Invers Kinematics: Key ideas

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Catching the idea 1 of 3

Call a simple Two Link

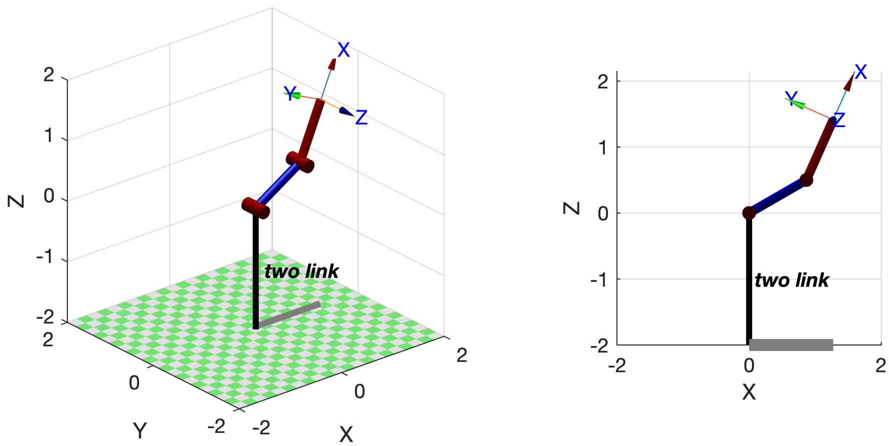
2R robot arm

```
clear
mdl_twolink
twolink
```

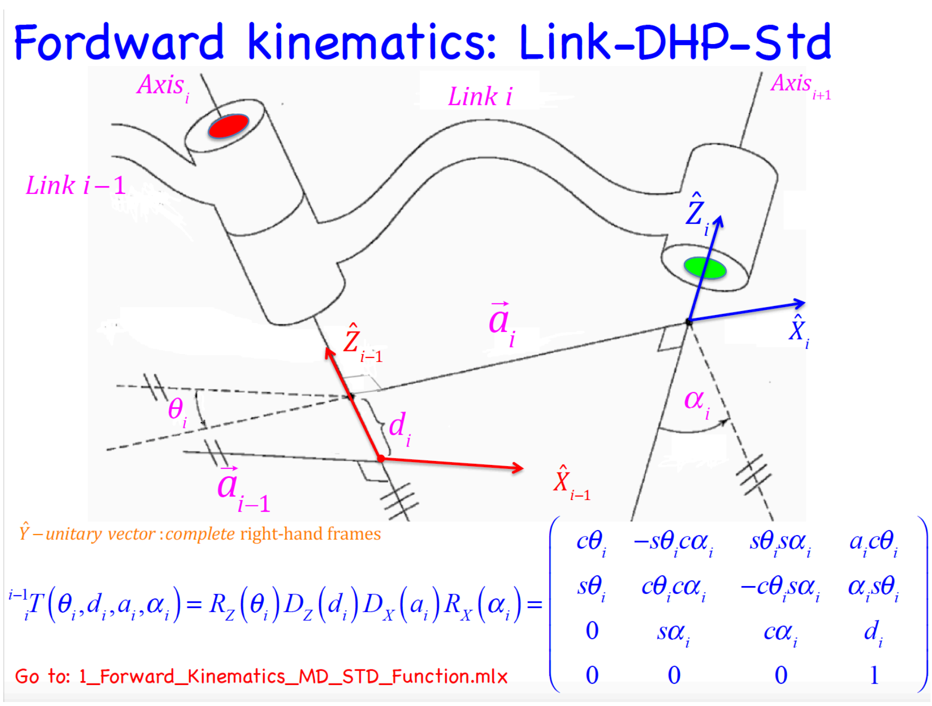
```
twolink =

two link:: 2 axis, RR, stdDH, slowRNE
 - from Spong, Hutchinson, Vidyasagar;
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|          0|          1|          0|          0|
|  2|         q2|          0|          1|          0|          0|
+---+-----------+-----------+-----------+-----------+-----------+
base:    t = (0, 0, 0), RPY/xyz = (0, 0, 90) deg
```

Plotting twolink

```
subplot(121)
twolink.plot([pi/6 pi/5],'view',[-40 25])
subplot(122)
twolink.plot([pi/6 pi/5])
axis([-0.5 2 -0.5 0.5 -0.5 2.5])
axis equal
view (0, 0)
```



Forward Kinematics of Two Link Std.




```
syms theta_1 theta_2 L_1 L_2 real
FK2L_Hand=simplify(troty(theta_1)*transl(L_1,0,0)*troty(theta_2)*transl(L_2,0,0))
```

```
FK2L_Hand =
```

$$\begin{pmatrix} \cos(\theta_1+\theta_2) & 0 & \sin(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2)+L_1\cos(\theta_1) \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_1+\theta_2) & 0 & \cos(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2)-L_1\sin(\theta_1) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
twolink.links(1, 1).a=L_1;
twolink.links(1, 2).a=L_2;
FK2L_RTB=simplify(twolink.fkine ([-theta_1 -theta_2]))
```

$$\begin{pmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & L_2\cos(\theta_1+\theta_2)+L_1\cos(\theta_1) \\ 0 & 0 & -1 & 0 \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & -L_2\sin(\theta_1+\theta_2)-L_1\sin(\theta_1) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

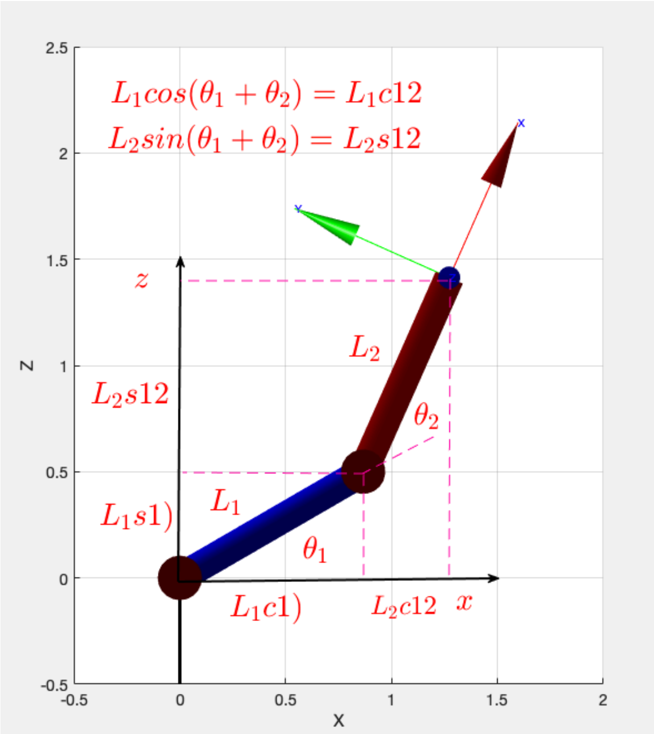
System equation to solve

```
syms x z real %position of the {EE}
e1 = x == FK2L_RTB.t(1)
```

e1 = $x = L_2\cos(\theta_1+\theta_2)+L_1\cos(\theta_1)$

```
e2 = z == FK2L_RTB.t(3)
```

e2 = $z = -L_2\sin(\theta_1+\theta_2)-L_1\sin(\theta_1)$



Invers Kinematics by hand.

Generate a known solution

```
clear
mdl_twolink
twolink.links(1, 1).a=1.5;
twolink.links(1, 2).a=1;
FD=twolink.fkine([pi/6 pi/5]);
XYZ=FD.t
```

```
XYZ = 3×1
    1.7058
         0
    1.6635
```

```
Rotation=FD.R
```

```
Rotation = 3×3
    0.4067   -0.9135         0
         0         0   -1.0000
    0.9135    0.4067         0
```

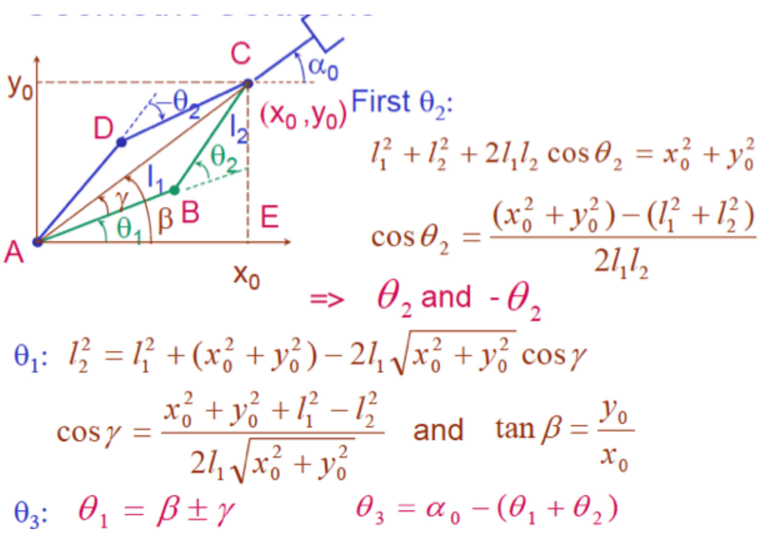
```
L1=twolink.links(1, 1).a
```

```
L1 = 1.5000
```

```
L2=twolink.links(1, 2).a
```

```
L2 = 1
```

Apply the theory




```
theta_2=acos(((XYZ(1)^2+XYZ(3)^2) -(L1^2+L2^2))/(2*L1*L2))
```

```
theta_2 = 0.6283
```

```
beta=atan2(XYZ(3),XYZ(1))
```

```
beta = 0.7729
```

```
gamma=acos((XYZ(1)^2+XYZ(3)^2+L1^2-L2^2)/(2*L1*sqrt((XYZ(1)^2+XYZ(3)^2))))
```

```
gamma = 0.2493
```

```
theta_1_1=beta+gamma
```

```
theta_1_1 = 1.0221
```

```
theta_1_2=beta-gamma
```

```
theta_1_2 = 0.5236
```

```
pi/6
```

```
ans = 0.5236
```

```
pi/5
```

```
ans = 0.6283
```

Testing solutions. See {EE}

```
XYZ
```

```
XYZ = 3×1
    1.7058
         0
    1.6635
```

```
FD=twolink.fkine([pi/6 pi/5]);
FD_1=twolink.fkine([theta_1_1 -theta_2])
```

```
FD_1 =
    0.9235   -0.3837         0     1.706
         0         0        -1         0
    0.3837    0.9235         0     1.664
         0         0         0         1
```

```
FD_2=twolink.fkine([theta_1_2 +theta_2])
```

```
FD_2 =
    0.4067   -0.9135         0     1.706
         0         0        -1         0
    0.9135    0.4067         0     1.664
         0         0         0         1
```

Invers Kinematics using RTB

At command windows:

help SerialLink/ikine

Generate a known solution

```
clear
mdl_twolink
```

```
FD=twolink.fkine([pi/6 pi/5])
```

```
FD = 
    0.4067   -0.9135         0     1.273
         0         0        -1         0
    0.9135    0.4067         0     1.414
         0         0         0         1
```

Invoque ikine function

At command windows: help SerialLink/ikine to know more...it is a recursive numerical approach

Elbow down

```
Qd=twolink.ikine(FD,'mask',[1 1 0 0 0 0 ], 'q0',[0 0])
```

```
Qd = 1×2
    0.5236    0.6283
```

```
[pi/6 pi/5]
```

```
ans = 1×2
    0.5236    0.6283
```

Elbow up

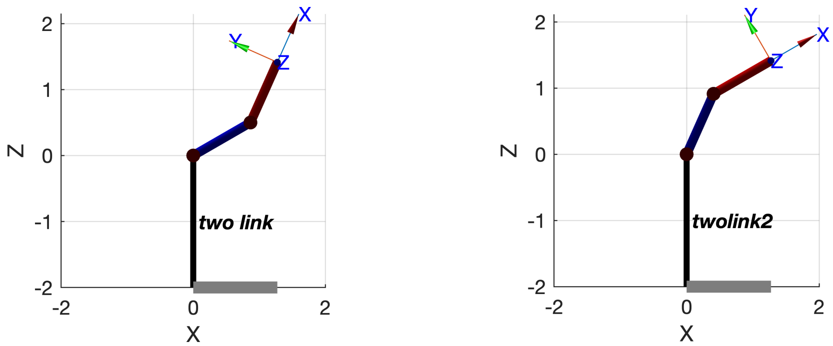
```
Qu=twolink.ikine(FD,'mask',[1 1 0 0 0 0 ], 'q0',[pi/2 0])
```

```
Qu = 1×2
    1.1519   -0.6283
```

Ploting both solutions

Clone the twolink

```
twolink2= SerialLink(twolink, 'name', 'twolink2');
subplot(121)
twolink.plot(Qd)
axis([-0.5 2 -0.5 0.5 -0.5 2.5])
axis equal
view (0, 0)
subplot(122)
twolink2.plot(Qu)
axis([-0.5 2 -0.5 0.5 -0.5 2.5])
axis equal
view (0, 0)
```



Inverse Kinematics of Puma560

At command windows:

help SerialLink/ikine6s

```
clear
mdl_puma560
bob= SerialLink(p560, 'name', 'bob'); % clone puma 560
qn
```

```
qn = 1×6
         0    0.7854    3.1416         0    0.7854         0
```

```
T1 = p560.fkine(qn)% Pose or Frame description
```

```
T1 =
         0         0         1    0.5963
         0         1         0   -0.1501
        -1         0         0  -0.01435
         0         0         0         1
```

```
qi1 = p560.ikine6s(T1,'ru')
```

```
qi1 = 1×6
   -0.0000    0.7854    3.1416   -0.0000    0.7854    0.0000
```

```
qi2 = p560.ikine6s(T1,'rd')
```

```
qi2 = 1×6
   -0.0000   -0.8335    0.0940   -3.1416    0.8312    3.1416
```

```
T2 = p560.fkine(qi1) % is equal to T1 !!!
```

```
T2 =
         0         0         1    0.5963
         0         1         0   -0.1501
        -1         0         0  -0.01435
         0         0         0         1
```

```
T3 = p560.fkine(qi2) % is equal to T1 !!!
```

```
T3 =
         0         0         1    0.5963
         0         1         0   -0.1501
        -1         0         0  -0.01435
         0         0         0         1
```

```
T1
```

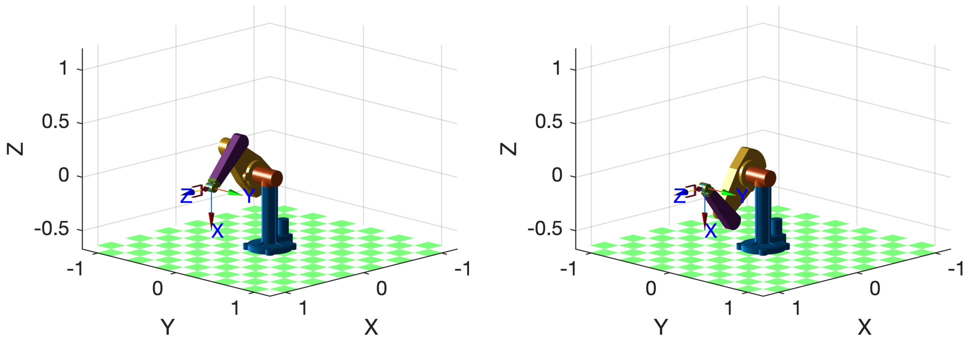
```
T1 =
         0         0         1    0.5963
         0         1         0   -0.1501
        -1         0         0  -0.01435
         0         0         0         1
```

```
figure
subplot(121)
bob.plot3d(qi1)
```

```
Loading STL models from ARTE Robotics Toolbox for Education  by Arturo Gil (http://arvc.umh.es/arte)......
```

```
subplot(122)
p560.plot3d(qi2)
```

```
Loading STL models from ARTE Robotics Toolbox for Education  by Arturo Gil (http://arvc.umh.es/arte)......
```



Practicing with ikine6s & p560.plot3d(qx)

Display the 8 solutions for qn (joint space). Use subplot 2x2 for elbown up/down and subplot 2x2 for {EE}