Introduction to Operation Management

030 **Deliver**

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3 Forecasting

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Demand Forecasting

Forecasting is one of three components of an organization's Demand Planning, Forecasting, and Management process. Demand Planning answers the question \What should we do to shape and create demand for our product?" and concerns things like promotions, pricing, packaging, etc. Demand Forecasting then answers \What should we expect demand for our product to be given the demand plan in place?" The final component, Demand Management, answers the question, \How do we prepare for and act on demand when it materializes?" This concerns things like Sales& Operations Planning (S&OP) and balancing supply and demand.

Within the Demand Forecasting component, you can think of three levels, each with its own time horizon and purpose. Strategic forecasts (years) are used for capacity planning, investment strategies, etc. Tactical forecasts (weeks to months to quarters) are used for sales plans, short-term budgets, inventory planning, labor planning, etc. Finally, operations forecasts (hours to days) are used for production, transportation, and inventory replenishment decisions. The time frame of the action dictates the time horizon of the forecast.

Forecasting methods can be divided into being subjective (most often used by marketing and sales) or objective (most often used by production and inventory planners). Subjective methods can be further divided into being either Judgemental (someone somewhere knows the truth), such as sales force surveys, Delphi sessions, or expert opinions, or Experimental (sampling local and then extrapolating), such as customer surveys, focus groups, or test marketing. Objective methods are either Causal (there is an underlying relationship or reason) such as leading indicators, etc. or Time Series (there are patterns in the demand) such as exponential smoothing, moving average, etc. All methods have their place and their role. We will spend a lot of time on the objective methods but will also discuss the subjective ones as well.

Regardless of the forecasting method used, you will want to measure the quality of the forecast. The two major dimensions of quality are bias (a persistent tendency to over- or under-predict) and accuracy (closeness to the actual observations). No single metric does a good job capturing both dimensions, so it is worth having multiple.

Forecasting Principles

1. Forecasts are always wrong { Yes, point forecasts will never be completely perfect. The solution is to not rely totally on point forecasts. Incorporate ranges into your forecasts. Also you should try to capture and track the forecast errors so that you can sense and measure any drift or changes.
2. Aggregated forecasts are more accurate than dis-aggregated forecasts { The idea is that combining different items leads to a pooling effect that will in turn lessen the variability. The peaks balance out the valleys. The coefficient of variation (CV) is commonly used to measure variability and is defined as the standard deviation over the mean. Forecasts are generally aggregated by SKU (a family of products versus an individual one), time (demand over a month versus over a single day), or location (demand for a region versus a single store).

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3. Shorter horizon forecasts are more accurate than longer horizon forecasts { Essentially this means that forecasting tomorrow's temperature (or demand) is easier and probably more accurate than forecasting for a year from tomorrow. This is not the same as aggregating. It is all about the time between making the forecast and the event happening. Shorter is always better. This is where postponement and modularization helps. If we can somehow shorten the forecasting time for an end item, we will generally be more accurate.

(Caplice, C., Ponce, E., 2024)

Forecasting metrics

Notation

- A_t: Actual value of observation t
- F_t: Forecast value of observation t
- e_t: Error for observation t, $e_t = A_t - F_t$
- n: number of observations
- μ: mean
- σ: standard deviation
- CV: Coefficient of Variation – a measure of volatility - $CV = \frac{\sigma}{\mu}$

Formulas

- Mean Deviation: $MD = \frac{\sum_{t=1}^{n} e^t}{n}$
- Mean Absolute Deviation: $MD = \frac{\sum_{t=1}^{n} e^t}{n}$
- Mean Squared Error: $MSE = \frac{\sum_{t=1}^{n} {e_t}^2}{n}$
- Root Mean Squared Error: $RMSE = \sqrt{\frac{\sum_{t=1}^{n} {e_t}^2}{n}}$
- Mean Percent Error: $MPE = \frac{\sum_{t=1}^{n} \frac{e_t}{A_t}}{n}$
- Mean Absolute Percent Error: $MAPE = \frac{\sum_{t=1}^{n} \frac{|e_t|}{A_t}}{n}$
- Statistical Aggregation of n Distributions of Equal Mean and Variance

$$\sqrt{{\sigma_1}^2 + {\sigma_2}^2 + {\sigma_3}^2 + \cdots + {\sigma_n}^2} = \sigma_{ind}\sqrt{n}$$

$$\mu_{agg} = \mu_1 + \mu_2 + \mu_3 + \cdots + \mu_n = n\mu_{ind}$$

$$CV_{agg} = \frac{\sigma\sqrt{n}}{\mu n} = \frac{\sigma}{\mu\sqrt{n}} = \frac{CV_{ind}}{\sqrt{n}}$$

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Time Series Analysis

Time Series is an extremely widely used forecasting technique for mid-range forecasts for items that have a long history or record of demand. Time series is essentially pattern matching of data that are distributed over time. For this reason, you tend to need a lot of data to be able to capture the components or patterns.

Business cycles are more suited to longer range, strategic forecasting time horizons. Three important time series models:

- Cumulative: where everything matters and all data are included. This results in a very calm forecast that changes very slowly over time { thus it is more stable than responsive.
- Naïve: where only the latest data point matters. This results in very nervous or volatile forecast that can change quickly and dramatically { thus it is more responsive than stable.
- Moving Average: where we can select how much data to use (the last M periods). This is essentially the generalized form for both the Cumulative (M = 1) and Naïve (M=1) models.

All three of these models are similar in that they assume stationary demand. Any trend in the underlying data will lead to severe lagging. These models also apply equal weighting to each piece of information that is included. Interestingly, while the M-Period Moving Average model requires M data elements for each SKU being forecast, the Naïve and Cumulative models only require 1 data element each.

(Caplice, C., Ponce, E., 2024)

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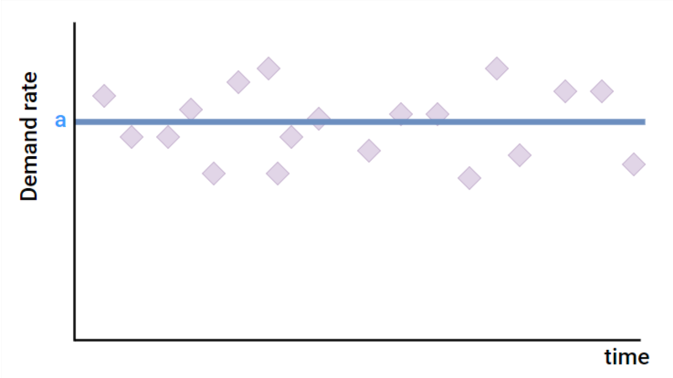
Components of Time Series

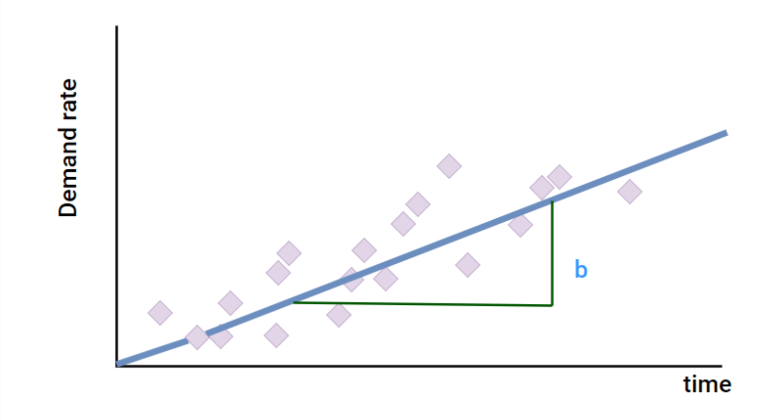
Time series data can be decomposed into several key components, each contributing to the overall pattern observed in the data:

- **Level (a)**: Represents the average value around which demand fluctuates. It captures the scale of the time series and remains constant in the absence of other patterns.



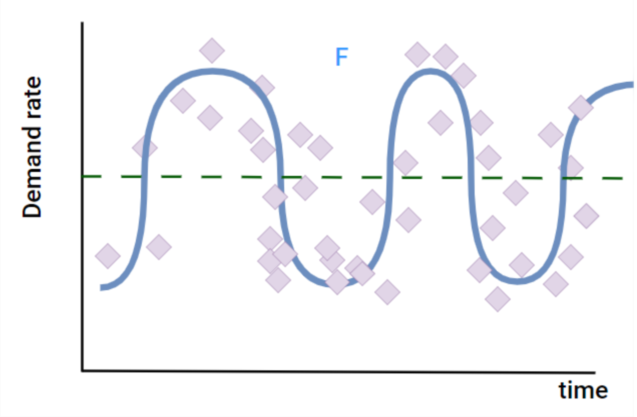
1. Level (a)

- **Trend (b)**: Indicates the rate of growth or decline in the data, reflecting a persistent movement in one direction. Trends can be linear, exponential, quadratic, or follow other mathematical forms, depending on the nature of the data.



2. Trend (b)

- **Seasonal Variations (F)**: These are recurring patterns that occur at regular intervals, such as hourly, daily, weekly, monthly, or quarterly cycles. Seasonal variations can be driven by natural factors (e.g., weather) or man-made factors (e.g., holidays,



promotions).

3. Season Variations (F)

- **Random Fluctuation (e or ε)**: This component accounts for irregular and unpredictable variations in the data, often referred to as "noise." These fluctuations are not explained by the level, trend, or seasonal components and represent the inherent uncertainty in the data.

Notation

- x_t: Actual demand in period t
- $\hat{x}_{t,\ t+1}$: Forecast for time $t+1$ made during time t
- a: Level component
- b: Linear trend component
- F_t: Season index appropriate for period t
- e_t: Error for observation $t, e_t = A_t - F_t$
- t: Time period $(0,1,2, \dots n)$
- Level Model: $x_t = a + e_t$
- Trend Model: $x_t = a + bt + e_t$
- Mix Level-Seasonality Model: $x_t = aF_t + e_t$
- Mix Level-Trend-Seasonality Model: $x_t = (a + bt)F_t + e_t$

Formulas

Time Series Models (Stationary Demand Only):

Cumulative Model: $\hat{x}_{t,t+1} = \frac{\sum_i^t x_i}{t}$

Naïve Model: $\hat{x}_{t,t+1} = x_t$

M-Period Moving Average Forecast Model: $\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^t x_i}{M}$

- If $M = t$, we have the cumulative model where all data is included
- If $M = 1$, we have the naïve model, where the last data point is used to predict the next data point.

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Exponential Smoothing

Exponential smoothing is a powerful forecasting technique used to predict future values based on past observations. Unlike other methods such as Cumulative, Naïve, and Moving Average, exponential smoothing assigns exponentially decreasing weights to older observations, giving more importance to recent data. This approach is particularly useful in time series forecasting where the most recent data points are more relevant for predicting future values.

Weighting of Observations:

- Exponential smoothing applies weights that decrease exponentially over time. This means that newer observations have a higher weight compared to older ones.
- The rate at which the weights decrease is determined by the smoothing parameter, often denoted as (α).

Smoothing Parameter (α):

- The smoothing parameter (α) ranges between 0 and 1.
- $(\alpha \rightarrow 1)$: The forecast becomes more sensitive to recent changes, making it more volatile and similar to a Naïve forecast.
- $(\alpha \rightarrow 0)$: The forecast becomes smoother and more stable, resembling a cumulative average.
- In practice, (α) is typically chosen between 0 and 0.3 to balance responsiveness and stability.

The simplest form of exponential smoothing, known as the Simple Exponential model, is based on the assumption that demand remains constant over time. In contrast, Holt's Model is an enhanced version that incorporates both the level and the trend of the data. This model introduces an additional smoothing parameter, similar to the one used for the level, to adjust for trends. Furthermore, exponential smoothing can be applied to dampen trend models, acknowledging that trends typically do not persist indefinitely. This approach helps in producing a more stable and reliable estimate of forecast errors.

Notation

- x_t: Actual demand in period t
- $x_{t,t+1}$ Forecast for time $t+1$ made during time t
- α: Exponential smoothing factor for <u>level</u> $0 \leq \alpha \leq 1$
- β: Exponential smoothing factor for <u>trend</u> $0 \leq \beta \leq 1$
- φ: Exponential smoothing factor for <u>dampening</u> $0 \leq \varphi \leq 1$
- ω: Mean Square Error trending factor $0.01 \leq \omega \leq 0.1$

Formulas

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Simple Exponential Smoothing Model (Level Only) is designed for situations where demand remains constant over time. In this model, the "new" information is simply the most recent observation, while the "old" information is represented by the latest forecast, which incorporates all previous data. The formula for updating the forecast is given by

$$\hat{x}t, t + 1 = \alpha x_t + (1 - \alpha)\hat{x}t - 1, t$$

Damped Trend Model with Level and Trend: This approach uses exponential smoothing to moderate a linear trend, reflecting the natural slowing of trends over time. The forecast for future periods is calculated as follows:

$$\hat{x}_{t,t+\tau} = \hat{a}t + \sum i = \phi^i \widehat{b_t}$$

- $\hat{a}_t = \alpha x_t + (1 - \alpha)(\hat{a}_t - 1 + \phi\hat{b}_{t-1})$
- $\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1 - \beta)\phi\hat{b}_{t-1}$

Exponential Smoothing with Holt-Winter: Exponential Smoothing for Level & Trend, also known as Holt's Method, is designed to handle data with a linear trend. The forecast for a future time period (t+k), made at time (t), combines the most recent estimates of both the level and the trend.

For the level component, the "new" information is the latest observation, while the "old" information is the most recent forecast for that period. This forecast is derived from the previous period's level estimate plus the trend estimate from that same period.

For the trend component, the "new" information is the difference between the latest level estimate and the one before it. The "old" information is simply the trend estimate from the last period.

The formulas used are:

$$\widehat{x_{t,t+\tau}} = \widehat{a_t} + \tau\widehat{b_t}$$

- $\hat{a}t = \alpha x_t + (1 - \alpha)(\hat{a}t - 1 + \widehat{b_{t-1}})$
- $\hat{b}t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1 - \beta)\hat{b}_{t-1}$

Mean Square Error Estimate: Exponential smoothing can also be applied to derive a more consistent and stable estimate of the mean square error for forecasts. The formula used is:

$$\mathrm{MSE}t = \omega(x_t - \hat{x}t - 1, t)^2 + (1 - \omega)\mathrm{MSE}_{t-1}$$

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Exponential Smoothing with Holt-Winter

- For multiplicative seasonality, consider each (F_i) as representing the "percentage of average demand" for a specific period (i).
- The total of all (F_i)) values across a complete season must equal a constant (P).
- It's crucial to regularly update the seasonality factors to prevent significant drift. This requires extensive record-keeping, which can be challenging to manage in a spreadsheet, but it's essential to grasp this concept.

Notation

- x_t: Actual demand in period t
- $x_{t,t+1}$ Forecast for time $t+1$ made during time t
- α: Exponential smoothing factor for <u>level</u> $0 \leq \alpha \leq 1$
- β: Exponential smoothing factor for <u>trend</u> $0 \leq \beta \leq 1$
- φ: Exponential smoothing factor for <u>dampening</u> $0 \leq \varphi \leq 1$
- F_t: Multiplicative seasonal index appropriate for period t
- P: Number of time periods within the seasonality (note: $\int_{i=1}^{P} \hat{F}_i = P$

Formulas

Holt-Winter Exponential Smoothing Model (Level, Trend, and Seasonality)

This model incorporates a linear trend along with a multiplicative seasonal effect on both the level and trend components.

- **Level Estimate**: The "new" information is the latest observation adjusted to remove seasonality, while the "old" information includes the previous estimates of both level and trend.
- **Trend Estimate**: This follows the same approach as in the Holt model.
- **Seasonality Estimate**: This is similar to the Double Exponential Smoothing model.

The forecast equation is:

$$\widehat{x_{t,t+\tau}} = \left(\hat{a}_t + \tau \hat{b}_t\right) \cdot \widehat{F_{t+\tau-P}}$$

- $\hat{a}t = \alpha\left(\frac{x_t}{\hat{F}_{t-P}}\right) + (1-\alpha)\left(\hat{a}_{t-1} + \widehat{b_{t-1}}\right)$
- $\hat{b}t = \beta(\hat{a}_t - \widehat{a_{t-1}}) + (1-\beta)\hat{b}_{t-1}$
- $\hat{F}_t = \varphi\left(\frac{x_t}{\hat{a}_t}\right) + (1-\varphi)\hat{F}_{t-P}$

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Double Exponential Smoothing (Seasonality and Level)

This model uses a multiplicative approach where the seasonality for each period is the product of the level and the seasonality factor for that period.

- **Level Estimate**: The "new" information is the latest observation adjusted to remove the seasonality factor, while the "old" information is the previous level estimate.
- **Seasonality Estimate**: The "new" information is the latest observation adjusted to remove the level factor, while the "old" information is the previous seasonality estimate for that period.

$$\widehat{x_{t,t+\tau}} = \hat{a}_t \cdot \widehat{F_{t+\tau-P}}$$

- $\hat{a}_t = \alpha(\frac{x_t}{\hat{F}_{t-P}}) + (1-\alpha)(\hat{a}_{t-1})$
- **Seasonality Estimate**: $\hat{F}_t = \gamma\left(\frac{x_t}{\hat{a}_t}\right) + (1-\gamma)\hat{F}_{t-P}$

Normalizing Seasonality Indices

After each forecast, it's important to normalize the seasonality indices to ensure they remain accurate and do not drift over time. If these indices are not updated, they can become significantly misaligned. Most software packages handle this automatically, but it's advisable to verify.

The normalization process is:

$$\widehat{F_t^{NEW}} = \widehat{F_t^{OLD}} \times \frac{P}{\sum_{i=t+1-P}^{t} \widehat{F_i^{OLD}}}$$

This adjustment ensures that the sum of the seasonality indices over a complete cycle remains consistent, maintaining the integrity of the model's seasonal adjustments.

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Application and Limitations

Time series models are particularly effective when the demand is stationary, meaning that the statistical properties of the series do not change over time. However, if there is a trend in the underlying data, these models may lag behind actual changes, as they assume equal weighting for all included data points. This can lead to inaccuracies if the demand pattern shifts significantly.

Despite these limitations, time series forecasting remains a powerful tool for businesses, especially when combined with other forecasting methods to account for non-stationary demand or when used in conjunction with qualitative insights. By understanding and leveraging the components of time series data, organizations can make more informed decisions, optimize inventory levels, and improve overall operational efficiency.

Enhancing Time Series Forecasting

To maximize the effectiveness of time series forecasting, businesses can employ several strategies and techniques:

1. **Data Enrichment**: Incorporating additional data sources can enhance the accuracy of time series models. This might include external factors such as economic indicators, weather data, or social media trends that could influence demand patterns.
2. **Hybrid Models**: Combining time series models with other forecasting techniques, such as causal models or machine learning algorithms, can help address the limitations of each method. Hybrid models can capture complex patterns and non-stationary demand more effectively.
3. **Adaptive Methods**: Implementing adaptive forecasting techniques allows models to adjust dynamically to changes in the data. Methods like exponential smoothing can provide more weight to recent observations, improving responsiveness to shifts in demand.
4. **Scenario Analysis**: Conducting scenario analysis can help businesses prepare for various potential future states. By simulating different scenarios, organizations can assess the impact of various factors on demand and develop contingency plans.
5. **Regular Review and Adjustment**: Continuously monitoring and adjusting forecasting models is crucial for maintaining accuracy. Regularly reviewing model performance and incorporating feedback can help identify areas for improvement and ensure that forecasts remain relevant.

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