

# Graph Theory

## Homework 1

1. Describe what a simple graph is.
2. Describe what is clique in a graph.
3. Determine whether a tree is bipartite.
4. (True or False) For every bipartite graph, the chromatic number is 2.
5. Based on Figure 1, determine whether graph  $G$  contains an Eulerian trail.
6. Based on Figures 1 and 2, determine whether graph  $T$  is an induced subgraph of graph  $G$ .

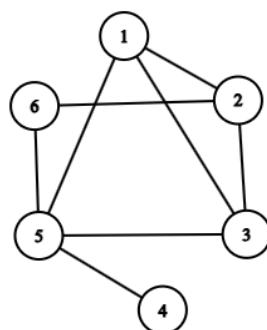


Figure 1. graph  $G$

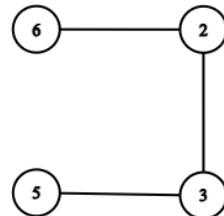


Figure 2. graph  $T$

7. Based on Figures 3 and 4, determine whether graph  $N$  is an isomorphism of graph  $M$ .

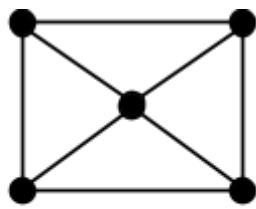


Figure 3. Graph  $M$

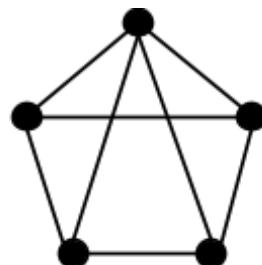


Figure 4. Graph  $N$

8. Describe the definition of self-complementary.
9. Describe the definition of girth.
10. Based on figure 5, the red directions can be called a,b-path.

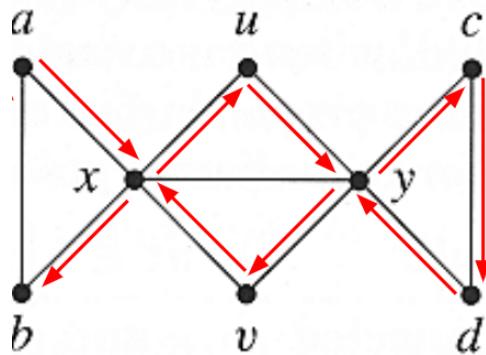


Figure 5.

11. (True or False) If an  $n$ -vertex graph has  $n$  components, then it is trivial.
12. (True or False) If a graph has a closed odd walk, then it can be bipartite.
13. Determine whether  $K_5$  has a Eulerian circuit.
14. (True or False) There exists a simple graph with 6 vertices, whose degrees are 5, 5, 4, 4, 3, 3.
15. (True or False) Every disconnected graph has an isolated vertex.
16. Given a graph  $G$  with 6 vertices and 9 edges, What is the average vertex degree of  $G$ ?
17. Prove that in a  $k$ -regular bipartite graph ( $k > 0$ ), the partite sets must have the same number of vertices.

18. Consider a simple **triangle-free** graph with 8 vertices. Find the maximum number of edges this graph can have.
19. Consider a simple **triangle-free** graph with 8 vertices, assuming the maximum number of edges is as calculated in Question 1. Describe a construction of a graph that achieves this maximum number of edges.
20. Consider a graph G with **7 vertices** and **4 edges**. What is the **maximum number of isolated vertices** that G can have?