Introduction to Scientific Computing, Homework #5

Problem 1 Solution

```
% Symbolic Math Toolbox must be installed first
clear;clc;
%% Derivative
syms x y z n;
fxy=cos(x)*sin(y)+y^3*log(x);
disp(diff(fxy,'x'));
%% Divergence
Fxyz=[2*(cos(x))^2 (sin(y))^3 2*x^2+y^2+4*z^2];
disp(divergence(Fxyz,[x,y,z]));
%% Gradient
fxyz=x^2+2*y^2+4*z^2;
disp(gradient(fxyz));
%% Integral
fx=x^2-5*x;
disp(int(fx));
fx=x^(2-n)-5*x;
disp(int(fx,x));
%% Triple Integration
fxyz=sin(x)*cos(y)*tan(z)+y*cos(x)+x*sin(y);
disp(simplify(int(int(int(fxyz,x),y),z))); %simplify for pretty answer
```

Problem 1 Output(Answers)

```
y^3/x - sin(x)*sin(y)

8*z - 4*cos(x)*sin(x) + 3*cos(y)*sin(y)^2

2*x
4*y
8*z

(x^2*(2*x - 15))/6

- (5*x^2)/2 - x^3/(x^n*(n - 3))

- log(tan(z) - 1i)*((cos(x)*sin(y))/2 + (y^2*sin(x)*1i)/4 - (x^2*cos(y)*1i)/4) - log(tan(z)
            + 1i)*((x^2*cos(y)*1i)/4 - (y^2*sin(x)*1i)/4 + (cos(x)*sin(y))/2)
```

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Problem 2 Solution(Conclusion included in CODE)

```
clear;clc;
%% Step1: ezplot for funciton image
p=ezplot('sin(x)+(cos(x^2-pi/7))^2',[-5*pi,5*pi]);
set(p, 'LineStyle', '-', 'Color', 'black');
xlabel('x');
ylabel('f(x)');
title('Plot for sin(x)+cos^2(x^2-\pi/7)');
%% Step2: using trapz() to estimate the integral
figure;
X = 5:1:500;
Y = zeros(1,496);
for div=5:1:500
    x=linspace(-5*pi,5*pi,div);
    y=sin(x)+(cos(x.^2.-pi/7)).^2;
    Y(div-4)=trapz(x,y);
end
plot(X,Y, 'b-', 'LineWidth',1);
axis auto;

%% Step3: compares to accurate answer generated by integral()
accurate_Y = integral(@(x)sin(x)+(cos(x.^2.-pi/7)).^2, -5*pi, 5*pi);
hold on;
plot(X, repelem(accurate_Y, length(X)));
title('trapz compares to integral');
legend('trapz', 'integral');
xlabel('divisions');
ylabel('result');
%% Conclusion
% From the figure we can derive that integral() is much accurate for
% computing integral in such cases; Also we can find that about 350
% divisions are needed for a converge.
```

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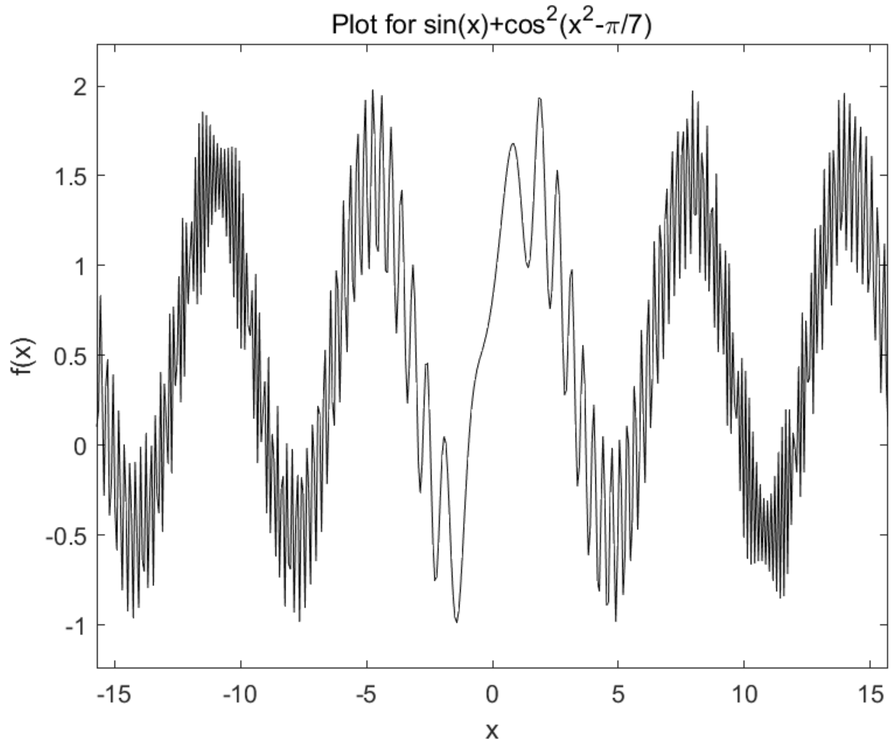
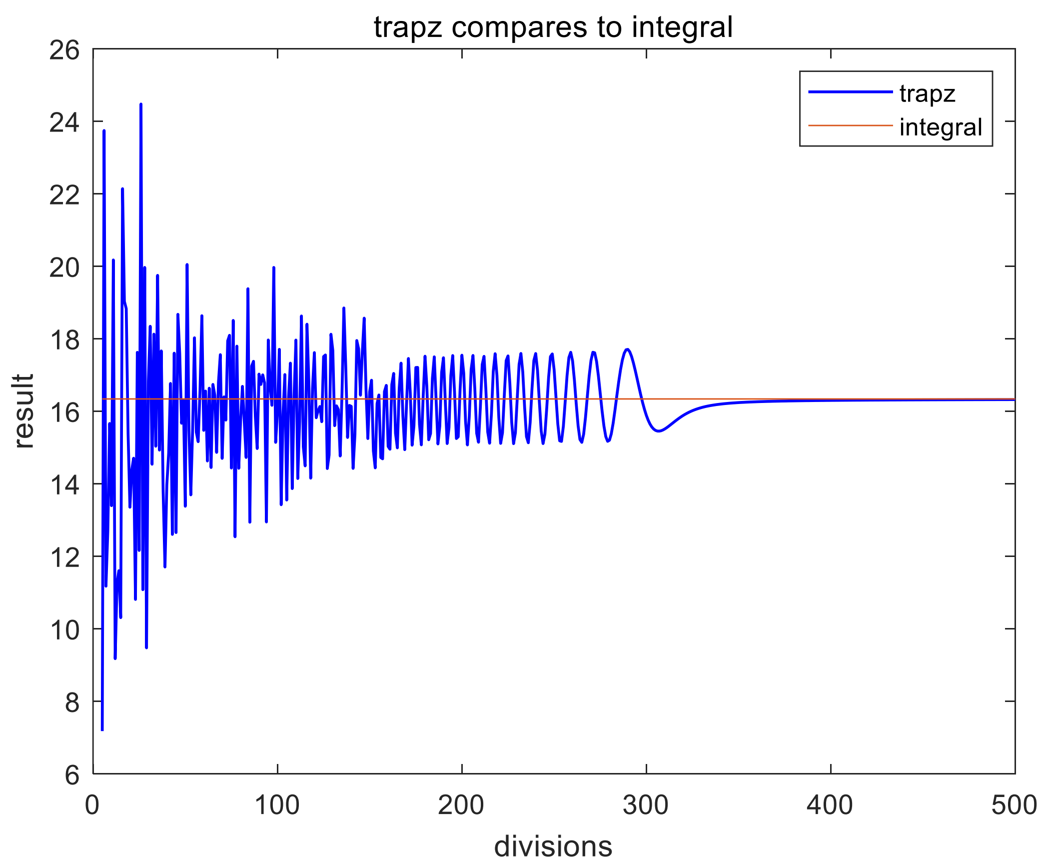
Problem 2 Plots





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Problem 3 Solution(Answer included in CODE)

```
clear;clc;
load poly.mat X Y;
[R2,N85,p]=discoverDegree(X,Y,0.85);
plot(X,Y, 'bo');
xlim([-10 10]);
ylim([-40 40]);
hold on;
plot(X,polyval(p,X),'r-','LineWidth',1.5); % Pretty plot
title('a fit for R^2\geq0.85'); % a title that fits for a fit
legend('Data', 'Polynomial Fit', 'Location','southeast');
% Required: legend
xlabel('x');ylabel('y'); % Required:label
[~,N95,~]=discoverDegree(X,Y,0.95);
disp([N85, N95]); % Generate answer for questions
%% Answer:
% For R^2≥0.85, 3 polynomial degree is needed;
% For R^2≥0.95, 5 polynomial degree is needed.
%%
function [R2out,pOrder,pCoeff] = discoverDegree(x,y,R2crit)
mean_y = mean(y);
pOrder = 1;
pCoeff=polyfit(x,y,pOrder);
R2out = 1-sum((y-polyval(pCoeff,x)).^2)/sum((y-mean_y).^2);
while R2out<R2crit
    pOrder = pOrder+1;
    pCoeff=polyfit(x,y,pOrder);
    R2out = 1-sum((y-polyval(pCoeff,x)).^2)/sum((y-mean_y).^2);
end
end
```

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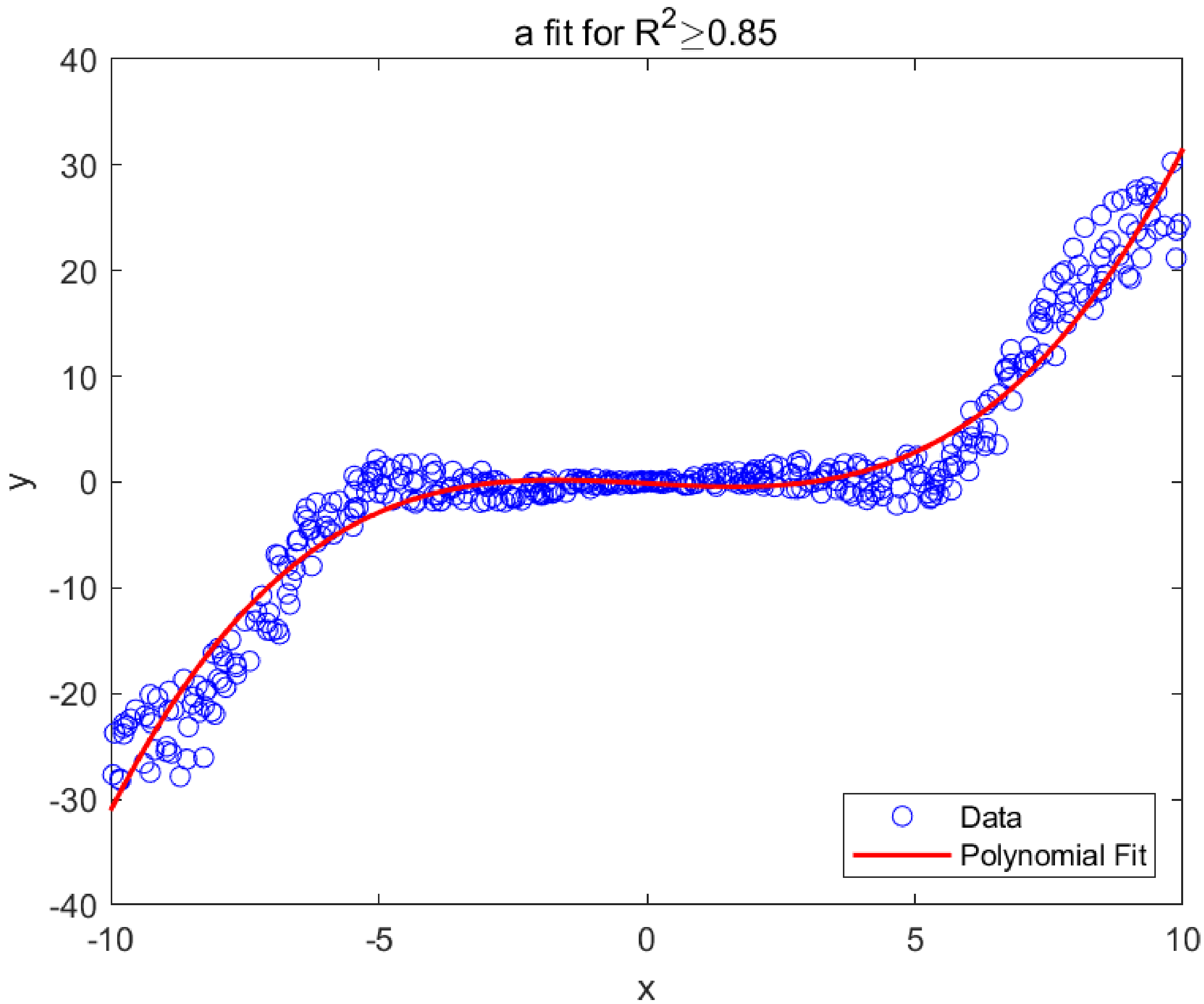
Problem 3 Plot



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Problem 4 Solution

```
clear;clc;
%% Initial conditions
g=9.8;
c=0.2;
x0=0;
y0=0;
low_x=0;
high_x=100;
vy=2.0;
time_step=0.01;
dis=10;
tol=0.01;
%% Calculation
disp(goldCornhole(g,c,x0,y0,low_x,high_x,vy,time_step,dis,tol));

function vxOpt = goldCornhole(g,c,x0,y0,vxa,vxb,vy,tstep,dis,tol)
%% Golden section search
vxOpt = inf;
vx1=vxb-0.618*(vxb-vxa); % Test point 1
vx2=0.618*(vxb-vxa)+vxa; % Test point 2
dis1 = abs(projectileSim(g,c,x0,y0,vx1,vy,tstep));
dis2 = abs(projectileSim(g,c,x0,y0,vx2,vy,tstep));
while abs(dis1-dis)>tol || abs(dis2-dis)>tol % Totally in the range
    if abs(dis1-dis) < abs(dis2-dis)
        % Choose the better, eliminate the worse
        vxOpt=vx1;
        vxb=vx2;
        vx2=vx1;dis2=dis1;
        vx1=vxb-0.618*(vxb-vxa);
        dis1 = abs(projectileSim(g,c,x0,y0,vx1,vy,tstep));
    else
        vxOpt=vx2;
        vxa=vx1;
        vx1=vx2;dis1=dis2;
        vx2=0.618*(vxb-vxa)+vxa;
        dis2 = abs(projectileSim(g,c,x0,y0,vx2,vy,tstep));
    end
end
end
function disX = projectileSim(g,c,x0,y0,vx0,vy0,tstep)
% Calculate landing point of the beanbag, where air drag considered.
EPS=1e-6;
disX = x0;
while y0>-EPS || vy0>0
    x0 = vx0*tstep + x0;
    y0 = vy0*tstep + y0;
    vy0 = vy0 - g*tstep - tstep*sign(vy0)*c*vy0^2;
    % be careful about the force direction
    vx0 = vx0 - tstep*sign(vx0)*c*vx0^2;
end
disX = x0 - disX;
end
```

Problem 5 Solution (Code is too long, hence no box placed)

```
clear;clc;
%% initial conditions
t=struct(); % create struct
x=struct();
dx=struct();
d2x=struct();
t.raw = [0, 1, 2.5, 5.0, 10.5, 12.5, 16, 20.5, 26.5, 30.5, 32];
x.raw = [0, 0.3, 1.2, 1.3, 1.6, 2.2, 2.4, 3.0, 3.6, 4.5, 4.6];
domain=linspace(0,32,100);

%% data generation
x.interp.linear = interp1(t.raw,x.raw,domain,'linear');
x.interp.spline = interp1(t.raw,x.raw,domain,'spline');
dx.interp.linear = diff(x.interp.linear)*100/32;
dx.interp.linear(end+1)=dx.interp.linear(end);
% 1 element is dropped after diff, must add an extra value
dx.interp.spline = diff(x.interp.spline)*100/32;
dx.interp.spline(end+1)=dx.interp.spline(end);

d2x.interp.linear = diff(dx.interp.linear)*100/32;
d2x.interp.linear(end+1) = d2x.interp.linear(end);

d2x.interp.spline = diff(dx.interp.spline)*100/32;
d2x.interp.spline(end+1) = d2x.interp.spline(end);

%% Linear
figure('units','normalized','outerposition',[0 0 1 1]);
% Full screen
set(gcf, 'Color', 'white');
ax1 = axes('Position', [.1  .69  .8  .23]);% Manually specific position
plot(domain,x.interp.linear, 'k-', 'LineWidth',1.5);
set(ax1, 'Xticklabel', [], 'LineWidth', 1, 'FontSize', 11); % Remove label
xlim(ax1, [0 32]);ylim(ax1, [0 6]);
ylabel(ax1, 'x (m)');
hold on;
plot(t.raw,x.raw, 'k.', 'MarkerSize',20); % Plot raw data as markers
hold off;

ax2 = axes('Position', [.1  .42  .8  .23]);
plot(domain,dx.interp.linear, 'k-', 'LineWidth',1.5);
set(ax2, 'Xticklabel', [], 'LineWidth', 1, 'FontSize', 11);
xlim(ax2, [0 32]);ylim(ax2, [0 1]);
ylabel(ax2, 'dx/dt (m/s)');

ax3 = axes('Position', [.1  .15  .8  .23]);
plot(domain,d2x.interp.linear, 'k-', 'LineWidth',1.5);
set(ax3, 'LineWidth', 1, 'FontSize', 11);
xlim(ax3, [0 32]);ylim(ax3, [-2 1]);
ylabel(ax3, 'd^2x/dt^2 (m/s^2)');
xlabel('Times (s)');
%% Spline
figure('units','normalized','outerposition',[0 0 1 1]);
```

```
set(gcf, 'Color', 'white');
ax1 = axes('Position', [.1  .69  .8  .23]);
plot(domain,x.interp.spline, 'k-', 'LineWidth',1.5);
set(ax1, 'Xticklabel', [], 'LineWidth', 1, 'FontSize', 11);
xlim(ax1, [0 32]);ylim(ax1, [0 6]);
ylabel(ax1, 'x (m)');
hold on;
plot(t.raw,x.raw, 'k.', 'MarkerSize',20);
hold off;

ax2 = axes('Position', [.1  .42  .8  .23]);
plot(domain,dx.interp.spline, 'k-', 'LineWidth',1.5);
set(ax2, 'Xticklabel', [], 'LineWidth', 1, 'FontSize', 11);
xlim(ax2, [0 32]);ylim(ax2, [-1 1]);
ylabel(ax2, 'dx/dt (m/s)');

ax3 = axes('Position', [.1  .15  .8  .23]);
plot(domain,d2x.interp.spline, 'k-', 'LineWidth',1.5);
set(ax3, 'LineWidth', 1, 'FontSize', 11);
xlim(ax3, [0 32]);ylim(ax3, [-2 1]);
ylabel(ax3, 'd^2x/dt^2 (m/s^2)');
xlabel('Times (s)');
```

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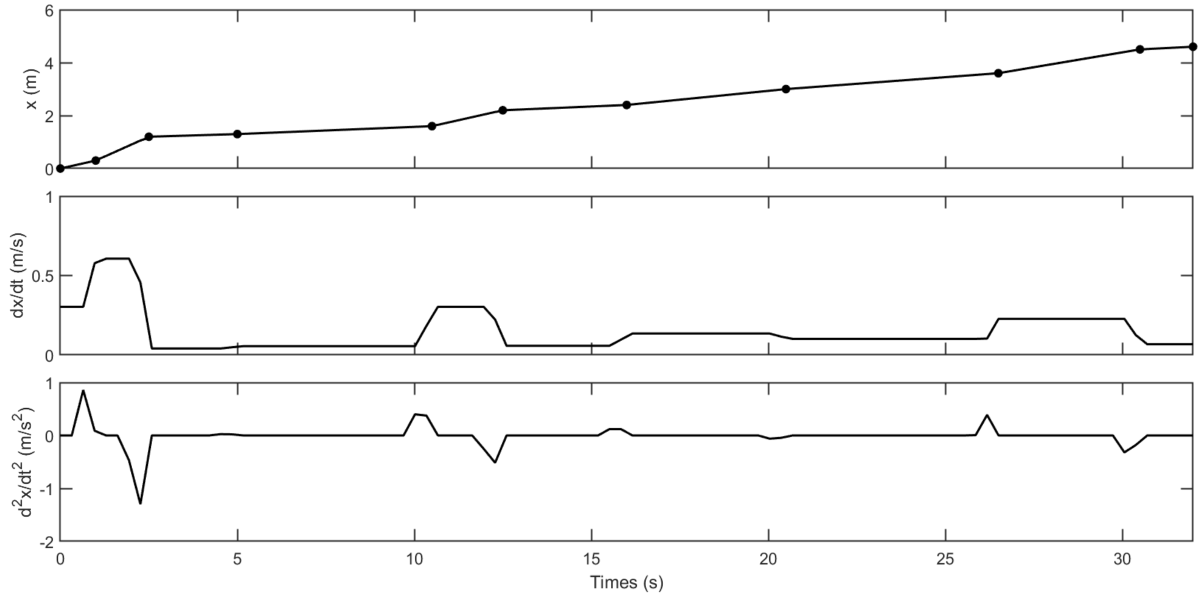
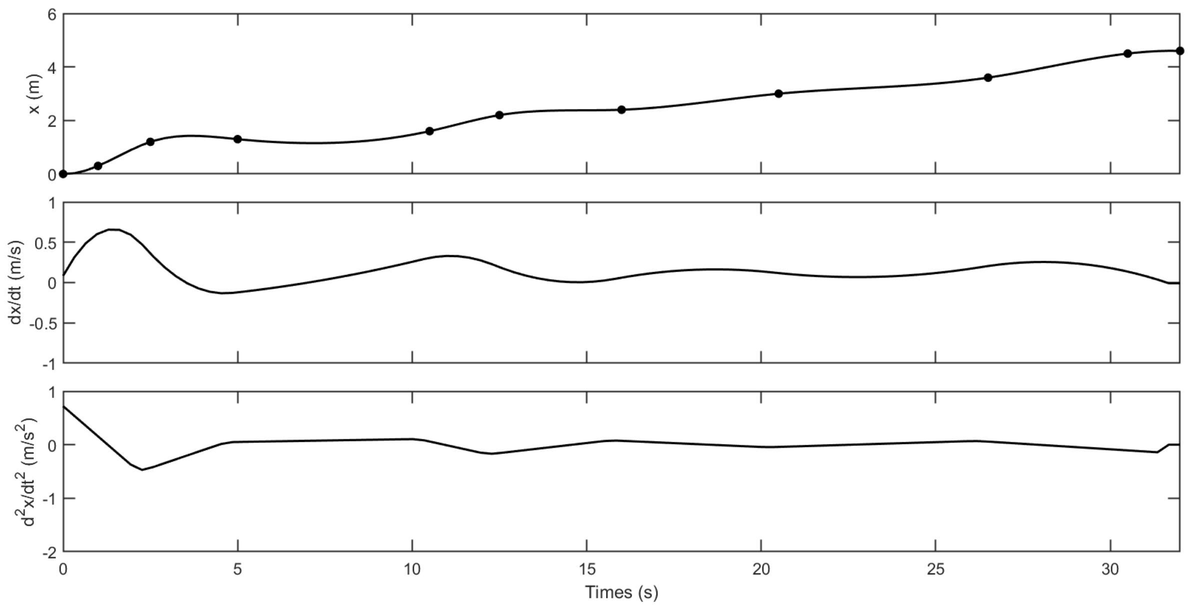
Problem 5 Plots(First:Linear Second:Spline)





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