Taylor Series

Taylor Series Expansion

Taylor series expansion represents an analytic function $f(x)$ as an infinite sum of terms around the expansion point $x = a$:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots = \sum_{m=0}^{\infty} \frac{f^{(m)}(a)}{m!} \cdot (x-a)^m$$

Taylor series expansion requires a function to have derivatives up to an infinite order around the expansion point.

Maclaurin Series Expansion

Taylor series expansion around x = 0 is called Maclaurin series expansion:

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots = \sum_{m=0}^{\infty} \frac{f^{(m)}(0)}{m!} x^m$$

Taylor expansion around a point

```
syms x
T = taylor(sin(x), x, 'ExpansionPoint', pi/4)
```

```
T =
```

$$\frac{\sqrt{2}\left(x-\frac{\pi}{4}\right)}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\left(x-\frac{\pi}{4}\right)^2}{4} - \frac{\sqrt{2}\left(x-\frac{\pi}{4}\right)^3}{12} + \frac{\sqrt{2}\left(x-\frac{\pi}{4}\right)^4}{48} + \frac{\sqrt{2}\left(x-\frac{\pi}{4}\right)^5}{240}$$

```
x=[0:0.1:pi]
```

```
x = 1×32
         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000 ...
```

```
fx_5=sqrt(2)*0.5*...
               (+1 ...
               +(x-(pi/4)) ...
               -0.5*(x-(pi/4)).^2 ...
               -(1/6)*(x-(pi/4)).^3 ...
               +(1/24)*(x-(pi/4)).^4 ...
               +(1/120)*(x-(pi/4)).^5)
```

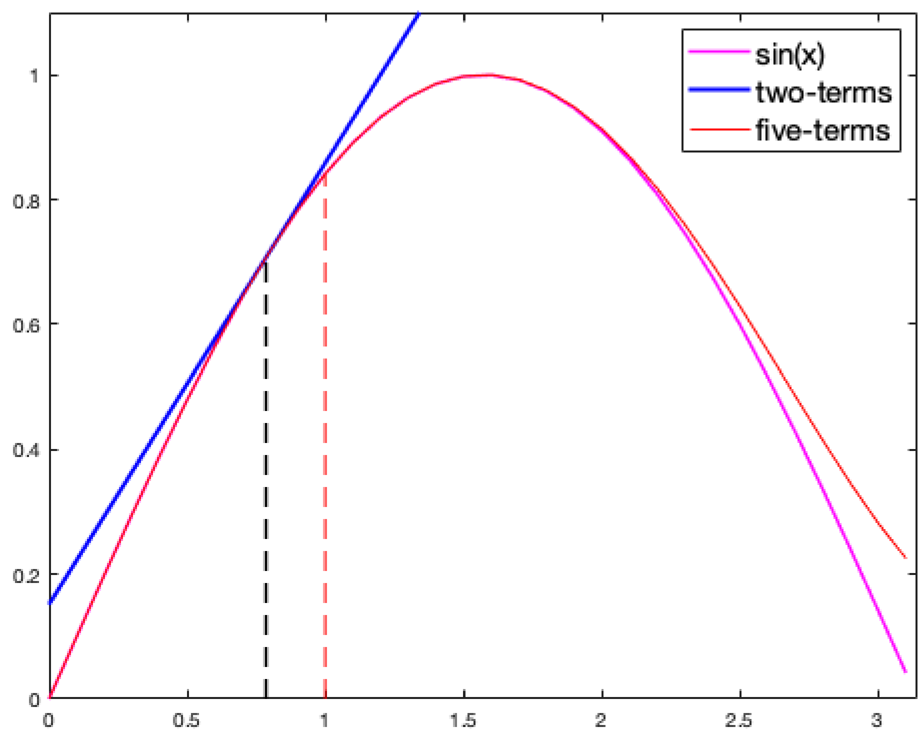
```
fx_5 = 1×32
    0.0002    0.0999    0.1987    0.2955    0.3894    0.4794    0.5646    0.6442 ...
```

```
fx_2=sqrt(2)*0.5*(+1+(x-(pi/4)))
```

```
fx_2 = 1×32
    0.1517    0.2225    0.2932    0.3639    0.4346    0.5053    0.5760    0.6467 ...
```

Plotting

```
figure
plot(x, sin(x), 'm','LineWidth',1.5)
hold on
plot(x,fx_2,'b','LineWidth',2)
plot(x,fx_5,'r')
line ([pi/4 pi/4],[0 sin(pi/4)],'Color','black','LineStyle','--','lineWidth',1.5)
line ([1 1],[0 sin(1)],'Color','red','LineStyle','--')
axis([0 pi 0 1.1])
legend ('sin(x)', 'two-terms','five-terms','Fontsize',15)
```



Comparing result

```
Value_No_Taylor= sin(1)
```

```
Value_No_Taylor = 0.8415
```

```
Value_Si_Taylor= sin(pi/4)+cos(pi/4)*(1-pi/4)
```

```
Value_Si_Taylor = 0.8589
```

```
Value_Matlab=sqrt(2)*0.5*(1+(1-(pi/4)))
```

```
Value_Matlab = 0.8589
```

Taylor expansion around '0'

```
syms x
T = taylor(sin(x), x, 'ExpansionPoint', 0)
```

T =

$$\frac{x^5}{120} - \frac{x^3}{6} + x$$

```
x=[0:0.1:pi]
```

x = 1×32

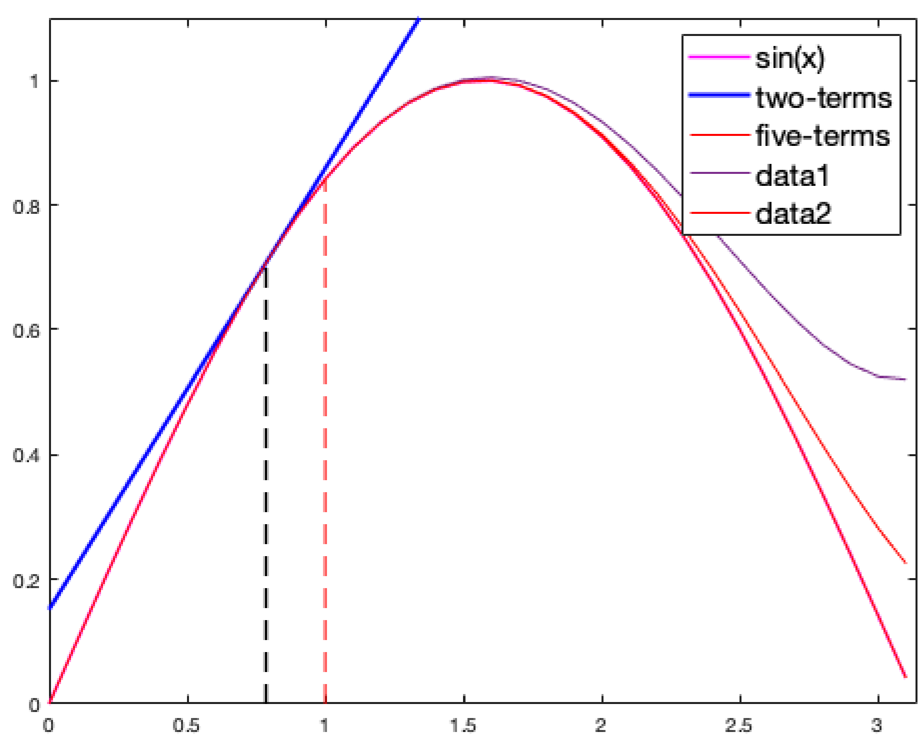
0 0.1000 0.2000 0.3000 0.4000 0.5000 0.6000 0.7000 ⋯

```
fx=x-(x.^3)/6+(x.^5)/120
```

fx = 1×32

0 0.0998 0.1987 0.2955 0.3894 0.4794 0.5646 0.6442 ⋯

```
plot(x,fx)
hold on
plot(x, sin(x), 'r')
```




```
syms x
T = taylor(sin(x), x, 'ExpansionPoint', pi/4)
```

T =

$$\frac{\sqrt{2}\,\left(x-\frac{\pi}{4}\right)}{2}+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\,\left(x-\frac{\pi}{4}\right)^2}{4}-\frac{\sqrt{2}\,\left(x-\frac{\pi}{4}\right)^3}{12}+\frac{\sqrt{2}\,\left(x-\frac{\pi}{4}\right)^4}{48}+\frac{\sqrt{2}\,\left(x-\frac{\pi}{4}\right)^5}{240}$$

```
x=[0:0.1:pi]
```

```
x = 1×32
         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000 ⋯
```

```
fx=sqrt(2)*0.5*...
               (+1 ...
               +(x-(pi/4)) ...
               -0.5*(x-(pi/4)).^2 ...
               -(1/6)*(x-(pi/4)).^3 ...
               +(1/24)*(x-(pi/4)).^4 ...
               +(1/120)*(x-(pi/4)).^5)
```

```
fx = 1×32
    0.0002    0.0999    0.1987    0.2955    0.3894    0.4794    0.5646    0.6442 ⋯
```

```
figure
plot(x,fx,'b')
hold on
plot(x, sin(x), 'r')
```