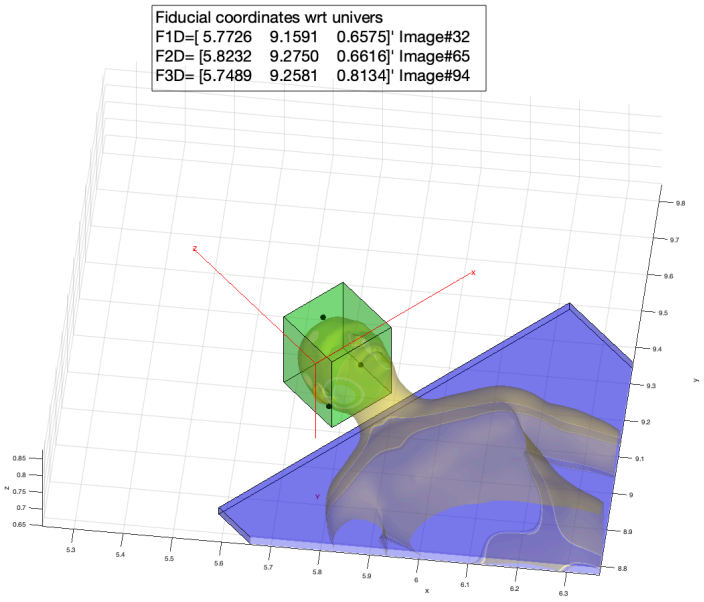
Second approach

Table of Contents

Given enviroment

Given the table with the patient in an arbitrary pose



Open the given figure

Double click on it or select and right click/evaluate

```
clear
open('3_Second_approach_Patient_pose.fig')
hold on
```

Robot Frame wrt {U}

The Robot Rosa is located near by the patient

Locating the Robot

```
mdl_puma560;
p560.base=transl(5.75,9.75,0)*trotz(-pi/4)
p560.plot(qr,'zoom',1.5)
axis([5 6.5 8.8 10 0 1.8])
T_R_U=p560.base.T
```

Fiducial wrt {U}

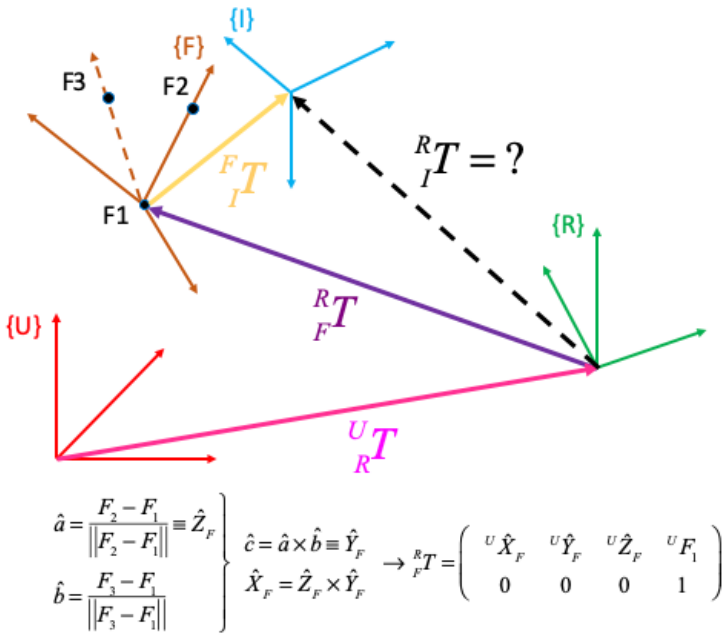
To emulate the action of measuring the fiducial wrt Robot Frame I gave you the fiducials coordinate wrt {U}.

```
F123=open ('Fiducial_U.mat');
F1= F123.F1;
F2= F123.F2;
F3= F123.F3;
```

Transform compound

We will need the following Reference Frames: {U}, {R}, {F}, {I}

and the Following Transformation: ${}^{U}_{R}T, {}^{R}_{F}T, {}^{F}_{I}T and {}^{R}_{I}T$



$$\left.\begin{array}{l}\hat{a}=\dfrac{F_2-F_1}{\|F_2-F_1\|}\equiv\hat{Z}_F \\ \hat{b}=\dfrac{F_3-F_1}{\|F_3-F_1\|}\end{array}\right\}\ \begin{array}{l}\hat{c}=\hat{a}\times\hat{b}\equiv\hat{Y}_F \\ \hat{X}_F=\hat{Z}_F\times\hat{Y}_F\end{array}\ \rightarrow {}^{R}_{F}T=\begin{pmatrix} {}^{U}\hat{X}_F & {}^{U}\hat{Y}_F & {}^{U}\hat{Z}_F & {}^{U}F_1 \\ 0 & 0 & 0 & 1\end{pmatrix}$$

It is clear that we will need the transformation Image to Robot, i.e. ${}^{R}_{I}T$ to correctly make the tumor surgery.

Remeber the doctor, based on the Dicom image locate the Tumor (i.e Reference Image {I}) .

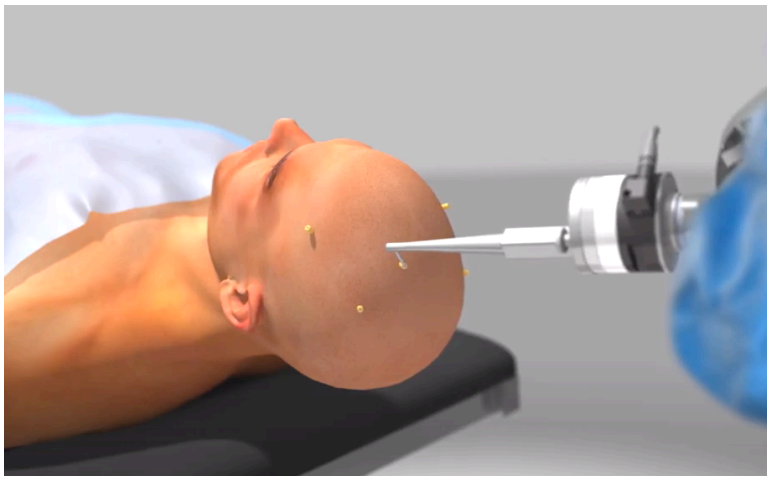
T_F_R Frame Description

Pay attention: and touch the fiducial to locate them wrt its own Reference Frame {R}.

You need to place the Robot Puma in RF {U} and infer the Fiducial locations wrt {R}

Fiducial in RF Robot {R}

Notice: You have fiducial in RF {U}={A } and you want it in RF{R}={B} ... So ${}^{B}P = {}^{A}_{B}T^{-1}{}^{A}P$



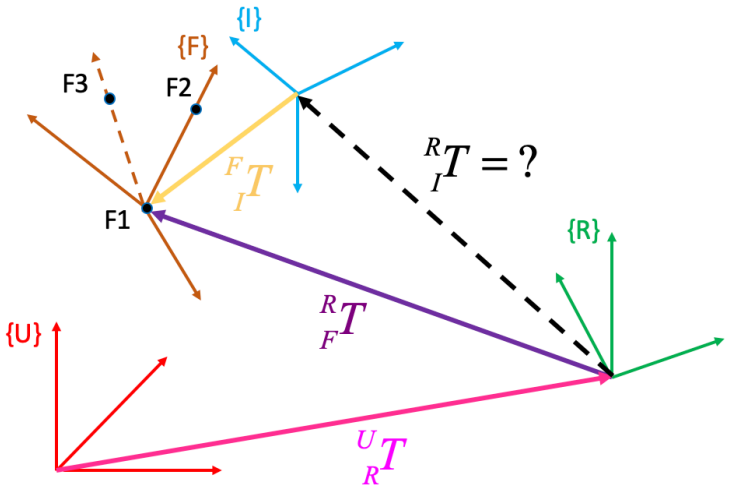
```
Fi_R=inv(T_R_U)*[F1 F2 F3;ones(1,3)]
```

```
Fi_R = 4×3
    0.4338    0.3876    0.3470
   -0.4018   -0.2841   -0.3486
    0.6575    0.6616    0.8134
    1.0000    1.0000    1.0000
```

RF Fiducial {F} wrt RF {R}

It is needed to use an auxiliary Reference Frame formed by the three fiducials becouse they are measured by the Robot using the teach Tool (See above section) and the Computed Tomography (Dicom images).

We will use the fiducials in RF {R}



$$\left.\begin{aligned}\hat{a}&=\frac{F_2-F_1}{\|F_2-F_1\|}\equiv\hat{Z}_F\\ \hat{b}&=\frac{F_3-F_1}{\|F_3-F_1\|}\end{aligned}\right\}\quad \hat{c}=\hat{a}\times\hat{b}\equiv\hat{Y}_F \quad \hat{X}_F=\hat{Z}_F\times\hat{Y}_F \quad \rightarrow {}^{R}_{F}T=\begin{pmatrix} {}^{U}\hat{X}_F & {}^{U}\hat{Y}_F & {}^{U}\hat{Z}_F & {}^{U}F_1\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Orientation wrt {R}

```
Yf = (Fi_R(1:3,2)-Fi_R(1:3,1))/norm(Fi_R(1:3,2)-Fi_R(1:3,1));
b=(Fi_R(1:3,3)-Fi_R(1:3,1))/norm(Fi_R(1:3,3)-Fi_R(1:3,1));
```

```
Zf = cross(Yf,b)/norm(cross(Yf,b));
Xf = cross(Yf,Zf)/norm(cross(Yf,Zf));
```

Frame Description

${}^{R}_{F}T$ take the Fiducial 1 as ${}^{A}P_{BORG}$ in our case A is RF{R}

```
T_F_R = [[Xf;0] [Yf;0] [Zf;0] [Fi_R(:,1)]]
```

```
T_F_R = 4×4
    0.3353   -0.3652    0.8684    0.4338
    0.1638    0.9304    0.3280   -0.4018
   -0.9278    0.0323    0.3718    0.6575
         0         0         0    1.0000
```

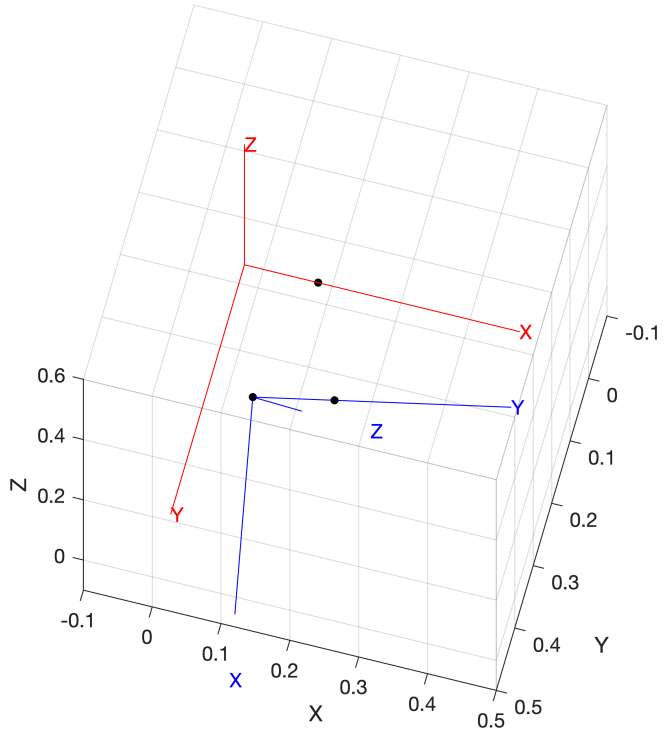
RF Fiducial {F} wrt RF {U}

The answer to this question is ${}^{U}_{F}T = {}^{U}_{R}T\,{}^{R}_{F}T$

```
T_F_U=T_R_U*T_F_R
```

```
T_F_U = 4×4
    0.3529    0.3996    0.8460    5.7726
   -0.1213    0.9161   -0.3822    9.1591
   -0.9278    0.0323    0.3718    0.6575
         0         0         0    1.0000
```

```
trplot(T_F_U,'Frame','F','color', 'b','length',0.4)
```



Check T_F_U

Going from {U} to {F}

Notice that the origen ${}^{U}P_{F1ORG}$ is coincident with the coordinate of fiducial 1 F1 in RF {U}

```
RPY=tr2rpy(T_F_R,'zyx')
```

```
RPY = 1×3
    0.0865    1.1884    0.4544
```

```
T_F1_U=T_R_U*transl(Fi_R(1:3,1))*trotz(RPY(3))*troty(RPY(2))*trotx(RPY(1))
```

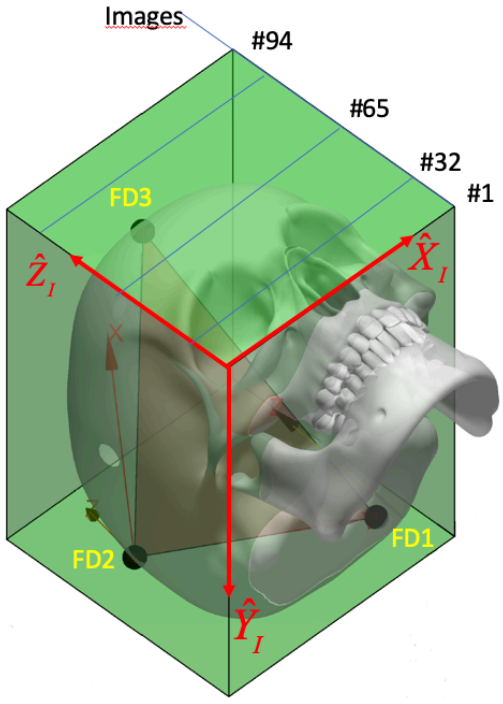
```
T_F1_U = 4×4
    0.3529    0.3996    0.8460    5.7726
   -0.1213    0.9161   -0.3822    9.1591
   -0.9278    0.0323    0.3718    0.6575
         0         0         0    1.0000
```

```
T_F_U
```

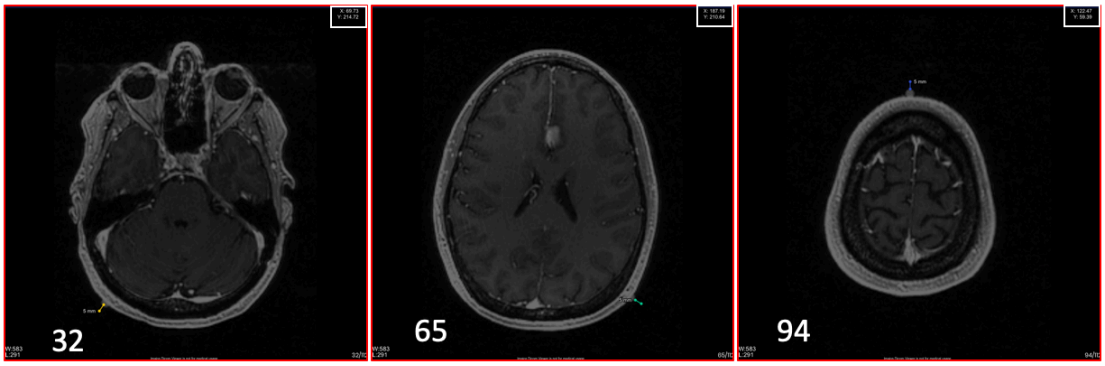
```
T_F_U = 4×4
    0.3529    0.3996    0.8460    5.7726
   -0.1213    0.9161   -0.3822    9.1591
   -0.9278    0.0323    0.3718    0.6575
         0         0         0    1.0000
```

Fiducials wrt {I}

From Dicom Images



Taking data from Dicom images



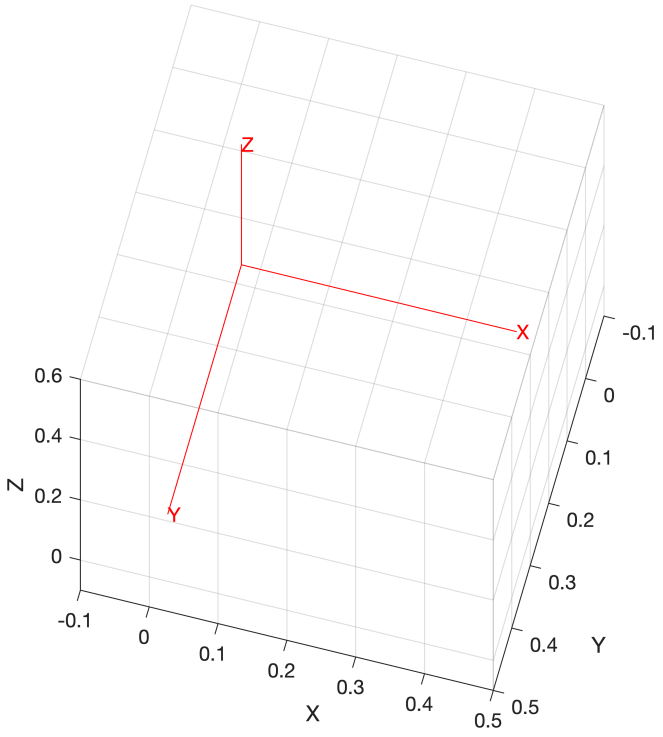

```
pitch =1.4; % Pitch among slices
F1D = [0.06973 0.21496 0.032*pitch]'; %image #32
F2D = [0.18743 0.21088 0.065*pitch]'; %image #65
F3D = [0.12295 0.05915 0.094*pitch]'; %image #94
```

Visualizing fiducials

Image Frame

Plotting Image Reference Frame at origen of Reference Frame Univers {U}

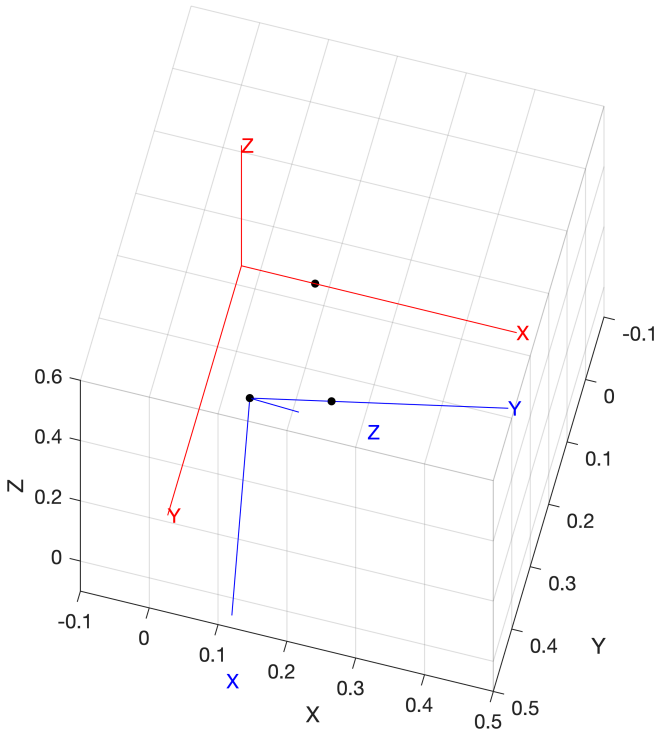
```
figure
trplot(eye(3),'Frame','I','color', 'r','length',0.4)
hold on
axis([-0.1 0.5 -0.1 0.5 -0.1 0.6])
view (-15,-65) % For better understanding
```



Fiducials in {I}

We use a sphere to represent a fiducial

```
[X,Y,Z] = sphere;
r = 0.005; % radius of the sphere
X2 = X * r;
Y2 = Y * r;
Z2 = Z * r;
surf(X2+F1D(1),Y2+F1D(2),Z2+F1D(3),'FaceColor',[0 1 0])
surf(X2+F2D(1),Y2+F2D(2),Z2+F2D(3),'FaceColor',[0 1 0])
surf(X2+F3D(1),Y2+F3D(2),Z2+F3D(3),'FaceColor',[0 1 0])
```



T_F_I - Frame Description

As we mention before It is needed to use the Fiducial as auxiliary Reference Frame

Extracted from the triangle F1D - F2D - F3D

Same procedure as before

Orientation

```
YfD = (F2D-F1D)/norm(F2D-F1D);
bD = (F3D-F1D)/norm(F3D-F1D);
ZfD = cross(YfD,bD)/norm(cross(YfD,bD));
XfD = cross(YfD,ZfD)/norm(cross(YfD,ZfD));
```

Frame Description

${}^{I}_{F}T$ take the Fiducial 1 as ${}^{A}P_{BORG}$ in our case A is RF{I}

```
T_F_I = [[XfD;0] [YfD;0] [ZfD;0] [F1D;1]]
```

```
T_F_I = 4×4
    0.1638    0.9304    0.3280    0.0697
    0.9278   -0.0323   -0.3718    0.2150
   -0.3353    0.3652   -0.8684    0.0448
         0         0         0    1.0000
```

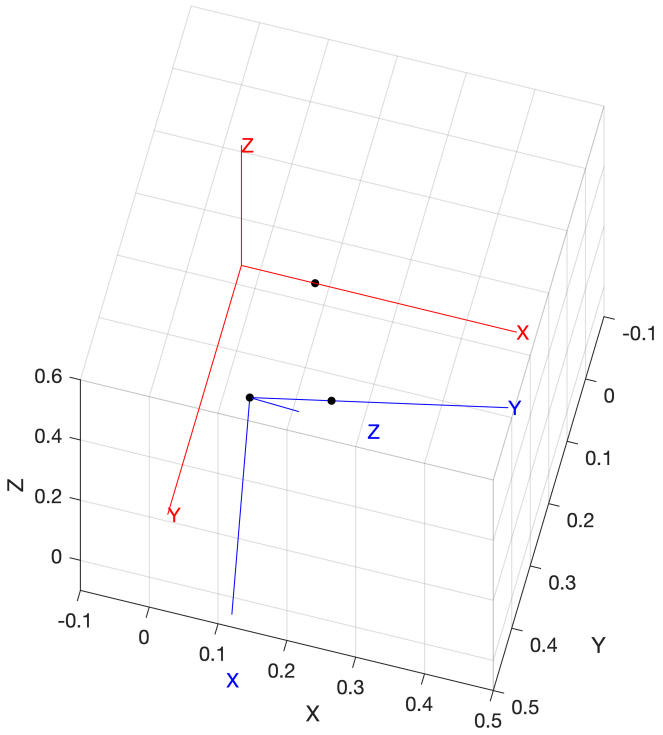
```
det(T_F_I)
```

```
ans = 1
```

Visualizing T_F_I

Fiducial Reference Frame wrt Reference frame {I} at origen of {U}

```
trplot(T_F_I,'Frame','F','color', 'b','length',0.4)
```



Check T_F_I as Going from {I} to {F}

```
RPY=tr2rpy(T_F_I,'zyx')
```

```
RPY = 1×3
```

```
    2.7435    0.3419    1.3961
```

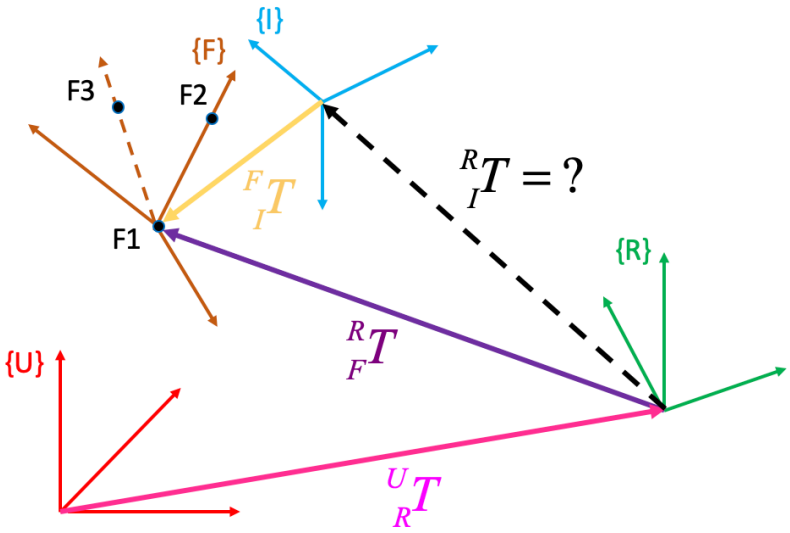
```
T_F_U2=transl(F1D)*trotz(RPY(3))*troty(RPY(2))*trotx(RPY(1))
```

```
T_F_U2 = 4×4
    0.1638    0.9304    0.3280    0.0697
    0.9278   -0.0323   -0.3718    0.2150
   -0.3353    0.3652   -0.8684    0.0448
         0         0         0    1.0000
```

```
T_F_I
```

```
T_F_I = 4×4
    0.1638    0.9304    0.3280    0.0697
    0.9278   -0.0323   -0.3718    0.2150
   -0.3353    0.3652   -0.8684    0.0448
         0         0         0    1.0000
```

RF Image {I} wrt RF {U}



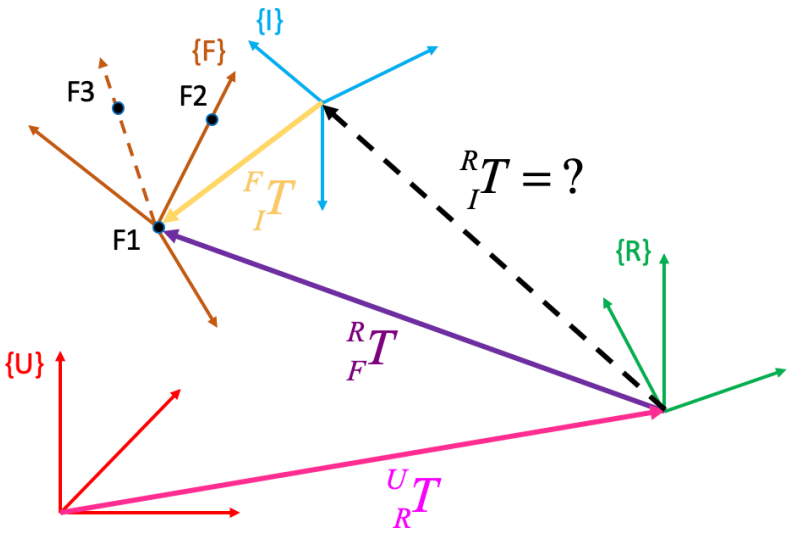
We have the Reference Frame {F} wrt {U} ${}^{U}_{F}T = {}^{U}_{R}T\,{}^{R}_{F}T$ and also ${}^{I}_{F}T$, so ... ${}^{U}_{I}T = {}^{U}_{F}T\,{}^{I}_{F}T^{-1}$

```
T_I_U=T_F_U*inv(T_F_I)
```

Ckeck that ${}^{U}_{I}T$ is coincident with the given Image Reference Frame

```
trplot(T_I_U,'Frame','I','color', 'cyan','length',0.7)
```

T_I_R System equation



$${}^{U}_{I}T = {}^{U}_{F}T\,{}^{I}_{F}T^{-1} = {}^{U}_{R}T\,{}^{R}_{I}T$$

$${}^{R}_{I}T = {}^{U}_{R}T^{-1}\,{}^{U}_{F}T\,{}^{I}_{F}T^{-1}$$

```
T_I_R=inv(T_R_U)*T_F_U*inv(T_F_I)
```

```
T_I_R = 4×4
    0.0000    0.0000   -1.0000    0.4786
    1.0000    0.0000    0.0000   -0.4716
    0.0000   -1.0000   -0.0000    0.8725
         0         0         0    1.0000
```

Overall ckecking

Fiducials in Robot Frame

As a matter of proof, ckeck that given Fiducials coordinates in Image Reference Frame we can obtain them in Robot Reference Frame

```
Fi_R2=T_I_R*[[F1D F2D F3D];ones(1,3)]
```

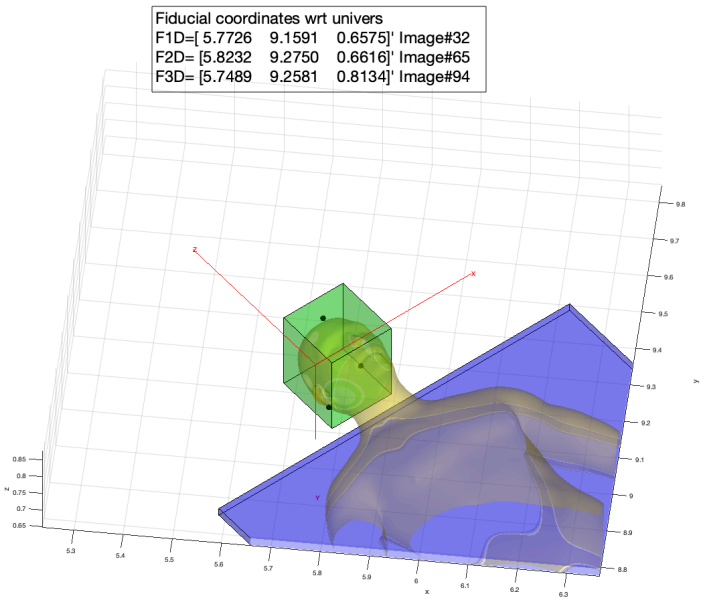
```
Fi_R2 = 4×3
    0.4338    0.3876    0.3470
   -0.4018   -0.2841   -0.3486
    0.6575    0.6616    0.8134
    1.0000    1.0000    1.0000
```

```
Fi_R
```

```
Fi_R = 4×3
    0.4338    0.3876    0.3470
   -0.4018   -0.2841   -0.3486
    0.6575    0.6616    0.8134
    1.0000    1.0000    1.0000
```

Fiducials in Univers Frame

As a matter of proof, ckeck that given Fiducials coordinates in Image Reference Frame we can obtain them in Univers Reference Frame.




```
Fi_R2=T_R_U*T_I_R*[[F1D F2D F3D];ones(1,3)]
```

```
Fi_R2 = 4×3
    5.7726    5.8232    5.7489
    9.1591    9.2750    9.2581
    0.6575    0.6616    0.8134
    1.0000    1.0000    1.0000
```