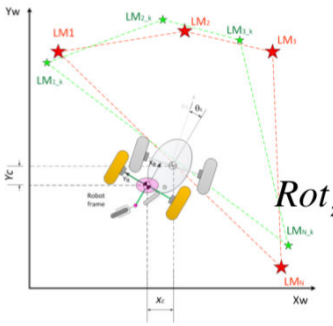
Similarity Transform



Similarity Transform

$$Rot_z(-\theta)L + transl(t_x, t_y) = U \rightarrow \begin{pmatrix} c\theta & s\theta & t_x \\ -s\theta & c\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_{3xN} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}_{3xN}$$

Least squared is used to estimate: rotation and translation errors

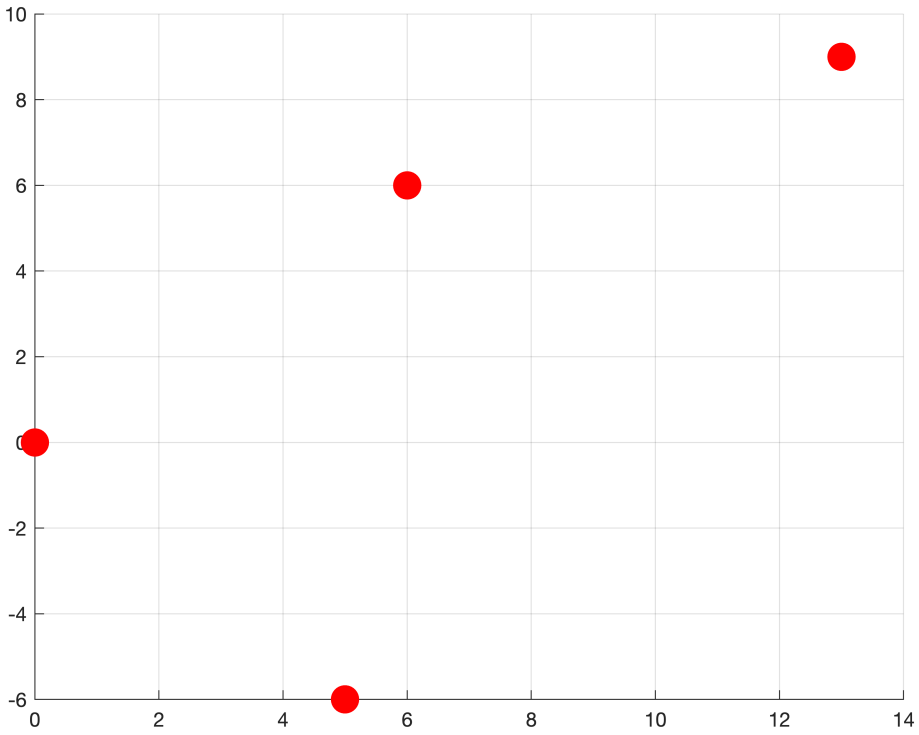
$$\underbrace{\begin{pmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \\ \dots & \dots & 1 & 0 \\ \dots & \dots & 0 & 1 \\ x_n & y_n & 1 & 0 \\ y_y & -x_n & 0 & 1 \end{pmatrix}}_{A_{2nx4}} \underbrace{\begin{pmatrix} c\theta \\ s\theta \\ t_x \\ t_y \end{pmatrix}}_{\hat{X}_{4x1}} = \underbrace{\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \dots \\ \dots \\ u_n \\ v_n \end{pmatrix}}_{B_{2nx1}} \rightarrow A\hat{X} = B \rightarrow \hat{X} = (A^T A)^{-1} A^T B \rightarrow \begin{cases} \theta = \tan^{-1} \dfrac{\hat{X}_2}{\hat{X}_1} \\ t_x = \hat{X}_3 \\ t_y = \hat{X}_4 \end{cases}$$

Table of Contents

Land Mark

They are known

```
LandMark = [  13,9; 6,6; 0,0;  5, -6]';
figure
axis ([-2 2 -8 10])
scatter(LandMark(1,:),LandMark(2,:),200, 'r','filled');
grid on
hold on
```



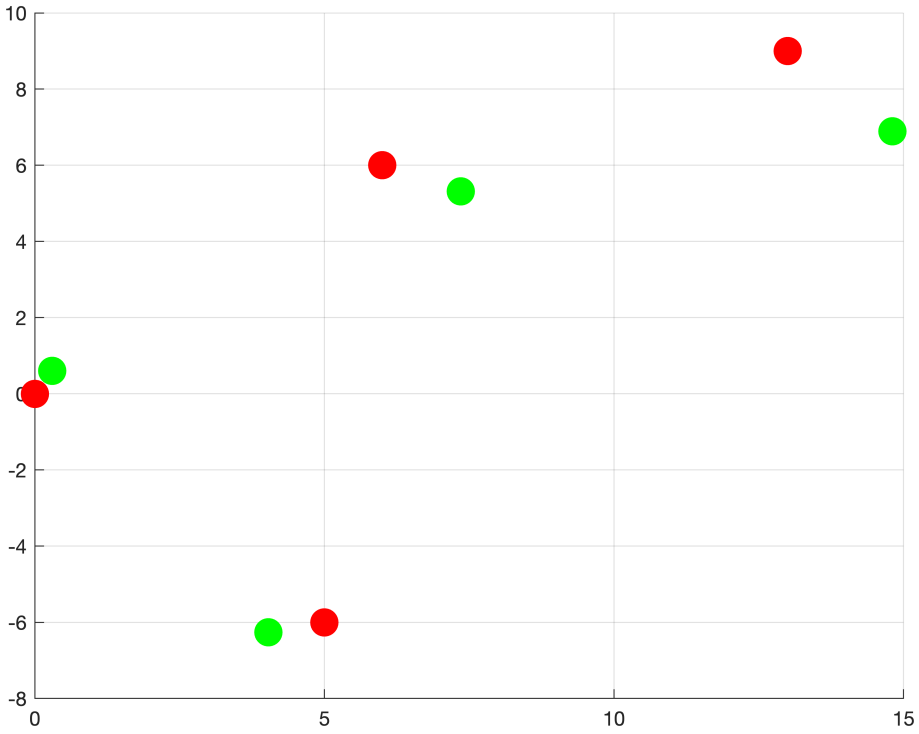
Simulating innovation

Using Postmultiplication: Translate and rotate the Land Marks

```
alpha = pi/16 % Rotate pi/16 rad --> 22.5 degrees
```

```
alpha = 0.1963
```

```
tx = 0.3; % Translate
ty = 0.6;
RotzTxy = [cos(alpha), sin(alpha), tx;...
          -sin(alpha), cos(alpha), ty;...
           0,           0,          1];
Innovation = RotzTxy*[LandMark;ones(1,4)];
hold on;
scatter(Innovation(1,:), Innovation(2,:),200, 'g','filled');
```



Building the Matrices

```
A = [];
for i=1:size(LandMark, 2)
    A = [A;[LandMark(1,i),LandMark(2,i),1,0]];
    A = [A;[LandMark(2,i),-LandMark(1,i),0,1]]
end
```

```
A = 2×4
    13     9     1     0
     9   -13     0     1
A = 4×4
    13     9     1     0
     9   -13     0     1
     6     6     1     0
     6    -6     0     1
A = 6×4
    13     9     1     0
     9   -13     0     1
     6     6     1     0
     6    -6     0     1
     0     0     1     0
     0     0     0     1
A = 8×4
    13     9     1     0
     9   -13     0     1
     6     6     1     0
     6    -6     0     1
     0     0     1     0
     0     0     0     1
```

```
     5    -6     1     0
    -6    -5     0     1
```

```
B = [];%Build Matrix B
for i=1:size(Innovation, 2)
    B = [B; Innovation(1,i); Innovation(2,i)]
end
```

```
B = 2×1
   14.8060
    6.8909
B = 4×1
   14.8060
    6.8909
    7.3553
    5.3142
B = 6×1
   14.8060
    6.8909
    7.3553
    5.3142
    0.3000
    0.6000
B = 8×1
   14.8060
    6.8909
    7.3553
    5.3142
    0.3000
    0.6000
    4.0334
   -6.2602
```

Finding the error in pose

```
X = inv((A'*A))*A'*B;
tx_ST = X(3)
```

```
tx_ST = 0.3000
```

```
ty_ST = X(4)
```

```
ty_ST = 0.6000
```

```
alpha_ST = atan2(X(2),X(1))
```

```
alpha_ST = 0.1963
```

Comparing

```
tx
```

```
tx = 0.3000
```

```
ty
```

```
ty = 0.6000
```

```
alpha
```

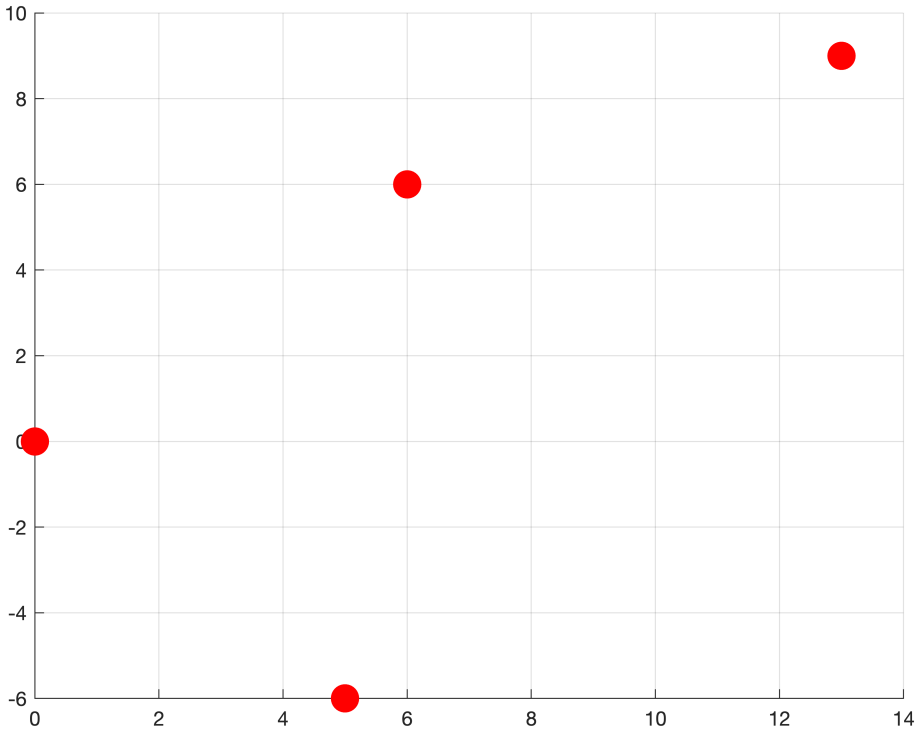
```
alpha = 0.1963
```

Using Laser data

Land Mark

They are known

```
LandMark = [  13,9; 6,6; 0,0; 5,-6]';
figure
axis ([-2 14 -8 10])
scatter(LandMark(1,:),LandMark(2,:),200, 'r','filled');
grid on
```



Robot Pose with uncertanty

```
Robot_pose_u=transl(2,-4,0)*trotz(pi/4)
xyz_u=transl(Robot_pose_u)
RPY_u=tr2rpy(Robot_pose_u)
```

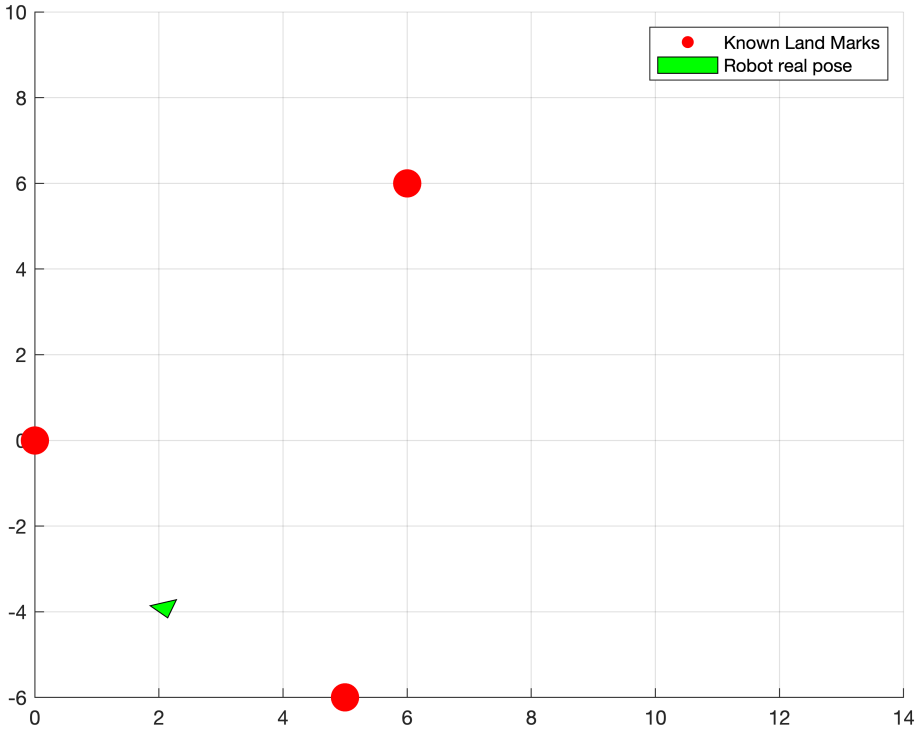
Robot shape

```
Robot.V= [0 -0.2 0 1;0.4 0 0 1;0 0.2 0 1]
```

```
Robot = struct with fields:
    V: [3×4 double]
    F: [1 2 3]
```

```
Robot.F = [1,2,3];
hr=[];
hold on

Robot_tr=Robot_pose_u*Robot.V';% moving the robot
hr=patch(Robot_tr(1,:), Robot_tr(2,:),'g');
legend('Known Land Marks','Robot real pose')
```



Laser measuring Land Marks

$$z = h(x, p_i) = \begin{bmatrix} \sqrt{(y_i - y_v)^2 + (x_i - x_v)^2} \\ \tan^{-1}(y_i - y_v)/(x_i - x_v) - \theta_v \end{bmatrix} + \begin{pmatrix} w_r \\ w_\beta \end{pmatrix}$$

```
z_laser=@(lm_xy,xy_r, th_r)...
    [sqrt((lm_xy(1)-xy_r(1))^2 +(lm_xy(2)-xy_r(2))^2) atan2((lm_xy(2)-xy_r(2)),(lm_xy(
```

```
z_laser = function_handle with value:
    @(lm_xy,xy_r,th_r)[sqrt((lm_xy(1)-xy_r(1))^2+(lm_xy(2)-xy_r(2))^2),atan2((lm_xy(2)-xy_r(2)),(lm_xy(1)-
```

```
z_lm_139=z_laser([13 9],xyz_u,RPY_u(3))
```

```
z_lm_139 = 1×2
   17.0294    0.0831
```

Laser data in Robot Frame

```
for i = 1:4
z_lm(i,:)=z_laser(LandMark(:,i),xyz_u,RPY_u(3));
lxy_rf_u(:,i)=[z_lm(i,1)*cos(z_lm(i,2)) z_lm(i,1)*sin(z_lm(i,2))];
end
```

Estimated Robot Pose (no noise)

Remember laser pose with noise [2 -4 pi/4]

```
Robot_pose=transl(4,-2,0)*trotz(pi/4)
```

```
Robot_pose = 4×4
    0.7071   -0.7071         0    4.0000
    0.7071    0.7071         0   -2.0000
         0         0    1.0000         0
         0         0         0    1.0000
```

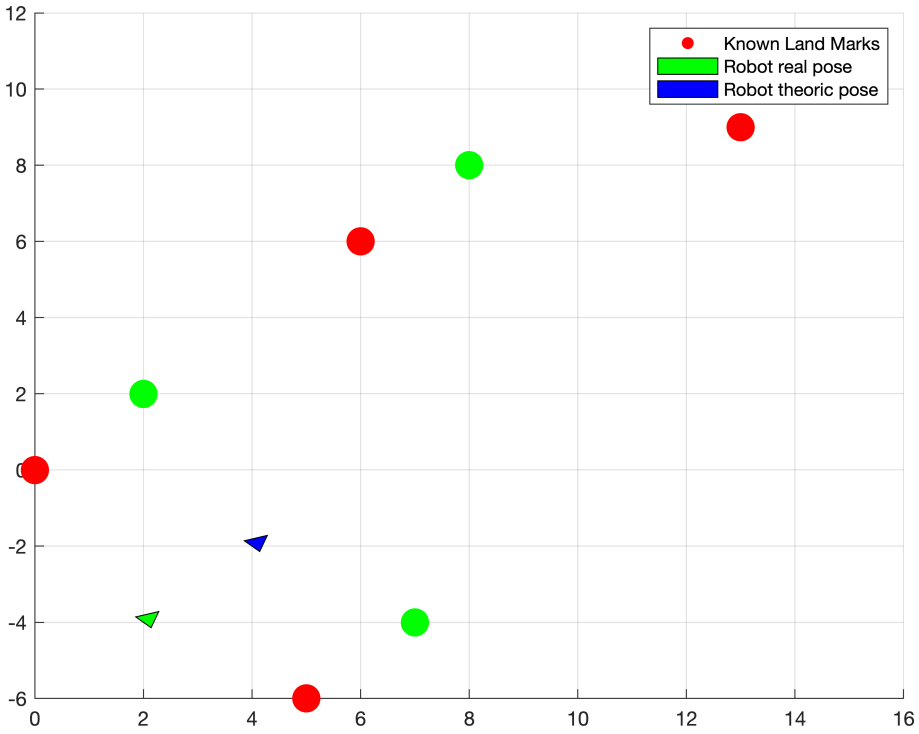
```
xyz=transl(Robot_pose)
```

```
xyz = 3×1
     4
    -2
     0
```

```
RPY=tr2rpy(Robot_pose)
```

```
RPY = 1×3
         0         0    0.7854
```

```
Robot_tr=Robot_pose*Robot.V';% moving the robot
hr=patch(Robot_tr(1,:), Robot_tr(2,:),'b');
legend('Known Land Marks','Robot real pose','Robot theoric pose')
```

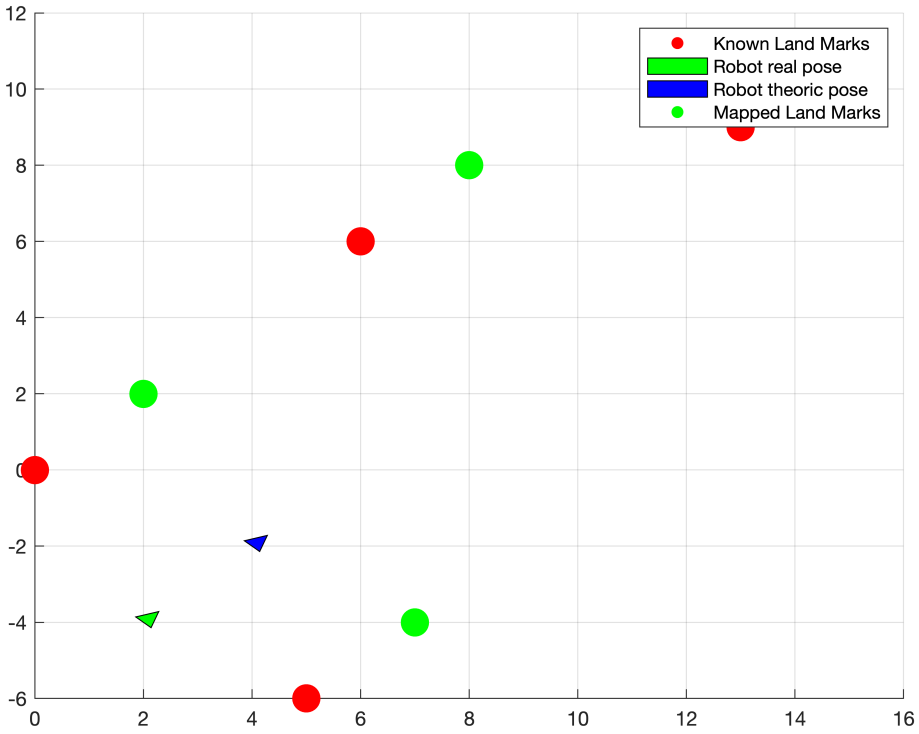


Mapped Land Mark

```
hold on
LandMark_u=Robot_pose*[lxy_rf_u; 0 0 0 0; 1 1 1 1]
```

```
LandMark_u = 4×4
   15.0000    8.0000    2.0000    7.0000
   11.0000    8.0000    2.0000   -4.0000
         0         0         0         0
    1.0000    1.0000    1.0000    1.0000
```

```
scatter(LandMark_u(1,:),LandMark_u(2,:),200, 'g','filled');
legend('Known Land Marks','Robot real pose','Robot theoric pose', 'Mapped Land Marks')
```



Repeating the example

```
B = [];%Build Matrix B
for i=1:size(LandMark_u, 2)
    B = [B; LandMark_u(1,i); LandMark_u(2,i)]
end
```

```
B = 2×1
   15.0000
   11.0000
B = 4×1
   15.0000
   11.0000
    8.0000
    8.0000
B = 6×1
   15.0000
   11.0000
    8.0000
    8.0000
    2.0000
    2.0000
B = 8×1
   15.0000
   11.0000
    8.0000
```

```
     8.0000
     2.0000
     2.0000
     7.0000
    -4.0000
```

```

```

Finding the error in pose

```
X = inv((A'*A))*A'*B;
tx_ST = X(3)
```

```
tx_ST = 2.0000
```

```
ty_ST = X(4)
```

```
ty_ST = 2.0000
```

```
alpha_ST = atan2(X(2),X(1))
```

```
alpha_ST = 2.1511e-16
```