3R Manipulator: Invers Kinematics

This function is not functional, it is just to show ideas

Table of Contents


```
clear
```

Use a 3R from RTB model's

```
mdl_3link3d
joint_pose=[pi/6 pi/4 -pi/5]
```

```
joint_pose = 1×3
    0.5236    0.7854   -0.6283
```

```
R3.plot(joint_pose)
```

FK, given $\theta's$ get pose

Note Pose is a SE3 object

```
Pose=R3.fkine(joint_pose)
```

```
Pose =
    0.8554   -0.1355    0.5000     3.791
    0.4938   -0.0782   -0.8660     2.189
    0.1564    0.9877         0     2.884
         0         0         0         1
```

Homogeneous transformation

```
HT=Pose.T
```

```
HT = 4×4
    0.8554   -0.1355    0.5000    3.7908
    0.4938   -0.0782   -0.8660    2.1886
    0.1564    0.9877    0.0000    2.8835
         0         0         0    1.0000
```

Translational part, i.e. position

```
Position=Pose.t
```

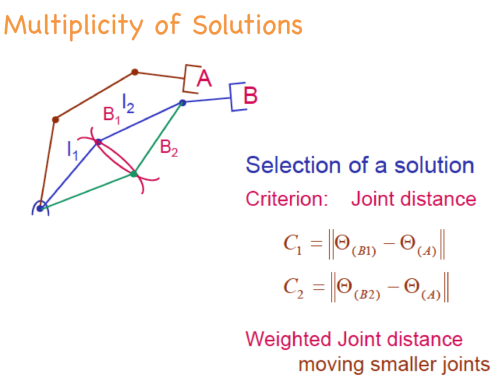
```
Position = 3×1
    3.7908
    2.1886
    2.8835
```

Orientation/Rotation matrix

```
Rotation=Pose.R
```

```
Rotation = 3×3
    0.8554   -0.1355    0.5000
    0.4938   -0.0782   -0.8660
    0.1564    0.9877    0.0000
```

IK, given pose get $\theta' s$



We impose the desired solution by telling to the function 'q0'

Elbown up solution

```
qq_up=R3.ikine(Pose,'mask',[1 1 1 0 0 0],'q0',[ 0 pi/2 0])
```

```
qq_up = 1×3
    0.5236    0.7854   -0.6283
```

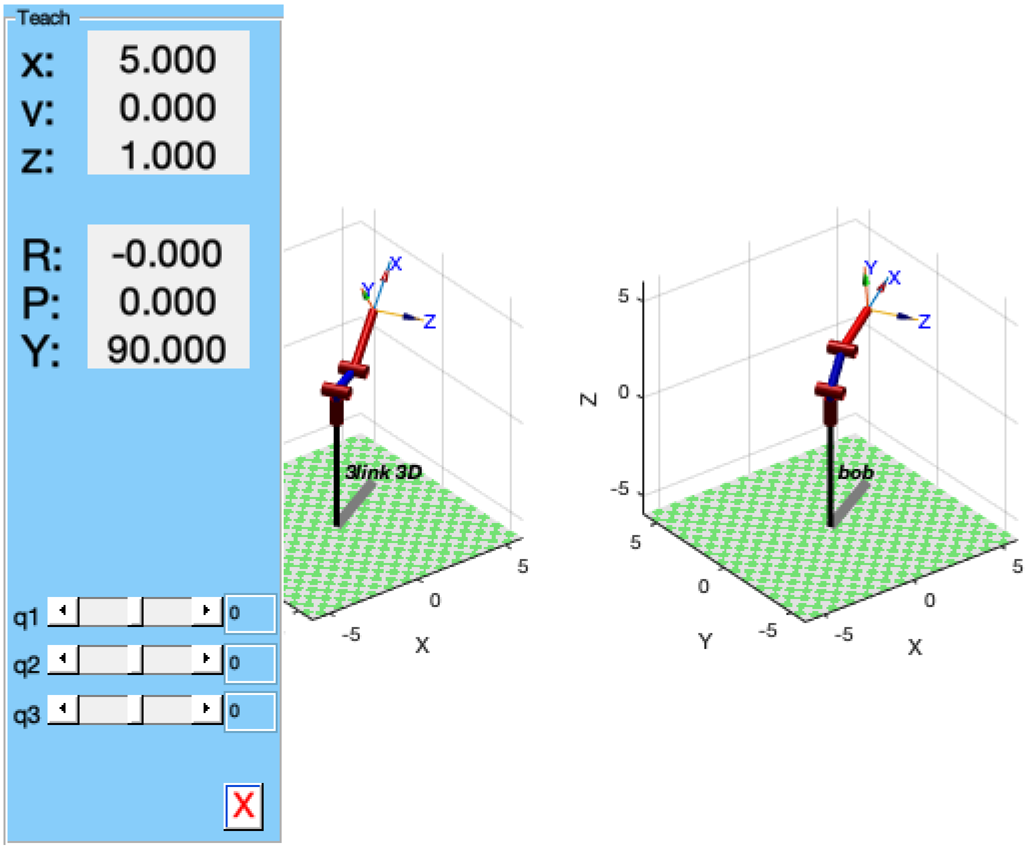
Elbown down solution

```
qq_down= R3.ikine(Pose,'mask',[1 1 1 0 0 0],'q0',[ 0 0 0])
```

```
qq_down = 1×3
    0.5236    0.0273    0.6283
```

Plotting elbown up/down solution

```
bob= SerialLink(R3, 'name', 'bob'); % clone R3
subplot (121)
R3.plot(qq_down)
subplot (122)
bob.plot(qq_up)
```



Ckecking position

Notice: position are the same, Orientation differs

```
Pose_down=R3.fkine(qq_down)
```

```
Pose_down =
    0.6865   -0.5280    0.5000     3.791
    0.3963   -0.3048   -0.8660     2.189
    0.6096    0.7927         0     2.884
         0         0         0         1
```

```
Pose
```

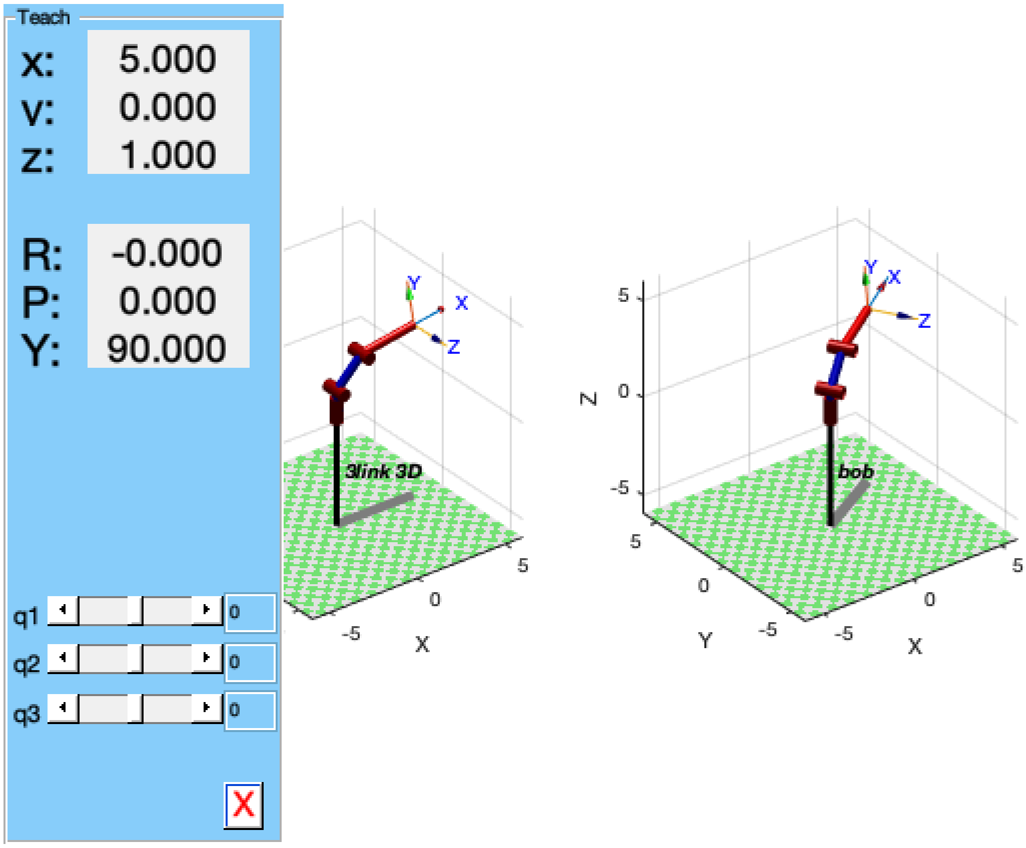
```
Pose =
    0.8554   -0.1355    0.5000     3.791
    0.4938   -0.0782   -0.8660     2.189
    0.1564    0.9877         0     2.884
         0         0         0         1
```

Ikine key idea: do it in RF {1}

Notice coordinate 'y' of the {EE} = 0 and pose of the {EE} is wrt Reference frame {1}

We will work in the X/Z plane

```
R3.plot(joint_pose -[qq_up(1) 0 0],'zoom',1.5)
```



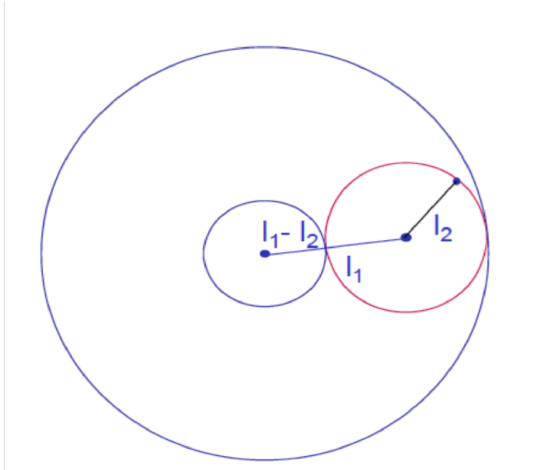

```
Pose_1=R3.fkine(joint_pose -[qq_up(1) 0 0])
```

```
Pose_1 =
    0.9877   -0.1564         0     4.377
         0         0        -1         0
    0.1564    0.9877         0     2.884
         0         0         0         1
```

Implement 'ikine' function and compare with RTB

Take into consideration what we mention in Theory

WorkSpace Solutions



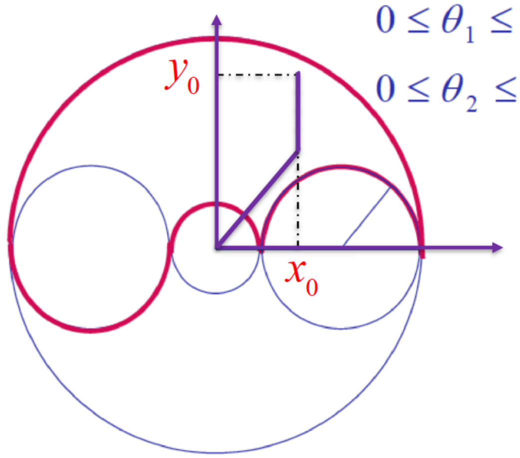
Solution if :

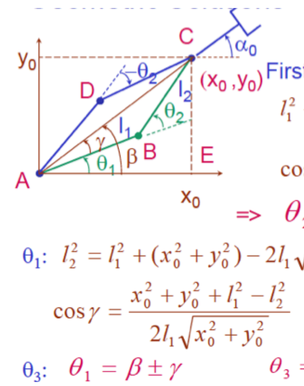
$$(l_1 - l_2)^2 \le (x_0^2 + y_0^2) \le (l_1 + l_2)^2$$

Joint Limits

$$0 \le \theta_1 \le 180^o$$

$$0 \le \theta_2 \le 180^o$$






```
Q_down=inv_3R(Pose_down,0) % the implemented ikine
```

```
L122 = 22.7082
L1L2 = 25
L1_L2 = 1
Pose Recheable
Q_down = 1×3
    0.5236    0.0273    0.6283
```

```
qq_down= R3.ikine(Pose,'mask',[1 1 1 0 0 0],'q0',[ 0 0 0]) % Using RTB
```

```
qq_down = 1×3
    0.5236    0.0273    0.6283
```

```
Q_up=inv_3R(Pose_down,1) % the implemented ikine
```

```
L122 = 22.7082
L1L2 = 25
L1_L2 = 1
Pose Recheable
Q_up = 1×3
    0.5236    0.7854   -0.6283
```

```
qq_up=R3.ikine(Pose,'mask',[1 1 1 0 0 0],'q0',[ 0 pi/2 0]) % Using RTB
```

```
qq_up = 1×3
    0.5236    0.7854   -0.6283
```

```
NoReach=SE3([7 2 3])
```

```
NoReach =
         1         0         0         7
         0         1         0         2
         0         0         1         3
         0         0         0         1
```

```
Q_up=inv_3R(NoReach,1)
```

```
L122 = 57
L1L2 = 25
L1_L2 = 1
Pose no Recheable
Q_up = NaN
```

Inverse Kinematics function

```
function q=inv_3R(pose,ud)
% if ud=1 -> elbown up; ud=0 -> elbown down
% There will be two solutions:
% 1.- [q1  (q2=beta+gamma) -q3]; elbow up
% 2.- [q1  (q2=beta-gamma) +q3]; elbow down
% Robot DH Parameters
mdl_3link3d;
L0=R3.links(1, 1).d;
L1=R3.links(1, 2).a;
L2=R3.links(1, 3).a;
xyz=[pose.t;1];
% q1 is a solution, then q(1) + pi is also a solution
theta_1=atan2(xyz(2), xyz(1));
% Express xyz in the reference frame 1, notice coordinates y = 0
T_1_0=transl(0,0,L0)*trotz(theta_1);
xyz=inv(T_1_0)*[pose.t;1];
% Now it is a twolink InversKinematics
% Recover: Invers_Kinematics_Key_ideas.mlx
L122=(norm(xyz(1:3)))^2
L1L2=(L1+L2)^2
L1_L2=(L1-L2)^2
theta_3=acos(((xyz(1)^2+xyz(3)^2)-(L1^2+L2^2))/(2*L1*L2));
beta=atan2(xyz(3),xyz(1));
gamma=acos((xyz(1)^2+xyz(3)^2+L1^2-L2^2)/(2*L1*sqrt((xyz(1)^2+xyz(3)^2))));
theta_2_1=beta+gamma;
theta_2_2=beta-gamma;
q_aux=zeros(1,3);

if (L122 >= L1_L2) && (L122 <= L1L2)
   disp('Pose Recheable')
  if ud==1
     q_aux=[theta_1 theta_2_1 -theta_3];
  else
  if ud==0
```

```
      q_aux=[theta_1 theta_2_2 theta_3];
   end
   end
 end
 if (L122>L1L2)||(L122<L1_L2)
     disp('Pose no Recheable')
     q_aux=nan;
 end
 q=q_aux;
 end
```