Weight trimming and weight smoothing methods in surveys

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Basic concepts

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- Goal: estimate the population total, $t_y = \sum_{i \in U} y_i$.

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- A sample s is selected according to a sampling design $p(s)$ with inclusion probabilities $\pi_1, \ldots, \pi_N$.
- We consider linear estimators of t_y of the form

$$\widehat{t}_y = \sum_{i \in s} w_i y_i,$$

where w_i is the survey weight attached to unit i.

Weighting in surveys

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- Calibration adjustment: The weights w_i^* are further adjusted to match known population counts.
  Final (or calibrated) weighting system: $\{w_i = w_i^* g_i; i \in s_r\}$.

$$\text{Steps: } d_i \xrightarrow{\text{nra}} w_i^* \xrightarrow{\text{calibration}} w_i$$

Classical estimators

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- Properties of the Horvitz-Thompson estimator:
  - design-unbiased for t_y: $E_p(\widehat{t}_{HT}) = t_y$.
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- We expect $\widehat{t}_{HT}$ to be very efficient if y is approximately proportional to π.

Classical estimators

- Suppose that a Q-vector of auxiliary variables $\mathbf{x} = (x_1, \ldots, x_Q)^\top$ is available for $i \in s$ and that the vector of population totals (benchmarks)
$$\mathbf{t_x} = (t_{x_1}, \ldots, t_{x_Q})^\top$$
is known, where $t_{x_q} = \sum_{i \in U} x_{qi}, \quad q = 1, \ldots, Q.$

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- In the absence of nonsampling errors, the calibrated weighting system $\{w_i = d_i g_i; i \in s\}$ leads to the calibration estimator:

$$\widehat{t}_C = \sum_{i \in s} w_i y_i,$$

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where the calibrated weights are such that
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- That is, the calibrated weighting system $\{w_i; i \in s\}$ ensures consistency with known population totals.

Classical estimators

- Properties of calibration estimators:
  - asymptotically design-unbiased for t_y: $E_p(\widehat{t}_C) \approx t_y$ provided that the sample size n is large enough.
  - If $y_i = \mathbf{x}_i^\top \boldsymbol{\beta}$ for a vector of constants $\boldsymbol{\beta}$ (i.e., there exists a perfect relationship between y and $\mathbf{x}$), then

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- The class of calibration estimators includes the Generalized REGression (GREG) estimator as a special case.

Classical estimators

- Simplest special case: $x_i = 1$ for all i and $t_x = N$. In this case, $\widehat{t}_C$ reduces to the Hàjek estimator

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- Properties of the Hàjek estimator:
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- We expect the Hàjek estimator to perform well if y_i and π_i are unrelated.

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  - High variation in the weights
  - Poor correlation between the weights and the study variables.
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- Combination of two factors can lead to inefficient estimators:
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  - Poor correlation between the weights and the study variables.
- Early references: Rao (1966) and Basu (1971).
- In surveys with multiple characteristics, it is not rare to end up with highly dispersed weights that are poorly related (or even unrelated) to some study variables.

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- At the nonresponse treatment stage, forming weighting classes may help preventing an extreme variation in the weights.
- At the calibration stage, it's possible to ensure that the calibration adjustment factors lie between pre-specified lower and upper bounds through an appropriate distance function $\rightarrow$ may also help preventing an extreme variation in the weights.

What can we do to limit the impact of dispersed weights?

- A number of techniques has been proposed in the literature:
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- A number of techniques has been proposed in the literature:
  - weight trimming methods;
  - weight smoothing methods;
- Different in nature but share a same objective:
  - modify the survey weights so that the resulting estimators have a lower mean square error than that of the usual estimators (e.g., the Horvitz-Thompson estimator)
  - Reduction of the mean square error is generally achieved at the expense of introducing a bias.
  - Treatment of survey weights: can be viewed as a compromise between bias and variance.

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- The concept of influential units is not always clearly defined in practice:
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  - Units with large weighted value $w_i y_i$?
  - Later, we define an appropriate influence measure called the conditional bias of a unit.

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  - Later, we define an appropriate influence measure called the conditional bias of a unit.
- We distinguish between two types of weight trimming methods:
  - the methods based only on the distribution of the survey weights
  - the methods based on the distribution of the weights but that also account for the study variables.

Methods based only on the distribution of the weights

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- The weights after trimming are defined as

$$\widetilde{w}_i = \begin{cases} w_0 & \text{if} \quad w_i \geq w_0 \\ \gamma w_i & \text{if} \quad w_i < w_0 \end{cases}$$

where γ is a rescaling factor that ensures that

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- The estimator of t_y after trimming is given by

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- Potter (1988, 1990) provides one of the first formal treatment of weight trimming.
- Modeling the distribution of the weights: it assumes that the reciprocal of the the scaled sampling weights follows a beta distribution:

$$f(w_i) = \frac{n\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1/(nw_i))^{\alpha-1}(1-1/(nw_i))^{\beta-1}$$

- The weights from the upper tail of the distribution, say where $1 - F(w_i) < .01$, are trimmed to w_0 such that $1 - F(w_0) = .01$.

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- The procedure is iterated until all the weights are less or equal to the threshold.
- The excess weight is redistributed among the non-trimmed units.
- Value of c: to be chosen empirically by examining the distribution of $nw_i^2 / \sum_{i \in S} w_i^2$.

Methods that account for the study variable

- Potter (1990) suggested to find the cutpoint w_0 that minimizes the estimated mean square error of the trimmed estimator $\widehat{t}_{trim}(w_0)$:

$$\widehat{MSE}(\widehat{t}_{trim}) = \widehat{V}(\widehat{t}_{trim}) + \left(\widehat{t}_{trim} - \widehat{t}_{HT}\right)^2 - \widehat{V}(\widehat{t}_{trim} - \widehat{t}_{HT}).$$

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- Unlike the previous ones, this method accounts for the y-variable being estimated.
- An optimal cutpoint w_0^{opt} for a given study variable may be far from optimal for another study variable.
- Minimizing the estimated mean square error is not an easy task, in general.

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- Dalén-Tambay winsorization: Dalén (1987) and Tambay (1988) considered the winsorization

$$\widetilde{y}_i = \begin{cases} y_i \text{ if } w_i y_i \leq K \\ \frac{K}{w_i} + \frac{1}{w_i}(y_i - \frac{K}{w_i}) \text{ if } w_i y_i > K \end{cases}$$

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- The Dalén-Tambay winsorized estimator is given by

$$\widehat{t}_{DT}(K) = \sum_{i \in s} w_i \widetilde{y}_i = \sum_{i \in s} \widetilde{w}_i y_i,$$

where

$$\widetilde{w}_i = 1 + (w_i - 1)\frac{min\left(y_i,\ \frac{K}{w_i}\right)}{y_i} \geq 1$$

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- If $min\left(y_i, \frac{K}{w_i}\right) = y_i$, then $\widetilde{w}_i = w_i$.
- For an influential unit: $\widetilde{w}_i < w_i$.
- Nice property: $\widetilde{w}_i$ cannot be smaller than 1.

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  - Should be determined using a more formal criterion.
- Second criterion: Determine K that minimizes the estimated mean square error of the winsorized estimator:
  - Requires historical information and/or a model for the study variable;
  - Often requires simplifying assumptions;
  - Complex to implement, in general;
  - Kokic and Bell (1994), Hulliger (1995) and Rivest and Hurtubise (1995).

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- Third criterion: find the value of K that minimizes the maximum estimated influence:
  - Does not require historical information or a model;
  - Does not require simplifying assumptions;
  - Easy to implement in practice;
  - see Beaumont, Haziza and Ruiz-Gazen (2013) and Favre-Martinoz, Haziza and Beaumont (2014).

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- So far, units exhibiting a large weight w_i (methods of Potter) or a large weighted value $w_i y_i$ (winsorized estimator) were considered as influential.
- However, these influence measures do not seem to account for
  - the sampling design;
  - the estimator used to estimate t_y;
  - the parameter to be estimated (total, ratio, etc);
- Alternative influence measure?

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  - (Revenue, Portion of the annual revenue for the first quarter, stratified simple random sampling, HT estimator).

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- An unit may be influential with respect to a configuration and have no influence with respect to another configuration.
- Modifying a single element of the configuration may have a drastic impact on the influence of an unit
- How measure the influence of an unit?

Measuring the influence: the conditional bias

- Let I_i be a sample selection indicator such that $I_i = 1$ if $i \in s$ and $I_i = 0$, otherwise.

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- Conditional bias B_{1i}: measure of influence of a sample unit.
- see Moreno-Rebollo, Munoz-Reyes and Munoz-Pichardo (1999) and Beaumont, Haziza and Ruiz Gazen (2014).

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- Let i be a nonsampled unit (i.e., $I_i = 0$). The conditional bias of unit i with respect to $\widehat{\theta}$ is defined as

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- Interpretation of B_{0i}: average of the sampling error taken among all samples that do not contain unit i.
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- Although, a nonsample unit can have a large influence, nothing can be done at the estimation stage because its y-value is not observed. At the estimation stage, only the influence of sample units can be curbed down.

Conditional bias: Horvitz-Thompson estimator

- Goal: estimate $t_y = \sum_{i \in U} y_i$
- Horvitz-Thompson estimator of t_y:

$$\widehat{t}_{HT} = \sum_{i \in s} d_i y_i.$$

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$$\begin{aligned} B_{1i}^{HT} &= E_p(\widehat{t}_{HT} - t_y | I_i = 1) \\ &= \sum_{j \in U} \left(\frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \right) y_j, \end{aligned}$$

where $\pi_{ij} = P(i \in s \;\&\; j \in s)$ denotes the joint inclusion probability of units i and j in the sample.

Remarks

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Conditional bias: special cases

- Simple random sampling without replacement: $\pi_i = n/N$ for all i and $\pi_{ij} = n(n-1)/N(N-1)$ for all $i \neq j$.

$$B_{1i}^{HT} = \frac{N}{(N-1)} \left(\frac{N}{n} - 1 \right) (y_i - \overline{Y}),$$

where $\overline{Y} = Y/N$ is the population mean.

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Consider a population of size $N = 100$ such that

$$y_1 = 0, \; y_2 = \cdots = y_{99} = 500 \text{ and } y_{100} = 1000 \Rightarrow \overline{Y} = 500.$$

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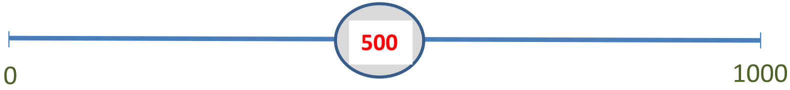
Poisson sampling: unit 1 has no influence, whereas unit 100 has a large influence.

$$B_{1i}^{HT} = (d_i - 1)(y_i - 0)$$

Poisson sampling vs. simple random sampling without replacement



Link between the conditional bias and the sampling error

- For Poisson sampling, the sampling error can be written as:

$$\widehat{t}_{HT} - t_y = \sum_{i \in s} B_{1i}^{HT} + \sum_{i \in U-s} B_{0i}^{HT}.$$

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- Interpretation: the conditional bias attached to unit i (sampled or not) is a measure of its contribution to the sampling error.
- For simple random sampling without replacement and high entropy sampling designs:

$$\widehat{t}_{HT} - t_y \approx \sum_{i \in s} B_{1i}^{HT} + \sum_{i \in U-s} B_{0i}^{HT}.$$

provided that the population size N is large.

Link between the conditional bias and the variance of $\widehat{t}_{HT}$

- For an arbitrary sampling design, the sampling variance of $\widehat{t}_{HT}$ is given by:

$$
\begin{aligned}
V_p\left(\widehat{t}_{HT}\right) &= \sum_{i\in U}\sum_{j\in U}\left(\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}\right)y_iy_j \\
&= \sum_{i\in U} B_{1i}^{HT} y_i
\end{aligned}
$$

Calibration estimators

- Conditional bias of $\widehat{t}_C$ attached to sample unit i :

$$
\begin{aligned}
B_{1i}^{C} &= E_p(\widehat{t}_C - t_y | I_i = 1) \\
&\approx \sum_{j \in U} \left(\frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \right) E_j,
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where $E_i = y_i - \mathbf{x}_i' \mathbf{B}$ denotes the census residual attached to unit i.

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- Recall: For $\widehat{t}_C$, the conditional bias of unit i was given by the previous expression with y_i instead of E_i.

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- Often, we have $\bar{E} = 0$.
- Conclusion: The conditional bias takes the type of estimator into account.

Method based on the conditional bias

- Proposed by Beaumont, Haziza and Ruiz-Gazen (2013).
- Consider a robust estimator of $\widehat{t}_{HT}$:

$$\widehat{t}_{BHR}(K) = \widehat{t}_{HT} - \sum_{i \in s} \widehat{B}_{1i}^{HT} + \sum_{i \in s} \psi_K\left(\widehat{B}_{1i}^{HT}\right)$$

where $\psi_K(\cdot)$ is the Huber function given by

$$\psi_K(x) = \begin{cases} K & \text{if } x > K \\ x & \text{if} |x| \leq K \\ -K & \text{if } x < -K \end{cases}$$

- Role of the ψ-function: reduce the influence of units that have a large influence.

Method based on the conditional bias

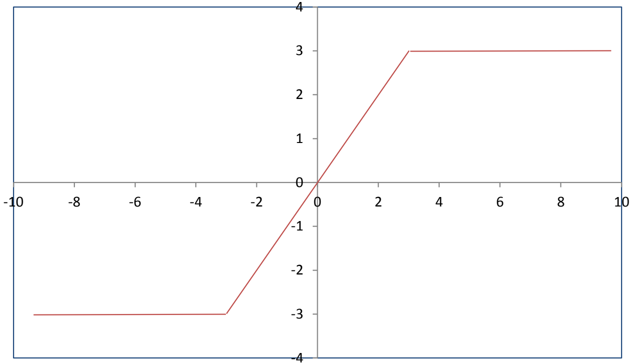
- The tuning constant K may be determined so that it minimizes the estimated mean square error of $\widehat{t}_{BHR}(K) \mapsto$ usually complex without simplifying assumptions.
- Alternative choice of K: determine K which minimizes $\max_{i \in s} \left\{ |\hat{B}^R_{1i}| \right\}$:

$$\widehat{t}_{BHR}(K_{opt}) = \widehat{t}_{HT} - \frac{1}{2}(\widehat{B}^{HT}_{min} + \widehat{B}^{HT}_{max}).$$

- Easy to implement.
- $\widehat{t}_{BHR}(K_{opt})$ is design-consistent for t_y.

Huber function $\psi_3(x)$ (with $K = 3$)

- Remark: $0 \leq \frac{\psi_K(x)}{x} \leq 1$.



Method based on the conditional bias

- The BHR estimator can be written as

$$\widehat{t}_{BHR}(K_{opt}) = \sum_{i \in s} \widetilde{d}_i y_i,$$

where

$$\widetilde{d}_i = d_i - \frac{\alpha_i}{y_i} \widehat{B}_{1i}^{HT},$$

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$$\alpha_i = 1 - \psi_K(\widehat{B}_{1i}^{HT}) / \widehat{B}_{1i}^{HT}.$$

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where $\phi_i = \alpha_i \left(\frac{y_i - \bar{Y}}{y_i} \right)$
  - If $\alpha_i = 0 \Rightarrow \phi_i = 0 \Rightarrow \widetilde{d}_i = N/n$
  - If $\alpha_i = 1$

$$\Rightarrow \widetilde{d}_i = 1 + \left(\frac{N}{n} - 1 \right) \frac{\overline{Y}}{y_i}.$$

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