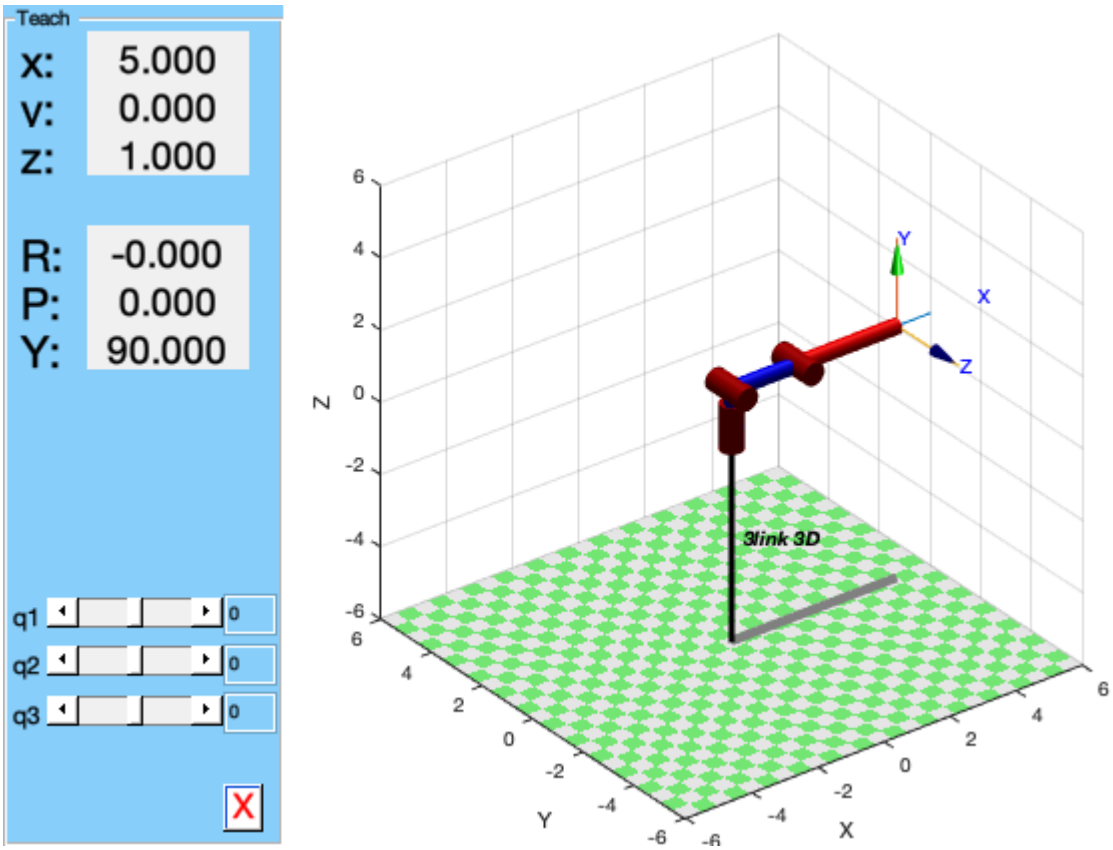
Solving 3R Forward kinematics

Reviewing 3R robot

```
clear
mdl_3link3d
R3
```

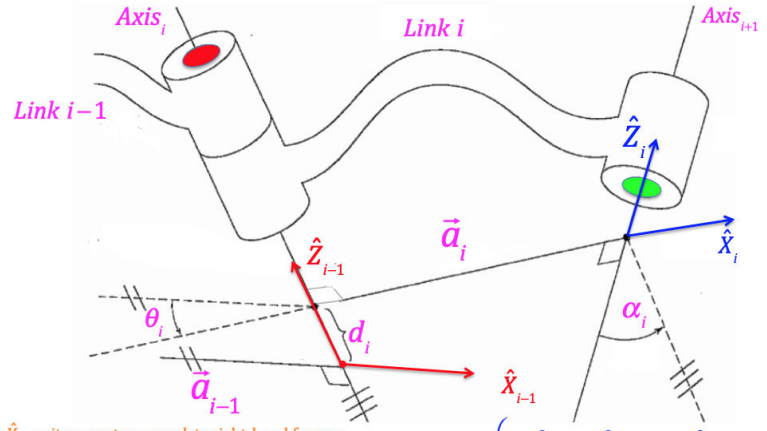
```
R3 =

3link 3D:: 3 axis, RRR, stdDH, fastRNE
 - Spong p106;
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|          1|          0|     1.5708|          0|
|  2|         q2|          0|          2|          0|          0|
|  3|         q3|          0|          3|          0|          0|
+---+-----------+-----------+-----------+-----------+-----------+
```

```
R3.plot([0 0 0])
R3.teach
```



Forward Kinematics DHP-Std by hand

Fordward kinematics: Link-DHP-Std



$\hat{Y}-unitary\ vector: complete$ right-hand frames

$$ {}^{i-1}_{i}T(\theta_i,d_i,a_i,\alpha_i) = R_Z(\theta_i)D_Z(d_i)D_X(a_i)R_X(\alpha_i) = \begin{pmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & \alpha_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

```
syms theta_1 theta_2 theta_3 L_0 L_1 L_2 real
FK_hand= simplify(transl(0,0,L_0)*trotz(theta_1)*trotx(pi/2)*trotz(theta_2)*transl(L_1
```

FK_hand =

$$ \begin{pmatrix} \cos(\theta_2+\theta_3)\cos(\theta_1) & -\sin(\theta_2+\theta_3)\cos(\theta_1) & \sin(\theta_1) & \cos(\theta_1)\,\sigma_1 \\ \cos(\theta_2+\theta_3)\sin(\theta_1) & -\sin(\theta_2+\theta_3)\sin(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)\,\sigma_1 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & L_0+L_2\sin(\theta_2+\theta_3)+L_1\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

where

$$ \sigma_1 = L_2\cos(\theta_2+\theta_3)+L_1\cos(\theta_2) $$

```
syms theta_1 theta_2 theta_3 L_0 L_1 L_2 real
FK_hand= simplify(trotz(theta_1)*transl(0,0,L_0)*trotx(pi/2)*trotz(theta_2)*transl(L_1
```

FK_hand =

$$ \begin{pmatrix} \cos(\theta_2+\theta_3)\cos(\theta_1) & -\sin(\theta_2+\theta_3)\cos(\theta_1) & \sin(\theta_1) & \cos(\theta_1)\,\sigma_1 \\ \cos(\theta_2+\theta_3)\sin(\theta_1) & -\sin(\theta_2+\theta_3)\sin(\theta_1) & -\cos(\theta_1) & \sin(\theta_1)\,\sigma_1 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & L_0+L_2\sin(\theta_2+\theta_3)+L_1\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

where

$$ \sigma_1 = L_2\cos(\theta_2+\theta_3)+L_1\cos(\theta_2) $$

Numerical example

```
FK_3R=R3.fkine([pi/6 0 0 ])
```

```
FK_3R =
    0.8660         0    0.5000      4.33
    0.5000         0   -0.8660       2.5
         0         1         0          1
         0         0         0          1
```

```
theta_1=atan2(FK_3R.t(2),FK_3R.t(1))
```

```
theta_1 = 0.5236
```

```
pi/6
```

```
ans = 0.5236
```

```
trotz(-theta_1)*FK_3R.T
```

```
ans = 4×4
    1.0000   -0.0000    0.0000    5.0000
    0.0000    0.0000   -1.0000    0.0000
         0    1.0000    0.0000    1.0000
         0         0         0    1.0000
```

Notice that

```
syms theta_1 theta_2 theta_3 L_1 L_2 real
FK= simplify(trotz(0)*trotx(pi/2)*trotz(theta_2)*transl(L_1,0,0)*trotz(theta_3)*transl
```

```
FK =
```

$$\begin{pmatrix} \cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) & 0 & L_2\cos(\theta_2+\theta_3)+L_1\cos(\theta_2) \\ 0 & 0 & -1 & 0 \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & L_2\sin(\theta_2+\theta_3)+L_1\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$