A

$$\begin{array}{c}\boldsymbol{y}\\ \begin{bmatrix} y_{1(\mathrm{f})}\\ y_{2(\mathrm{f})}\\ y_{3(\mathrm{f})}\\ y_{4(\mathrm{m})}\\ y_{5(\mathrm{m})}\\ y_{6(\mathrm{m})}\end{bmatrix}\end{array} = \begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\mu + \begin{array}{c}\boldsymbol{h}\\ \begin{bmatrix}1\\1\\1\\0\\0\\0\end{bmatrix}\end{array}\mu_s + \begin{array}{c}\boldsymbol{X}\\ \begin{bmatrix}X_1 & X_1\\ X_2 & X_2\\ X_3 & X_3\\ X_4 & 0\\ X_5 & 0\\ X_6 & 0\end{bmatrix}\end{array}\begin{bmatrix}\beta\\ \beta_{g\times s}\end{bmatrix} + \boldsymbol{u}_g + \boldsymbol{u}_f + \boldsymbol{u}_m + \boldsymbol{e}_f + \boldsymbol{e}_m$$

$$\text{where } \boldsymbol{u}_g \sim N\left(0, \boldsymbol{\sigma}_g^2 \boldsymbol{K}_g\right),\ \boldsymbol{u}_f \sim N\left(0, \boldsymbol{\sigma}_{g,f}^2 (\boldsymbol{K} \circ \boldsymbol{h}\boldsymbol{h}^T)\right)$$

$$, \boldsymbol{u}_m \sim N\left(0, \boldsymbol{\sigma}_{g,m}^2 (\boldsymbol{K} \circ (1-\boldsymbol{h})(1-\boldsymbol{h})^T)\right)$$

$$, \boldsymbol{e}_f \sim N\left(0, \boldsymbol{\sigma}_{e,f}^2 (\boldsymbol{I} \circ \boldsymbol{h}\boldsymbol{h}^T)\right), \boldsymbol{e}_m \sim N\left(0, \boldsymbol{\sigma}_{e,m}^2 (\boldsymbol{I} \circ (1-\boldsymbol{h})(1-\boldsymbol{h})^T)\right)$$

$$\boldsymbol{K} = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} & r_{1,5} & r_{1,6}\\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} & r_{2,5} & r_{2,6}\\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} & r_{3,5} & r_{3,6}\\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} & r_{4,5} & r_{4,6}\\ r_{5,1} & r_{5,2} & r_{5,3} & r_{5,4} & r_{5,5} & r_{5,6}\\ r_{6,1} & r_{6,2} & r_{6,3} & r_{6,4} & r_{6,5} & r_{6,6}\end{bmatrix}$$

$$\boldsymbol{K} \circ (\boldsymbol{h}\boldsymbol{h})^T = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & 0 & 0 & 0\\ r_{2,1} & r_{2,2} & r_{2,3} & 0 & 0 & 0\\ r_{3,1} & r_{3,2} & r_{3,3} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\boldsymbol{K} \circ (1-\boldsymbol{h})(1-\boldsymbol{h})^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & r_{4,4} & r_{4,5} & r_{4,6}\\ 0 & 0 & 0 & r_{5,4} & r_{5,5} & r_{5,6}\\ 0 & 0 & 0 & r_{6,4} & r_{6,5} & r_{6,6}\end{bmatrix}$$

B

Females

$$\begin{bmatrix} y_{1(\mathrm{f})}\\ y_{2(\mathrm{f})}\\ y_{3(\mathrm{f})}\end{bmatrix} = \mu_{\mathrm{f}}\begin{bmatrix}1\\1\\1\end{bmatrix} + \beta_{\mathrm{f}}\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix} + \boldsymbol{\nu}_{\mathrm{f}} + \boldsymbol{\varepsilon}_{\mathrm{f}}$$

$$\text{where } \mu_{\mathrm{f}} = \mu + \mu_s, \beta_{\mathrm{f}} = \beta_{\mathrm{g\times s}} + \beta$$

$$\boldsymbol{\nu}_{\mathrm{f}} \sim N\left(0, (\boldsymbol{\sigma}_{\mathrm{g}}^2 + \boldsymbol{\sigma}_{\mathrm{g,f}}^2)\boldsymbol{K}_{\mathrm{f}}\right)$$

$$\boldsymbol{\varepsilon}_{\mathrm{f}} \sim N\left(0, \boldsymbol{\sigma}_{\mathrm{e,f}}^2 \boldsymbol{I}\right)$$

$$\boldsymbol{K}_{\mathrm{f}} = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3}\\ r_{2,1} & r_{2,2} & r_{2,3}\\ r_{3,1} & r_{3,2} & r_{3,3}\end{bmatrix}$$

Males

$$\begin{bmatrix} y_{4(\mathrm{m})}\\ y_{5(\mathrm{m})}\\ y_{6(\mathrm{m})}\end{bmatrix} = \mu_{\mathrm{m}}\begin{bmatrix}1\\1\\1\end{bmatrix} + \beta_{\mathrm{m}}\begin{bmatrix}X_4\\X_5\\X_6\end{bmatrix} + \boldsymbol{\nu}_m + \boldsymbol{\varepsilon}_{\mathrm{m}}$$

$$\text{where } \mu_{\mathrm{m}} = \mu + \mu_s, \beta_{\mathrm{m}} = \beta_{\mathrm{g\times s}} + \beta$$

$$\boldsymbol{\nu}_{\mathrm{m}} \sim N\left(0, (\boldsymbol{\sigma}_{\mathrm{g}}^2 + \boldsymbol{\sigma}_{\mathrm{g,m}}^2)\boldsymbol{K}_{\mathrm{m}}\right)$$

$$\boldsymbol{\varepsilon}_{\mathrm{m}} \sim N\left(0, \boldsymbol{\sigma}_{\mathrm{e,m}}^2 I\right)$$

$$\boldsymbol{K}_{\mathrm{m}} = \begin{bmatrix} r_{4,4} & r_{4,5} & r_{4,6}\\ r_{5,4} & r_{5,5} & r_{5,6}\\ r_{6,4} & r_{6,5} & r_{6,6}\end{bmatrix}$$

C

$$\hat{\beta}_{\mathrm{f}} \quad SE\left(\hat{\beta}_{\mathrm{f}}\right)$$

$$\hat{\beta}_{\mathrm{m}} \quad SE\left(\hat{\beta}_{\mathrm{m}}\right)$$

Female-specific Test

Random Effects Model Meta Analysis

Male-specific Test