(iii) Where proofs are given, they should be carefully developed, with emphasis on the deductive processes used;

(iv) Attaining competence in mathematical skills and techniques requires many examples, given as teaching illustrations and as exercises to be undertaken independently by the student;

(v) Since the course is to be ‘useful for concurrent studies of science, industrial arts and commerce’ students could be given some experience in applying mathematics to problems drawn from such areas. Realistic problems should follow the attainment of skills, and

techniques of problem solving should be continually developed.

# Gospel Values Integrated into Mathematics Programs

**There is no such thing as ‘Catholic Science’ or ‘Catholic Maths’ or ‘Catholic English’ but, more importantly, there is no subject in our school’s curriculum, including mathematics, that is ‘values free’. Catholic educators are, in fact, charged with the responsibility of ‘…bringing to their courses, their pedagogies, their structures, their relationships – to the very core of their school culture – a Gospel perspective on what it means to be a human being living in Australia…’ (A Sense of the Sacred – Foundations; p17 Section 3).**

There are a multiplicity of opportunities that teachers of mathematics can take to infuse gospel values that are relevant to the life, learning and contemporary culture of their students into programme development and classroom teaching and learning activities. Treating social justice and other values issues appropriately in this academic discipline can add to the intrinsic relevance and value of the course.

***Key points to consider -***

1. A process for developing a unit of work in Mathematics that includes Gospel Values – In the planning stages teachers should identify the knowledge, skills and values outcomes for the unit. The values outcomes should be more than those listed in the syllabus. They should focus on the issues that evolve from integrating the maths concepts with real life situations. For example: in a unit on *Earning Money* for the Stage 6 General mathematics course, the values outcomes could include -

*For the students to demonstrate their ability to:*

* *Question the ethics surrounding people who earn money as piece workers and the prices the goods are then sold for in shops.*
* *Explore the idea that people have a right to meaningful and appropriately paid work.*
* *Identify the organisations that protect people’s rights in the area of economic justice (where labour takes precedence over both capital and technology in the production process.).*
* *Investigate and discuss the responsibilities societies around the world have taken on to protect the dignity of those not able to work.*

Once the values outcomes are identified teachers need to ask the critical questions:

* What gospel values / issues need to be emphasised?
* What secular values / issues need to be challenged?
* What strategies will provide opportunities for the students to demonstrate the values outcomes?
* What resources can be used or developed to enhance these strategies?

This exercise will ensure that gospel values are articulated and integrated into maths programmes in a purposeful and relevant manner.   
  
2. The problem solving approach is much more suited to values integration and is a model for sound teaching and learning. This approach provides a framework for thinking, creative and active participants to consider a common problem and find solutions. Questions that can be raised include: why, how, and who? Participants are active by describing, analysing, suggesting, deciding and planning.   
  
3. When information is needed to demonstrate a concept, a teacher can use materials that are not value neutral; for example, in the topic of statistics teachers often direct their students to perform statistical calculations on activities like – how many people walked past the shops over certain periods of time? Rather, statistics related to the availability of clean water in various countries is more suitable for a values infusion learning strategy. Once the students have examined the information for the mathematical content and skills application, the teacher can lead the students to consider such things as the reasons for inequities between countries and the response it calls us to make. This type of discussion allows students to see the relevance of mathematics in a meaningful way ie: mathematics skills – statistics – provides the opportunity for people to gain easy access to information about real life situations by summarising it in a simple manner.

***Reference: Catholic Education Office, Sydney 1998. A Sense of the Sacred; Foundations Document***

**Assessment for Learning**

Assessment in Mathematics is the process gathering, judging and interpreting information about student’s achievement, knowledge, skills, values and awareness of their learning processes in order to:

* provide reliable information that can be used to inform teaching and learning
* provide feedback to students about progress
* generate information to be used in the reporting process

As well as:

* being a learning tool in itself
* providing a source of student motivation

In short, Assessment in Mathematics should not only be Assessment of student’s achievement it should be Assessment for learning.

*Assessment for learning in Mathematics is designed to enhance teaching and improve learning. It is assessment that gives students opportunities to produce the work that leads to development of their knowledge, skills and understanding. Assessment for learning involves teachers in deciding how and when to assess student achievement, as they plan the work students will do, using a range of appropriate assessment strategies including self-assessment and peer assessment.*

*Assessment for learning:*

* *is an essential and integrated part of teaching and learning*
* *reflects a belief that all students can improve*
* *involves setting learning goals with students*
* *helps students know and recognise the standards they are aiming for*
* *involves students in self-assessment**and peer assessment*
* *provides feedback that helps students understand the next steps in learning and plan how to   
   achieve them*
* *involves teachers, students and parents reflecting on assessment data.*

Assessment of learning takes place both informally in the context of everyday classroom activities, as well as formal, planned assessment events.

Assessment in Mathematics aims to:

* be manageable for teachers to administer
* be inclusive of all learners
* encourage students to strive for personal excellence, whilst recognizing that mistakes are a normal part of the learning process
* provide regular and meaningful feedback to students on their achievements, knowledge, skills, strengths and areas in need of improvement in relation to the outcomes of the teaching program that is constructive
* provide stimulating activities and exercises that engage students in their learning
* reflect a view of learning in which assessment helps students learn better, rather than just achieve a better mark
* help students to take responsibility for their own learning
* include provisions for assessment events that are:
  + informal: day to day inspection of homework, classroom observations, topic tests, quick quizzes, assignments that are used for class work, etc.
  + formative: over the period of a semester, largely descriptive, and very specific information about student learning in terms of strategies/ areas for improvement and achievement (progress towards achievement) of specific outcomes
  + summative: measurement of overall teaching and learning at the end of a semester arising as a natural development of the formative assessment of the semester, feedback of which has been given to parents and students
* provide meaningful feedback for teachers on the effectiveness of teaching and learning

**Formal Assessment Tasks**

As per the College Assessment Policy, Formal Assessments Tasks should be chosen that:

* meet Board of Studies guidelines (where relevant), as set out in the BOS Subject Handbooks, in terms of timing and syllabus components, and thus satisfy requirements of accreditation
* encourage/ enhance and assist in the learning process
* cover all course outcomes
* are appropriate to the learning experiences of students
* cover a range of assessment modes
* assess the stated educational outcomes for which they are devised, and accurately indicate students achievements of outcomes
* measure student achievement in terms of standards reached
* are fair reward, in terms of marks/ grades allocated, for the effort that the average student expends to gain a reasonable result
* are not onerous on staff in terms of preparation, conduct and/ or marking
* provide, where applicable, effective positive discrimination between the students of a course
* are of a reasonable number and distribution throughout the course
* are sufficiently flexible to provide for students/ occasions that are atypical
* are sensitive to issues of socio-economic background, culture, disability, gender or race

Formal Assessment Tasks in Mathematics should:

* be set by a teacher of the course
* be checked by each teacher of the course.
* clearly state on the front cover: (Cover sheet pro-former is available in the Maths folder on the College Network)
  + St Pius X College Assessment Task
  + The name of the course (eg. Year 11 Extension 1 Mathematics)
  + Instructions for students
  + The date of the assessment task
  + The time allowed for the task
* have marks assigned to each question (where applicable) and be labelled to allow students to use and develop ‘time management’ strategies
* have an outcomes/ marking/ grading scheme worked out **before** students sit for the task
* be photocopied at least 2 days prior to the date of the assessment task

# If teachers are having difficulty in completing the task in time they should speak to the Mathematics Coordinator so that assistance can be organised.

**Informal Assessment Tasks**

Informal Assessment Tasks are a vital part of the process of Assessment for Learning in Mathematics.

Possible sources of information for assessment purposes include the following:

* samples of students’ work
* explanation and demonstration to others
* questions posed by students
* student-produced overviews or summaries of topics
* practical tasks such as measurement activities
* investigations and/or projects
* students’ oral and written reports
* short quizzes
* pen-and-paper tests involving multiple choice, short-answer questions and questions requiring longer responses, including interdependent questions (where one part depends on the answer obtained in the preceding part)
* open-book tests
* comprehension and interpretation exercises
* student-produced worked examples
* teacher/student discussion or interviews
* observation of students during learning activities, including listening to students’ use of   
   language
* observation of students’ participation in a group activity
* consideration of students’ portfolios
* students’ plans for and records of their solutions of problems
* students’ journals and comments on the process of their solutions.

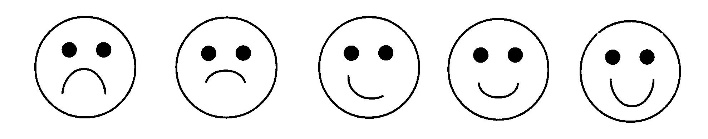
# St Pius X College Chatswood

# Student Unit Evaluation: MATHEMATICS

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Class: \_\_\_\_\_\_\_\_\_\_ Teacher: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Topic: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Circle the face, which shows best how you feel about this unit.



Name a new thing you learnt in the unit.

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Write down three things you liked in the unit and three things you disliked.

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| --- | --- | --- |
| THINGS I LIKED |  | THINGS I DISLIKED |
|  |  |  |
|  |  |  |
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Did you do well in this unit? How?

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What could be done to improve this unit?

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| **Unit 1 - Probability** | | **TEACHER** |
| **CLASS** |
| **Focus**: This unit of work uses examples from every day activities to explore probability involving both single and multiple events.  Time Allocation: 2 Weeks Syllabus Reference: 3.1 – 3.3 | | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range of contexts  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of probability to solve problems  **H9** communicates using mathematical language, notation, diagrams and graphs | Content Description  3.1 Random experiments, equally likely outcomes; probability of a given result.  3.2 Sum and product of results.  3.3 Experiments involving successive outcomes; tree diagrams. | |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 13  Fitzpatrick 2U Chapter 5  Understanding Year 12 Maths (Accelerated Maths learning) pg 154-156  New Senior Mathematics  2U Mathematics Book 2  Excel HSC  Dice  Counters | | |

| **Content (3.1 – 3.3 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
| --- | --- | --- |
| Know the meaning of terms such as event, sample space, random, equally likely, mutually exclusive. |  |  |
| Be familiar with:   * a common pack of cards * coins * dice * raffles and lotteries |  |  |
| Use:  P ( E ) =  and 0 ≤ P ( E ) ≤ 1 |  | Students should be encouraged to list and use appropriate counting techniques. |
| Use dot diagrams |  | A captain and vice captain are to be chosen from 5 prefects, Anne, Bob, Carol, Doug, Eve. Find the probability that:  a) Eve is captain and Bob is vice captain  b) Carol is chosen  **STRESS NEED TO CHECK DIAGONAL.** |
| Use tree diagrams |  | Questions should include lottery type questions, replacement and non-replacement.  Probability should be given in terms of percentages and ratio. |

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| **PROBABILITY** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Discuss the meaning of key probability terms such as event, sample space, random, equally likely, mutually exclusive. * Discuss common misconceptions about probability. * Evaluate media statements involving Probability and discuss the use of Probability by Governments and Companies e.g Demography, Insurance, Planning for roads. * Review questions involving simple one stage probability. * Simulate probability experiments using the random number generator function on the calculator. * Discuss the Monty Hall Problem * [The Monty Hall Problem](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\The%20Monty%20Hall%20Problem.doc) * [The Monty Hall Problem Simulation](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\Probability\Monty%20Hall%20Simulation.exe) * [The Monty Hall Recording Sheet](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\The%20Monty%20Hall%20Problem%20Recording%20Sheet.doc) * Play games such as [Sums and Differences Games](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\Sums%20and%20Differences%20Investigations.doc), [Unders and Overs](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\Unders%20and%20Over1.doc)[[1]](#footnote-1), discuss the fairness of each game in terms of probability and use tables and tree diagrams to plan the best strategy for playing the games. * Play Tossing Two Coins – An Interesting Game with class using “[cheat sheet](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Probability\TOSSING%20TWO%20COINS%20an%20interesting%20game.doc)” and the have students prove the probabilities on the cheat shee * Use tables, tree diagrams and dot plots to solve compound probability questions. * Solve problems involving common pack of cards, coins, dice, raffles and lotteries. * Complete Examination Style questions on Probability[[2]](#footnote-2) |
| * Review the different counting techniques, including lists, tables, permutations and combinations. * Use the different counting techniques to solve practical probability questions. * Work in groups to solve practical probability questions. * Investigate the probability of being dealt each hand in a game of Poker.[[3]](#footnote-3) * [Have students complete questions on Probability](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Probability\MIF3U25probability.doc) * Complete Exam Style Questions on Probability. |

| Syllabus Topics | Additional Information | Text References |
| --- | --- | --- |
| 3.1  Random experiments, equally likely outcomes; probability of a given result. | Students should be familiar with the common terms used in popular activities and games, hence examples should be given in pastimes such as playing cards, monopoly and backgammon as well as in gaming activities such as lotteries and raffles, the tossing of coins and the throwing of dice.  3.1 Use everyday examples of ‘random experiments’, such as coin tossing, throwing dice, drawing raffles or lotteries, dealing cards, to introduce the ideas of *outcomes* of experiments, the notion of *equally likely* outcomes, and the idea that for experiments having a finite number n of *equally likely,*  *mutually exclusive* outcomes *E*1, …,*En*, the probability *P*(*A*) of a single *result* A is given by  . | Text pages 252 – 254  Exercise 13.1  Text pp 260 – 261  Exercise 13.4 |
|  | In particular, since one of *E*1,...,*En* must occur,  *P*(*E*1) + ... + *P*(*En*) = 1,  and for *any* result *A*,  0 ≤*P*(*A*) ≤*1*.  **Examples**  (i) An ordinary die is thrown. Find the probabilities that (a) 1 is shown, (b) an odd number is shown.  (a) Since the 6 outcomes are equally likely, the probability of the outcome ‘1 is shown’ is 1/6.  (b) An odd number is shown if and only if any of the outcomes 1, 3, 5 occur. Hence the probability is 3/6 = 1/2.  (ii) A pair of dice are thrown. What is the probability that they show a total of three?  Solution. Each outcome of the first die is equally likely to occur with each outcome of the second die. The total number of possible outcomes is 6 6 = 36, each occurring with a probability 1/36. The outcomes producing a total of three and a 1 and a 2, or a 2 and a 1.  Hence the probability of a total of three is  (iii) In a raffle, 30 tickets are sold and there is one prize. What is the probability that someone buying 5 tickets wins the prize?  Solution. The probability that a given ticket wins the prize is 1/30. Hence the probability of winning with 5 tickets is 5/30 = 1/6. | Text pp. 258-260  Exercise 13.3  Roulette – Exercise 13.2 pp 255-266  Two dice – Exercise 13.4 pp 260-261  Craps – Exercise 13.5 pp 261-262 |
| 3.2  Sum and product of results. | (iv) A card is drawn randomly from a standard pack of 52 cards. What is the probability that it is an even-numbered card?  Solution. Of the 52 equally likely outcomes, the drawing of a 2, 4, 6, 8 or 10 of clubs, diamonds, hearts or spades are the outcomes producing the result. The probability of the result is thus (5 4)/52 = 5/13.  Practice should be given in calculating the probabilities of various types of result (both composite and simple) from a knowledge of the probabilities of the possible outcomes of an experiment.  The complementary result (*A* does *not* occur, or ‘not *A*’) to *A* should be defined, the relation P( ) + *P* (*A*) = 1 derived, and use made of it in simple examples.  In the case of mutually exclusive outcomes *A*1, *A*2, the probability that *A*1or A2 occurs is the sum of the probabilities that A1, A2 each occur.  Denoting the result ‘*A*1 or *A*2’ by *A*1 *A*2 (called the *sum* of *A*1, and *A*2),  *P*(*A*1 *A*2) = *P*(*A*1) + *P*(*A*2), (1)  and generally, for mutually exclusive outcomes *A*1, …, *An* ,  *P*(*A*1 … *An*) = *P*(*A*1) + … + *P*(*An*). | Text pp 269-270  Exercise 13.9  Text pp 270- 272  Exercise 13.10 |
| 3.3  Experiments involving successive outcomes; tree diagrams | Sometimes, two results may occur together (ie they are not mutually exclusive). For example, in randomly selecting a digit from the digits, 1, 2,  3, 4, 5, 6, 7, 8, 9, if A is the result ‘an even digit is selected’ and B is the result ‘a digit less  than 5 is selected’, then *A* and *B* will both occur if 2 or 4 is selected. Denoting the result ‘*A* and *B*’, called the product of *A* and *B*, by *AB*,  *P*(*AB*) = 2/9.  *A* *B* occurs if 1, 2, 3, 4, 6 or 8 is selected, thus  *P*(*A* *B*) = 6/9 = 2/3,  and in this experiment,  *P*(*A* *B*) ≠*P*(*A*) + *P*(*B*) (= 4/9 + 4/9 = 8/9).  This inequality holds because *A* and *B* are not mutually exclusive results. For general results *A*, *B*, the formula (1) must be replaced by  *P*(*A* *B*) = *P*(*A*) + *P*(*B*) – *P*(*AB*), (2)  and it is readily verified that (2) holds in the above example. Practice in the use of (2) should be given, but formal proofs of (1) or (2) are not required.  Examples should be given which illustrate the difference between, say, successive tosses of a coin (where the probabilities of successive outcomes  do not depend on previous outcomes) and the drawing of names from a hat (where probabilities depend on previous outcomes).  Tree diagrams should be used to trace the possible outcomes of two or three stage experiments, and hence to calculate the probabilities of certain  final results. Explanation of all steps in the diagram should be given, so that students can construct diagrams, as shown below, directly from given  information. | Text pp 262-265  Exercise 13.6  Text pp 266-269  Exercise 13.8 |
|  | **Examples**  (i) 5 boys’ names and 6 girls’ names are in a hat. Find the probability that in two draws a boy’s name and a girl’s name are chosen. (No replacement of names after a draw.)  (*Refer tree diagram in Syllabus page 33)*  Required probability = *P*(*GB* *BG*) = 3/11 + 3/11 = 6/11.  (ii) In a raffle, 30 tickets are sold and there are two prizes. What is the probability that someone buying 5 tickets wins at least one prize?  In this example, we simplify the tree diagram by considering carefully what it is we are required to find. The required result is obtained in exactly two exclusive ways: either first prize is won, in which case it does not matter whether second prize is won or not, or first prize is not won but second prize is won.  (*Refer tree diagram in Syllabus page 34)*  Required probability = *P* (*W* *LW*)  = 1/6 + 25/174 = 9/29.  An alternative method is to notice that the probability of winning at least one prize is the complement of winning no prizes, ie;  Required probability = 1 – *P* (*LL*)  = 1 – 25/30 24/29  = 1 – 20/29  = 9/29.  (iii) In a mixture of red and white pebbles in a gravel, red and white pebbles occur in the ratio of 3 to 7. Find the probability that if 3 pebbles are chosen from the mixture, (a) exactly two are red; (b) at least one is white. |  |
|  | Let p (=0.3) and q (=0.7) be respectively the probabilities of choosing a red or a white pebble. The tree diagram is as follows:  (*Refer tree diagram in Syllabus page 34)*  (a) Required probability = *P* (*RRW**RWR**WRR*)  = 3*p*2*q* = 189/1000  (b) Required probability  = *P* (*RRW**RWR**RWW**WRR**WRW**WWR**WWW*)  = 3*p*2*q* + 3*pq*2 + *q*3  = 973/1000.  In examples such as (b), it is important to realise that the result is more readily obtained by calculating the complementary probability (that none is white) and subtracting from 1. Here, the probability of obtaining 3 red pebbles is *p*3, hence the probability of obtaining at least one white pebble is 1 – *p*3 = 973/1000. |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 2 – Geometrical applications of differentiation** | | **TEACHER** |
| **CLASS** |
| **Focus**: This unit of work provides a link between the work already done on both functions and differentiation as part of the Preliminary course. The relationship between the graph of a function and the graph of its derivative is an important theme. Practical applications are also considered.  Time Allocation: 3 Weeks Syllabus Reference: 10.1 – 10.8 | | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range of contexts  **H2** constructs arguments to prove and justify results  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of calculu and geometry to solve problems  **H6** uses the derivative to determine the features of the graph of a function  **H7** uses the graph to deduce information about the derivative  **H9** communicates using mathematical language, notation, diagrams and graphs | Content Description  10.1 Significance of the sign of the derivative.  10.2 Stationary points on curves.  10.3 The second derivative. The notations f’’(x), , y’’.  10.4 Geometrical significance of the second derivative.  10.5 The sketching of simple curves.  10.6 Problems on maxima and minima.  10.7 Tangents and normals to curves.  10.8 The primitive function and its geometrical interpretation.  . | |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Chs. 11,16  Autograph  Graphics calculators  Maths in Focus Chapter 2  Excel HSC  Neap Revision by Topic |  | |

| **Content (10.1 – 10.8 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
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| Students should know the geometrical significance of the sign of the  first derivative. |  |  |
| Students should know the definitions of (monotonic) increasing and  decreasing in terms of the first derivative. |  |  |
| Apply the above to problems including determination of whether or  not a function is increasing or decreasing, finding values of a formula  which is a function that is increasing or decreasing etc. |  | \* Show that , x ≠ 0 is always  decreasing.  \* For what values of *x* is  either increasing  or decreasing ? |
| The definition of a stationary point and be able to distinguish between maximum and minimum turning points and points of horizontal inflexion by considering the sign of the first derivative. |  | In setting out students **MUST CLEARLY** indicate that the first derivative is being considered. |
| Sketch curves using the previous information and know the difference between absolute and local minimum and maximum |  |  |
| Students should find the second derivatives of functions and know the  geometrical significance of the second derivative. |  |  |
| Use the second derivative to distinguish between stationary points. |  | Second derivative **MUST** be clearly indicated. |
| Students should be aware of the limitations of the above method and use the first derivative test when the second derivative is inappropriate. |  | \*  \* when the second derivative is too  complicated to find. |
| The definition of a point of inflexion (a point where the curve changes concavity)  i.e. changes sign |  | **STRESS** = 0 is not the definition of a point of inflexion. |
| Find stationary points and points of inflexion and sketch the curve. Curves should include cubics, higher polynomials and simple rational functions.    Students should consider symmetry about the axes, behaviour for very large positive and negative values of *x*, domain and range, and points where the function is undefined. |  | For the curve  a) Find the stationary points and determine  their nature.  b) Find the points of inflexion and justify  your answer.  c) Hence sketch the curve for –2 ≤ *x* ≤ 3  , |
| Maximum and minimum value problems. |  |  |
| a) Students should be given practice at constructing functions from  data given in words or on a diagram. |  |  |
| b) Identify turning points and distinguish between local and   absolute maximum and minimum values over fixed domains. |  |  |
| **STRESS** students should recognize quadratic functions and find the vertex to determine maximum and minimum values without the use of **CALCULUS.** |  |  |

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| **GEOMETRICAL APPLICATIONS OF DIFFERENTIATION** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Watch the Video “Off on a tangent” (The Concepts of Calculus) to review the concepts relating to differentiation * Define key terms such as *function, curve, sketch, monotonic, increasing, decreasing, stationary, local, absolute, maximum, minimum, horizontal, inflexion, turning point.* * Explore how the gradient of the curve changes at different points – in particular examining whether the first and second derivatives are –ve, 0 or +ve. * Discuss the how the 1st and 2nd derivatives tell us whether a function is increasing at a decreasing/constant/increasing rate, or decreasing at a decreasing/constant/increasing rate. Relate this to curves and practical problems. * Explore how the second derivative can be used to classify stationery points. * [Use Geometry SketchPad file to explain the effects of the 2nd derivative](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\2nd%20derivative.gsp) * Discuss by way of examples when it is better to use the first derivative test to classify turning points. * Discuss how to use the 2nd derivative to find P.O.I – placing emphasis on the fact the finding where *y*’’ = 0 is not enough to classify this as a P.O.I. * Review curve sketching skills (such as finding asymptotes, intercepts, determining whether a function is odd or even, etc) from functions unit of work and incorporate these skills in sketching graphs. * Explore by way of example the way to solve maxima and minima problems. * [Use Geometry SketchPad file to examine volume of a Box using calculus](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\Box%20Volume.gsp) * [Use Geometry SketchPad file to examine volume of a Square Box using calculus](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\Max%20Min%20square%20box.gsp) * [Explore the Graph of a function showing the volume of a square based rectangular prism](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\maxima%20and%20minima%20-%20square%20based%20rect%20prism.gsp) * [Explore the Graph of a function to find the maximum area of a rectangle with a fixed perimeter](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\Maximum%20Area%20given%20perimeter.gsp) * [Explore Maximizing the Area of a Rectangular Pen Fenced in on Three Sides](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\maxpen.gsp) * [Complete questions on Geometric Applications of Calculus](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Geometric%20Applications%20of%20Differentiation\GEOMETRICAL%20APPLICATIONS%20OF%20CALCULUS.doc) * Complete HSC Style questions. |

| Syllabus Topics | Additional Information | Text Reference |
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| 10.1  Significance of the sign of the derivative. | This is a continuation of the exploration of the relationship between geometrical properties of functions and analytic properties of functions begun in Topic 4 and developed further in Topic 8. In particular, it will be found useful to consider  examples in which the graphs of *y = f(x)* and  *y = f '(x)* are drawn so that visual transfer of information occurs easily. In addition, the interpretation of *f '(x)* as the gradient of the tangent at *(x, f(x))* is usefully retained by drawing tangent lines to *y = f(x)* at appropriately chosen values of *x*.  The geometrical significance of the sign of *f '(x)* is to be understood, including the determination of whether or not *f(x)* is increasing or decreasing. | Text pp. 232-233 |
| 10.2  Stationary points on curves.  10.3  The second derivative.  The notations *f’’(x), , y’’.*  10.4  Geometrical significance of the second derivative. | A *stationary point* of *f(x)* is defined to be a point on  *y* = *f(x)* where the tangent is parallel to the *x*–axis.  At such a point, = 0.  A *turning point* of *f(x)* is a point where the curve *y = f(x)* is locally a maximum or a minimum. For differentiable functions *f*, all turning points are stationary; but there are stationary points of some functions at which the tangent ‘crosses the curve’ and which are not turning points. (The term *inflexion* might be deferred for the present to avoid the mistaken idea that all inflexions are ‘horizontal’). Thus the criterion for a turning point is the change in sign of *f '(x)* as *x* passes through the abscissa of the point, and the identification of type of stationary point should be made by considering the sign of *f '(x)* on either side of the point.  The distinction between a local maximum and the greatest value of a function for a given domain of the variable should be made clear.  The definition of the second derivative and the notations *f"(x)*, , *y"*.  Geometrical significance of the sign of the second derivative:  if > 0 at *P*, the curve is concave upwards at *P*;  if < 0 at *P*, the curve is concave downwards at *P*.  At a point of inflexion, vanishes and its sign changes on passing through the point. | Text pp 233 – 237  Exercises 11.6, 11.7  Text pp 320 – 321  Exercise 16.1  Text pp 321 – 322  Exercise 16.2  Text pp 322 – 323  Exercise 16.3 |
| 10.5  The sketching of simple curves.  10.6  Problems on maxima and minima. | The second derivative may be used to distinguish between maximum and minimum turning points. The criterion should be used with caution, since  the condition  = 0, ≠0,  is sufficient but not necessary. For example, *y = x*4 has a minimum point at the origin where  = = 0.  On the other hand, the curve *y = x*(3*x* – 3 – *x*2) has a point of inflexion at *x* = 1.  The sketching of curves such as quadratics, cubics and higher polynomials and simple rational functions. After computing some values (which may include points where *x* = 0 and where *y* = 0), the determination of the stationary points is frequently very useful. Other considerations are  symmetry about the axes, behaviour for very large positive and negative values of *x*, and the points at which functions such as:  are defined.  **Examples should include both horizontal and vertical asymptotes,**  **eg**  For all students the techniques developed in this topic should be applied to examples involving functions introduced later in their courses.  Problems on maxima and minima should include the identification of turning points for curves, the finding of maximum and minimum values of given functions over different intervals and over their domains, and the treatment of problems for which the appropriate function to be analysed is to be constructed from data given in words or on a diagram. | Text pp 324  Exercise 16.4  Text pp 325 – 328  Exercise 16.5  Text pp 237 – 240  Exercise 11.8  Text pp 328 – 330  Exercise 16.6 |
| 10.7  Tangents and normals to curves.  10.8  The primitive function and its geometrical interpretation. | The equations of tangents and normals to curves should be found for simple curves. Curves in which the differentiation involves heavy mechanical work should be avoided.  Given *f '(x)*, the question ‘What is *f(x)*?’ naturally arises. Particular examples will make plausible, and a proof will show, that *f(x)* is not uniquely determined but that two functions which have the same derivative can only differ by a constant.  Geometrically, given the gradient function of a curve, the curve is not fixed but it is one of a ‘family’ of similar curves. For example, if  = 2*x*, then *y = x*2 + *c* and for different values of *c* a family of parabolas is obtained.  The term integration need not be used at this stage. It is in fact preferable to avoid it. Primitive function is a correct and suitable term.  The following are typical examples.  (i) Given = 2, find *s* in terms of *t* if = 10 and *s* = 0 when *t* = 0.  (ii) The gradient function of a curve is 3*x*2 – 1 and the curve passes through the point (4, 1). Find its equation. | Text pp 231- 232  Exercise 11.5  Text pp 330 – 333  Exercise 16.7  Exercise 16.8 |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 3 - Integration** | | TEACHER |
| CLASS |
| Focus: The application of integration techniques to the calculation of areas and volumes is important in this topic. Conversely, the ability to approximate the value of a definite integral through various methods of approximating areas is introduced.  Time Allocation: 3 Weeks Syllabus Reference: 11.1 – 11.5 | | |
| Targeted Outcomes (Board of Studies):  H1 seeks to apply mathematical techniques to problems in a wide range of contexts  H2 constructs arguments to prove and justify results  H4 expresses practical problems in mathematical terms based on simple given models  H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems  H8 uses techniques of integration to calculate areas and volumes  H9 communicates using mathematical language, notation, diagrams and graphs.  HE6 determines integrals by reduction to a standard form through substitution | Content Description  11.1 The definite integral.  11.2 The relation between the integral and the primitive function.  11.3 Approximate methods: trapezoidal rule and Simpson’s rule.  11.4 Applications of integration: areas and volumes of solids of revolution.  E 11.5 Methods of integration, including reduction to standard forms by very simple substitutions. | |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 18  New Senior Mathematics Three Unit course (Fitzpatrick) Ch. 24  Maths in Focus Blackline masters  Edudata  [www.ies.co.jp/math/java/calc/](http://www.ies.co.jp/math/java/calc/)  Microsoft Excel | Maths in Focus Chapter 3  Excel HSC  Neap Revision by Topic | |

| **Content (11.1 – 11.4 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
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| Students should be aware of the development and definition of the definite integral. |  |  |
| Know the relationship between the primitive function and the definite integral. |  |  |
| Find indefinite and evaluate definite integrals involving: |  |  |
| a) polynomial functions |  |  |
| b) expressions which can be simplified to  include |  | , , |
| c) |  | , |
| Find approximate values of integrals using:    a) Trapezoidal Rule |  |  |
| b) Simpson’s Rule |  |  |
| Find the area:  a) bounded by the *x* axis |  |  |
| b) bounded by the *y* axis |  |  |
| c) below the *x* axis |  |  |
| d) between two curves |  |  |
| Find the volume of revolution about:  a) the *x* axis |  | Approximate methods can be used to evaluate an integral and are therefore applicable to |
| b) the *y* axis |  | volume questions. Standard results for cone and sphere should be derived. |
| Undertake integration using substitution. |  | **In all questions, the substitution will be given.**  Such as :  \* using  \* using |

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| **INTEGRATION** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Watch appropriate sections of the the Video: Off on a tangent “The Concepts of Calculus” as an introduction to Integration * Discuss the progression of Integral Calculus from Archimedes- Newton & Leibniz- present * Discuss the relationship between the primitive function and the indefinite integral. * Discuss the difference between definite and indefinite integrals. * Evaluate indefinite and indefinite integrals * Conduct Internet Research on: Why find Areas and Volumes? Where is this used in today’s world * [Use Geometry SketchPad file to explore Trapezoidal Rule](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\TrapezoidalAccumulation.gsp) * Use [Excel Demonstration](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\Integration\Integration.htm) to introduce Simpson’s Rule and Trapezoidal Rule to find to find an approximation of an area * Explore the use of Simpson’s Rule and Trapezoidal Rule to find to find an approximation of an area to a range of problems * Use [Excel Demonstration](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\Integration\Integration.htm) to introduce the concept of finding the area under a curve using integration. * [Explore Geometry SketchPad file on Area under a curve](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\Area%20under%20a%20curve.gsp) * [Explore Geometry SketchPad file on Definate Integrals](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\DefiniteIntegral.gsp) * Use integration to find the area under a variety of curves bounded by the *x* axis. * Use integration to find the area under a variety of curves bounded by the *y* axis. * Use integration to find the area below two curves * Discuss how to find the area between two curves using integrations and the fact that area. * Use Graphics Calculators to help visualize the areas that are to be found. * Discuss the process for using the sum of two area’s and when this is necessary * Discuss the difference between finding the area under a curve and evaluating a definite integral. * Explore the use of the Standard Table of Integrals in exams. * Use [Excel Demonstration](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Integration\SolidsOfRevolution\SolidsOfRevolution.htm) to introduce the concept of finding the volume of revolution about the *x* axis and the *y* axis. * Complete problems involving finding the volume of revolution. * Complete HSC Style Questions on the topic of Integration. * Explore the process of using a substitution to evaluate “complicated” indefinite and definite integrals. * [Complete questions on integration by substitution](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Integration\Year%2012%20Ext%20INTEGRATION%20BY%20SUBS.doc) * [Complete Ext 1 Integration questions](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Integration\MIF3U16integration.doc) * Complete HSC Style questions involving substitution. * Complete more difficult examples of Mathematics (2 unit) Integration Questions. |

| Syllabus Topics | Additional Information | Text Reference |
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| 11.1  The definite integral | The notion of limit underlies both the differential calculus and the integral calculus. In the former, the intuitive geometrical notion of a tangent to a curve is formalised by the definition of gradient; if the gradient *f '(x)* exists at  *(x, f(x))*, the slope of the tangent at the point is defined to be *f '(x)*. The intuitive geometrical notion of area under a curve provides the basis for the development of the integral calculus. Its formal development requires that this area be defined as the limit (if it exists) of a certain sum of approximating rectangles, but this development is complicated by two factors which do not arise in the case of the gradient. Firstly, the appropriate limit is more difficult to specify, and secondly, having specified the limit, the specification usually does not allow the value of the limit to be calculated easily. | J & C pp 352 – 355  Exercise 18.1  Investigation of area under curve using either Excel or Java applet |
|  | For these reasons, the development of this topic will not follow that used for the differential calculus. Although the ideas leading to a formal definition of an area under a curve are to be discussed and illustrated with examples, the formal definition itself is not required. It is to be assumed that for common functions (and certainly for all continuous functions) there is an analytical formulation, in terms of a limit operation, of the intuitive idea of an area under a curve resulting in the *definite integral* becoming defined, when it exists, as the measure of this area.  Fortunately, the discovery that the differential calculus and the integral calculus are related provided a much simpler method of evaluating definite integrals of most of the common functions. This discovery (the ‘*fundamental theorem of the calculus*’) can be presented in a simple manner (provided the existence of area is assumed) and forms the basis of much theoretical and practical work involving integration.  We complete this section by briefly describing a suggested method of introducing the topic. We suppose *y = f(x)* is defined for values of *x*  including all *x* in *a* ≤*x* ≤*b*, is positive for these values of *x*, and has an easily drawn graph:  Intuitively, there is an area enclosed by the *x*-axis, the ordinates at *x = a* and *x = b*, and the curve. The problem is to calculate the size *A*, say, of this area. We know how to calculate areas of rectangles, triangles and of simple polygons. If *h,, H* are respectively the minimum and maximum values of *f(x)* in *a* ≤*x* ≤*b* (and examples should be chosen so that these occur both inside and at the endpoints), then  *h(b –a)* ≤*A* ≤*H(b – a)*. | Note: This method is discussed on pp 353 – 354 of text. An Excel spreadsheet could be used to enhance this discussion. |
|  | If we now split *a* ≤*x* ≤*b* into two subintervals, and , take minimum and maximum values *h*1, *H*1 in the first subinterval and  *h*2, *H*2 in the second, then the diagram shows that  *h(b – a)* ≤(*h*1 + *h*2) ≤*A* ≤(*H*1 + *H*2) ≤*H(b – a)*,  and usually the inequalities are strict, so that the new area sums are closer to *A* than the original bounds.  Taking three, four, … , *n* equal subdivisions of  *a* ≤*x* ≤*b* and forming the corresponding area sums, gives closer and closer bounds for *A*. This is to be done for a simple function such as *y = x*2 for 0 ≤*x* ≤1, the sums to be written down and evaluated using a calculator.  Intuitively, as the number *n* of subdivisions increases, the approximating sums approach the value *A*. Supposing *n* large, a typical rectangle in such a sum has a small base of length *dx* (‘*dx*’ or ‘*x*’ are notations used for a very small length) and a height which is close to *f(x)* for *any* value of *x* lying in its base. This is so because *f continuous* means that all values *f(x)* are close together if the values *x* are close together. Thus the area of a typical rectangle is *f(x)dx* and the sum of these areas is represented symbolically by *f(x)dx*.  The limiting value of this sum as *n* increases (and *dx* decreases) was denoted symbolically by  where the large S stood for ‘limiting sum’ and the bounds *x = a* and *x = b* indicated the interval over which the sum was taken. As time went by the S became elongated and modern notation for the same limit is , which is called the *definite integral of the function f(x) between x = a and x = b* and whose value *A* is the size of the area under the curve *y = f(x)* between  *x = a* and *x = b*. |  |
| 11.2  The relation between the integral and the primitive function | 11.2 Recall that a *primitive function* of a given function *f(x)* is any function *F(x)* such that  *F'(x) = f(x)*. Any two primitive functions of *f(x)* differ by a constant function.  Let *y = f(x)* be a continuous positive curve defined for values of *x* including all *x* *a*..    If *c* > *a*, let *A(c)* denote the area between *a* and *c*, and *A(x)* denote the area between *a* and *x*. Then *A(x) – A(c)* denotes the shaded area, which is  approximately that of a rectangle of height *f(x)* and base *x – c*, and so of area *f(x) (x – c)*. Thus the ratio is approximately equal to *f(x)*. If the maximum and minimum values of *f(t)* for *c* ≤*t* ≤*x* are *M*, *m* respectively, then precise bounds for this ratio are *m*≤≤*M*, and these inequalities also hold if *x* < *c*.  As *x* approaches *c*, both *m* and *M* approach *f(c)*. Hence and (by the definition of the derivative) is also equal to *A'(c)*. Hence  *A'(c) = f(c) (c > a)*, and this equation, being true for all *c* > *a*, says that the two functions *y = f(x)* and *y = A'(x)* are equal for all *x* > *a*. Hence *A(x)* is a primitive function of *f(x)* for *x* > *a*. If we know already a primitive function *F(x)* of *f(x)*, then  *A(x) = F(x) + C* for some constant *C*. | J & C pp 356 – 364  Exercise 18.2 – 18.5 |
|  | As *x* approaches *a*, *A(x)* approaches 0, *F(x)* approaches *F(a)* and consequently the value of *C* is *–F(a)*. Thus  *A(x) = F(x) – F(a) (x > a)*.  If *b* > *a*, then  *A(b)* = *f(x)dx = F(b) – F(a)*,  and hence the area and the definite integral can both be found using the known primitive function *F(x)*. The problem of evaluating definite integrals  is therefore solved if we can find a primitive function. For example, if *f(x) = x*2, we may choose *F(x)* = , and  This is an appropriate time to introduce and discuss the following extensions.  (i) The case where *f(x)* is negative or changes sign.  For example, if *f(x) = –x*3, draw *y = f(x)* for *x* –1 and evaluate, say,  and  and using the primitive function . From this, the idea that area is evaluated  as positive or negative according as f is positive or negative over the interval, and also that when *f* changes sign the integral gives the net area  (positive + negative) should be developed.  (ii) The result that if *a* <*b* <*c*,  obtained by writing down values in terms of a primitive *F(x)*, should be related to equations involving the corresponding signed areas. |  |
| 11.3  Approximate methods: trapezoidal rule and Simpson’s rule. | (iii) If *F, G* are primitives of *f, g* respectively, then *F + G* is a primitive of *f + g*. Thus  The practical importance of this result is to be emphasised; the integral of any sum is a sum of the integrals, so any polynomial may be integrated by finding primitives of each term, etc.  There is no need to discuss the case , where  *b* < *a*, immediately, but it should be introduced at an appropriate time, with simple examples.  11.3 There are quite simple functions whose definite integrals cannot be found exactly in terms of common functions. For such cases, for problems involving more complex functions, and also because of the ease with which computers can execute the necessary calculations to a high degree of accuracy, this section is devoted to two of the simple methods of approximate integration based on the idea of approximating an exact area by a sum of areas of shapes whose areas can be calculated. As well as using simple polygons such as rectangles or trapezia, one may use any curve *y = f(x)* whose area can be calculated explicitly; for example, quadratic polynomials (parabolas) or cubic polynomials.  Examples should be confined to problems in which the resulting calculations are easily performed on a calculator, but should include functions whose integrals are discussed or occur later in this syllabus when examples involving them may appropriately be introduced. | J & C pp 370 – 374  Exercise 18.9 |
|  | (i) **Trapezoidal rule**  If we approximate the curve *y = f(x)* on the interval *a* ≤*x* ≤*b* by the straight line passing through *(a, f(a))* and *(b, f(b))*, we may estimate the area under the curve by the area under the line:  *(Refer diagram in Syllabus page 64)*  This approximates by the area of a trapezium, which may be calculated as  *(f(a) + f(b))*.  If *f(x) = x*2, *a* = 1 and *b* = 2, = 2.33…,  while the above approximation gives  (*f*(1) + *f*(2)) = = 2.5.  By dividing the base into two equal subintervals, and using a linear approximation to *f(x)* on each subinterval, the area is approximated by two trapezia of total area , which for the given example yields the better approximation (1 + 9/2 + 4) / 4 = 19/8 = 2.375.  *(Refer diagram Syllabus page 64)*  The method extends to dividing the interval into *n* equal subintervals of length *h = (b – a)*/*n*, and using a trapezium approximation on each subinterval. For instance, using *n* = 5 (so *h* = 0.2) in our example  *(Refer diagram Syllabus page 65)*    = 2.34. |  |
|  | (ii) **Simpson’s rule**  If we approximate the curve *y = f(x)* on *a* ≤*x* ≤*b* by a quadratic function (ie by a parabola) agreeing with *f(x)* at the three points *(a, f(a))*, and *(b, f(b))*, then, as a diagram will show, we would expect to increase the accuracy of the area approximation by using the area under the parabola. The resulting approximation, known as Simpson’s rule, is .  Applied to the same integral as before, we obtain  *,* which is of course exact because the approximating parabola is in this case the same function *y = x*2. (In fact, Simpson’s rule is *exact* for *f(x)* equal to any quadratic or cubic polynomial.)  An example such as *y* = for 1 ≤*x* ≤2 should now be chosen, where students cannot find a primitive function, and the integral approximated by the trapezoidal rule with *n* = 1 and *n* = 2, and by Simpson’s rule applied first to the whole interval and then to the two subintervals 1  It is important to emphasise that any definite integral may be evaluated approximately by such rules, and that estimates of the size of possible error can often be calculated. Later, when integrals are used to find volumes or velocities, etc, examples should be chosen to obtain approximate values of the relevant quantities. | J & C pp 374 – 378  Exercise 18.10 |
| 11.4  Applications of integration: areas and volumes of solids of revolution | (i) **Areas**  Only very simple functions can be considered at this stage. Examples should include the calculation of areas bounded by a curve and the *x*-axis, by a curve and the *y*-axis, and between two curves. Simple examples where the function changes sign in the interval of integration should also be included. All problems should be accompanied by a clear sketch showing the required area.  (ii) **Volumes of solids of revolution**  Standard results for the cone and the sphere should be derived. Other problems should involve only simple curves, but revolution about the *y*-axis should also be considered.  (iii) Other applications occur in later topics, and care should be taken to ensure that examples and problems in the later topics are used to reinforce integration techniques.  **11.5 3 Unit students should be prepared to undertake some harder integration, as exemplified below.**  **(i) Find , using the substitution**  ***u = x*2 + 1.**  **(ii) Use the substitution *t = u*2 – 1 to evaluate**    **In all examples the substitution is to be given.** | J & C pp 364 – 369  Exercises 18.6 -18.8  J & C pp 379 – 386  Exercise 18.11  Fitzpatrick pp 82 – 89  Exercise 24(e) and 24(f) |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 4 – Coordinate Methods in Geometry** | | **TEACHER** |
| **CLASS** |
| **Focus**: In this unit students focus on using coordinate geometry methods to prove geometrical properties and relationships.  Time Allocation: 1 Week Syllabus Reference: 6.8 | | |
| Targeted Outcomes (Board of Studies):  **H2** constructs arguments to prove and justify results  **H5** applies appropriate techniques from the study of geometry to solve problems  **H9** communicates using mathematical language, notation, diagrams and graphs. | Content Description  6.8 Coordinate methods in geometry | |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 10  Excel HSC  Neap Revision by Topic |  | |

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| **Content (6.8 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
| Students should be given practice at applying the previous working to:  a) geometrical problems |  | Such as:  Prove P ( 1, 4 ), Q ( 5, 2 ), R ( 2, -4 ) and  S ( -2, -2 ) form a rectangle.  **Note:** Drawing a diagram is to be stressed in  solving these problems |
| b) multiple step problems involving  inter-related parts |  | See HSC questions |
| Sketch regions, the intersection and union of regions representing inequalities. |  | Such as:  Sketch the region where the following inequalities hold simultaneously  *x* – *y* ≤ 1, *x* + 3*y* ≥ 1 and 5*x* + 3*y* ≤ 19  Final answer must be clearly indicated.  ( cross hatching is not sufficient ) |

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| **COORDINATE METHODS IN GEOMETRY** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms such as union, intersection, inequalities, geometrical bisect, simultaneously. * Review important concepts in geometry and apply these to coordinate geometry. * Complete HSC Style Questions. |

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| Syllabus Topics | Additional Information | Text References |
| 6.8  Coordinate methods in geometry | Examples, illustrating the use of coordinate methods in solving geometrical problems, are to be restricted to problems with specified data. The following are typical problems.  (i) Show that the triangle whose vertices are (1, 1), (–1, 3) and (3, 5) is isosceles.  (ii) Show that the four points (0, 0), (2, 1), (3, –1), (1, –2) are the corners of a square.  (iii) Given that *A, B, C* are the points (–1, –2), (2, 5) and (4, 1) respectively, find *D* so that *ABCD* is a parallelogram.  (iv) Find the coordinates of the point *A* on the line *x* = –3 such that the line joining *A* to *B* (3, 5) is perpendicular to the line  2*x* + 5*y* = 12. | J&C pp. 213 – 215  Exercise 10.10 |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 5 – Applications of Geometrical Properties** | **TEACHER** | |
| **CLASS** | |
| **Focus**: This unit of work requires students to work with generalized applications of geometrical properties to solve problems.  Time Allocation: 1 Week Syllabus Reference: 2.5 | | |
| Targeted Outcomes (Board of Studies):  **H2** constructs arguments to prove and justify results  **H5** applies appropriate techniques from the study of geometry to solve problems  **H9** communicates using mathematical language, notation, diagrams  and graphs | | Content Description  2.5 Application of 2.1 – 2.3 to simple theoretical problems requiring one or more steps of reasoning. |
| Resources:  New Senior Mathematics 2 Unit (Fitzpatrick) Ch. 4  Maths in Focus Chapter 1  Excel HSC  Neap Revision by topic  Autograph  Geometers Sketchpad | |  |

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| **Content (2.5 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** | |
| Use the geometric properties from earlier an earlier topic and apply them to questions with or without a diagram |  | Problems mainly have the diagrams supplied though practice should be given in sketching a diagram from a given set of data. Problems may be either numerical or general. In all cases a geometrical justification for each step will be required where it is appropriate. |

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| **APPLICATIONS OF GEOMETRICAL PROPERTIES** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms such as points, angles, lines, parallel, intervals, polygons, perpendicular, congruent, similar, produced, angle sum, triangles, quadrilaterals, ratio. * Review important concepts in geometry and apply these to harder problems. * Complete HSC Style Questions. |

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| Syllabus Topics | Additional Information | Text References |
| 2.5  Application of 2.1 – 2.3 to simple theoretical problems requiring one or more steps of reasoning. | Problems for both 2 and 3 Unit students may involve the application of any of the properties treated above. For 2 Unit students, problems should mainly have diagrams supplied although practice should be given in sketching a diagram from a given set of data. In all cases, a geometrical justification for each step will be required where it is appropriate. | 2 Unit (Fitzpatrick) pp. 100 – 117  Exercise 4(e)  Exercise 4(h) (from Q 16) |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 6 – Logarithmic and Exponential Functions** | **TEACHER** |
| **CLASS** |
| **Focus**: Students learn about the algebraic and practical aspects of logarithmic and exponential functions. The application of calculus to these functions is an important element of this topic.  Time Allocation: 2 Weeks Syllabus Reference: 12.1 – 12.5 | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range of contexts  **H2** constructs arguments to prove and justify results  **H3** manipulates algebraic expressions involving logarithmic and exponential functions  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of calculus  **H6** uses the derivative to determine the features of the graph of a function  **H8** uses techniques of integration to calculate areas and volumes  **H9** communicates using mathematical language, notation, diagrams and graphs. | Content Description  12.1 Review of index laws, and definition of *ar* for *a* > 0, where r is rational.  12.2 Definition of logarithm to the base *a*. Algebraic properties of logarithms and exponents.  12.3 The functions *y = ax* and *y =* log*ax* for *a* > 0 and real *x*. Change of base.  12.4 The derivatives of *y = ax* and *y =* log*ax.* Natural logarithms and exponential function.  12.5 Differentiation and integration of simple composite functions involving exponentials and logarithms. |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 19  New Senior Mathematics 2 Unit (Fitzpatrick) Ch. 13  Maths in Focus blackline masters  Excel Year 10 Advanced workbook  Edudata | Maths in Focus Chapter 4  Excel HSC  Neap Revision by Topic |

| **Content (12.1 – 12.5 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
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| Use the key, exponential key and log and ln keys on their calculators. |  | Find the values of ,…. ,  for varying values of *x* |
| Sketch the graphs of logarithmic and exponential functions. |  | Sketch the graphs of:  *y* = *ln x* , *y* = *ln* 2*x* , *y* = 1 + *ln x* |
| Recall the definition of and evaluate simple logarithmic expressions using this definition.  ⇔ |  | Simplify the following:  \* \*  \* |
| Draw graphs of *y* = *ax* and *y* = log*ax* |  |  |
| Use change of base formula. |  |  |
| Solve simple equations. |  | Solve the following for *x*:  \* \*  \* |
| Write a log statement as an index and vice-versa. |  |  |
| Use the rules for logarithms and use them to simplify numerical and algebraic expressions. |  | Evaluate:  \* |
| The rules for logarithms can be proven for better students  Rules for logarithms:  \* log *x* + log *y* = log *xy*  \* log *x* - log *y* = log  \* log = *n* log *x* \*  \* \* |  | \* |
| Differentiate:  a) |  | Students should be shown the derivatives of and using a calculator. |
| b) ln[ f(*x*)] |  | **STRESS**: different notations for |
| c) |  |  |
| Apply the above derivatives to previous calculus problems. |  | Such as : curve sketching, tangent, normal, maximum and minimum value problems. |
| Integrate  a) |  | These should be done by “inspection”. |
| b) log ( a*x* + b ) |  | No formal change of variable is required in the Mathematics course. |
| c) |  |  |
| Apply the above integration to previous calculus problems |  | Such as: area, volume |

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| **LOGARITHMIC AND EXPONENTIAL FUNCTIONS** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms and notations such as logarithm, exponential, base, index, log, ln. * Introduce exponential functions through examples such a s population growth, bacteria growth, etc. * Explain that *ax* is an exponential function and that *ex* is The exponential function as the gradient of *y* = *ex* is the same as the *y-value*. * Discuss exponential and logarithmic functions ate inverse functions. * Use graphics calculators to explore the shapes of different exponential and logarithmic functions. * Use [Excel Demonstration](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\Resource%20Shortcuts\Excel%20Demonstations%20-%20Stella%20Maris%20(paul).lnk) to explore the shapes of different exponential and logarithmic functions. * Explore how to differentiate and integrate exponential and logarithmic functions. * Apply the rules of exponential and logarithmic functions to HSC Style Problems. * Complete HSC Style questions. * Discuss how to use standard table of integrals. * Discuss how to use MATH-O-MAT * [Complete questions on Exponential and Logarithmic Functions](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Logarithmic%20and%20Exponential%20Functions\EXPONENTIAL%20AND%20LOGARITHMIC%20FUNCTIONS.doc). |
| * Apply techniques of substitution for the integration of exponential functions. * [Have students answer questions involving Exponential and Logarithmic Functions (Ext 1)](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Logarithmic%20and%20Exponential%20Functions\MIF3U17exponentialandandlogs.doc) |

| Syllabus Topics | Additional Information | Text Reference |
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| 12.1  Review of index laws, and definition of *ar* for *a* > 0, where r is rational. | Keys for the exponential and logarithmic functions appear on most scientific calculators and students should be familiar with their use.  12.1 The introduction of scientific calculators into the classroom in the junior school eliminates the necessity of practical work with logarithms to the base of 10. Even though familiarity with index notation is expected of the  users of such calculators, it may be necessary to revise or to develop that notation and the index laws for integer exponents. The relationship of indices to multiplication of repeated factors and the introduction of zero and negative integer indices should be understood, as should the fact that the index laws (obtained initially for positive integer indices) remain valid for arbitrary integer indices. | J&C pp 7 – 8  Exercise 1.6 |
| 12.2  Definition of logarithm to the base *a*.  Algebraic properties of logarithms and exponents.  12.3  The functions *y = ax* and *y =* log*ax* for *a* > 0 and real *x*. Change of base. | If *a* > 0 and *r* = > 0, then *ar* is defined as the *q*th root of *ap*. This should be approached via simple values of *r* (eg , ).  The index laws should be verified for simple cases (such as ) and stated to be true for general rational indices (a general proof is not required). The extension to negative rational indices follows as for the case of integer indices.  Calculators should be used to verify numerical examples of index notation and index laws. It is expected that algebraic computations involving the index rules will be tested in the examination.  The words *exponent*, *base* and *logarithm* should be introduced and understood. The notations log*ax*, log10*x* should be known. There should be some examples of cases where the logarithm is integral and rational, and  other examples using a calculator. The algebraic properties of logarithms and exponents (including in  particular, the identities log*a*1 = 0, log*axy* = log*ax* + log*ay*, and log*axc* = *c* log*ax*) should be derived from the appropriate index laws.  For a fixed *a* > 0, calculators should be used to draw graphs of *y = ax* and *y* = log*ax*, the cases *a* < 1, *a* = 1, *a* > 1 to be discussed. [Some discussion could be introduced here, or into 12.2, regarding the problem of defining *ax* when *x* is real but not rational, and it could be pointed out that the method used in 12.1 to define, say, simply does not extend to cover the case of attaching a meaning to  Say or *a*. The idea of using rational approximations to real exponents (rational and non-rational) could be explored using calculators.] | J&C p8  Exercise 1.7  New Senior Mathematics 2 Unit  (Fitzpatrick) pp 281-288  Exercise 13(a)  Exercise 13(b)  New Senior Mathematics 2 Unit  (Fitzpatrick) pp 290-295  Exercise 13(c)  New Senior Mathematics 2 Unit  (Fitzpatrick) pp 288-298  Exercise 13(d)  J&C pp. 397-400  Exercise 19.7 |
| 12.4  The derivatives of *y = ax* and *y =* log*ax.* Natural logarithms and exponential function. | If *a* > 1, it should be stated that *y = ax* is an increasing function which takes each positive real value once only. Thus to each positive *y* there is a unique real *x* such that *y = ax* and hence such that log*ay = x*. The change of base formulae follow from the index laws. Computational examples of the algebraic properties of logarithms and change of base should be set, and may be tested in the examination.  (a) To find the gradient of the curve *y* = 10*x*.  (i) Let *f(x)* = 10*x*. At any value *x*, we know that  *f'(x)*= .  (ii) Evaluate for various values of *h*, using a calculator, eg *h* = 0.1,0.01, 0.001  = 2.6, 2.33, 2.31.  It appears that approaches a limit (call it ). This is also intuitively obvious, since is the gradient of the curve *y* = 10*x* at *x* = 0.  (b) Let = 2.3. If *y* = 10*x*, then by (a) (i),  If *z* = 10 *x* = *y* , then  Now choose so that = 1 (ie 0.43) and write  *e* = 102.7.  Then *z = ex* and = *ex*.  (If calculators are used, the argument may be applied to *ax* for other *a*, such as *a* = 2.) | New Senior Mathematics 2 Unit  (Fitzpatrick) pp 406-407  New Senior Mathematics 2 Unit  (Fitzpatrick) p 408 |
| 12.5  Differentiation and integration of simple composite functions involving exponentials and logarithms. | (c) Alternative methods of calculating *e* are:  (i) put *H* = 10*h* – 1 so that *h* = log10(1 + *H*) and *H* 0 as *h* 0. Then ;  (ii) put so that and *n* as *h* 0. Since , it follows from (b) that *e* = 10/ = .  (d) If *y* = *ex*, so  *x* = log*ey* = , etc.  (e) Teachers may prefer to use for able students the approach, based on , which first defines ln *x* and then defines *ex* as the function inverse to ln *x.*  The notations ln *x* and log *x* for the natural logarithm log*ex* should be known. For historical reasons some books and calculators use log *x* for log10*x*. This practice should be discouraged.  (i) Differentiation of *eax+b*, log*e(ax + b)* and the corresponding integrations.  (ii) Differentiation of log*ef(x)* for simple functions *f(x)*.  (iii) Integration of *f’(x)*/*f(x)* by inspection, without formal change of variable. | J&C pp 408-409  J&C pp. 400-401  **Note: This should be pointed out to students when discussing J&C p.401.**  J&C pp. 392-397  Exercise 19.4 - 19.6  J&C pp. 402-403  Exercise 19.8  J&C pp. 403-405  Exercise 19.9  **Derivative of *y = ax***  J&C p. 406  Exercise 19.10  J&C pp 409 -410  Exercise 19.13 |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 7 – Series and Series Applications** | | **TEACHER** |
| **CLASS** |
| **Focus**: This Unit explores both arithmetic and geometric series and considers their application to practical problems, especially in relation to financial mathematics. Extension students are introduced to the techniques of proof requiring inductive reasoning.  Time Allocation: 4 Weeks Syllabus Reference: 7.1 – 7.5 | | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range   of contexts  **H2** constructs arguments to prove and justify results  **H3** manipulates algebraic expressions involving logarithmic functions  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of series to solve problems  **H9** communicates using mathematical language, notation, diagrams and graphs.  **HE2** uses inductive reasoning in the construction of proofs | Content Description  7.1 Arithmetic series. Formulae for the *n*th term and sum of *n* terms.  7.2 Geometric series. Formulae for the *n*th term and sum of *n* terms.  7.3 Geometric series with a ratio between –1 and 1. The limit of *xn*, as *n* → ∞, for | *x* | < 1, and the concept of limiting sum for a geometric series.  E 7.4 Mathematical induction. Applications.  7.5 Applications of arithmetic series.  Applications of geometric series: compound interest, simplified hire purchase and repayment problems.  Applications to recurring decimals. | |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 14  New Senior Mathematics 2 Unit (Fitzpatrick) Ch. 11  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 23  Edudata | Maths in Focus Chapter 8  Excel HSC  Neap Revision by Topic | |

| **Content (7.1 – 7.5 Summarised)**  **Students should be able to :** | | **✓** | **Further Explanation** |
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| Know the difference between a sequence and a series |  | |  |
| Find the general term from a pattern of numbers |  | | Find the nth  term of 1, 4, 9, ……. |
| Find the terms of a sequence given the general term. |  | | Write down the first 3 terms and 10th term of |
| Determine whether a term is a member of the sequence and if it is, which term it is. |  | | Which term of the sequence { *n* ( *n* + 3 ) } is 460 |
| Use the result Tn = Sn - Sn-1 |  | | Find the nth term of the series if the sum of the first ***n*** terms is |
| Arithmetic Progression    Students should know and be apply to apply:  a) the general arithmetic progression  *a*, *a* + *d*, *a* + 2*d*, … *a* + ( *n* – 1 ) *d* |  | |  |
| b) the test for an Arithmetic Progression  **T**2 – **T**1 = **T**3 - **T**2  = … = *d* |  | | Prove that 1, 4, 7, … is an Arithmetic Progression |
| c) the nth term of an Arithmetic  Progression  **T**n = *a* + ( *n* – 1 ) *d* |  | | Find the general term of the Arithmetic progression 1, 4, 7, … |
| d) the sum to ***n*** terms of an Arithmetic  Progression    where *l* = *a* + ( *n* – 1 )*d*  The above results should be derived |  | | \* Find the sum to ***n*** terms of 1, 4, 7, …  \* Evaluate 1 + 4 + 7 + … + 67  \* The seventh and twelfth terms of an  Arithmetic Progression are 13 and 28  respectively. Find the nth term and sum to  *n* terms. |
| Geometric Progression  Students should know and be able to apply:  a) the general geometric Progression |  | |  |
| b) the test for a Geometric Progression |  | | Show that 1, 4, 16, … is a Geometric Progression. |
| c) the nth term of a Geometric  Progression |  | | Find the nth and 50th term of 1, 4, 16, … |
| d) the sum to ***n*** terms of a Geometric  Progression        The above results should be derived. |  | | Find the sum of the first ***n*** terms of  1, 4, 16, …  Find the value of -9 + 3 – 1 + … + |
| Find the limiting sum of a Geometric Progression and the condition for that sum to exist. |  | | Concept of limiting sum can be illustrated by using the calculator.  \* Evaluate  \* Find *x* if has a limiting  sum |
| Use sigma notation for both Arithmetic and Geometric Progressions. |  | | \* Evaluate  \* Evaluate |
| Find the sum of the series of the form for arbitrary values of ***n***… and determine if a limiting sum exists and find it if it does. |  | | (1 ) – ( 2 )    as ***n*** → ∞ → ∞  ∴ no limiting sum |
| Find the general term and sum of series of composite types. |  | | \* Evaluate |
| Applications  Students should be able to apply all the previous work to:  a) problem type questions |  | | A farmer harvests 500 bushels of wheat in his first year on a farm. He harvests 550 bushels in the second year. Each year his harvests 10% greater than that of his previous year. What does he harvest in his 10th year ? How much wheat can he sell altogether in his first 10 years ? |
| b) superannuation  **students should derive the series**  **and not simply quote formula.** |  | | A person pays $ 1200 at the beginning of each year into a superannuation fund. If the fund pays 8%p.a. interest compounded yearly, find to the nearest dollar:  a) the value of the original investment after  35 years  b) the total value of their investment after 35  years. |
| c) Time Repayment  **students should derive the series and**  **sum it and not simply quote formula** |  | | A couple borrow $ 30 000 to buy a house and agree to repay the loan in equal monthly instalments over 20 years. If the interest rate is 15% p.a. calculated on the amount owing at the beginning of each month, find their monthly repayment. |
| d) Harder Time Repayment |  | | In the example above, the couple inherit $8000 and make a payment of $ 8000 off the loan after 10 years. How long will it now take them to repay the loan. |
| e) General problems of a non –  commercial nature |  | | A hospital patient receives a 10mg dose of medicine at intervals of 4 hours. During each interval the amount of medicine in the body reduces to 75% of the amount present at the start. Prove that there will always be less than 40 mg of medicine in the body, irrespective of the length of treatment. |
| Students should be familiar with and be able to apply the method of proof by Mathematical Induction. |  | | The proof by induction consists of 2 steps :   1. verification 2. assumption   It is important to realize that **both** steps of the method of induction must be made.  \* 1 + 3 + 5 + … + ( 2*n* – 1 ) =  \* *n* ( *n* – 1 )( *n* + 1 ) is divisible by 6  \* for *n* > 1  \*  \* Geometrical facts |
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| **SERIES AND SERIES APPLICATIONS** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms and notation such as sequence, series, common difference, infinite, arithmetic progression, geometric progression, general term, time payment, superannuation, AP, GP, limiting sum, Tn, Sn, , nth term.  * Introduce arithmetic sequences through the story of Gauss and his discovery of the pattern for the sum of an AP. * Derive the formula’s for the nth term of an AP and the sum of an AP. * Complete practical problems involving AP’s. * Introduce the concept of a GP and the formula for finding the nth term of a GP. * Discuss the concept of a limiting sum providing explanations with examples about why no limiting value exists for any geometric series in which .  * Introduce the method for finding the sum of a GP by looking for the pattern using PowerPoint presentation. * Use the pattern for finding the sum of a GP to find the limiting sum of a GP. * Complete practical problems involving GP’s. * Watch [Powerpoint Presentation on Sequences and Series](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\sequences%20and%20series\Sequences%20and%20Series%20Powerpoint\Sequences%20and%20Series.ppt). * Discuss process for setting out Mathematical Induction solutions. * [Have students complete questions on Mathematical Induction\](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Mathematical%20Induction\hscinduction.doc) * Complete HSC Style Questions. |

| Syllabus Topics | Additional Information | Text Reference |
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| 7.1 Arithmetic series. Formulae for the *n*th term and the sum of *n* terms. | This topic might be introduced by a general discussion on series, including aspects of notation such as  12 + 22 + 32 + ... + *N*2 =  There should also be justification of the topic in terms of the practical examples given below. The definitions of ‘series’, ‘term’, ‘*n*th term’ and ‘sum to *n* terms’ should be understood.  The definition of an arithmetic series and its common difference should be understood. The formulae for the *n*th term and the sum to *n* terms should be derived. | J&C pp 274-281  Sequences: Exercise 14.1  General term: Exercise 14.2  Arithmetic sequence: Exercise 14.3  *n*th term of AP: Exercise 14.4  Sum of *n* terms: Exercise 14.5  Sigma notation: Exercise 14.6 |
| * 1. Geometric series. Formulae for the *n*th term and the sum of *n* terms.   7.3 Geometric series with a ratio between –1 and 1. The limit of *xn,* as , for , and the concept of the limiting sum of a geometric series.  Applications to recurring decimals.  7.5 Applications of arithmetic series.  Applications of geometric series: compound interest, simplified hire purchase and repayment problems. | The definitions of a geometric series and its common ratio should be understood, and the formulae for the *n*th term and sum to *n* terms derived.  Using a calculator, or otherwise,  should be derived. The case *r*1 should be discussed.  For a geometric series whose ratio r satisfies *r*< 1, it follows that *Sn* approaches a limiting value *S* as *n* increases:    No limiting value exists for any geometric series in which *r*1.  Applications to recurring decimals  Recurring decimals should be expressed as rational numbers, eg  Other examples should also be discussed.  Applications of arithmetic series.  Applications of arithmetic series should include problems of the type ‘A clerk is employed at an initial salary of $10 200 per annum. After each year of service he receives an increment of $900. What is his salary in his ninth year of service, and what will be his total earnings for the first nine years?’ | J&C pp 281 – 287  Geometric sequence: Exercise 14.7  *n*th term of GP: Exercise 14.8  Sum of *n* terms: Exercise 14.9  J&C pp. 291 – 293  Exercise 14.13  2 Unit (Fitzpatrick) pp 238 – 241  Exercise 11(b) |
|  | Applications of geometric series should include the following types.  The compound interest formula *An = P(*1 + *r*/100*)n*,  where *P* is the principal (initial amount), *r* % the rate of interest per period, and *An* the amount accumulated after *n* periods, should be understood.  (i) Superannuation  ‘A man invests $1000 at the beginning of each year in a superannuation fund. Assuming interest is paid at 8% per  annum on the investment, how much will his investment amount to, after 30 years?’Obviously the first $1000 is invested at 8% compound interest for 30 years, the next $1000 for 29 years, and the last $1000 for 1 year. Thus his investment after 30 years is (in dollars)  1000 (1.0830 + 1.0829 + … + 1.08).  This is a geometric series of 30 terms, with first term 1080 and common ratio 1.08, so that this sum is    = $122 346, to the nearest dollar.  (ii) Time Payments  ‘A woman borrows $3000 at 1 % per month reducible interest and pays it off in equal monthly instalments. What should her instalments be in order to pay off the loan at the end of 4 years?’  Let $*An* be the amount owing after *n* months.  After one month and paying the first instalment $*M*, she will owe 3000 1.015 – *M = A*1.  Similarly, *A*1 1.015 – *M = A*2, and after *n* months,  *An = An*–1 1.015 – *M*,  = 3000 (1.015)*n – M* (1 + 1.015 + … + 1.015*n*–1).  But *A*48 = 0.  *M* (1 + 1.015 + … + 1.01547) = 3000 1.01548  *M* = = 3000 1.01548,  so that *M* == 88.12 | J& C pp.287 – 291  Exercise 14.10  Exercise 14.11  Exercise 14.2  2 Unit (Fitzpatrick) Exercise 11 (d) |
| \*7.4 Mathematical induction. Applications. | The instalment amount should be $88.12.  Students should understand the difference between the reducible interest rate, and the rate published by finance companies. The published rate in this case is the equivalent simple interest rate on $3000 for 4 years, ie  *R*=  The method of proof known as ‘proof by induction’ makes use of a test for a set to contain the set of positive integers. This test, called the *principle of mathematical induction*, is an assumption concerning the positive integers and may be stated as follows.  ‘If a set of positive integers  (a) contains the positive integer 1, and  (b) can be proved to contain the positive integer *k* + 1 whenever it contains the positive integers 1, 2, …, *k*,  then the set contains all positive integers.’  The use of this method of proof is often suggested when a problem of the following kind arises. From given information and perhaps by experiment, we obtain a statement *S(n)*, depending on the positive integer *n*, which we wish to prove true for every positive integer *n*. We let *S* denote the set of positive integers *n* for which *S(n)* is true. We now try to prove:  (i) that *S* contains 1 (ie that *S*(1) is true), and  (ii) that if *S* contains 1, 2, …, *k*, then *S* contains *k* + 1  (ie if *S*(1), *S*(2), …, *S(k)* are true, then *S*(*k* +1) is true).  If we manage to prove (i) and (ii), then by our test, *S* contains all positive integers (ie *S(n)* is true for every positive integer *n*).  It frequently happens that we may be able to prove (ii) by using only the assumption that *S(k)* is true, instead of the full assumption that *S*(1), *S*(2), …, *S(k)* are all true. | J&C pp294 – 296  Exercise 14.15  3 Unit (Fitzpatrick) pp 64 – 67  Exercise 23(c)  Numerous worked examples and extensive guided practice are essential to student confidence. |
|  | Sometimes, we may guess that *S(n)* is true only for positive integers *n* *M*, a given positive integer. In that case we replace (i) by ‘*S* contains *M*’ and (ii) by ‘if *S* contains *M, M* + 1, …, *k*, then *S* contains *k* + 1’. The test  enables us to conclude that *S* contains every positive integer greater than or equal to *M*.  Below are several applications illustrating the use of proof by induction.  1. Consider the results.  1 = 12 We call this statement *S*(1).  1 + 3 = 22 We call this statement *S*(2).  1 + 3 + 5 = 32 We call this statement *S*(3).  1 + 3 + 5 + 7 = 42 We call this statement *S*(4).  We may now guess that the following statement *S(n)* is true for every integer *n*.  Statement *S(n)*: 1 + 3 + 5 + … + (2*n* – 1) = *n*2  The *proof* by induction of this statement consists of two steps.  Step 1. Verification that *S*(1) is a true statement; this is easy since *S*(1) is merely the statement 1 = 1.  Step 2. We assume that *S*(1), *S*(2), …, *S(k)* are true. We then attempt to deduce logically that *S*(*k* + 1) must also be true. In the present instance, our  assumption supposes that the following is true:  *S(k)*: 1 + 3 + 5 + … + (2*k* – 1) = *k*2,  and using this we try to show that *S*(*k* + 1) is true, ie, that  1 + 3 + 5 + … + (2(*k* + 1) – 1) = (*k* + 1)2.  By adding 2*k* + 1 to each side of the (by assumption) true statement *S(k)*, we obtain  1 + 3 + 5 + … + (2*k* – 1) + (2*k* + 1) = *k*2 + (2*k* + 1),  ie 1 + 3 + 5 + … + (2*k* + 1) = (*k* + 1)2,  which is *S*(*k* + 1). Thus, from the assumption that *S(k)* is true, we have deduced that *S*(*k* + 1) is true.  We have satisfied the conditions of the test for proof by induction, hence we may conclude that *S(n)* is true for every *n* |  |
|  | It is important to realize that *both* steps of the method of induction must be made, before the proof is valid. This can be illustrated vividly by ‘proofs’ of false results, for example, the ‘proof by induction’ that all successive integers are equal to each other (let *S(n)* be the statement *n = n* + 1, then *S(k* + 1) follows logically from *S(k)*, but *S*(1) is not true).  2. The standard notation for the sum of a series should be introduced, and induction used to prove results such as:  (i)  (ii)  3. If we construct a triangle *ABC* and measure its external angles we find *p + q + r* 360°.  If we construct a plane *convex* quadrilateral and measure its external angles, we again find  *p + q + r + s* 360°.  Generally, if we construct a plane convex polygon with *n* sides, and calculate the sum of the measures of its external angles, we expect the answer to be 360° for the following reason. If we were to stand at a given vertex, facing along one of its edges, and if we were to walk once around the polygon until we returned to the original vertex, facing in the original  direction, then we have turned through one complete revolution (360°), and this is composed of turns of each external angle at each vertex. |  |
|  | To prove this by induction, we suppose *n* 3 and let *S(n)* be the statement ‘the sum of the exterior angles of an *n*-sided plane convex polygon is 360°’. We now  verify steps (i) and (ii).  (i) We must prove *S*(3) is true. Referring to the figure, we must prove that *p + q + r* = 360°. Using the angle sum property of a triangle, we have  ie (180° – *r*) + (180° – *p*) + (180° – *q*) = 180°,  or 360°= *p + q + r*.  Thus *S*(3) is true.  (ii) We now suppose *S*(3), *S*(4), …, *S(k)* are true, and prove *S*(*k* + 1) is true. Let *A*1*A*2 … *A*k+1 be the vertices (in order) of a (*k* + 1)-sided plane convex polygon, with exterior angles *p*1, *p*2, …, *pk*+1 respectively. We must prove *p*1 + *p*2 + … + *pk*+1 = 360°.  Join *Ak* to *A*1 and apply *S(k)* to the *k*-sided plane convex polygon *A*1*A*2 … *Ak*. The sum of its exterior angles is therefore 360°. But  exterior angle at *A*1 = *+ p*1  exterior angle at *A*k = *ß + pk*  and at the other vertices *A*2, …, *Ak*–1, the exterior angle is the same as the exterior angle of the original polygon. Hence  ( *+ p*1) + *p*2 + … + *pk*–1 + *(ß +pk)* = 360°,  ie *p*1 + *p*2 + … + *pk*–1 + *(* *+ ß)* = 360°. (i)  But in *A*1*A*k*Ak*+1,  exterior angle at *Ak*+1 = sum of interior opposite angles,  ie *pk*+1 = *+ ß* (since the angle at *A*1 = ).  Substituting this into (1) gives the result:  *p*1 + *p*2 + … + *pk + pk*+1 = 360°.  Thus *S*(*k*+1) is true. Steps (i) and (ii) are both completed, hence we may conclude *S(n)* is true for every *n* 3.  *(Refer Syllabus page 49 for diagram)* |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 8 – Trigonometric Functions** | **TEACHER** | |
| **CLASS** | |
| **Focus**: Students are introduced to radian measure of angles and the calculus of trigonometric functions.  Time Allocation: 3 Weeks Syllabus Reference: 13.1 – 13.7 | | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range of contexts  **H2** constructs arguments to prove and justify results  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of calculus and trigonometry to solve problems  **H6** uses the derivative to determine the features of the graph of a function  **H7** uses the graph to deduce information about the derivative  **H8** uses techniques of integration to calculate areas and volumes  **H9** communicates using mathematical language, notation, diagrams and graphs.  **HE6** determines integrals by reduction to a standard form through substitution | | Content Description  13.1 Circular measure of angles. Angle, arc, sector.  13.2 The functions sin *x*, cos *x*, tan *x*, cosec *x*, sec *x*, cot *x* and their graphs.  13.3 Periodicity and other simple properties of the functions sin *x*, cos *x* and tan *x*.  13.4 Approximations to sin *x*, cos *x*, tan *x*, when *x* is small.  The result .  13.5 Differentiation of cos *x*, sin *x*, tan *x*.  13.6 Primitive functions of sin *x*, cos *x*, sec2*x*.  E Primitive functions of sin2*x* and cos2*x*.  13.7 Extension of 13.2 – 13.6 to functions of the form *a* sin(*bx* + *c*), etc. |
| Resources:  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 20  New Senior Mathematics 2 Unit (Fitzpatrick) Ch. 17  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 22, 24 | | Autograph  Geometers Sketchpad  Maths In Focus Chapter 5  Excel HSC |

| **Content (13.1 – 13.7 Summarised)**  **Students should be able to :** | | **✓** | **Further Explanation** |
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| Understand the definition of radian measure for angles. | |  |  |
| Be able to convert from:  a) degrees to radians | |  | Use πc = 180° |
| b) radians to degrees | |  |  |
| Evaluate:  a) exact value expressions | |  | \* sin \* cos |
| b) radian expressions in the calculator | |  | \* sin 2  **STRESS NOTATION sin 2 means sin 2c** |
| Find:  a) length of an arc  *l* = *r* θ | |  | **STRESS** θ is in radians |
| b) area of a sector | |  |  |
| c) area of a segment | |  | Area of sector – Area of a triangle |
| Graph trigonometry functions as examples of periodic graphs | |  | Such as:  *y* = sin *x*, *y* = cos *x*, *y* = tan*x*,  *y* = cosec *x*, *y* = sec *x*, *y* = cot *x*, *y* = -2sin*x*,  *y* = cos 2*x* + 1, *y* = 4 – 2sin 3*x*  for varying domains.  Graphs should be drawn by adjusting the scales of the axes and translating the axes for basic trigonometric graphs. |
| Find the period and amplitude of trigonometric functions. | |  |  |
| Find the natural domain and range of trigonometric functions. | |  |  |
| Know the approximations to sin*x*, cos*x*, tan*x* where *x* is small. | |  |  |
| Know the result | |  |  |
| Differentiate sin*x*, cos*x*, tan*x*. | |  | **STRESS** *x* must be in radians  \* \*  \* |
| Apply derivatives of trigonometric functions to previous calculus work. | |  | finding equations of tangents and normals |
| Integrate trigonometric functions. | |  | Such as:  \* sin ( a*x* + b), \* cos ( a*x* + b )  \* sec( a*x* + b ) |
| Apply trigonometric functions to previous integration work. | |  | Finding area and volume |
| Solve simple trigonometric equations in radians. | |  | Such as : \* cos A = 0.7  \* 13 sinA + 10 = 0 \* cos *x* =  **domain under consideration should always be given** |
| Use addition of ordinates to graph functions. | |  | Such as : \* *y* = *x* + sin 2*x*  \* *y* = 2 sin *x* + cos 2*x* |
| Use graphs to solve equations. | |  | Such as sin 2*x* + x = 1 |
| Perform integrations involving sin*x* and cos*x*. |  | | Examples should include primitive functions, definite and indefinite integrals.  Such as :  \*  \* |

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| **TRIGONOMETRIC FUNCTIONS** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Watch video on Trigonometric Functions. * Define key terms such as radians, arc, sector, minor segment, periodic, period, amplitude. * Discuss the advantages of radian measures. * Explore method for converting degrees to radians and vice versa. * Derive formula’s for arc length, area of sector, area of segment for angles measured in degrees and radians. * Use the graphics calculator to explore the graphs of trigonometric functions. * Explore how to use MATH-O-MAT to aid in the process of sketching trigonometric functions. * [Explore the gradient of trigonometric functions using Geometry SketchPad](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Trigonometric%20Functions\Gradient%20of%20Trig%20Functions.gsp) * Use graphics calculator to examine differential of trig functions.   Eg. Use graphics calculator to sketch *y* = sin2*x*, and the differential of *y* = sin2*x*. Students can then observe that the derivative is  *y* = 2cos2*x*.   * Discuss how to use the standard table of integrals. * Explore method of addition and subtraction of ordinates. * [Complete questions on trigonometric functions](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Trigonometric%20Functions\TRIGONOMETRIC%20FUNCTIONS.doc) * Complete HSC Style Questions. |
| * Integrate trigonometric functions involving substitution. * Explore method for performing integrations involving sin*x* and cos*x*.  * [Have students complete questions on trigonometric functions (Ext 1)](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Trig%20Functions\MIF3U18trigfunctions.doc) |

| Syllabus Topics | Additional Information | Integrated teaching, learning and assessment |
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| 13.1 Circular measure of angles. Angle, arc, sector. | Just as natural logarithms are preferable to logarithms to other bases (including 10) for the study of logarithmic and exponential functions, it turns out that degrees are not a satisfactory measure of angle size in further work involving the trigonometric functions. A suitable measure of angle size is the length of arc subtended by the angle when it is at the centre of a unit circle. An angle is of measure 1 if the arc it subtends on a unit circle has unit length. This new unit of measure is called a *radian*. | J&C pp. 412-413 |
| 13.2 The functions sin *x,* cos *x*, tan *x*, cosec *x*, sec *x*, cot *x* and their graphs. | The number may be defined as the length of a semi-circular arc of a unit circle. It then follows that  radians = 180°,  and the formulae for conversion from degrees to radians and vice-versa now follow. Familiarity with both measures is expected. Practice should be given so that exact equivalents are known for common angle sizes and so  that accuracy is developed in approximating sizes given in one measure by sizes in the other.  The formula for the length of an arc subtending an angle at the centre of a circle of radius *r*, should be derived, as also should the formula *A* = *r*2 for the area of the corresponding sector.  The relations treated in Topic 5.2 should be revised using radian measure, **as should Topic 5.9 for 3 Unit students**.  Using radian measure, sine and cosine are now defined as functions of a *real* variable: for each real number *x*, sin *x* is defined as the sine of an angle of size *x* radians, cos *x* as the cosine of this angle. Thus the functions *y* = sin *x*,  *y* = cos *x* are defined for all real *x* and graphs should be drawn of them. The functions tan *x* etc, may not be defined in terms of sin *x* and cos *x*, their domains of definition are to be found, and graphs drawn of them. | J& C pp. 13 – 415  Exercise 20.1  J&C pp. 415 – 419  Exercise 20.2  2 Unit (Fitzpatrick) pp. 385 – 389  Exercise 17(c)  3Unit (Fitzpatrick) pp. 41 – 56  *Omit Type 5 – Example 12*  Exercise 22(a)  Exercise 22(b)  Exercise 22(c)  Exercise 22(e)  J&C pp. 419 – 423 |
| 13.3 Periodicity and other simple properties of the functions sin *x*, cos *x* and tan *x.*  13.4 Approximations to sin *x*, cos *x*, tan *x*, when *x* is small.  The result . | The graphs of sin *x*, cos *x* and tan *x* should be known and their periodicity noted. Graphs of functions such as  *y* = 3 cos 2*x, y* = sin *x* or *y* = 1 – cos *x* should be drawn, and the main features noted.  Some practice is to be given in using graphs to solve simple equations such as .  13.4 and 13.5 It should be noted that sin *h* 0 and  cos *h* 1 as *h* 0. The limit as *h* 0, should be obtained. This can be tentatively derived on a calculator, or the following geometrical proof can be used.  *(Refer diagram in Syllabus page 69)*  *AB* is an arc of the unit circle, *h* is in radians. *O* is the centre of the circle and *AT* is the tangent at *A*.    sin *h* < *h* < tan *h*.    As *h* 0, cos *h* 1. Hence 1 as *h* 0.  This procedure can be repeated with *h* in degrees to illustrate the reason for introducing radians. Moreover,  .  Thus as *h* 0. | Exercise 20.3  Exercise 20.4  Exercise 20.5  J&C pp.423 – 424  Exercise 20.7  J&C p. 424 |
| 13.5 Differentiation of cos *x*, sin *x*, tan *x*. | The evaluation of (sin *x*) given below uses these limits and the formula for sin *(x + h)*. A simple derivation of this formula, valid for 0 < *x* < and for small positive values of *h* is as follows.  *(Refer diagram in Syllabus page 70)*  In *ABC*, Area *ABC* = *cb* sin *A*;  = *cb* sin(*x+h*);  = *kp* + *kq*.  Hence sin *(x + h)* = ,  = sin *h* cos *x* + sin *x* cos *h*.  Proofs of results in 13.5 are not examinable and alternative methods of proof may be used at the teacher’s discretion.  also be given.  **Extension 1 students should be able to use the result**  **Example: given , find .**  The derivative of sin *x*.  As *h* 0, 1 and – 0.  Hence sin *x*=  Since cos *x* = sin(/2 – *x*), the function of a function rule gives  cos *x* = –cos(/2 – *x*),  = –sin *x*,  and finally (tan *x*) = (sin *x*/cos *x*) = sec2*x*. | 3 Unit (Fitzpatrick) pp. 81 – 82  Exercise 24(d)  J&C pp.424 – 427  Exercise 20.8 |
| 13.6 Primitive functions of sin *x*, cos *x*, sec2 *x.*  **Primitive functions of sin2*x* and cos2*x***.  13.7 Extension of 13.3 – 13.6 to functions of the form *a* sin(*bx + c*), etc. | **Extension 1 students should be able to perform integrations of the types**  **2 cos2*x dx*, cos *x* sin2*x dx***  sin*(ax + b)* = *a* cos*(ax + b)*, hence  cos*(ax + b)dx* = sin*(ax + b) (a* 0*)*.  Other pairs should be derived similarly. Examples giving practice in differentiation and integration of these functions and related to items discussed in topics 10 and 11 should also be given. | J&C pp. 427 – 429  Exercise 20.9  3 Unit (Fitzpatrick) pp. 90 – 94  Exercise 24(g)  2 Unit (Fitzpatrick) pp. 391 – 393, 397 – 398, 400  Exercise 17(d)  Exercise 17(g)  J&C pp. 429 – 430  Exercise 20.10  (Miscellaneous). |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 9 – Iterative Methods for Estimation of Roots**  (Polynomials) | | **TEACHER** |
| **CLASS** |
| **Focus**: In this unit of work students extend the treatment of polynomials, begun in the Preliminary course, to include methods of estimating the zeros of polynomials.  Time Allocation: 2 Weeks Syllabus Reference: 16.4 | | |
| Targeted Outcomes (Board of Studies):  **HE7** evaluates mathematical solutions to problems and communicates them in appropriate form. | Content Description  16.4 Iterative methods for numerical estimation of the roots of a polynomial equation. | |
| Resources:  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 28  Maths I n Focus Chaper 9 |  | |

| **Content (16.4 Summarised)**  **Students should be able to :** | | **✓** | **Further Explanation** |
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| **Iterative methods for the estimation of**  **roots of polynomials**  Discussion may be confined to the following two methods:  1. Halving the interval |  | | Both methods may be applied to find approximate roots of equations involving other types of functions.  `Suppose we have 2 values of *x*, say *x* = *x*1 and *x* = *x*2 , such that the polynomial P ( *x* ) is positive for *x* = *x*1 and negative for *x* = *x*2 . Since P (*x* ) is a continuous function, there is a root of P ( *x* ) in the interval *x*1 < *x* < *x*2 |
| 2. Newton’s Method |  | | Newton’s Method is generally faster than halving the interval however some care must be taken in applying it. |

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| **ITERATIVE METHODS FOR ESTIMATION OF ROOTS** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms such as *roots, zeros, polynomials, coefficients, degree, quotient, remainder, divisor, factor*. * Discuss two methods Halving the interval and Newton’s Method for estimation of roots of polynomials. * Complete HSC Style Questions. |

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| Syllabus Topics | Additional Information | Text Reference |
| 16.4 Iterative methods for numerical estimation of the roots of a polynomial equation. | Discussion may be confined to the following two methods.  (i) Halving the interval. Suppose we have two values of *x*, say *x = x*1 and *x = x*2, such that the polynomial *P(x)* is positive for *x = x*1, ie *P(x*1*)* > 0, and is negative for  *x = x*2, ie *P(x*2*)*< 0. Since *P(x)* is a continuous function, there is a root of *P(x)* in the interval *x*1 < *x* < *x*2.  Now compute the midpoint *x*3 = (*x*1 + *x*2) and the corresponding polynomial value *P(x*3*)*. If *P(x*3*)* = 0, *x*3 is the desired root. If *P(x*3*)* > 0, we replace *x*1 by *x*3 and repeat the process using *x*3 and *x*2. If *P(x*3*)* < 0, we replace *x*2 by *x*3 and repeat the process. | 3 Unit (Fitzpatrick) pp. 174 – 179  Exercise 27(e) |
|  | (ii) Newton’s method. Suppose *z* is close to a root of  *P(x)* = 0. The tangent to *y = P(x)* at *x = z* has the equation y – *P(z) = P'(z) (x – z)*. This tangent intersects the *x*-axis at *x = z* – *P(z)*/*P'(z)*. If the original value of *z* was sufficiently close to the desired root, and if certain  other conditions are satisfied, the new value *x* is even closer. We repeat the process to converge in general to the desired root. Newton’s method is in principle faster (requires fewer steps for a given accuracy) than halving the interval, but some care must be exercised in applying it. A check, that the values obtained do appear to be approaching a root, should be made by calculating the  corresponding function values.  Both of these methods may be applied to find approximate roots of equations involving other types of functions. |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 10 – Inverse Functions and Inverse Trig Functions** | | **TEACHER** |
| **CLASS** |
| **Focus**: This unit explores the concept of inverse functions, with particular reference to trigonometric functions.  Time Allocation: 3 Weeks Syllabus Reference: 15.1 – 15.5 | | |
| Targeted Outcomes (Board of Studies):  **HE1** appreciates interrelationships between ideas drawn from different areas of mathematics  **HE4** uses the relationship between functions, inverse functions and their derivatives  **HE6** determines integrals by reduction to a standard form through substitution  **HE7** evaluates mathematical solutions to problems and communicates them in appropriate form. | Content Description  15.1 Discussion of inverse function. The functions *y* = log*ax* and *y* = *ax* as inverse functions. The relation  15.2 The inverse trigonometric functions.  15.3 The graphs of sin–1*x*, cos–1*x*, tan–1*x*.  15.4 Simple properties of the inverse trigonometric functions.  15.5 The derivatives of sin–1(*x/a*), cos–1(*x/a*), tan–1(*x/a*), and the corresponding integrations. | |
| Resources:  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 26  3 Unit Mathematics Book 1 (Jones & Couchman) Ch. 27  Maths in Focus Chapter 7 |  | |

| **Content (15.1 – 15.5 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** |
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| Graph a function or relation and its inverse |  | Note reflection about *y* = *x*. |
| Define and use the notation to represent the inverse of *f* ( *x* ).  Inverse - |  |  |
| Understand the concept of restricting the domain in order to get an inverse function. |  |  |
| Know and understand the relationship: |  |  |
| Know that is *x* = sin *y* with domain between –1 and 1 and range between and . |  | Careful distinct should be made between and . |
| Know that is *x* = cos *y* and its domain and range. |  |  |
| Know that is *x* = tan *y* and its domain and range. |  |  |
| Draw the graphs of sin-1x, cos-1x, tan-1x |  |  |
| Know the properties of inverse trigonometric functions :  a) use definitions to evaluate problems |  |  |
| b) |  |  |
|  |  |  |
| d) |  |  |
| Use the general solution to trigonometric equations.  a) sin θ = *b* is θ = *n*π + |  | **Note :** in all solutions to equations, *n* is any integer. |
| b) cos θ = *b* is  θ = 2*n*π ± |  |  |
| c) tan θ = *b* is θ = *n*π + |  |  |
| Differentiate inverse trigonometric functions.s |  | Applications to tangents, normals, curve sketching, etc |
| Integrate to inverse trigonometric functions. |  | Applications to areas, volumes, etc |

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| **INVERSE FUNCTIONS AND INVERSE TRIG FUNCTIONS** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Define key terms such as *function, inverse, increasing, domain, differentiable, mutually inverse functions.* * Discuss the graph a function or relation and its inverse * [Have students complete worksheet on inverse functions](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Inverse%20Functions\invfns.doc) * Use graphics calculator to explore the graphs of inverse functions. * Derive inverse trig functions and integrate to inverse trig functions applying this to problems. * Discuss how to use the standard table of integrals in dealing with inverse trig functions. * [Have students complete questions on Inverse Functions and Inverse Trig Functions](\\\\stpiusx.local\\shares\\Staff\\Staff\\Mathematics Department\\2016\\2016 Programs\\2016 Stage 6 Programs\\Stage 6 Resources\\ext 1\\Inverse Functions\\MIF3U20inversefunctions.doc) * Complete HSC Style Questions. |

| Syllabus Topics | Additional Information | Text References |
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| **15.1 Discussion of inverse functions. The functions and as inverse functions. The relation** | Suppose that *y = f(x)* is a continuous *increasing* function in the domain (ie *fb)* > *f(* *)* for all *b* > in this domain). To each value *y* in there corresponds a *unique* value (namely, the *x* such that  *y = f(x)*) and we may write *x = g(y)*, where the function *g* has domain .  The two relations *y = f(x)* and *x = g(y)* are equivalent and are represented by the same graph in the *x, y* plane. These two relations are *mutually inverse functions* in the sense that  for *, g(f(x)) = g(y) = x*,  for *, y = f(x) = f(g(y))*;  accordingly the notation *f*–1 is commonly used for the function inverse to *f*.  If we express the function *f*–1 in the conventional form  *y* = *f*–1*(x)*, its graph is obtained from that of *y = f(x)* by reflection in the line *y = x*. The domain of *y = f*–1*(x)* is the range of *y = f(x)* and vice-versa. | 3 Unit (Fitzpatrick) pp.133 – 137 |
| * 1. **The inverse trigonometric functions.**   **15.3 The graphs of , , .** | Care must be taken to distinguish *f*–1*(x)* from *(f(x))*–1 = .  Simple examples of mutually inverse functions, which could be used to introduce this topic, are:  (i) *y = x*3, all real *x; y =* , all real *x*.  (ii) *y = e*x, all real *x; y* = log*ex*, *x* > 0.  The problem of defining an inverse function when the equation *y = f(x)* has more than one solution *x* for a given *y* should be discussed; the case *y = x*2 is a useful illustration.  *Differentiation*. If in addition *f* is a differentiable function of *x* then (since a tangent line to *y = f(x)* is also a tangent line to *x = g(y)*) *g* is a differentiable function of *y*, and (since the relevant angles of inclination are complementary) or = 1, which may be formally obtained from the definition of the derivative.  **Example** If *y = x*3, = 3*x*2= . Hence  The inverse function y = sin–1*x* should be defined with domain and range . The general solution of the equation sin *= b*, for , is then expressible as The functions  *y* = cos –1*x* () and *y* = tan –1*x* (< *y* < ) should also be defined, and the general solution of the equations cos *= b (*) and tan *= b* should be obtained. | J&C 3 Unit Bk 2 pp.189 – 191  Exercise 27.2  3 Unit (Fitzpatrick) pp.139 – 147  Exercise 26(b) |
| **15.4 Simple properties of the inverse trigonometric functions.**  **15.5 The derivatives of , , , and the corresponding integrations.** | For example, properties such as:  (i) sin–1 *(–x)* = –sin–1*x*, cos–1 *(–x)* = – cos–1*x*,  tan–1 *(–x)* = –tan–1*x*;  (ii) sin–1*x* + cos–1*x* = .  The derivatives of sin–1 *(x*/*a)*, cos–1 *(x*/*a)* and tan–1 *(x*/*a)* should be obtained. The corresponding integrations should be known. | J&C 3 Unit Bk 2 pp.198 – 201  Exercise 27.4  3 Unit (Fitzpatrick) pp. 147 - 156  Exercise 26(c)  Exercise 26(d) |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 11 – Binomial Theorem** | **TEACHER** |
| **CLASS** |
| **Focus**: The binomial expansion is introduced and students learn to prove the Pascal triangle relations and implement them in the solution of problems.  Time Allocation: 3 Weeks Syllabus Reference: 17.1 – 17.3 | |
| Targeted Outcomes (Board of Studies):  **HE1** appreciates interrelationships between ideas drawn from different areas of mathematics  **HE2** uses inductive reasoning in the construction of proofs  **HE7** evaluates mathematical solutions to problems and communicates them in appropriate form. | Content Description  17.1 Expansion of (1 + *x*)*n* for *n* = 2, 3, 4 …  Pascal Triangle.  Proof of the Pascal Triangle relations.  Extension to the expansion (*a* + *x*)*n*.  17.2 Proof by Mathematical Induction of the formula for (also denoted  (also denoted by .)  17.3 Finite series and further properties of binomial coefficients. |
| Resources:  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 28  Maths in Focus Chapter 10 |  |

| **Content (17.1 – 17.3 Summarised)**  **Students should be able to :** | **✓** | **Further Explanation** | |
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| Construct Pascal’s triangle and use it in finding coefficients in the expansion of |  | |  |
| Know factorial notation |  | | *n* ! = *n* ( *n* – 1 )( *n* – 2 )( *n* – 3 ) … 3 . 2 . 1 |
| Use the notation , and define as the coefficients of in |  | |  |
| Use the above results to expand |  | |  |
| Prove the results , and |  | |  |
| Know the formula for |  | | Prove by Mathematical Induction that:  =  and hence = |
| Use the Binomial Theorem |  | | Using the general term of the expansion to find coefficients, terms independent of *x*, the *k*th term, the largest coefficient, etc |
| Combinatorial interpretation of the Binomial coefficients. |  | |  |

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| **BINOMIAL THEOREM** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Introduce Binomial Theorem by examining Pascal’s triangle and the history of the development. * [View Binomial Theorem PowerPoint](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Binomial%20Theorem\Binomial%20Theorem.ppt) * [View examples of Binomial Theorem Questions](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Binomial%20Theorem\M1-Binomial.pdf) * Write Pascal’s triangle using combinatorial notation. * Define key terms and notation such as binomial, Pascal triangle, polynomial, coefficients, factorials, , .  * Apply binomial theorem to expansion of and other related expansions.  * Relate binomial theorem to the probability. * Prove various binomial facts by Mathematical Induction. * [Have students complete questions on Binomial Theorem](\\\\stpiusx.local\\shares\\Staff\\Staff\\Mathematics Department\\2016\\2016 Programs\\2016 Stage 6 Programs\\Stage 6 Resources\\ext 1\\Binomial Theorem\\MIF3U24binomialtheorem.doc) * Complete HSC Style Questions. |

| Syllabus Topics | Additional Information | Text References |
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| **17.1 Expansion of for *n* = 2, 3, 4, …**  **Pascal Triangle.**  **Proof of the Pascal Triangle relations.**  **Extension to the expansion of .** | 17.1 The binomial expansion is introduced by example, using *n* = 1,2,3,4. The Pascal triangle is constructed.  We observe that for any integral n, no matter how large, (1 + *x*)*n* is a polynomial of degree *n* in the variable *x*. The (so far unknown) coefficients in this polynomial must be labelled by some symbol; we choose ‘*C*’ for ‘coefficient’ and write the power of x as a right subscript *k*, the power *n* as a left superscript, ie *nCk* is by definition the coefficient of *xk* in *(*1 + *x)n*:  (1 + *x*)*n* = *nC*0 + *nC*1*x + nC*2*x2* + … *nCnxn*.  We extend this identity to an expansion of *(a + u)n* by putting *x = u*/*a*, and multiplying both sides by *an*; the result is:  *(a + u)n* = *nC*0*an* + *nC*1*an*–1*u* + *nC*2*an*–2*u*2 + … + *nCnun*. | 3 Unit (Fitzpatrick) pp. 200 – 201 |
| **17.2 Proof by mathematical induction of the formula for nCk (also denoted by ).** | **Proof of the Pascal triangle relations**  There are two separate relations, (i) for the outer coefficients, and (ii) for the others.  In the above expansion for *(a + u)n*, put *a* = 1, *u* = 0 on both sides to get *nC*0 =1. Put *a* = 0, *u* = 1 on both sides to get *nCn* = 1.  Next we write down *(*1 + *x)n*–1 and below it, the same expression multiplied by *x*; adding them together, and collecting terms of the same degree in *x*, we obtain  (1 + *x*)*n*–1 + *x* (1 + *x*)*n*–1 = *n*–1*C*0 + (*n*–1*C*0 + *n* –1*C*1)*x* +  (*n*–1*C*1 + *n*–1*C*2)*x*2 + … + (*n*–1*Cn*–2 + *n*–1*Cn*–1)*xn*–1 + *n*–1*Cn*–1*xn*.  However, the left side is obviously equal to (1 + *x*)*n*. We now use the fact (which we have proved as a theorem only for the special case of second degree polynomials) that two polynomials are equal for all values of *x* if and only if all corresponding coefficients are equal. Comparing the right side above with the earlier expansion of (1 + *x*)*n*, we immediately deduce:  *nCk = n*–1*Ck* + *n*–1*Ck -1* for 1 ≤*k* ≤*n* – 1.  Let us consider (1 + *x*)4 = 1 + 4*x* + 6*x*2 + 4*x*3 + *x*4. The ratios of successive coefficients are: 4/1 = *n*/1; 6/4 = 3/2 = (*n* – 1)/2; 4/6 = 2/3 = (*n* – 2)/3; and finally, 1/4 = (*n* – 3)/4. Thus the coefficients in this formula are, going  from left to right,  and  This leads us to guess at the general formula:  statement *S(n): nCk*= for 1 ≤*k* ≤*n*.  The remaining coefficient, *nC*0, has already been shown to equal 1 in 17.1. It should also be noted at this stage that our guess gives the correct, and already proved, answer for *nCn*. We now use mathematical induction to  prove this result. | 3 Unit (Fitzpatrick) pp. 201  3 Unit (Fitzpatrick) p. 202 |
| 17.3 Finite series and further properties of binomial coefficients. | *Step 1*. Statement *S* (2) is true, by inspection of (1 + *x*)2.  *Step 2.* We assume statement *S*(*n* – 1) to be true, and deduce from it the truth of *S(n)*.  The easiest case is *k* = 1. We assume the truth of  *n*–1*C*1 = (*n* – 1)/1, which is part of statement *S*(*n* –1) and we already know, from 17.1, that *n*–1*C*0 = 1.  Adding these two, and using the Pascal triangle relation proved in 17.1, we obtain *nC*1 = 1 + (*n* – 1) = *n = n*/1, in agreement with statement *S(n)*.  For *k* 2, we have *k* – 1 1 and we assume the truth of the relations:  *n*–1*Ck* = and  *n*–1*Ck-1* = ,  both of which are part of statement *S*(*n*-1). We add these two numbers and use the Pascal triangle relation to get  *nCk = n*–1*Ck*–1 + *n*–1*Ck*=  Since the square bracket equals *n*/*k*, this is precisely the same as statement *S(n)*, which is hereby proved for all  *k* 2, and has been proved for *k* = 1 above. The proof by induction is therefore complete.  The student should be able to use the general formula  *(a + b)n* =  For example, find the coefficient of *x*3 when (*x*2 – 2/*x*)3*N* is expanded in powers of *x*, given that *N* is a positive integer.  The typical term is *x*2*k* (–2/*x*)3*N–k*, in which the power of *x* is 3*k* – 3*N*. Taking *k = N* + 1, we find that the required coefficient is (–2)2*N*–1. | 3 Unit (Fitzpatrick) pp. 203 – 210  Exercise 29(a) |
|  | When *a* and *b* are positive, consideration of the ratio of successive terms in the expansion of *(a + b)n* makes it easy to determine the greatest term in that expansion, eg  find the greatest coefficient in the expansion of (3 + 5*x*)20.  Writing (3 + 5*x)*20 = , we have ,  which exceeds one for 1 ≤*k* ≤12 and is less than one thereafter. Hence the greatest coefficient is  *t*13= 51337.  The substitution, *x* = 1, in the expansion of (1 + *x*)*n* gives the formula 2*n* = . (This can, of course, be given a direct combinatorial interpretation as the equality of two methods of enumerating the subsets of a set of *n* elements.) We may use functional properties of (1 + *x*)*n* to sum other finite series involving binomial coefficients. For example, consideration of the coefficient of *xn* on each side of the identity, (1 + *x*)*n*(1 + *x*)*n* (1 + *x*)2*n*, gives the formula .  Differentiation or integration is also possible, eg differentiate both sides of the identity  and show that . |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 12 - Applications of Calculus to the Physical World** | **TEACHER** |
| **CLASS** |
| **Focus**: The relationship between the derivative and the concept of rate of change is the basis of this unit. Various practical applications are considered, especially those related to motion. The treatment for Extension students is deeper and includes the consideration of simple harmonic motion and projectile motion.  Time Allocation: 3 Weeks Syllabus Reference: 14.1 - 14.4 | |
| Targeted Outcomes (Board of Studies):  **H1** seeks to apply mathematical techniques to problems in a wide range of contexts  **H2** constructs arguments to prove and justify results  **H3** manipulates algebraic expressions involving logarithmic and exponential functions  **H4** expresses practical problems in mathematical terms based on simple given models  **H5** applies appropriate techniques from the study of calculus and trigonometry to solve problems  **H9** communicates using mathematical language, notation, diagrams and graphs.  **HE3** uses a variety of strategies to investigate mathematical models of situations involving, projectiles, simple harmonic motion, or exponential growth and decay.  **HE5** applies the chain rule to problems including those involving velocity and acceleration as functions of displacement  **HE7** evaluates mathematical solutions to problems and communicates them in appropriate form. | Content Description  14.1 Rates of change as derivatives with respect to time.  The notation etc.  14.2 Exponential growth and decay; rate of change of population; the equation , where *k* is the growth constant.  E The equation = where *k* is the population growth constant, and *P* is a population constant.  14.3 Velocity and acceleration as time derivatives. Applications involving:  (i) the determination of the velocity and acceleration of a particle given its distance from a point as a function of time;  (ii) the determination of the distance of a particle from a given point, given its acceleration or velocity as a function of time together with appropriate initial conditions.  E Velocity and acceleration as functions of *x*.  Applications in one and two dimensions (projectiles).  E 14.4 Description of simple harmonic motion from the equation  *x* = *a* cos (*nt* + α), *a* > 0, *n* > 0.  The differential equation of the motion. |
| Resources:  New Senior Mathematics 2 Unit (Fitzpatrick) Ch. 19  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 25  3 Unit Mathematics Book 2 (Jones & Couchman) Ch. 22  Maths in Focus Chapter 6  Cambridge Mathematics 3 Unit Year 12 |  |

| **Content (14.1 – 14.3 Summarised)**  **Students should be able to :** | | **✓** | **Further Explanation** |
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| **Motion**  a) use appropriate notation | |  | Emphasis throughout this topic should be on qualitative deduction about the nature of the motion rather than upon lengthy and involved algebraic or numerical work. |
| b) use differentiation to find expressions  for velocity and acceleration given the  displacement. | |  | Examples should include all functions that students can differentiate.  Such as : \*  \* |
| c) evaluate the above expressions for  given values of *t*. | |  |  |
| d) use integration to determine  expressions and values of velocity and  displacement given acceleration and  specific conditions. | |  | Such as :  \* , *v* = 0, *t* = 0  \* = 3 sin *t* – 4 cos *t* if *v* = 0, *x* = 0 when  *t* = 0  **Stress: the constant in integration** |
| e) describe the motion of particles in both  the above cases. | |  | Descriptions of motion should include initial conditions; where and when the particle stops; whether the particle is speeding up or slowing down; final conditions. |
| f) draw and interpret  i) displacement – time graphs | |  | **Stress :** relationship between acceleration and resultant force. |
| ii) velocity – time graphs | |  |  |
| iii) acceleration – time graphs | |  |  |
| **Rates of Change**  a) express rates as derivatives in terms of  time. | |  |  |
| b) integrate to find the quantity when  given an expression for the rate. | |  | **Stress:**  constant of integration.  A valve is slowly turned in a pipeline such that the volume flow rate R varies with time according to the relation R = *kt*, *t* > 0 where *k* is a constant. Calculate the total volume of water through the valve in the first 10 seconds if *k* = 1.3 m |
| 1. differentiate to find the rate given to an   expression for the quantity in terms of *t*. | |  | The number of bacteria in a culture is given by where *t* is in minutes. Find the rate of increase in the number of bacteria after 10 minutes. |
| **Exponential Growth and Decay**  a) graph functions of the form  for various values of *A* and *k*, where *k*  may be positive or negative. | |  |  |
| b) convert verbal information such as  “ the rate at which the population N of  a town increases is proportional to the  population present “ to a differential  equation. | |  |  |
| c) understand the significance of *k* > 0  and *k* < 0, *k* = 0 to population growth | |  | **Stress:** Growth rate per hour indicate that time is in hours and is **NOT** the average rate of change |
| d) understand the terms growth rate,  death rate, birth rate. | |  |  |
| e) verify by substitution that is  a solution of | |  | Students should know A represents the initial value of the quantity. |
| f) use alternative function notation | |  | Such as : \*  \* |
| g) answer problem type questions  involving population change,  radioactive decay and so on. | |  | The growth rate per hour of a population of bacteria is 10% of the population. At *t* = 0 the population is one million. Sketch the curve of population against time and determine the population after 3.5 hours, correct to 4 significant figures. |
| h) graph and interpret graphs of  population against time and so on. | |  |  |
| i) distinguish between exponential growth  and decay and rates of  change | |  |  |
| Consider the equation , when dealing with exponential growth and decay questions.  Both *k* and *P* are constants. |  | | First note that one solution of this equation is *N* = *P*. Direct substitution shows that the solution of this equation may be written in the form where A is an arbitrary constant.  Some numerical examples should be undertaken, determining A and/or *k* from given initial data. |
| Write velocity and acceleration as functions of *x*.  The result |  | | **Care should be taken to use the approach emphasized in the syllabus NOT that taken by some texts e.g. Jones and Couchman as it is contrary to what the students are expected to do in the HSC exam.** |
| In relation to **Simple Harmonic Motion**:  a) define Simple Harmonic Motion |  | | Simple Harmonic Motion occurs when the acceleration of a particle is directed towards a fixed point O and is proportional to its distance from that point. |
| 1. Begin with and work through :   *x* = *a* cos ( *n*t + α ) obtain  = -*an* sin ( *n*t + α ) and  and |  | |  |
| c) define amplitude, frequency, period |  | |  |
| d) solve problems from first principles  and not rely on learning results by rote |  | |  |
| e) use the case  describing Simple Harmonic Motion  about position *x* = *b*. |  | |  |
| In relation to **Projectile Motion :**   1. consider motion with vertical and horizontal components of v sinθ 2. and, v cosθ, initial velocity v and angle of projection θ to the horizontal. |  | |  |
| Derive the equations for motion of a particle from initial conditions. |  | | Horizontal Vertical  = 0 = -*g*  = v cos θ = -*gt* + v sin θ  *x* = v*t* cos θ v*t* sin θ |
| Use the equations of motion to find:   * cartesian equation * maximum height * range, maximum range * time of flight * angle at which the projectile strikes the ground |  | |  |
| Work all problems from first principles choosing suitable origin and axes |  | | Problems should include projectiles launched from ground level and from a height.  Such as: top of a cliff |

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| **APPLICATIONS OF CALCULUS TO THE PHYSICAL WORLD** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Introduce the concept of exponential growth and decay through examples such as population growth, bacteria growth, etc. * Define key terms and notation such as motion, exponential growth, exponential decay, initial, population, at rest, displacement, velocity, acceleration, rate, .  * Discuss the difference between distance and displacement, speed and velocity. * Discuss the fact that velocity is the rate of change of displacement and acceleration is the rate of change of velocity with **respect to time.** * Draw graphs showing displacement, velocity and acceleration of a particle. * Solve practical problems. * [Have students complete questions on Applications of Calculus to the Physical World](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\2%20unit\Applications%20of%20Calculus%20to%20the%20Physical%20World\APPLICATIONS%20OF%20CALCULUS%20TO%20THE%20PHYSICAL%20WORLD.doc) * Have students complete HSC style questions |
| * Define key terms and notation such as *motion, exponential growth, exponential decay, initial, population, rest, displacement, velocity, acceleration, rate, simple harmonic motion, motion of a projectile.* * [Explore Geometry SketchPad File on SHM](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Applications%20of%20Calculus%20to%20the%20Physical%20World\Simple%20Harmonic%20Motion.gsp). * Complete HSC Style Questions. * Watch [Powerpoint Presentation on Motion](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\Resources%20for%20All\Extension%201%20Mathematics\Features%20of%20Motion.ppt). |

| Syllabus Topics | Additional Information | Text Reference |
| --- | --- | --- |
| 14.1 Rates of change as derivatives with respect to time.  The notation etc. | The rate of change of some physical quantity *Q* is defined as . (An alternative notation is ). This can be justified by considering . The rate of change of the population *P* of a town is defined, for example, as while the rate of change of the volume of water is a container would be defined as . | 2 Unit (Fitzpatrick) pp. 426 – 428  Exercise 19(a)  Note: notation not covered at this stage |
|  | In doing questions on rates of change students should be encouraged to draw sketches of *Q* and as functions of *t* whenever this is possible. In particular the relationship between an integral and the area under a curve is relevant here.  Examples should be kept as mathematically simple as possible, with the emphasis on understanding the behaviour of the system. 2 Unit students will not be expected to derive an equation from given information; in all examples, an equation will be given.  **Examples**  (i) A valve is slowly opened in a pipeline such that the volume flow rate *R* varies with time according to the relation *R = kt (t* > 0*)*, where *k* is a constant.  Calculate the total volume of water that flows through the valve in the first 10 seconds if *k* = 1.3m3*s*–2.  *R* = = *kt*.  . Since *V* = 0 at *t* = 0, *c* = 0.  When *t* = 10, *v* = = 65 m3.  65 m3 flows through in the first ten seconds.  **The following are examples of harder problems.**  **(ii) A spherical balloon is being deflated so that the radius decreases at a constant rate of 10 mm per second. Calculate the rate of change of**  **volume when the radius of the balloon is 100 mm.**  **Let *V* be the volume of the balloon. Then , and we are given that = –10. Using the chain rule,**    **Now at *R* = 100,** | 3 Unit (Fitzpatrick) pp. 95 – 101  Exercise 25(a) |
| 14.2 Exponential growth and decay; rate of change of population; the equation , where *k* is the growth constant. | **(iii) A spherical bubble is expanding so that its volume increases at the constant rate of 70 mm3 per second. What is the rate of increases of its surface area when the radius is 10 mm?**  **Let *V* be the volume of the bubble.**  **Then .**  **Now ; (1)**  **Also ; (2)**  **Thus ,**    **Now when *R* = 10, = 14 mm2 per second.**  **This example illustrates the elimination of between (1) and (2) but the simple calculation of itself from (1) should not be overlooked.**  Students initially should sketch the curve *y = Aekx* for the various values of *A* and *k*, both positive and negative.  Let *N* be a population. Note that *N(t)* is a function of time. Assume that the birth and death rates at any one time are proportional to *N*, so that the rate  of change of population is given by  = *kN*. (3)  *k* is assumed to be constant and is called the *growth rate*; it might be different for different species or locations. The rate is often given as a percentage, but in (3) it should be expressed as a decimal or fraction. If *k* > 0, the birth rate is larger than the death rate, while if *k* < 0 the birth rate is smaller than the death rate. If for a particular species the birth and death rates are equal, then  *k* = 0, *N* is constant and the population is static. | 2 Unit (Fitzpatrick) pp. 429 – 435  Exercise 19(b)  OR  J&C 3Unit Bk 2 pp. 85 – 90  Exercise 22.7 |
|  | Derivation of the solution of (3) is not required. Use direct substitution of *N = Aekt* (*A* fixed) to demonstrate that it satisfies the equation for every choice of *A*, so that *A* fulfils the role of a constant of integration. The idea of an ‘initial population’ *N*(0) should be introduced. It is then clear that *A = N*(0), so that *A* is completely determined by the initial population. Since the subsequent population at a time *t* later is given by  *N(t) = N(0)ekt*,  it is also completely determined.  Populations *N(t)* should be graphed as functions of *t* for various values of *A* and *k*, as shown in the following examples.  (i) The growth rate per hour of a population of bacteria is 10% of the population. At *t* = 0 the population is 1.0 × 106. Sketch the curve of population after 3 hours, correct to 4 significant figures.  In this case = 0.1 *N* so that *N = Ae*0.1*t*, where *t* is in hours.  At *t* = 0, *N* = 106, so that *A* = 106.  *(Refer diagram in Syllabus page 73)*  After 3 hours the population is *N* = 106 × *e*0.35  = 1.419 × 106.  **Note:** The term ‘growth rate per hour’ is a widely used method of indicating that the time is to be measured in hours. The value of *k* is *not* a measure of an average rate of increase over a period of one hour (*N*(1) ≠1.1*A*); rather, it indicates the instantaneous rate of increase of the population. |  |
| **The equation , where *k* is the population growth constant, and *P* is a population constant.** | (ii) On an island, the population in 1960 was 1732, and in 1970 it was 1260. Find the annual growth rate to the nearest per cent assuming it is proportional to the population. In how many years will the population be half that in 1960?  In this case *N = Aekt*. At *t* = 0, *N* = 1732, therefore *N* = 1732*ekt*. At *t* = 10, *N* = 1260, and hence  *e*10*k* = 1260/1732,  *k* = log*e* (1260/1732)  = –0.0318  = –3% (to the nearest percent.)  The population has halved at the time *T* years if *ekT* = 0.5. Thus *kT* = loge0.5 and  *T* =  = 21.79.  the population has halved in about 22 years.  The concept of exponential growth has been applied above to populations but could equally well be applied to depletion of natural resources, industrial production, inflation etc.  **(iii) For 3 Unit students, consider also the equation**  **= *k(N – P)*,**  **where *k* and *P* are constants. First note that one solution of this equation is *N = P*. Direct substitution shows that the solution of this**  **equation may be written in the form**  ***N = P + Aekt*,**  **where *A* is an arbitrary constant. Some numerical examples should be undertaken, determining *A* and/or *k* from given initial conditions. It should be noted that whenever**  ***k* < 0, the population *N* tends to the limit *P* as**  **, irrespective of the initial conditions. The case *k* > 0 should also be discussed.** | 3 Unit (Fitzpatrick) pp. 125 – 130  Exercise 25(e)  OR  J&C 3Unit Bk 2 pp. 90 – 96  Exercise 22.8 |
| * 1. Velocity and acceleration as time derivatives.   Applications involving:  (i) the determination of the velocity and acceleration of a particle given its distance from a point as a function of time.  (ii) the determination of the distance of a particle from a given point, given its acceleration or velocity as a function of time together with appropriate initial conditions. | Velocity is defined as the rate of change of displacement, and acceleration as the rate of change of velocity. The notations  should be introduced and used. Examples should concentrate on simple applications including physical descriptions of the motion of a particle given its distance from an origin, its velocity or its acceleration as a function of time. The significance of negative displacements, velocities and accelerations should be clearly understood. Some examples illustrating these points are now given.  (i) The acceleration *a* ms–2 of a moving particle is given after *t* seconds by *a* = –3*t*. Initially the particle is located at *x* = 0 and its velocity *v* = 2 ms–1. Find the velocity *v* and displacement *x* as functions of time. Determine when the particle is at rest and when it returns to the origin at 0. Sketch *x* as a function of time. Describe the motion.  If *a* = –3*t, v* =∫ *a dt* = – *t*2 + *A*.  At *t* = 0, *v* = 2 and therefore *A* = 2.  *v* = – *t*2 + 2.  Now *x* = ∫*v d*t = ∫ (– *t*2 + 2)*dt*.  *x* = –*t*3 + 2*t* + *C*.  But at *t* = 0, *x* = 0, *C* = 0.  *x* = – *t*3 + 2*t*.  The particle is at rest when *v* = 0, ie when *t*2 = , so that *t* = seconds. | J&C 3 Unit Book 2 pp. 76 – 80  Exercise 22.1  Exercise 22.2  J&C 3 Unit Book 2 pp. 80 – 83  Exercise 22.3 |
|  | The particle returns to 0 when *x* = 0.  *t*(2 – *t*2) = 0  *t*(2 – *t*) (2 + *t*) = 0.  Thus *x* = 0 at *t* = 0, ± 2 and the particle returns to the origin after 2 seconds (since *t* = –2 occurs before the start of the motion).  *(Refer diagram in Syllabus page 75)*  Note that at *t* = , *x* = .  The particle was initially located at the origin, was travelling right (since *v* > 0) and was slowing down (since *v* > 0, *a* < 0). It stopped after seconds, to the right of 0. It then travels back to the left (since *v* < 0 when *t* > ) with increasing speed (since *v* < 0, *a* < 0), passing through the origin after 2 seconds. It continues to travel left (since *v* ≠ 0 for all other values of *t*).  (ii) A particle moves in a straight line. At time *t* seconds its distance *x* metres from a fixed point *O* in the line is given by *x* = 2 – 2 cos 2*t*.  Sketch the graph of *x* as a function of *t*. Find the times when the particle is at rest and the position of the particle at those times. Describe the motion.  *(Refer diagram in Syllabus page 75)*  The velocity is given by *v* = = 4 sin 2*t*.  *v* = 0 when *t* = 0, ,, , … and at these times, *x* = 0, 4, 0, 4, …. |  |
| **14.3 Velocity and acceleration as functions of *x.*** | The particle is initially at 0 and is at rest. It starts to travel to the right with increasing speed for seconds. It then slows down and stops after seconds at a position 4 units to the right of 0. It travels back to the left arriving at 0 after an additional seconds. It continues to oscillate between *x* = 0 and *x* = 4 taking seconds for one complete oscillation. (It should be noted that the above description may be obtained directly from the displacement-time graph).  **Note:** It is suggested that students be led gradually into a full description of the particle’s motion by first attempting to describe the motion only from consideration of the displacements of the particle  at various times.  If it is also possible to give *v* as a function of *x*, then, using the function of a function rule,  but = *v*, so  .  Problems involving the solutions of equations of the forms  = *f(x)* and = = *g(x)*  should be considered. Eg, find *x* as a function of *t* given that  = – *e–x, v* = 1, *x* = 0 at *t* = 0.  By integration, *v*2 = *e–x + C*.  But *v* = 1 when *x* = 0. *C* = 0.  *v*2 = *e–x*,  *v* = ± . | 3 Unit (Fitzpatrick) pp. 101 – 105  Exercise 25(b) |
| **Applications in one and two dimensions (projectiles).**  **14.4 Description of simple harmonic motion from the equation** | To decide which expression for *v* is relevant for this motion it will be necessary to examine the given data.  At *x* = 0, *v* = 1 > 0.  But > 0  *v* = .  Hence = .  Using the rule for derivatives of inverse functions (see Topic 15.1) = , hence *t* = 2 – 2 and *x* = 2 log*e*(1 + *t*/2).  The *Motion of a Projectile*. The equations of motion of a particle projected vertically upwards should be derived.  The two-dimensional motion of a projectile with the initial conditions that at *t* = 0, *x = y* = 0,  *u*= = *V*cos , *v*= = *V*sin , results in the  expressions that, at time *t, x = Vt* cos ,  *y = Vt* sin – *gt*2. This pair of equations gives a *parametric representation* of the ‘flight parabola’. Here *V* is the initial speed and the angle of projection. The cartesian equation of the flight parabola is  *y = x* tan – *gx*2/*(V*2 cos2 *)*.  The range should be derived for a projectile fired on a horizontal plane. The maximum range on a horizontal plane is *V*2/*g* when = 45°.  It follows immediately from the given equation that  *v* = = *–an*sin *(nt +* *)*  and that  = = *–an*2cos*(nt +* *)* = *–n*2*x*. | 3 Unit (Fitzpatrick) pp. 115 – 125  Exercise 25(d)  3 Unit (Fitzpatrick) pp. 105 – 114  Exercise 25(c) |
|  | Graphs of *x*, and as functions of *t* should be sketched and the relationships between zero, minimum and maximum values of the three quantities noted. The physical significance of the parameters *a, n* and should be understood, as should the terms amplitude, frequency, period and phase.  The differential equation of the motion may be interpreted as describing the motion of a particle acting under a force directed towards the origin *O* and proportional to the distance from *O*. This occurs in practice where a particle oscillates about an equilibrium position (as, for example, in the motion of a pendulum bob or of a mass attached to a spring, or the bobbing motion of a buoy).  Note that from the expression for *x* and *v*,  *v*2 + *n*2*x*2 = *a*2*n*2,  a positive constant (ie independent of *t*). Notice also that if *n* is given, then *a* and may be determined from a knowledge of *x* and *v* at any given time *t*0: a directly from the above equation and from either of the two expressions for *x* and *v*.  Extension to the case  *x = b + a* cos *(nt +* *)*  and the corresponding equation  = –*n*2 *(x–b)*  describing simple harmonic motion about the position  *x = b*. |  |
| **The differential equation of the motion.** | An alternative treatment of simple harmonic motion begins with the differential equation  = –*n*2*x*  and then uses the inverse trigonometric functions to derive a solution of the form *x* = *a* cos *(nt +**)*. A rigorous treatment along these lines is difficult because *x* is not an invertible function of *t* (and *v* is not a function of *x*) unless the range of *x* is restricted. It can be observed that  *x = x*0 cos*(n(t – t*0*))* + sin*(n(t – t*0*))*  is a solution of the equation, which satisfies the additional conditions *x (t*0*)* = *x*0, *v(t*0*)* = *v*0. Questions of uniqueness of solution are outside the  scope of the syllabus. |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
| **Outcomes Achieved:**  *According to Board of Studies guidelines, both the  “Knowledge and Skills” and “Working Mathematically” outcomes are to be covered.*  *If all appropriate outcomes for your set have been achieved indicate “ALL” – otherwise list what has not been achieved for particular students or the class as a whole)* | | | | | | | | | | | |  | | | | | | | | |
| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

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| **Unit 13 – Further probability** | | **TEACHER** |
| **CLASS** |
| **Focus**: This unit completes the earlier work on probability by considering the application of the binomial theorem in relevant contexts.  Time Allocation: 2 Weeks Syllabus Reference: 18.2 | | |
| Targeted Outcomes (Board of Studies):  **HE3** uses a variety of strategies to investigate mathematical models of situations involving binomial probability  **HE7** evaluates mathematical solutions to problems and communicates them in appropriate form. | Content Description  18.2 Binomial probabilities and the binomial distribution. | |
| Resources:  New Senior Mathematics 3 Unit (Fitzpatrick) Ch. 29  Maths in Focus Chapter 9 |  | |

| **Content (18.2 Summarised)**  **Students should be able to:** | | **✓** | **Further Explanation** |
| --- | --- | --- | --- |
| Include binomial probabilities:  P ( success ) = p and  P ( failure ) = q where  p + q = 1 |  | |  |
| Include probabilities and the binomial expansion P ( *x* = *r* ) =  is the probability of *r* successes in *n* events. |  | |  |

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| **FURTHER PROBABILITY** |
| **Suggested Teaching, Learning and Assessment Ideas** |
| * Review the different counting techniques, including lists, tables, permutations and combinations. * Use the different counting techniques to solve practical probability questions. * Work in groups to solve practical probability questions. * Introduce Binomial Theorem Probability by using Multiple Choice Example.[[4]](#footnote-4) * [Binomial Probabilty Theorem investigation](file:///\\stpiusx.local\shares\Staff\Staff\Mathematics%20Department\2016\2016%20Programs\2016%20Stage%206%20Programs\Stage%206%20Resources\ext%201\Probability\A%20WAY%20TO%20INTRODUCE%20BINOMIAL%20PROBABILITY%20DISTRIBUTION.doc) * Explore Binomial Probability Theorem and its uses to solve probability problems.. * Complete Exam Style Questions on Probability. |

| Syllabus Topics | Additional Information | Text References |
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| 18.2 Binomial probabilities and the binomial distribution. | Suppose that a coin gives heads (H) with probability *p* and tails (T) with probability *q* = 1 – *p*. Representing the outcomes of three tosses on a tree diagram spreading vertically downwards, we find that the bottom row has  the entries HHH, HHT, HTH, HTT, THH, THT, TTH, TTT with respective probabilities *p*3, *p*2*q*, *p*2*q*, *pq*2, *p*2*q*, *pq*2, *pq*2, *q*3. The number *X* of heads appearing in these three tosses has a frequency distribution given by  *P*(*X*=0) = *q*3, *P*(*x* = 1) = 3*pq*2, *P*(*X* = 2) = 3*p*2*q*,  *P*(*x* = 3) = *p*3 and it is easy (and helpful) to draw a histogram for various values of *p*. | 3 Unit (Fitzpatrick) pp. 210 – 219  Exercise 29(b) |
|  | It is straightforward to generalise to the case of n tosses, where the outcomes are represented by strings of length n using the letters H, T and each string arises with probability *prqn–r*, where *r* is the number of times *H*  appears. Because there are strings with *r* occurrences of H and *n – r* occurrences of T, we see that  *P(X = r)* = *prqn–r,* where *x* denotes the number of heads which appear in *n* tosses. *X* is said to have a *binomial distribution*. (Observe that the statement  can be rewritten in the form )  The binomial distribution can be used to model repeated trials of any experiment with precisely two outcomes. The probability *p* may be known in advance, eg sampling with replacement from a box with two red and three white balls (*p* = 2/5 if H represents drawing a red ball); or estimated from frequency considerations, eg guessing the sex of a baby (*p* is slightly greater than if H represents the birth of a girl).  The following are typical examples.  It is known that *x*% of the bolts produced by a machine are faulty. What is the probability that in a random sample of 4 bolts:  (a) no bolts are defective?  (b) precisely one bolt is defective?  (c) at most, two bolts are defective?  (Express all answers in the form of 10–8*R(x)*, where *R* is a polynomial which need not be simplified.)  Let *p = x*/100 denote the probability that a bolt is defective and *q* = 1 – *p*. |  |
|  | The following are typical examples.  It is known that *x*% of the bolts produced by a machine are faulty. What is the probability that in a random sample of 4 bolts:  (a) no bolts are defective?  (b) precisely one bolt is defective?  (c) at most, two bolts are defective?  (Express all answers in the form of 10–8*R(x)*, where *R* is a polynomial which need not be simplified.)  Let *p = x*/100 denote the probability that a bolt is defective and *q* = 1 – *p*.  Then the required probabilities are respectively *q*4, *pq*3, and the sum of the first two together with  *p*2*q*2. Thus we can write the answers as  (a) 10–8 (100 – *x*)4,  (b) 10–8 4*x* (100 – *x*)3  (c) 10–8 {(100 –*x*)4 + 4*x*(100 – *x*)3 + 6*x*2 (100 – *x*)2}.  On the average, batsmen in a certain cricket team make a scoring shot on every third ball. Estimate how many six ball overs with precisely two scoring shots occur in a thousand overs of batting by that team.  We take *p* =  to represent the probability of a scoring shot on a given ball The probability that a random over contains precisely 2 scoring shots is . We multiply this by 103 and (round off to the nearest  integer) estimate that there are 329 overs. |  |

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| Crest St Pius Small | | **St Pius X College Mathematics Department**  **Topic Evaluation** | | | | | | | | | | | | | | | | | | |
| Year: |  | | Unit: | |  | Topic: | |  | | | Teacher: | | |  | | Class: | | | |  |
| Date Started: | | |  | | | Date Finished: | | |  | | No. of Lessons: | | | | | |  | | | |
| Recommended Time  (*please circle*) | | | | weeks | | was: | Too Short | | | Satisfactory | | | | | Too Long | | | | | |
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| **Learning Experiences:**  *List any activities used not already in this programme (besides the textbook):*   * *worksheets* * *technology or websites used* * *group work*   *Comment on what worked well or not and upload an electronic copy of each to the departmental drive.* | | | | | | | | | | | |  | | | | | | | | |
| **Teacher Reflection**  *Comments on your teaching practice:*   * *what worked* * *what didn’t work* * *what you will change next time* | | | | | | | | | | | |  | | | | | | | | |
| **Assessment Tool used:**  *If Assessment task other than pen and paper test, please describe and upload  an electronic copy to the departmental drive.* | | | | | | | | | | | |  | | Date: | | | |  | | |
| Teacher’s Signature: | | | | | | | | | | | |  | Today’s Date: | | | | | |  | |

1. pp 13 - 21Motivating Maths Series – Probability, Mahoney [↑](#footnote-ref-1)
2. pp 10-12 Motivating Maths Series – Probability, Mahoney [↑](#footnote-ref-2)
3. pp38 – 40 Motivating Maths Series – Probability, Mahoney [↑](#footnote-ref-3)
4. pp48 – 49 Motivating Maths Series – Probability, Mahoney [↑](#footnote-ref-4)