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CS3233 Competitive Programming

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Week 03 – Problem Solving Paradigms

(Complete Search; Divide & Conquer; Greedy)

Outline

- Mini Contest 2 + Break + Discuss Last Week's C, Today's A/B
- Admins
- Complete Search
  - Iterative: (Nested) Loops, Permutations, Subsets
  - Recursive Backtracking
  - Some Tips
- Greedy Algorithm (we will discuss this first before D&C)
  - Ingredients and some examples
- Divide and Conquer (D&C)
  - Focus on **Binary Search the Answer**

Iterative: (Nested) Loops, Permutations, Subsets

Recursive Backtracking

Some Tips

COMPLETE SEARCH

Iterative Complete Search (1)

- UVa 725 – Division
  - Find two 5-digits number s.t. → **abcde** / **fghij** = N
  - **abcdefghij** must be all different, 2 <= N <= 79
- Iterative Complete Search Solution (Nested Loops):
  - Try all possible **fghij** (one loop)
  - Obtain **abcde** from **fghij * N**
  - Check if **abcdefghij** are all different (*another* loop)

Iterative Complete Search (2)

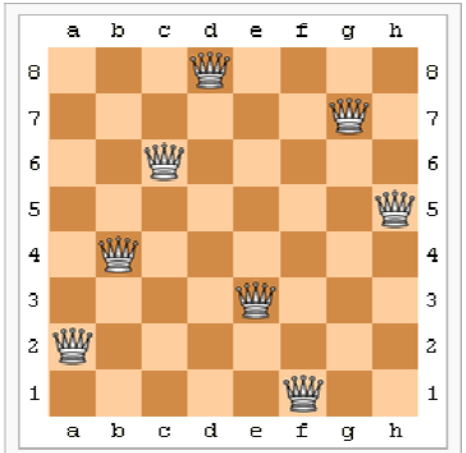
- UVa 11742 – Social Constraints
  - There are $0 < \mathbf{n} \leq 8$ movie goers
  - They will sit in the front row with **n** consecutive open seats
  - There are $0 \leq \mathbf{m} \leq 20$ seating constraints among them, i.e. **a** and **b** must be at most (or at least) **c** seats apart
  - How many possible seating arrangements are there?
- Iterative Complete Search Solution (Permutations):
  - Set counter = 0 and then try all possible **n! permutations**
  - Increase counter if a permutation satisfies all **m** constraints
  - Output the final value of counter

Iterative Complete Search (3)

- UVa 12346 – Water Gate Management
  - A dam has $1 \leq \mathbf{n} \leq 20$ water gates to let out water when necessary, each water gate has **flow rate** and **damage cost**
  - Your task is to manage the opening of the water gates in order to get rid of *at least* the specified **total flow rate** condition that the **total damage cost** is minimized!
- Iterative Complete Search Solution (Subsets):
  - Try all possible $\mathbf{2^n}$ subsets of water gates to be opened
  - For each subset, check if it has sufficient flow rate
    - If it is, check if the total damage cost of this subset is smaller than the overall minimum damage cost so far
      - If it is, update the overall minimum damage cost so far
  - Output the minimum damage cost

Recursive Backtracking (1)

- UVa 750 – 8 Queens Chess Problem
  - Put 8 queens in 8x8 Chessboard
  - No queen can attack other queens
- Naïve way (Time Limit Exceeded)
  - Generate 8^8 = 17M possible 8 queens position and filter the infeasible ones…



One solution.

Recursive Backtracking (2)

- Better way, recursive backtracking
  - Insight 1: **all-different constraint** for row (or col)
    - Search space goes down from 8^8 = 17M to 8! = 40K!
  - Insight 2: diagonal check
    - Another way to prune the search space
    - Queen A (i, j) attacks Queen B (k, l) iff `abs(i - k) == abs(j - l)`
- Scrutinize the sample code of recursive backtracking!

Tips

1. Filtering versus Generating
2. Prune Infeasible Search Space Early
3. Utilize Symmetries
4. Pre-Computation a.k.a. Pre-Calculation
5. Try Solving the Problem Backwards
6. Optimizing Your Source Code
7. Use Better Data Structure & Algorithm ☺

More Search Algorithms…

- Depth Limited Search
- Iterative Depth Limited Search
- A*
- Iterative Deepening A* (IDA*)
- Memory Bounded A*
- Branch and Bound (BnB)
- Maybe in Week12 ☺...

Ingredients and Non Classical Example

GREEDY ALGORITHM

Coin Change (Special Case)

- Given many coins of different denominations = {25, 10, 5, 1} cents, make X cents with the *least number of coins* used!
  - If X = 42, solution: 25+10+**5+1+1** = 42 (5 coins)
  - If X = 17, solution: 10+**5+1+1** = 17 (4 coins)
  - If X = 7, solution: **5+1+1** = 7 (3 coins)
- Observe "optimal sub-structure" and "Greedy property"!

Greedy Algorithm – Basic Idea

- Always make **locally optimal** choice that looks best at the moment, hoping that this choice will lead to a **globally optimal** solution
- Does not always work, but sometimes it does
  - When it does, you have a very efficient solution that can be coded quickly

Coin Change (General Case)

- Given many coins of different denominations = {4, 3, 1} cents, make X cents with the *least number of coins* used!
  - If X = 6, greedy solution: 4+1+1 = 6 (3 coins)
  - But the optimal solution is: 3+3 = 6 (2 coins)
- This problem will be revisited in "Dynamic Programming" topic

Few Classic Greedy Problems

- Coin Change *certain denominations only*
- Fractional Knapsack
- Load Balancing
- Greedy Assignment
- Interval Covering/Activity Selection
- Huffman's Coding
- Minimum Spanning Tree
- +ve Weighted Single-Source Shortest Paths

Designing Greedy Algorithm (5)

- Tips (Not rule of thumb!)
  - In ICPC, if the input size is too large* for Complete Search or even DP, Greedy may be the (only) way
    - But if the input size is small, avoid greedy!
      Try complete search or DP first!
      - If our Complete Search or DP always select one particular (best) sub problem at each step, greedy strategy may works
  - If no obvious greedy strategy seen, try to sort the data first or introduce some tweaks
    - See if greedy strategy emerges

Greedy in Other Algorithms

- Greedy strategy is embedded in more complex algorithms:
  - +ve Weighted Single-Source Shortest Path
    - Dijkstra's
  - Minimum Spanning Tree
    - Kruskal's
    - Prim's
  - Combined with binary search the answer
    - See the next section
  - Etc

Interesting Usages of Binary Search

DIVIDE AND CONQUER

Divide and Conquer Paradigm

- Idea:
  - Divide: Split original problem into sub-problems
  - Conquer: Solve sub-problems and then use some (or all) the results of sub-problems to solve the original problem
- Data Structures/Algorithms with D&C flavor:
  - Merge/Quick/Heap Sort, findKthIndex, Binary Search, BST/Heap/Segment/Fenwick Tree, etc
- See if your problem can be solved using D&C!

Focus on Binary Search the Answer

- Ordinary Usage
  - On Static Sorted Array
    - C++ STL algorithm::lower_bound (Java Collections.binarySearch)
- Not so common (read textbook)
  - On weighted tree with root-to-leaf path sorted!
- Bisection (Numerical) Method (read textbook)
  - Find root x of a function s.t. f(x) = 0
    - Let computer do the work
- Binary Search the Answer (discussed next)

Binary Search the Answer (1)

- UVa 714 – Copying Books
  - Given m books with different pages p_i $\{p_1, p_2, \ldots, p_m\}$
  - Make a copy of each book with k scribes
  - Each book can only be assigned to a single scriber
  - Every scriber must get a continuous sequence of books, i.e. $0 = b_0 < b_1 < b_2 < \ldots < b_{k-1} < b_k = m$
  - Minimize the maximum number of pages assigned to a scriber
  - $1 \leq k, m \leq 500$, $1 \leq p_i \leq 10000000$
- Example:
  - m = 9, k = 3, p = {100,200,300,400,500,600,700,800,900}
  - Answer: 1700 pages: 100,200,300,400,500|600,700|800,900

Binary Search the Answer (2)

- Can we "guess" this answer in binary search fashion?
  - Yes
  - If we guess answer = 1000 pages, we have no solution
    - Greedy assignment!
      - Give as many consecutive books to current scribe until "overflow"
    - 100,200,300,400|500|600|**{700,800,900} (unassigned)**
    - Answer is too low, we have to increase the workload per each scriber
  - If we guess answer = 2000 pages, we have a solution + slack
    - 100,200,300,400,500|600,700|800,900 **(slack 500|700|300)**
    - Answer is too high, we can decrease the workload per each scriber
  - And so on until we hit answer = 1700 pages (optimal)

Summary

- We have seen:
  - Complete Search: Iterative and Recursive Backtracking
  - Greedy
  - Divide and Conquer: Focus on Binary Search the Answer
- Next week, we will see (revisit) the fourth paradigm:
  - Dynamic Programming

References

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  - Steven, Felix ☺
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  - Cormen, Leiserson, Rivest, Stein
- **Algorithm Design**, p115-208
  - Tardos, Kleinberg
- **Algorithms**, p127-155
  - Dasgupta, Papadimitirou, Vazirani