#### Elements Of Data Science - S2022

Week 7: Model Evaluation and Hyperparameter Tuning

3/1/2022

#### **TODOs**

- Readings:
  - **PDSH Chapter 5: In-Depth: Decision Trees and Random Forests**
  - HOML: Chapter 6
  - HOML: Chapter 7
  - PDSH Chapter 5: Feature Engineering
- Quiz 7, due Monday March 7th, 11:59pm ET
- HW2, out this week Fri March 11th, 11:59pm ET
- Project
  - March 22nd presentation
  - Details will be released on Canvas

# Today

- Model Evaluation and Selection
- Hyperparameter Tuning
- Regularization

Questions?

# **Environment Setup**

In [1]:

```
import numpy
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

from mlxtend.plotting import plot_decision_regions
sns.set_style('darkgrid')
%matplotlib inline
```

# Model Evaluation and Hyperparameter Tuning

- How well are any of our models working?
- How can we compare different models?
- How do we decide on hyperparameter settings?
- How can we keep our models from "overfitting"?
- How do we do all this both for Regression and Classification?

# How well are our models performing?

#### Regression

- Mean Squared Error (MSE) and Root Mean Squared Error (RMSE)
- R<sup>2</sup> or R^2
- (Adjusted R<sup>2</sup> will talk about during Feature Selection)

#### Classification

- Accuracy
- Precision/Recall/F1
- ROC Area Under the Curve (AUC)

#### Data Setup for Regression

```
In [2]:
zscore = lambda x: (x - x.mean()) / x.std()
df wine = pd.read csv('../data/wine dataset.csv',
               usecols=['alcalinity_of_ash','magnesium','alcohol','ash','proline','hue','class'])
numeric cols = ['alcalinity of ash','magnesium','alcohol','ash','proline','hue']
df_wine[numeric_cols] = df_wine[numeric_cols].apply(zscore) # standardize numeric feature cols
X = df wine[['proline', 'hue', 'ash']]
y_r = df_wine['alcohol']
                                          # regression target
In [3]:
df_wine.info()
 <class 'pandas.core.frame.DataFrame'>
 RangeIndex: 178 entries, 0 to 177
Data columns (total 7 columns):
                                      Non-Null Count Dtype
        Column
  #
        alcohol
                                      178 non-null
                                                               float64
                                      178 non-null
                                                              float64
        ash
        alcalinity_of_ash
                                                              float64
                                     178 non-null
        magnesium
  3
                                      178 non-null
                                                               float64
```

4 hue 178 non-null float64
5 proline 178 non-null float64
6 class 178 non-null int64

dtypes: float64(6), int64(1)

memory usage: 9.9 KB

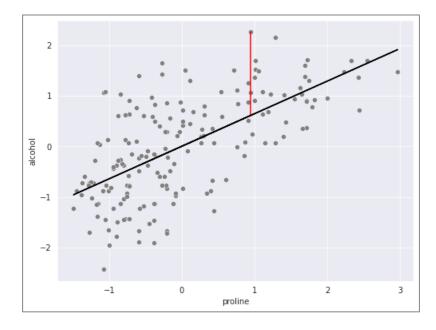
# Regression with Simple Linear Model

In [4]:

```
from sklearn.linear_model import LinearRegression

lr = LinearRegression().fit(X[['proline']],y_r)
argmax_y_r = np.argmax(y_r)
y_pred = lr.predict(X[['proline']])

fig,ax = plt.subplots(1,1,figsize=(8,6))
sns.scatterplot(x=X.proline, y=y_r, color='grey');
ax.plot(X.proline,y_pred,color='k');
ax.vlines(X.proline.iloc[argmax_y_r],y_r.iloc[argmax_y_r],y_pred[argmax_y_r],color='r');
```



#### How Good is This Fit? MSE and RMSE

• Mean Squared Error:  $\frac{1}{n} \sum_{i} (y_i - \hat{y}_i)^2$ 

In [5]:

```
from sklearn.metrics import mean_squared_error

lr_mse = mean_squared_error(y_r,y_pred)
print(f'{lr_mse = :0.2f}')
```

$$lr_mse = 0.58$$

- But this is the squared error! (alcohol^2)
- Root Mean Squared Error:  $\sqrt{\frac{1}{n}\sum_{i}{(y_i \hat{y}_i)^2}}$

In [6]:

```
def root_mean_squared_error(y_true, y_pred):
    return np.sqrt(mean_squared_error(y_true,y_pred))

lr_rmse = root_mean_squared_error(y_r,y_pred)
print(f'{lr_rmse = :0.2f}')
```

$$lr\_rmse = 0.76$$

## Is this good? Need a Baseline Comparison

- What's a baseline to compare against?
- Simple one for Regression: always predict the mean of the targets

In [7]:

```
from sklearn.dummy import DummyRegressor

dummyr = DummyRegressor(strategy='mean') # default strategy
dummyr.fit(X[['proline']],y_r)

dummy_rmse = root_mean_squared_error(y_r,dummyr.predict(X[['proline']]))
print(f'{dummy_rmse = :0.2f}')
```

 $dummy_rmse = 1.00$ 

#### Comparing against the mean: R2

• the proportion of variance explained by the model

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y}_i)^2}$$

- maximum value of 1
- a value below 0 means the model is predicting worse than just predicting the mean
- sklearn uses R<sup>2</sup> as the default for regression scoring

```
In [8]:
```

```
r2_lr = lr.score(X[['proline']],y_r)
r2_dummyr = dummyr.score(X[['proline']],y_r)
print(f'{r2_dummyr = :0.2f}\n{r2_lr = :0.2f}')
```

#### Can we do better?

In [9]:

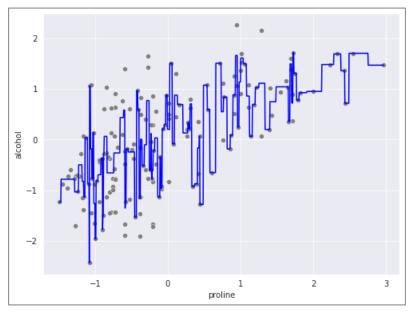
```
from sklearn.tree import DecisionTreeRegressor

dtr = DecisionTreeRegressor(max_depth=10)
dtr.fit(X[['proline']],y_r)
r2_dtr = dtr.score(X[['proline']],y_r)
print(f'{r2_lr = :0.2f}\n{r2_dtr = :0.2f}')
```

```
r2_{lr} = 0.41
r2_{dtr} = 0.76
```

In [10]:

```
X_proline_sorted = X[['proline']].sort_values(by='proline').values
X_query = np.linspace(X_proline_sorted.min(),X_proline_sorted.max(),1000).reshape(-1,1)
y_pred = dtr.predict(X_query)
fig,ax = plt.subplots(1,1,figsize=(8,6))
sns.scatterplot(x=X.proline, y=y_r,color='gray');
ax.plot(X_query,y_pred,color='b');
```



#### But is this what we want? Interpretation vs Prediction

#### Always good to ask:

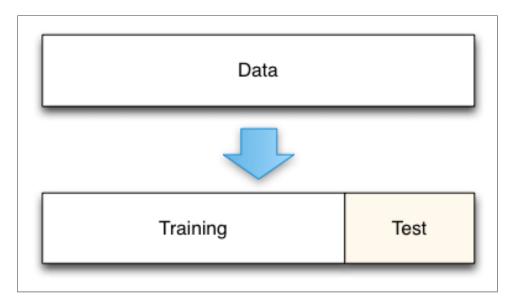
- do we want our model to very closely fit our data for interpretation?
- do we want our model to predict well on new, unseen data?

#### **Generalization:**

- how well will model predict on data that it hasn't seen yet?
- But we used all of our data to train?
- Need to do a **Train/Test Split** to create a held-aside set

## Train/Test Split

- Training Set: portion of dataset used for training
- Test/Held-Aside/Out of sample: portion of dataset used for evaluation
- Want the test set to reflect the same distribution as training



From https://www.researchgate.net/figure/Train-Test-Data-Split\_fig6\_325870973

### Train/Test split with Sklearn

In [11]:

```
n_total = 178
n_train = 133
n_test = 45
```

- How big should test be?
  - Large enough to capture variance of dataset.
  - Depends on the dataset and the models being trained

# Training and Evaluate on Different Data

```
In [12]:
```

```
dummyr = DummyRegressor().fit(X_train_r,y_train_r)
lr = LinearRegression().fit(X_train_r,y_train_r)
dtr = DecisionTreeRegressor(max_depth=10).fit(X_train_r,y_train_r)

r2_dummyr = dummyr.score(X_test_r,y_test_r)
r2_lr = lr.score(X_test_r,y_test_r)
r2_dtr = dtr.score(X_test_r,y_test_r)

print(f'{r2_dummyr = : 0.2f}\n{r2_lr = : 0.2f}\n{r2_dtr = : 0.2f}')
```

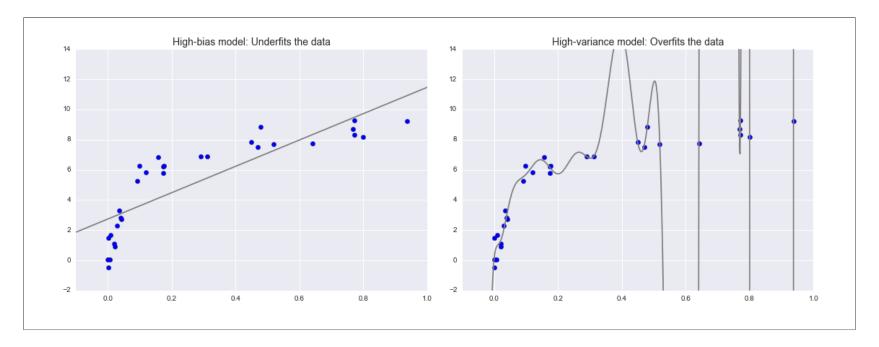
```
r2_dummyr = -0.03
r2_lr = 0.28
r2_dtr = -0.31
```

• DecisionTree model is doing worse than the Dummy model on the test set!

# Overfitting and Underfitting

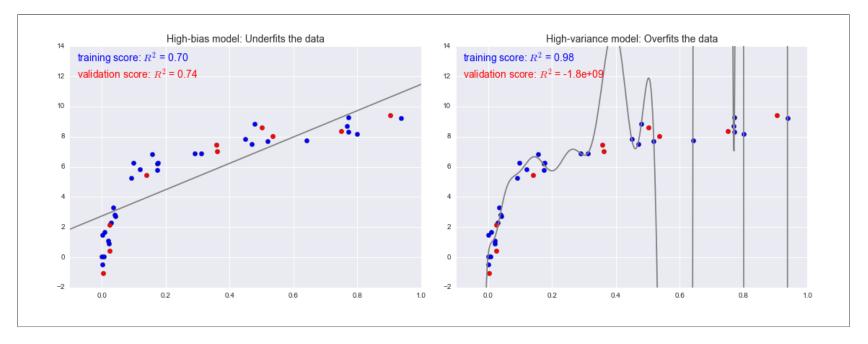
- Overfitting: poor generalization due to complexity
  - learning noise in training data
- Underfitting: poor generalization due to simplicity
  - not flexibile enough to learn concept
- Need to find a balance between simplicity and complexity
- Need to find a balance between bias and variance

### Bias-Variance Tradeoff



From PDSH

#### Bias-Variance Tradeoff



From PDSH

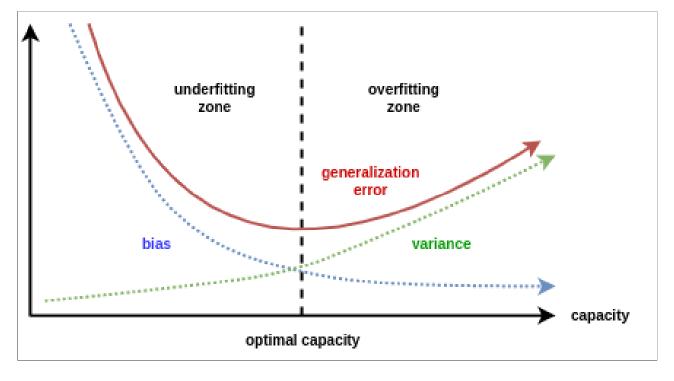
#### Bias-Variance Tradeoff Continued

- How close is the model to the underlying concept?
- How sensitive is the model to the training set?



Fig. 1: Graphical Illustration of bias-<u>variance trade</u>-off , Source: Scott Fortmann-Roe., Understanding Bias-Variance Trade-off

#### Bias-Variance Tradeoff Continued



- We'd like to:
  - reduce the Bias (use a model complex enough to capture the concept)
  - without introducing too much Variance (overfit the data)
  - all in order to minimize Generalization Error

## Overfitting/Underfitting Revisited

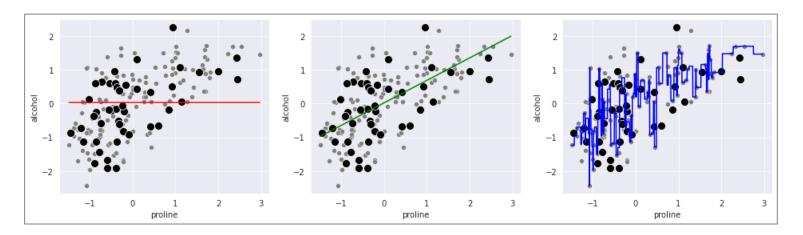
- Overfitting: poor generalization due to complexity
  - learning noise in training data
  - model has high variance and low bias
- Underfitting: poor generalization due to simplicity
  - not flexibile enough to learn concept
  - model has high bias and low variance

# Avoiding Overfitting/Underfitting

- Never train and evaluate on the same set of data!
  - train test split
  - cross-validation
- Keep the model as simple as possible (Occom's Razor)

#### In [13]:

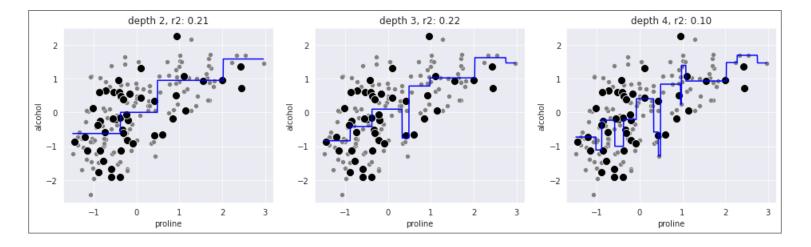
```
fig,ax = plt.subplots(1,3,figsize=(16,4))
for i in range(3):
    sns.scatterplot(x=X_train_r.proline,y=y_train_r,color="gray",ax=ax[i])
    sns.scatterplot(x=X_test_r.proline,y=y_test_r,color="black",s=100,ax=ax[i]);
ax[0].plot(X_query,dummyr.predict(X_query),color='r');
ax[1].plot(X_query,lr.predict(X_query),color='g');
ax[2].plot(X_query,dtr.predict(X_query),color='b');
```



### Overfitting? Simplify the model

In [14]:

```
max_depths = [2,3,4]
fig,ax = plt.subplots(1,3,figsize=(16,4))
for i in range(3):
    dtr = DecisionTreeRegressor(max_depth=max_depths[i]).fit(X_train_r,y_train_r)
    sns.scatterplot(x=X_train_r.proline,y=y_train_r,color="gray",ax=ax[i])
    sns.scatterplot(x=X_test_r.proline,y=y_test_r,color="black",s=100,ax=ax[i]);
    ax[i].plot(X_query,dtr.predict(X_query),color='b');
    ax[i].set_title(f'depth {max_depths[i]}, r2: {dtr.score(X_test_r,y_test_r):0.2f}')
```



- But now we might be overfitting on the test set!
- How to choose hyperparameters: **Cross-Validation**

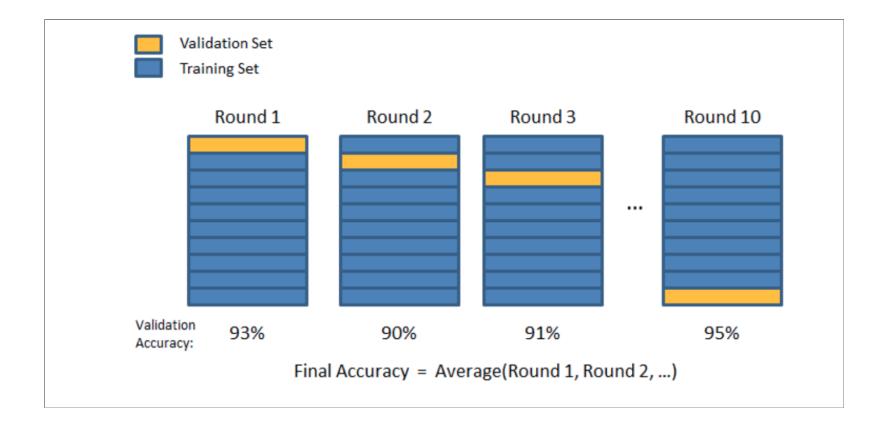
## Aside: Hyperparameters

- parameter: something learned by the model itself (eg. coefficient in linear model)
- hyperparameter: something we set by hand (eg. decision tree max depth)

#### k-Fold Cross-Validation

- 1. split dataset into k equal sized subsets (folds)
- 2. for each subset (fold)
  - train on the other k-1 subsets combined
  - test on this subset to get a score
- 3. average across all scores
- Result is a set of samples of model performance
- Can use to set hyperparameters without overfitting on train or test
- Can also use to estimate range of generalization performance

# Example: 10-Fold Cross-Validation



#### k-Fold Cross-Validation Continued

- Can be used for:
  - tuning hyperparameters
  - model selection
  - any time we need estimate of model performance
- Issue: each fold requires training the model
  - Training time can be an issue for large k
- What values can k take?
  - min: 2
  - max: n, the size of the dataset (aka Leave-One-Out CV)

#### k-Fold Cross-Validation in sklearn

0.35 + - 0.18

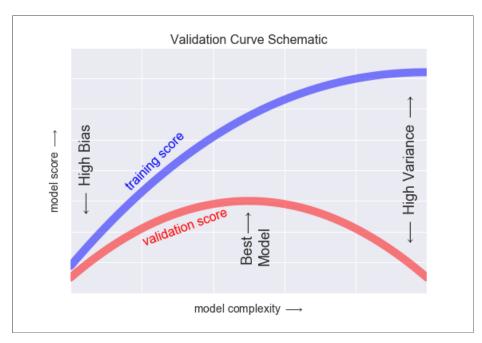
### Tuning Hyperparameters with CV

```
In [17]:
mean_scores = []
for depth in [1, 2, 3, 5, 10]:
   dtr = DecisionTreeRegressor(max_depth=depth)
   scores = cross val score(dtr,X train r,y train r,cv=5)
   mean scores.append( (depth, scores.mean().round(3)) )
for depth, mean score in mean scores:
   print(f'{depth = :2d} : {mean score: 0.3f}')
 depth = 1 : 0.289
depth = 2 : 0.346
 depth = 3 : 0.341
 depth = 5 : 0.050
 depth = 10 : -0.142
In [18]:
# find the depth that gives best score (highest R^2)
sorted(mean scores, key=lambda x:x[1],reverse=True)[0] # sorted is ascending by default
Out[18]:
 (2, 0.346)
```

### Visualize Tuning: Validation Curve

#### **Validation Curve**

- Show model complexity vs model performance on both train and test/validation
- Want to find point where performance on validation set begins to decline (overfitting)



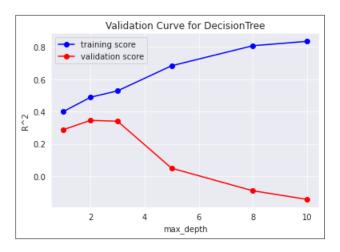
From PDSH

### Validation Curve in sklearn

In [19]:

In [20]:

```
fig,ax = plt.subplots(1,1)
ax.plot(depth, mean_train_scores, 'o-', color='b',label='training score');
ax.plot(depth, mean_test_scores, 'o-', color='r', label='validation score');
ax.set_xlabel('max_depth'), ax.set_ylabel('R^2'); ax.set_title('Validation Curve for DecisionTree');
ax.legend();
```



## More Than One HyperParameter? Grid Search

**Grid Search:** Search over a 'grid' of hyperparameter settings Example: KNN "number of neighbors" and "distance metric"

```
In [21]:

distance_metrics = ['euclidean', 'manhattan']
n_neighbors = [1,3,5]

grid = []
for d in distance_metrics:
    for k in n_neighbors:
        print([d,k])

['euclidean', 1]
['euclidean', 3]
['euclidean', 5]
['manhattan', 1]
['manhattan', 1]
['manhattan', 3]
['manhattan', 5]
```

### Grid Search in sklearn

In [22]:

```
{'metric': 'euclidean', 'n_neighbors': 5}
```

```
In [23]:
```

```
scores = cross_val_score(gscv.best_estimator_,X_train_r,y_train_r,cv=5)
print(f'{np.mean(scores):0.2f} +- {2*np.std(scores):0.2f}')
```

0.35 + - 0.21

### Review So Far

- Regression Metrics
  - MSE and RMSE
  - $\blacksquare$  R<sup>2</sup>
- Model Selection
  - Comparison to Baseline Model
  - Underfitting/Overfitting, Bias/Variance
  - Train/Test Split
- Hyperparameter Tuning
  - Cross-Validation
  - Grid Search
  - Validation Curve

In [24]:

### Data Setup for Classification

```
idx_binary = df_wine['class'].isin([0,1])
                                                       # reduce to binary classification
X_bc = df_wine.loc[idx_binary,['alcalinity_of_ash','magnesium']] # only 2 features for ease of plotting
                                           # pull out classification target [0,1]
y bc = df wine.loc[idx binary,'class']
X train bc,X test bc,y train bc,y test bc = train test split(X bc,
                                                            y_bc,
                                                            stratify=y bc, # maintain label proportions
                                                            random state=0
pd.DataFrame({'train':y train bc.value counts(),'test':y test bc.value counts()}).sort index()
Out[24]:
           test
0 44
   53
          18
In [25]:
X mc = df wine.loc[:,['alcalinity of ash', 'magnesium']]
                                                             # multiple features for multiclass classification task
y_mc = df_wine.loc[:,'class']
                                                             # pull out classification target [0,1,2]
X_train_mc,X_test_mc,y_train_mc,y_test_mc = train_test_split(X_mc,
                                                            stratify=y_mc, # maintain label proportions
                                                            random_state=123
pd.DataFrame({'train':y_train_mc.value_counts(),'test':y_test_mc.value_counts()}).sort_values(by="train")
Out[25]:
    train
           test
```

	train	test
2	36	12
0	44	15
1	53	18

### Default Metric in Classification: Accuracy

• Accuracy: out of all the observations, how many did I get right?

In [26]:

```
from sklearn.dummy import DummyClassifier
from sklearn.tree import DecisionTreeClassifier
dummyc = DummyClassifier(strategy='prior').fit(X_train_bc,y_train_bc) # works like 'most-frequent'
dtc = DecisionTreeClassifier(max_depth=2).fit(X_train_bc,y_train_bc)

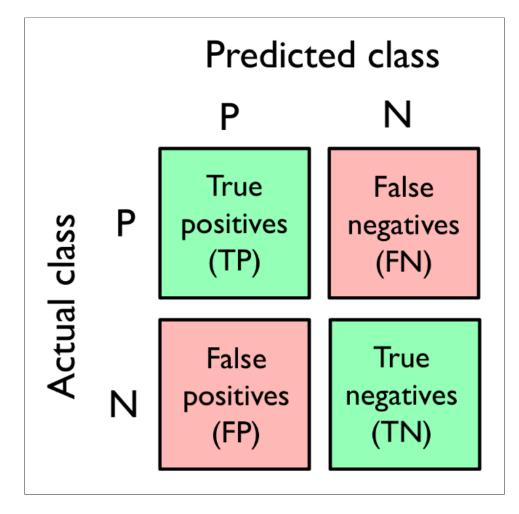
print(f'{dummyc.score(X_test_bc,y_test_bc) = :0.2f}') # default classification score is accuracy
print(f'{dtc.score(X_test_bc,y_test_bc) = :0.2f}')
```

```
dummyc.score(X_test_bc,y_test_bc) = 0.55
dtc.score(X_test_bc,y_test_bc) = 0.79
```

- But what if the cost of calling a negative a positive is different from calling a positive a negative?
- Examples:
  - disease testing
  - medical product failures
  - incarceration

### Errors in Classification

• There are different kinds of error in classification

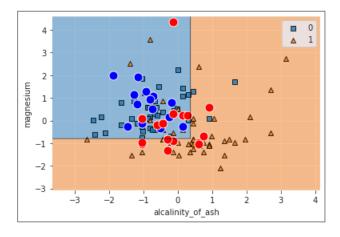


From PML

## Visualizing Errors with a Confusion Matrix

In [27]:

```
fig,ax = plt.subplots(1,1,figsize=(6,4));
plot_decision_regions(X_train_bc.values,y_train_bc.values,dtc,ax=ax);
sns.scatterplot(x=X_bc.columns[0],y=X_bc.columns[1],data=X_test_bc[y_test_bc == 0],color="blue",s=120);
sns.scatterplot(x=X_bc.columns[0],y=X_bc.columns[1],data=X_test_bc[y_test_bc == 1],color="red",s=120);
```



In [28]:

```
from sklearn.metrics import confusion_matrix

print('training set error\n', confusion_matrix(y_train_bc,dtc.predict(X_train_bc)))
print()
print('test set error\n', confusion_matrix(y_test_bc,dtc.predict(X_test_bc)))
```

```
training set error
[[41 3]
```

[14 39]]

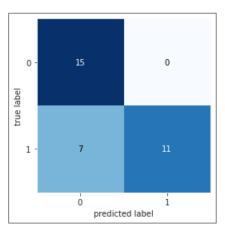
test set error [[15 0] [ 7 11]]

### Plot Confusion Matrix with mlxtend

In [29]:

```
from mlxtend.plotting import plot_confusion_matrix

fig,ax = plt.subplots(1,1,figsize=(4,4))
plot_confusion_matrix(confusion_matrix(y_test_bc,dtc.predict(X_test_bc)),axis=ax);
```



## Weighing Errors: Precision vs. Recall

#### **Precision**

• Out of the observations I predicted positive (TP+FP), how many are truly positive (TP)?

precision = 
$$\frac{TP}{TP+FP}$$

#### Recall

• Out of the truly positive (TP+FN), how many obsersevations did I predict positive (TP)?

$$recall = \frac{TP}{TP+FN}$$

### Using Other Measures in sklearn

In [30]:

```
dummyc_precision_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='precision')
dummyc_recall_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='precision')
print(f'dummy precision: {np.mean(dummyc_precision_scores):0.2f} +- {2*np.std(dummyc_precision_scores):0.2f}')
print(f'dummy recall : {np.mean(dummyc_recall_scores):0.2f} +- {2*np.std(dummyc_recall_scores):0.2f}')
print()

dtc_precision_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='precision')
dtc_recall_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='recall')

print(f'dtc_precision_scores):0.2f} +- {2*np.std(dtc_precision_scores):0.2f}')
print(f'dtc_recall_scores):0.2f} +- {2*np.std(dtc_precision_scores):0.2f}')
```

```
dummy precision: 0.55 +- 0.04
dummy recall : 1.00 +- 0.00
```

```
dtc precision : 0.85 +- 0.36
dtc recall : 0.66 +- 0.38
```

## How do we decide if something is positive or negative?

Usually set a threshold:

$$\hat{y}_i = \begin{cases} 1 & \text{if } P(y_i = 1 | x_i) > \text{threshold,} \\ \\ 0 & \text{o.w.} \end{cases}$$

Usually, threshold = .5, but it doesn't have to be. What happens if we change it?

- High threshold → High Precision, Low Recall
- Low threshold → High Recall, Low Precision

## Combining Precision and Recall: F1-score

Usually, we just want one number to optimize  $F_1$ -score: harmonic mean of precision and recall

eg. weighted average of the precision and recall

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

### Note that F<sub>1</sub> ignores True Negatives!

```
In [31]:
```

```
dummyc_f1_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='f1')
dtc_f1_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='f1')
print(f'dummyc f1 = {np.mean(dummyc_f1_scores):0.2f} +- {2*np.std(dummyc_f1_scores):0.2f}')
print(f'dtc f1 = {np.mean(dtc_f1_scores):0.2f} +- {2*np.std(dtc_f1_scores):0.2f}')
```

```
dummyc f1 = 0.71 +- 0.03
dtc f1 = 0.72 +- 0.31
```

## Paying attention to True Negatives: ROC

### Receiver Operating Characteristic

displays FPR vs TPR

```
False Positive Rate (FPR) = \frac{FP}{FP+TN} = \frac{\text{negatives we got wrong}}{\text{all negatives}}

True Positive Rate (TPR) = Recall = \frac{TP}{TP+FN} = \frac{\text{positives we got right}}{\text{all positives}}
```

How do these change as we move our threshold?

## Plotting ROC Curves

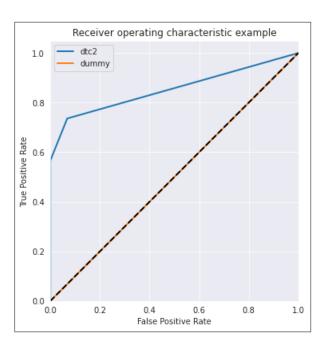
In [33]:

```
def plot_roc(curves):
    fig,ax = plt.subplots(1,1,figsize=(6,6))
    lw = 2
    for fpr,tpr,model_name in curves:
        l1, = ax.plot(fpr, tpr, lw=lw, label=model_name)
    ax.plot([0, 1], [0, 1], color='k', lw=lw, linestyle='--')
    ax.set_xlim([0.0, 1.0])
    ax.set_ylim([0.0, 1.05])
    ax.set_ylabel('False Positive Rate')
    ax.set_ylabel('True Positive Rate')
    ax.set_aspect('equal', 'box')
    ax.set_title('Receiver operating characteristic example')
    ax.legend()
```

# Plotting ROC Curves

In [34]:

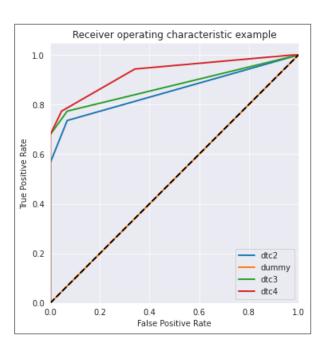
```
curves = [(fpr_dtc,tpr_dtc,'dtc2')]
fpr_dummyc, tpr_dummyc, _ = roc_curve(y_train_bc, dummyc.predict_proba(X_train_bc)[:,1]) # Compare dummy
curves.append((fpr_dummyc,tpr_dummyc,'dummy'));
plot_roc(curves);
```



# Plotting ROC Curves

In [35]:

```
for depth in [3,4]:
    fpr, tpr, _ = roc_curve(y_train_bc,DecisionTreeClassifier(max_depth=depth).fit(X_train_bc,y_train_bc).predict_proba(X_train_bc)[:,1]
        curves.append((fpr,tpr,'dtc'+str(depth)))
plot_roc(curves);
```



### **ROC AUC**

- But again, we'd like one number to optimize
- ROC Area Under the Curve
  - How much area falls under the ROC curve?

#### In [36]:

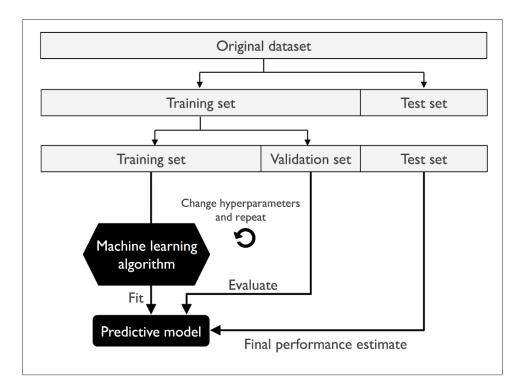
```
dummyc_rocauc_scores = cross_val_score(dummyc,X_train_bc,y_train_bc,cv=5,scoring='roc_auc')
dtc_rocauc_scores = cross_val_score(dtc,X_train_bc,y_train_bc,cv=5,scoring='roc_auc')

print(f'dummyc_rocauc = {np.mean(dummyc_rocauc_scores):0.2f} +- {2*np.std(dummyc_rocauc_scores):0.2f}')
print(f'dtc_rocauc = {np.mean(dtc_rocauc_scores):0.2f} +- {2*np.std(dtc_rocauc_scores):0.2f}')
```

```
dummyc rocauc = 0.50 +- 0.00
dtc rocauc = 0.78 +- 0.22
```

### Review: Steps to Choosing a Model

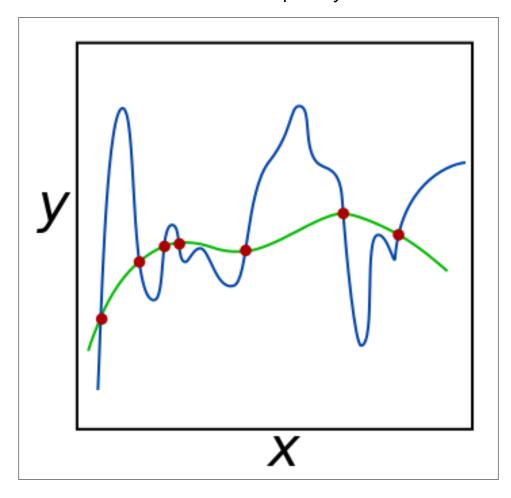
- 1. Create Held-Aside Set (Train/Test Split)
- 2. Determine Metric (or combination of metrics)
- 3. Get a Baseline for comparison
- 4. Use Cross-Validation to fit Hyperparameters and Choose Model
- 5. Evaluate Chosen Model on Held-Aside Set



From PML

# Avoiding Overfitting in Linear Models: Regularization

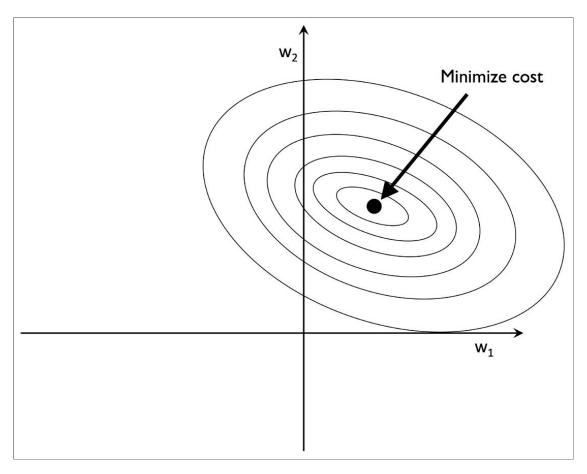
- Use to avoid overfitting in linear models
- Idea: can we reduce complexity of our linear model by minimizing weights?



From <a href="https://www.wikiwand.com/en/Regularization">https://www.wikiwand.com/en/Regularization</a> (mathematics)

## Regression: Finding the Weights

- Linear models learn by finding weights that minimize a cost.
- Can we get close to the solution while still keeping weights small (simpler model)?



### From PML

## Regularization: Add a cost for large weights

Penalizing extreme weights (w)

If the original cost function looks like:

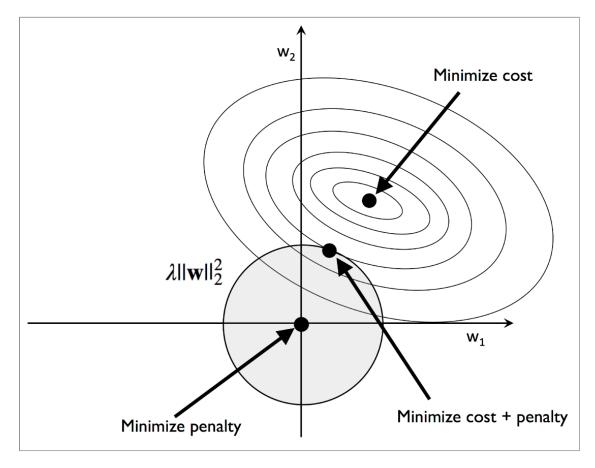
 $arg min_w C(f(w, x), y)$ 

we can add a **regularization term**:

 $arg min_w C(f(w, x), y) + \lambda g(w)$ 

# Regularization: Ridge

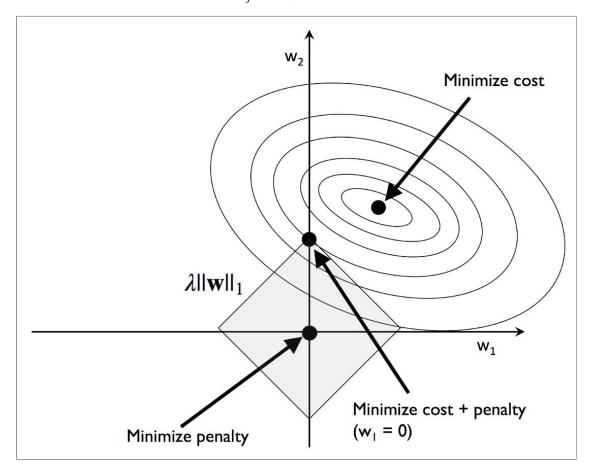
- Coefficients are encouraged to be "small"
- Uses  $L_2$  norm:  $\|w\|_2 = \sqrt{\sum_{j=1}^m w_j^2}$



From PML

# Regularization: LASSO

- Coefficients are (likely) driven to zero
- Uses  $L_1$  norm:  $\|\mathbf{w}\|_1 = \sum_{j=1}^m |\mathbf{w}_j|$



From PML

# Regularization: ElasticNet

- Mixture of L<sub>1</sub> and L<sub>2</sub>
- $\alpha L_1 + (1 \alpha)L_2$
- introduces a new hyperparameter α or l1\_ratio
- 11\_ratio = 1 is LASSO ( $L_1$ )
- $11_{\text{ratio}} = 0$  is Ridge (L<sub>2</sub>)

### Regularization in sklearn

```
In [37]:
from sklearn.linear model import LogisticRegression
logr = LogisticRegression(penalty='12', # default
                             # weight on regularization, 1/lambda above
                      C=1.0,
                      11 ratio=None # only used when penalty is 'elasticnet'
In [38]:
for C in [.001,.1,1,10,1000]:
   logr = LogisticRegression(penalty='12', # default
                                   # weight on regularization, 1/lambda above
                     ).fit(X train bc,y train bc)
   print(f'{str(C):5s} : {logr.coef [0].round(2)}')
0.001 : [ 0.02 - 0.02]
 0.1 : [0.69 - 0.57]
     : [ 1.09 -0.93]
 10 : [ 1.19 -1.02]
 1000 : [ 1.2 -1.03]
In [39]:
for C in [.001,.1,1,10,1000]:
   logr = LogisticRegression(penalty='l1',
                                     # weight on regularization, 1/lambda above
                         solver='liblinear'
                     ).fit(X_train_bc,y_train_bc)
   print(f'{str(C):5s} : {logr.coef_[0].round(2)}')
```

```
0.001 : [0. 0.]
0.1 : [ 0.5 -0.37]
1 : [ 1.08 -0.92]
10 : [ 1.19 -1.02]
1000 : [ 1.2 -1.03]
```

### GridSearchCV with Regularization

```
In [40]:
```

```
param_grid = {'l1_ratio':[0,.5,1],
              'C': [.001,.01,1,10]}
logr gscv = GridSearchCV(estimator=LogisticRegression(penalty='elasticnet',solver='saga'),
                         param_grid=param_grid,
                         cv=3,
                         n jobs=-1).fit(X train bc,y train bc)
print(f'best parameter setting found: {logr gscv.best params }')
print(f'best coefficients found : {logr gscv.best estimator .coef [0]}')
print(f'best training score found : {logr gscv.best score .round(3)}')
logr_gscv_test_score = logr_gscv.score(X_test_bc,y_test_bc)
logr noreg test score = (LogisticRegression(penalty='none')
                         .fit(X_train_bc,y_train_bc)
                         .score(X test bc,y test bc)
print()
print(f'logr gscv test score : {logr gscv test score.round(3)}')
print(f'logr noreg test score : {logr noreg test score.round(3)}')
```

```
best parameter setting found: {'C': 1, 'l1_ratio': 1}
best coefficients found : [ 1.09869298 -0.9278305
8]
```

best training score found : 0.825

logr\_gscv test score : 0.818
logr noreg test score : 0.818

### **ElasticNetCV**

In [41]:

```
from sklearn.datasets import make regression
from sklearn.linear model import ElasticNetCV
X_synth,y_synth = make_regression(n_samples=100,
                                  n features=200,
                                  n informative=10,
                                  random state=123
X synth train, X synth test, y synth train, y synth test = train test split(X synth, y synth, random state=123)
dummy synth = DummyRegressor(strategy='mean').fit(X synth train,y synth train)
lr synth = LinearRegression().fit(X synth train, y synth train)
en synth = ElasticNetCV(alphas=[.01,.1,1,100]).fit(X synth train,y synth train)
print(f'found alpha: {en synth.alpha }, found l1 ratio: {en synth.l1 ratio }\n')
print(f'{dummy synth.score(X synth train,y synth train) = : 0.2f}')
print(f'{lr synth.score(X synth train,y synth train) = : 0.2f}')
print(f'{en synth.score(X synth train,y synth train) = : 0.2f}\n')
print(f'{dummy synth.score(X synth test,y synth test) = : 0.2f}')
print(f'{lr_synth.score(X_synth_test,y_synth_test) = : 0.2f}')
print(f'{en synth.score(X synth test,y synth test) = : 0.2f}')
```

found alpha: 1.0, found l1\_ratio: 0.5

```
dummy_synth.score(X_synth_train,y_synth_train) = 0.00
lr_synth.score(X_synth_train,y_synth_train) = 1.00
en_synth.score(X_synth_train,y_synth_train) = 0.95
```

```
dummy_synth.score(X_synth_test,y_synth_test) = -0.00
lr_synth.score(X_synth_test,y_synth_test) = 0.16
en_synth.score(X_synth_test,y_synth_test) = 0.24
```

Questions?