REAR
a Regional ElAstic Rebound calculator

User Manual for version 1.0

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1 Introduction

REAR (acronym of "Regional ElAstic Rebound calculator") is a Fortran 90 program for computing the response of a SNREI (Solid, Non–Rotating, Elastic, Isotropic) Earth model to surface loading. REAR is free software and is released under the GPLv3 license (see *http://www.gnu.org/copyleft/gpl.html* for details).

REAR has been specifically designed for studying the regional deformations induced by variations of the cryospheric loads. As such, it only accounts for the direct effect of the surface load. On a global scale, this approximation could lead to significant errors and the use of a more realistic tool as a sea–level equation solver is recommended (see program SELEN by Spada and Stocchi 2007, available from *http://geodynamics.org/cig/software/selen/*).

While other tools exist for the computation of the elastic response of the Earth (see in particular the TABOO calculator of Spada 2003), REAR offers the advantage of being much more flexible and versatile. Furthermore, it is optimized for the particularly challenging task of the computation of deformations of very high harmonic degrees, which is essential for the interpretation of geodetic observations in regions subject to small–scale surface mass variations.

REAR obtains the solution in two distinct steps. In the first step, REAR computes the response to a finite–sized disc load of unitary thickness with a Heaviside load history, or, equivalently, the surface rates of displacement associated to a unit rate of mass variation. In order to compute these "Green Functions" (GFs), REAR needs a set of load–deformation coefficients (LDCs). Some sample LDCs sets are provided with REAR. Of course, the user can employ any other preferred set.

In a second step, by means of the superposition principle, REAR combines the GFs computed in the fist step in order to obtain maps of geodetic observables associated to a user–supplied mass balance model. This model must be discretized in disc–shaped elements with the same radius of those used to construct the GF. In the current version (1.0), REAR can compute vertical and horizontal surface displacements and geoid height variations (or their rates of variation). Due to its simple structure, REAR can be easily modified to deal with additional geodetic quantities.

For the complete theory background of REAR, the reader is referred to Farrell (1972), Spada (2003) and Spada et al. (2011). A condensed description of the theory is given below in Section 4. The method of computation of the Love numbers and LDCs is given by Gegout et al. (2010). A pre–release version of REAR has been recently employed by Spada et al. (2012b) in order to solve the "Regional Elastic Rebound" problem for the present melting of the Greenland Ice Sheet (GIS), based on the results of Sørensen et al. (2011).

2 Acknowledgements

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3 Reference

The seminal reference to REAR is:

Melini D., Gegout P., Spada G, King M. (2014) REAR - a regional ElAstic Rebound calculator. User manual for version 1.0, available on–line at: *http://hpc.rm.ingv.it/rear*.

4 Theory

The elastic response to a disc load is computed in a straightforward way according to the theory presented by Farrell (1972). Throughout this manual, the response to this disc load will be referred to as "Green's Function" (GF), although strictly speaking this name denotes the solution for a perfectly localized and impulsive load (see Farrell 1972).

Assuming that the load is turned on at time $t = 0$ and its amplitude is kept constant, REAR implements the formula:

$$\left\{ \begin{array}{c} u \\ v \\ n \end{array} \right\} (\psi) = \frac{3\rho_i}{\rho_e} \, H \sum_{\ell=\ell_{min}}^{\ell_{max}} \left\{ \begin{array}{c} h'_\ell \\ l'_\ell \\ 1 + k'_\ell \end{array} \right\} \frac{f_\ell(\alpha)}{2\ell+1} \left\{ \begin{array}{c} 1 \\ \frac{\partial}{\partial\psi} \\ 1 \end{array} \right\} P_\ell(\cos\psi), \tag{1}$$

where $u(\psi)$ and $v(\psi)$ are the GFs for vertical and horizontal displacement, $n(\psi)$ is the GF for the geoid height variation, defined as:

$$n(\psi) = \frac{\Phi(\psi)}{\gamma}, \tag{2}$$

where Φ is the total variation of the Earth's gravity potential, γ is the reference gravity field at the surface, and $(h'_\ell,\ l'_\ell,\ k'_\ell)$ – the LDCs – are the proportionality factors between the cause (the surface load) and its mechanical consequences in terms of gravitational potential (k'), radial displacement (h'), and tangential displacement (l'). In (1), ψ is the Earth's center angle separating the observer from the load center ($\psi = 0$), ρ_i is the mass density of the surface load, H is the change of the thickness of the surface load, ρ_e is the Earth's average density, ℓ is the harmonic degree, $P_\ell(\cos\psi)$ is the Legendre polynomial of degree ℓ, and f_ℓ is a non–dimensional shape factor depending on the load geometry. It is important to note that in (1) u is positive upward, and that a positive (negative) value of v denotes horizontal motion *away from* (*towards*) the center of the load, respectively.

In REAR, the default shape factor

$$f_\ell = \left\{ \begin{array}{ll} 0 & \text{if } \ell = 0 \\ \dfrac{P_{\ell+1}(\cos\alpha) - P_{\ell-1}(\cos\alpha)}{1 + \cos\alpha} & \text{if } \ell \geq 1, \end{array} \right. \tag{3}$$

describes a disc load uniformly distributed for co–latitude $0 \leq \psi \leq \alpha$, and exactly compensated by a complementary load of constant thickness acting on $\alpha < \psi \leq \pi$ so that there is no net variation of the total mass of the Earth (Spada et al., 2011). In Spada (2003), this particular surface load is dubbed *"compensated disc load"*. Other choices are possible, corresponding to different load geometries and assumptions regarding mass conservation. These are illustrated in detail in Spada et al. (2011), so an easy implementation in REAR is straightforward. We note that for a compensated surface load there is no response of the Earth at degree $\ell = 0$, since $f_0 = 0$. For this reason, REAR only employs LDCs for degrees $\ell \geq 1$ in order to retrieve the solution.

Since the three GFs (1) scale with the variation of the load thickness H (this is a direct consequence of the assumption of perfect elasticity), rates of change of the GFs for relevant geodetic quantities $(\dot{u}, \dot{v}, \dot{n})$ are simply obtained from Eq. (1), with H substituted by its time–derivative $\dot{H}$. Therefore, REAR can equivalently compute absolute variation of observables or their rate of variation. By convention, a value $H > 0$ indicates an instantaneous increase in the load thickness; while $\dot{H} > 0$ denotes a positive rate of variation. Of course, the source of REAR can be easily modified in order to build other GFs of interest, such as for example those pertaining to free–air gravity field variations, using appropriate combinations of LDCs.

In REAR, the response of the Earth to the load exerted by a realistic surface mass distribution is constructed by a superposition of GFs, each corresponding to one of many discs that discretize the load itself. Accordingly, at a given location $P(r, \theta, \lambda)$ on the Earth's surface, the total response is

$$\left\{ \begin{array}{c} u_r \\ u_\theta \\ u_\lambda \\ N \end{array} \right\} (\theta, \lambda) = \sum_{k=1}^{N_d} \left\{ \begin{array}{c} u_k \\ \cos X \ v_k \\ \sin X \ v_k \\ n_k \end{array} \right\} (\psi_k), \tag{4}$$

where $(u_r, u_\theta, u_\lambda)$ are the components of surface displacement in spherical coordinates, N is the geoid height variation, (r, θ, λ) denote the radius, co–latitude and longitude, N_d is the number of disc elements, (u_k, v_k, n_k) are the GFs relative to the elementary k–th surface load (these are given by Eq. (1), assuming that the load thickness changes by H_k), and ψ_k is the co–latitude of the observer relative to the center of the k–th disc. According to usual conventions in spherical geometry, u_r is positive upward, u_θ is positive southward, and u_λ is positive eastward.

With the aid of the cosine and sine theorems in spherical geometry, it can be easily established that

$$\begin{aligned} \cos X &= \frac{\cos\theta_k - \cos\theta\cos\psi_k}{\sin\theta\sqrt{1-\cos^2\psi_k}} \\ \sin X &= \frac{\sin(\lambda-\lambda_k)\sin\theta_k}{\sqrt{1-\cos^2\psi_k}}, \end{aligned} \tag{5}$$

with

$$\cos\psi_k = \cos\theta\cos\theta_k + \sin\theta\sin\theta_k\cos(\lambda-\lambda_k), \tag{6}$$

where θ_k and λ_k are the co–latitude and the longitude of the k–th disc, respectively (for details, the reader is referred to page 78 of Spada 2003).

In REAR, all the computations rely upon input data expressed in terms of latitude and longitude, hence all formulas above are directly implemented. Loading or unloading data that are given on projected grids need to be unprojected before running REAR. Raw data may therefore need re–sampling in order to ensure that loading and unloading data have consistent geometry (e.g. disc sizes).

5 Running REAR

5.1 Installation and prerequisites

REAR can be run on any UNIX–like system (including Linux and Mac OS X); on Windows computers, it can be run within the Cygwin environment[1]. REAR program units are parallelized with OpenMP directives, so that the code can take advantage of modern multi–core CPUs.

REAR has been tested with both the open–source GNU `gfortran` compiler[2] and the commercial Intel Fortran compiler; however, REAR can be used with any standards–compliant Fortran 90 compiler.

[1] The Cygwin environment can be obtained at `http://www.cygwin.org/`

[2] The `gfortran` compiler is part of the GNU compiler collection, available at `http://gcc.gnu.org/`

The Generic Mapping Tools (GMT) public domain mapping software[3] (Wessel and Smith, 1998), while not strictly needed to run REAR, it is useful to plot the results of examples provided with the REAR distribution.

REAR incudes routines from SHTOOLS[4] for the numerical evaluation of the Legendre polynomials. It also takes advantage of the icosahedron–based method[5] for pixelizing the sphere intruduced by Tegmark (1996).

REAR does not require an installation procedure; it is sufficient to extract the REAR distribution package into any folder, configure the program components and run them in place. The contents of the distribution package is described in Table 1. T1

In Section 6, the compilation and the execution of REAR will be validated by means of an example.

5.2 Configuration and execution

REAR program units `make_gf.f90` and `make_map.f90` are configured by editing the Fortran "include" files `input_data_gf.inc` and `input_data_map.inc`, respectively. These two program units accomplish the first and the second execution steps described in the Introduction, respectively. Since the configuration files are included into the program source at compile time, when editing them the user must follow the Fortran 90 syntax, and any change in configuration requires re-compiling the corresponding program units.

The user can manually compile the program units or, alternatively, use the `Makefile` provided with the REAR package. The `Makefile` is configured to invoke the GNU `gfortran` compiler; different compilers can be employed by editing the `Makefile` and changing the `FC` and `FOPTS` variables. Alternatively, it is possible to override the default values for `FC` and `FOPTS` on the `make` command line. For instance, `make FC=ifort FOPTS=-openmp` will instruct REAR to use the Intel Fortran compiler.

In the following sections, we give the configuration data for the regional rebound due ice mass loss in Greenland, and discuss the meaning of the various options of REAR.

5.3 Computing the Green's Functions

The Fortran include file `input_data_gf.inc` contains configuration data for the calculation of the GFs. Below, we briefly illustrate the meaning of each section of this file.

5.3.1 Constants and parameters

```
!
! Include data for ther Green's Functions calculator (make_gf.f90)
! ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
!
 real*8, parameter :: PI=3.14159265358979323840d0 ! Pi
!
! ##### Earth model parameters
 real*8, parameter :: &
        ggg=6.67384d-11, & ! Newton's constant (SI units)
        radius=6371d0, & ! Radius of the Earth (km)
        radiusm=radius*1d3, & ! Radius of the Earth (m)
```

[3] GMT can be downloaded from: http://gmt.soest.hawaii.edu/
[4] The home of SHTOOLS is: http://shtools.ipgp.fr
[5] The theory and a the code are available from: http://space.mit.edu/home/tegmark/icosahedron.html

```
        grav=9.8046961d+00, & ! Surface gravity (m/s/s)
        rhoear=3d0*grav/4d0/ggg/pi/radiusm ! Average Earth density (kg/m^3)
!
```

In this first section of the GF configuration file `input_data_gf.inc`, some general constants are defined: π, the Newton's constant G, the average Earth radius a, the surface gravity acceleration γ, and the average Earth density ρ_e. Note that REAR employs SI units throughout. The REAR settings in this section of `input_data_gf.inc` generally do not need to be edited. However, the values of the constants must be consistent with those employed to compute the LDCs. Since these are assumed to be known and stored in folder `DATA`, the user should check the consistency before running the program.

5.3.2 Surface load

```
!
! ##### Disc load parameters
! NOTE: The thickness variation is of +1 m/yr regardless of the load size
! real*8, parameter :: &
        rhoice = 917d0,  & ! Ice density (kg/m^3)
        alfa=0.02537d0 ! Half-amplitude of the load (deg)
!
```

This section of `input_data_gf.inc` contains the definition of the disc load parameters for the computation of the GFs: the ice density ρ_i and the half–amplitude of the disc load α, expressed in decimal degrees (the meaning of parameter α is explained in Section 4). The value of α given here must match the one employed to discretize the ice model in Section 5.4 below.

5.3.3 Green's function grid

```
!
! #### 1D Grid (two options available)
!
 integer, parameter :: grid_opt = 1 ! type of grid (1/2)
!
 real*8, parameter :: theta_min = 0d0 ! Min.colatitude (deg) both grids
 real*8, parameter :: theta_max = 60d0 ! Max colatitude (deg) both grids
!
! ##### grid_opt=1: constant stepsize
 real*8, parameter :: theta_inc = alfa/5d0  ! Increment (deg)
!
! ##### grid_opt=2: stepsize increases with colatitude
 integer, parameter :: ngrid= 1001 ! number of points
 real*8, parameter :: spac_min= alfa/5d0 ! min. grid spacing (deg)
```

Here various options for the computation of the GFs are given. For `grid_opt=1`, the GFs will be computed on equally–spaced points at angular distances from the load center between `theta_min` and `theta_max`, with spacing `theta_inc`. For `grid_opt=2`, a profile with `ngrid` points will be generated, with spacing between adjacent points that increases with distance from the axis of symmetry of the disc element. In this case, `spac_min` defines the minimum spacing, i.e. that between the pair of grid points that are closer to the center of the load ($\theta = 0$). This second option is useful to save computation time by sampling the GF with a finer grid only near the load, where the geodetic variables are expected to show significant gradients. When `grid_opt=2` is selected, the parameters $\theta_{min} \equiv$ `theta_min`, $\theta_{max} \equiv$ `theta_max`, $\Delta\theta_{min} \equiv$ `spac_min` and $n_{grid} \equiv$ `ngrid` must satisfy the relation $\Delta\theta_{min} < (\theta_{max} - \theta_{min})/(n_{grid} - 1)$. If this condition is not verified, the execution of `make_gf` stops with an error message.

In the lines above, a uniformly spaced grid is configured between the center of the load and an angular distance of $60°$, with points spaced by $\alpha/5$, where α is the load half–amplitude. When configuring the grid parameters, it is important to set theta_max to a value larger than the largest load-observation distance (otherwise make_map.f90 will fail).

5.3.4 LDCs and harmonic synthesis

```
!
! ##### Max harmonic degrees for synthesis
 integer, parameter :: &
         lmin=1, & ! lmin
         lmax=32768 ! lmax
!
! ##### File name for Load-deformation coefficients (input)
 character*100, parameter :: &
               file_love_numbers="./DATA/REF_6371_loading_love_numbers.txt"
  integer, parameter :: nh=1 ! Number of header lines in file_love_numbers
!
```

This section of file input_data_gf.inc defines the parameters for the harmonic synthesis of the GFs. The range of harmonic degrees (minimum and maximum values) is set with lmin and lmax, while the LDCs are read from the multi–column ASCII text file file_love_numbers. Here, the user can provide the name of one of the LDC databases distributed with REAR (see Table 1) or T1
any other LDCs set.

The sets of LDCs made available in folder DATA in the REAR package are expressed in the reference frame of the center of mass (CM) of the whole Earth (this implies that the gravity potential LDC is $k'_1 = -1$). From these, the users can easily derive sets of LDCs referred to the system of the center of mass of the solid Earth (CE). These are related to those written in the CM by ${h'_1}^{CE} = 1 + h_1$, ${l'_1}^{CE} = 1 + l_1$, ${k'_1}^{CE} = 1 + k_1 = 0$, and ${h'_\ell}^{CE} = h'_\ell$, ${l'_\ell}^{CE} = l'_\ell$, ${k'_\ell}^{CE} = k'_1 = 0$ for $\ell > 1$ (see e.g. Greff–Lefftz and Legros 1997).

Since REAR ensures that the mass of the system (Earth+Load) is conserved (see Eq. 3), the REAR results are unaffected by the value of the degree $\ell = 0$ LDCs and the minimum harmonic degree effectively available for the computations is $\ell = 1$.

LDCs must be stored in a free–format text file, with each row containing the harmonic degree, vertical, horizontal and gravity potential LDCs, respectively $(\ell, h'_\ell, l'_\ell, k'_\ell)$. The variable nh must correspond to the number of header lines in file file_love_numbers.

The set of LDCs in file REF_6371_loading_love_numbers.txt corresponds to the seismological "REF model" STW105 (Kustowski et al., 2007), which is an update of PREM (Dziewonski and Anderson, 1981) that provides a description of the propagation of seismic waves inside the Earth. The elastic rheology of the Earth derives from the waves speed and the density, with the top three kilometers of oceanic water replaced by underneath rock materials. The model defines the Earth (hydrostatic) equilibrium, the pressure and the gravity inside the Earth; numerical computations allow to take compressibility into account. For reference, the structure of the REF model is shown in Figure 1, while Figure 2 shows the LDCs as a function of harmonic degree ℓ. Some numerical F1,F2
values are given in Table 2. T2

5.3.5 Output file for the GFs

```
!
! ##### File name for gridded Green's Function (output)
 character*30, parameter :: file_gf="gf_Greenland.dat"
```

```
!
```

The parameter `file_gf` contains the name of the output file that will be created by `make_gf`. By default, REAR will provide the outputs in multi–column ASCII text files. In this case, the GFs will be stored into `gf_Greenland.dat`.

5.4 Creating deformation maps

The Fortran include file `input_data_map.inc` contains configuration data for the computation of geodetic quantities. Below, we shortly illustrate its features.

5.4.1 Constants and parameters

```
!
! ###########################################
! Input parameters for program "make_map.f90"
! ###########################################
!
! >>>> INPUT PARAMETERS <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
!
 real*8, parameter :: RADIUS = 6371d0 ! Earth radius (km)
 real*8, parameter :: PI=3.1415926535897932384d0
 real*8, parameter :: D2R=PI/180.D0
 real*8, parameter :: R2D=180.D0/PI
```

The first block of the configuration file contains general constants: the Earth average radius, the numerical value of π, and constants used to convert angles between degrees and radians. These settings should not be edited.

5.4.2 Load model

```
!
! >>>> ICE MODEL
!
 character*50, parameter :: file_ice="./DATA/greeM3R.dat" ! File of the ice model
 integer, parameter :: NH_ICE=10 ! Header lines in file_ice
```

This section contains the definition of the ice model. The parameter `file_ice` contains the name of the ice model file; in this case REAR is configured to use file `greeM3R.dat`, which is derived from the M3 present-day ice balance model for Greenland by Sørensen et al. (2011). Parameter `NH_ICE` contains the number of header lines in `file_ice`.

File `file_ice` must be a free–format text file, with each row containing k, λ_k, ϕ_k, α_k, h_k (ice element number, longitude and latitude of its center, angular half–amplitude of the disc, thickness variation). Disc half–amplitudes α_k must be all equal to the value used to compute the GFs in Section 5.3.2 above. The `make_map` program checks the values of disc half–amplitudes in the ice model and GFs, and exits with an error message if inconsistent values are found.

5.4.3 Green's functions

```
!
! >>>> GREEN'S FUNCTION
!
 character*30, parameter :: file_gf="gf_Greenland.dat" ! gridded Green function
 integer, parameter :: GRID_OPT=1 ! grid style (1=uniform, 2=increasingly sparse)
```

```
 integer, parameter :: NH_GF=13 ! Header lines in file_gf
```

In this section the user specifies the GFs to be used to compute the geodetic quantities. The parameter `file_gf` contains the name of the GF file; `GRID_OPT` contains the type of grid spacing (uniform or "increasingly sparse") and must correspond to the value set in `input_data_gf.inc`; `NH_GF` contains the number of header lines in `file_gf`.

5.4.4 Output options

```
!
! >>>> SELECTION of the OUTPUT <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
!
! IWHERE=1 ---> Rates of displacement are on a 2D grid
! IWHERE=2 ---> Rates of displacement on isolated points
!
  integer, parameter :: iwhere = 1
!
! -------------------------------------------------------------
! >>>> PARAMETERS for IWHERE = 1 (2D grid) <<<<<<<<<<<<<<<<<<<<
! -------------------------------------------------------------
!
 REAL*8, PARAMETER :: GRD_TYPE=2 ! Grid type (1=Tegmark, 2=lon/lat)
!
 REAL*8, PARAMETER :: PIXEL_AREA=100d0 ! Pixel area for GRD_TYPE=1 (km^2)
 REAL*8, PARAMETER :: GRD_SPACING=0.5D0 ! Grid spacing for GRD_TYPE=2 (deg)
!
 REAL*4, PARAMETER :: LONMIN_GR=285.0, LONMAX_GR=350.0 ! Region boundaries (lon) (for both types)
 REAL*4, PARAMETER :: LATMIN_GR= 58.0, LATMAX_GR= 86.0 ! Region boundaries (lat)
!
 CHARACTER*30, PARAMETER :: FILE_OUT_GRID="uvg.dat" ! Output filename (for both types)
!
! -------------------------------------------------------------
! >>>> PARAMETERS for IWHERE = 2 (isolated points) <<<<<<<<<<<<<
! -------------------------------------------------------------
!
 CHARACTER*30, PARAMETER :: FILE_IN_POINTS="GPS-points.dat" ! Input Filename (points)
 integer, parameter :: NH_IP=6 ! Header lines
!
 CHARACTER*30, PARAMETER :: FILE_OUT_POINTS="GPS-out.dat" ! Output filename (points)
!
! END of FILE
!
```

Here the user defines where `make_map.f90` computes predictions of surface displacements and geoid changes. The option `IWHERE` controls whether the program will use a 2–D grid (suitable for plotting a map), or a set of user–provided isolated points (suitable, e.g., for computing predictions at the location of GPS or tide gauge sites).

For `IWHERE=1`, REAR will compute observables on a 2D grid. In this case, the parameter `GRD_TYPE` specifies if the grid is registered on a icosahedron-based equal-area pixelization of the sphere (`GRD_TYPE=1`, see Tegmark (1996) for details) or on a regular grid (`GRD_TYPE=2`). For the icosahedron–based grid, the user provides the pixel area through the parameter `PIXEL_AREA`, while for a regular grid the spacing in both latitude and longitude is specified by `GRD_SPACING` (the output will be on a pixel–registered grid). For both grid types, parameters `LONMIN_GR`, `LONMAX_GR`, `LATMIN_GR` and `LATMAX_GR` define the extents of the grid. Finally, `FILE_OUT_GRID` contains the name of the output file that will be generated by `make_map.f90`. In the example above, REAR is configured for a regular grid spacing of $0.5°$ and extents $285° \leq \lambda \leq 350°$, $58° \leq \phi \leq 86°$ in longitude and in latitude, respectively.

For IWHERE=2, REAR will compute geodetic quantities on a set of user–supplied points. The name of the file containing the points is specified with FILE_IN_POINTS and must contain, for each point, a line with point number, longitude, latitude and (optionally) station name. An example of FILE_IN_POINTS is in the DATA folder (see Table 1). Parameter NH_IP contains the number of T1
header lines in FILE_IN_POINTS. Finally, FILE_OUT_POINTS contains the name of the output file that will be generated.

For both IWHERE=1 and IWHERE=2, the output file will contain the following columns: longitude and latitude of the observation point, displacement (or velocity) along the radial, co–latitude and longitude directions, and geoid change (or variation rate). For IWHERE=2, a column with station name is also present.

6 Validation

By a simple example, here we will demonstrate the use of REAR to compute the present–day elastic rebound of Greenland using the ice mass balance of Sørensen et al. (2011). The corresponding configuration files for this case have been illustrated in Section 5. Once the configuration parameters are set, the program units must be compiled and the resulting executables can be run. This task can be done manually, or the Makefile provided with REAR can be used.

First, we need to copy the configuration files described above from the EXAMPLES folder into the working directory:

```
$ cp EXAMPLES/input_data_gf.inc .
$ cp EXAMPLES/input_data_map.inc .
```

The configuration of the REAR Makefile can be checked by typing the make command at the shell prompt:

```
$ make

  <<<<<<<< REAR: a Regional ElAstic Rebound calculator >>>>>>>>

  Use:
        make gf to compute Green's Functions
        make map to compute predictions of geodetic observables

  Fortran compiler: gfortran
  Compiler options: -fopenmp -O
```

As a default, REAR is configured to use the GNU gfortran compiler with OpenMP enabled. We are now ready to compute the GFs by typing make gf at the shell prompt:

```
$ make gf

  <<<<<<<< REAR: Regional ElAstic Rebound calculator >>>>>>>>

  ---> Compiling make_gf.f90 ...


  ---> Running make_gf.exe ...

 make_gf: Reading the Love numbers from file: ./DATA/REF_6371_loading_love_numbers.txt
 make_gf: Computing the harmonic coefficients of the load ...
 make_gf: Computing displacements and geoid height at the grid points ...
 make_gf: The Green function grid has a uniform spacing
```

```
make_gf: The Green functions are written on file: gf_Greenland.dat
```

After the execution, the GFs are stored in the text file `gf_Greenland.dat`, which can be examined with a text editor:

```
 - Love numbers from file:./DATA/REF_6371_loading_love_numbers.txt
 - Green functions for MAX degree: 32768
 - Ice density (kg/m^3): 917.00000000000000
 - Earth density (kg/m^3): 5505.0609557847711
 - Load half-amplitude (deg): 2.5370000000000000E-002
 - Ice thickness variation: +1 m/year
 - ======= Grid properties (the grid has costant spacing) =======
 - Min and Max colatitudes (deg): 0.0000000000000000 60.000000000000000
 - Colatitude increment(deg): 5.0740000000000004E-003
 - Number of grid points: 11825
#
# Line colat, deg vert. vel., mm/yr hor. vel., mm/yr geoid vel., mm/yr
#
     1 0.00000000E+00 -0.50871721E+00 0.00000000E+00 0.10955602E+00
     2 0.50740000E-02 -0.50733845E+00 -0.17094634E-01 0.10926704E+00
     3 0.10148000E-01 -0.49089914E+00 -0.36009384E-01 0.10581934E+00
     4 0.15222000E-01 -0.45985367E+00 -0.52474962E-01 0.99308204E-01
     5 0.20296000E-01 -0.41580245E+00 -0.71728450E-01 0.90069331E-01
     6 0.25370000E-01 -0.32076983E+00 -0.84531181E-01 0.70136357E-01
     7 0.30444000E-01 -0.22986821E+00 -0.74395911E-01 0.51069059E-01
[...]
```

The header section of `gf_Greenland.dat` contains, for the User's convenience, a summary of the options used for the generation of the GFs. Starting from line 14, the file contains the response to a unit load rate, computed along the user-specified grid. Columns represent, respectively: line number, co–latitude (i.e., angular distance from the center of the disc), velocities along the radial and co–latitude directions, geoid variation rate. This set of GFs can be plotted with the `plot_greenfunctions.gmt` GMT script, which is provided with REAR in the `EXAMPLES` folder[6]. The resulting plot is shown in Figure 3. Since the load used to compute the GFs represents a loading, the elastic response corresponds to a subsidence below the disc, while the geoid is shifted upwards because of the mass excess associated with the load. Horizontal velocities are maximum at the disc edge and directed towards the disc center.

In order to check that `make_gf` has run correctly, the GF file produced by this example can be compared with a "reference" file `gf_Greenland_REF6371.dat`, distributed with the REAR package in the `EXAMPLES` folder. The results can be conveniently compared with the `compare_gf.sh` script, stored in the `EXAMPLES` folder:

```
$ compare_gf.sh gf_Greenland.dat EXAMPLES/gf_Greenland_REF6371.dat

Comparing gf_Greenland.dat and EXAMPLES/gf_Greenland_REF6371.dat ...

Min/max ABSOLUTE differences:
  - Vertical component: -1.0000e-11 / 1.0000e-11
  - Horizontal component: -1.0000e-12 / 1.0000e-11
  - Geoid: -1.0000e-10 / 1.0000e-11

Min/max RELATIVE differences:
  - Vertical component: -9.4029e-08 / 9.9015e-08
  - Horizontal component: -7.6562e-07 / 4.6191e-07
  - Geoid: -4.0250e-07 / 2.3291e-06
```

[6] The `plot_greenfunctions.gmt` script is compatible with version 4.5 of GMT. Users of GMT version 5 should use the `plot_greenfunctions.gmt5` script.

In this example, the negligible differences between the two files are likely the result of numerical roundoffs, so make_gf is working correctly.

Once the GFs are computed, program make_map can be used to compute observables corresponding to a given ice model. To compile and execute make_map, we type make map at the shell prompt:

```
$ make map

  <<<<<<<< REAR: Regional ElAstic Rebound calculator >>>>>>>>

  ---> Compiling make_map.f90 ...


  ---> Running make_map.exe ...

 make_map: Reading ice model information
 make_map: Found 70850 ice elements in file ./DATA/greeM3R.dat
 make_map: Reading the Green Function file: gf_Greenland.dat
 make_map: Reading 11826 points from the Green Function file
 make_map: Grid data: min colat = 0.0000000000000000
 make_map: Grid data: max colat = 60.000050000000002
 make_map: Number of points: = 11826
 make_map: Option 1: Gridded output
 make_map: Using a uniform lon/lat grid
 make_map: Spacing of the 2D GRID of observers (deg): 0.50000000000000000
 make_map: Number of GRID points within the region: 7467
 make_map: Number of pixels (observers) within the region: 7467
 make_map: Pre-computing trigonometric functions
 make_map: Computing displacements at the requested points
 make_map: END of computation
 make_map: Writing the output
 make_map: The output is reported on file: uvg.dat
```

After the execution, predictions of observables on the grid points are stored in the text file uvg.dat:

```
 # Lon, Lat, UrDOT, UthetaDOT, UlambdaDOT, NDOT (mm/yr)
 # Green function from file: gf_Greenland.dat
  285.0000 58.0000 0.465931E+00 0.485907E-01 -.438666E-01 -.179354E+00
  285.5000 58.0000 0.470530E+00 0.500897E-01 -.451277E-01 -.184710E+00
  286.0000 58.0000 0.475209E+00 0.516127E-01 -.463986E-01 -.190182E+00
  286.5000 58.0000 0.479968E+00 0.531624E-01 -.476745E-01 -.195773E+00
  287.0000 58.0000 0.484819E+00 0.547334E-01 -.489495E-01 -.201491E+00
  287.5000 58.0000 0.489748E+00 0.563409E-01 -.502422E-01 -.207336E+00
  288.0000 58.0000 0.494754E+00 0.579815E-01 -.515479E-01 -.213309E+00
  288.5000 58.0000 0.499862E+00 0.596577E-01 -.528631E-01 -.219418E+00
[...]
```

The file contains an header section with the name of the GFs file used to compute the observables. Data columns correspond, respectively, to longitude, latitude, surface velocity along the radial (r), co–latitudinal (θ) and longitudinal (λ) directions, and geoid variation rate. A map of the uplift for this run can be obtained with the GMT script plot_greenland_maps.gmt, provided with the REAR package in the EXAMPLES folder[7]. The resulting map is shown in Figure 4. F4

As shown for the computation of GFs, it is possible to validate the results of make_map by comparing the uvg.dat file produced by this run with the uvg_REF6371.dat file located in the EXAMPLES folder. The comparison can be done either manually or through the compare_map.sh script, distributed with REAR in the EXAMPLES folder:

[7]The plot_greenland_maps.gmt script is compatible with version 4.5 of GMT. Users of GMT version 5 should use the plot_greenland_maps.gmt5 script.

```
$ compare_map.sh uvg.dat EXAMPLES/uvg_REF6371.dat

Comparing uvg.dat and EXAMPLES/uvg_REF6371.dat ...

Min/max ABSOLUTE differences:
  - UrDOT: 0.0000e+00 / 0.0000e+00
  - UthetaDOT: -1.0000e-07 / 1.0000e-07
  - UphiDOT: -1.0000e-06 / 1.0000e-10
  - NDOT: 0.0000e+00 / 0.0000e+00

Min/max RELATIVE differences:
  - UrDOT: 0.0000e+00 / 0.0000e+00
  - UthetaDOT: 0.0000e+00 / 4.9830e-06
  - UphiDOT: -1.9613e-06 / 1.5299e-06
  - NDOT: 0.0000e+00 / 0.0000e+00
```

If the differences between the two files are null or negligible, the REAR setup is working correctly.

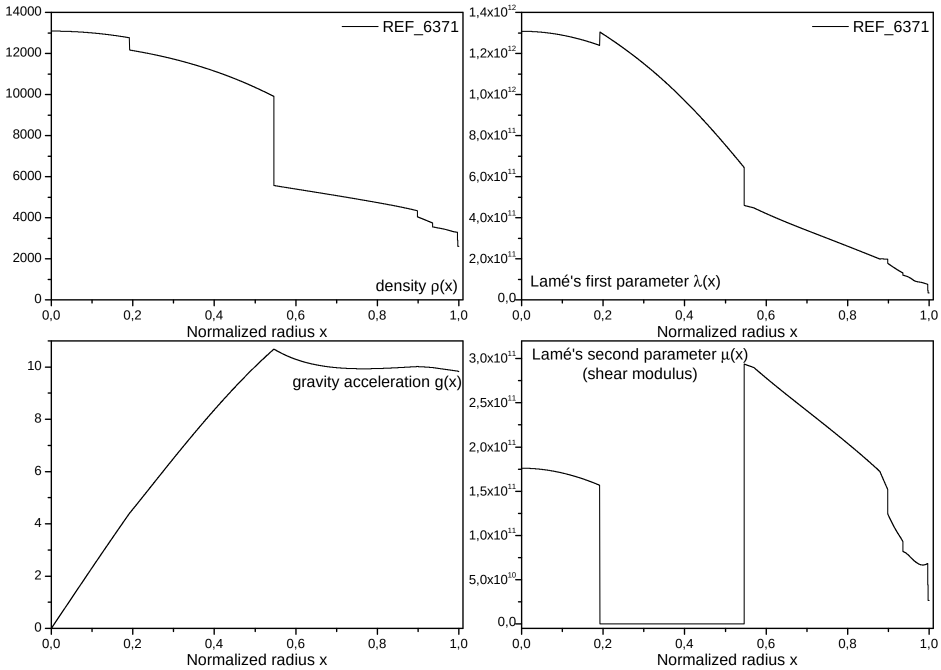

Figure 1: Structure of the REF6371 model, showing density, the first Lame parameter (λ), gravity acceleration, and the second Lame parameter (μ) as a function of x, the normalized Earth's radius.

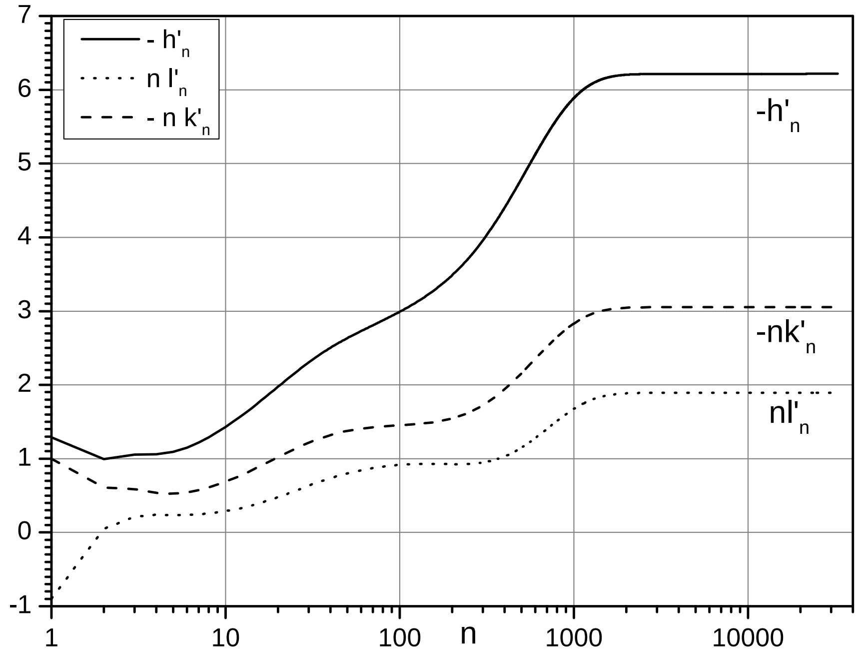

Figure 2: LDCs $(h'_\ell, l'_\ell, k'_\ell)$ for model REF6371, as a function of harmonic degree (note that in this figure $n = \ell$).

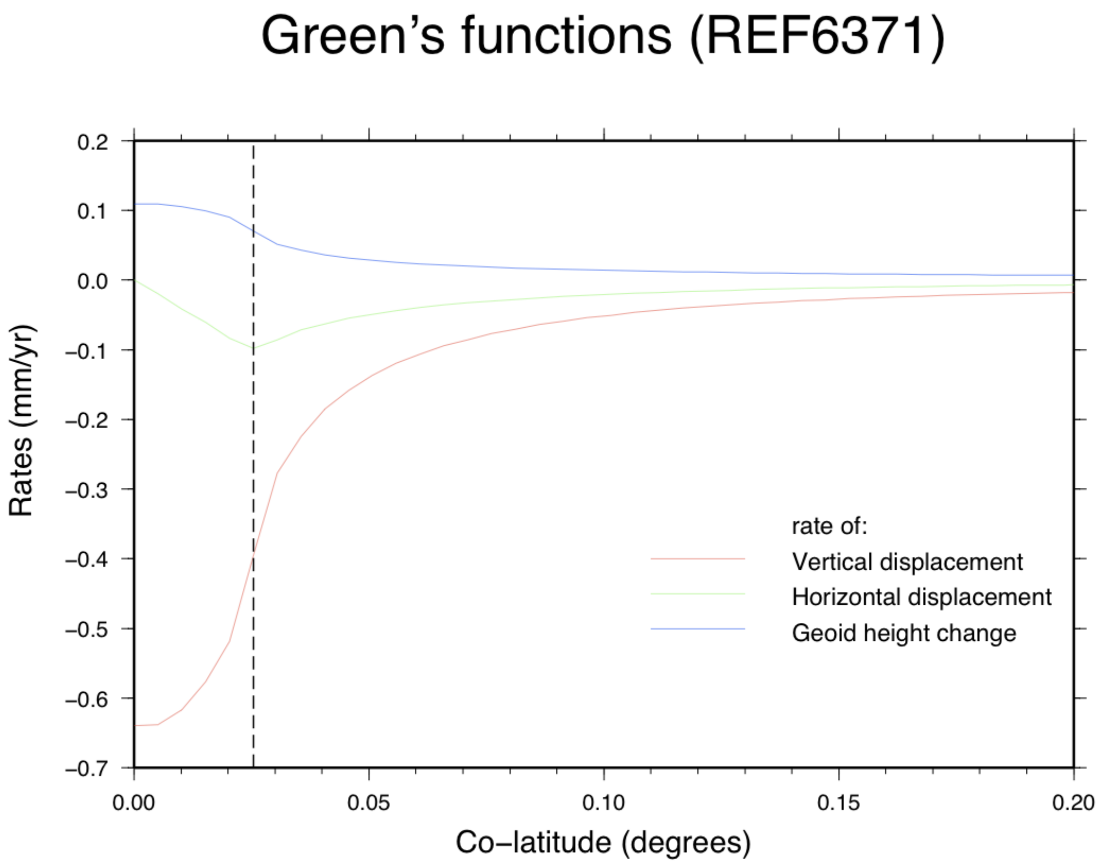

Figure 3: GFs computed with the example configuration discussed in Section 6. Plotted quantities show the rate of change of the GFs for a disc load of angular half–amplitude $\alpha = 0.02537°$, with a thickness changing at the rate of 1 m/yr. The dashed line marks the margin of the disc.

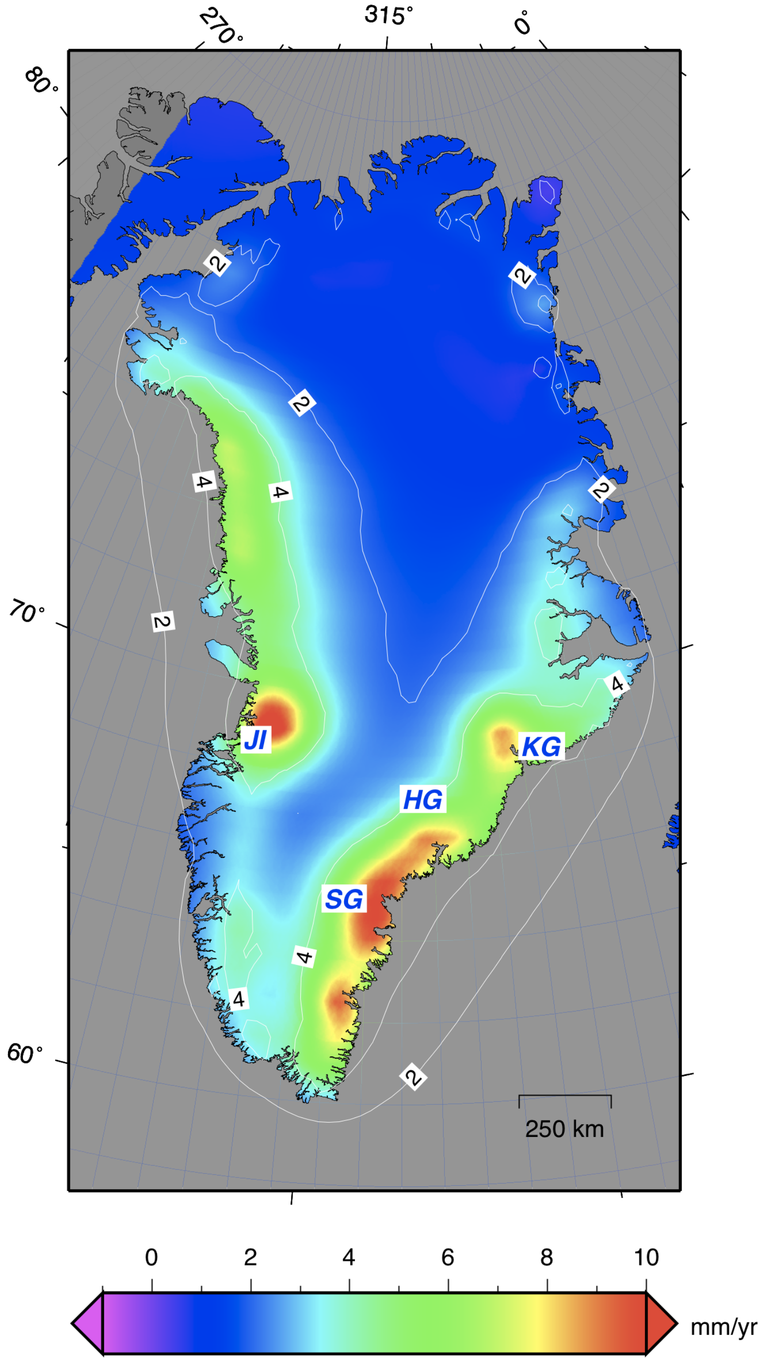

Figure 4: Map showing the rate of vertical uplift across the GIS according to the mass balance model M3 derived by ICESat data by Sørensen et al. (2011), for the time period 2003–2008. The map is obtained by the REAR example described in the text. Labels mark the location of the glaciers mostly contributing to mass loss: Kangerdlugssuaq Glacier (KG), Jakobshavn Isbræ (JI), Helheim Glacier (HG) and Southeast Glaciers (SG).

File	Description
`Makefile`	"Makefile" for `make_gf` and `make_map`
`make_gf.f90`	Source code for `make_gf`
`make_map.f90`	Source code for `make_map`
`hrm.f90`	Utility subroutines used by `make_gf` and `make_gf`
`GPL.txt`	A copy of the GPLv3 license
`REAR1.0-User-Guide.pdf`	This document
`EXAMPLES/`	
`compare_gf.sh`	Shell script for Green's Functions comparison
`compare_map.sh`	Shell script for geodetic observables comparison
`input_data_gf.inc`	Example configuration file for `make_gf.f90`
`input_data_map.inc`	Example configuration file for `make_map.f90`
`gf_Greenland_REF6371.dat`	Green's Functions for the Greenland example
`uvg_REF6371.dat`	Geodetic observables the Greenland example
`plot_greenfunctions.gmt`	Example GMT script for the Green's Functions plot
`plot_greenland_maps.gmt`	Example GMT script for the Greenland uplift map
`plot_greenfunctions.gmt5`	Example GMT5 script for the Green's Functions plot
`plot_greenland_maps.gmt5`	Example GMT5 script for the Greenland uplift map
`DATA/`	
`Load_Love2_CM.dat`	LDCs from the NASA Atmospheric Pressure Loading Service $(0 \leq \ell \leq 1,024)$
`REF_6371_loading_love_numbers.txt`	LDCs for model REF6371 $(1 \leq \ell \leq 32,768)$.
`REF_6371_potential_love_numbers.txt`	Love numbers for model REF6371 $(1 \leq \ell \leq 32,768)$.
`greeM3R.dat`	Greenland ice mass balance model from Sørensen et al. (2011).
`GPS-Bevis2012.dat`	Coordinates of Greenland GPS sites from Bevis et al. (2012).

Table 1: Contents of the REAR distribution package.

ℓ	h'_ℓ	l'_ℓ	k'_ℓ	$-h'_\ell$	$\ell l'_\ell$	$-\ell k'_\ell$
1	-1.287113	-0.895496	-1.000000	1.287113	-0.895496	1.000000
2	-0.994587	0.024113	-0.305770	0.994587	0.048225	0.611541
3	-1.054653	0.070855	-0.196272	1.054653	0.212565	0.588817
4	-1.057784	0.059587	-0.133791	1.057784	0.238349	0.535162
5	-1.091186	0.047026	-0.104762	1.091186	0.235131	0.523809
6	-1.149254	0.039408	-0.090350	1.149254	0.236449	0.542097
7	-1.218363	0.034994	-0.082057	1.218363	0.244958	0.574401
8	-1.290474	0.032251	-0.076523	1.290474	0.258010	0.612188
9	-1.361848	0.030386	-0.072393	1.361848	0.273471	0.651536
10	-1.430982	0.029023	-0.069078	1.430982	0.290226	0.690777
20	-1.975466	0.023899	-0.050927	1.975466	0.477972	1.018545
30	-2.304458	0.021158	-0.040503	2.304458	0.634734	1.215100
40	-2.502887	0.018419	-0.032997	2.502887	0.736753	1.319881
50	-2.633749	0.016026	-0.027515	2.633749	0.801321	1.375768
60	-2.730019	0.014065	-0.023449	2.730019	0.843906	1.406926
70	-2.807682	0.012470	-0.020363	2.807682	0.872915	1.425409
80	-2.874634	0.011164	-0.017966	2.874634	0.893121	1.437272
90	-2.935055	0.010080	-0.016064	2.935055	0.907204	1.445727
100	-2.991305	0.009169	-0.014526	2.991305	0.916862	1.452572
200	-3.486737	0.004632	-0.007734	3.486737	0.926495	1.546760
300	-3.958810	0.003164	-0.005749	3.958810	0.949282	1.724819
400	-4.401433	0.002577	-0.004853	4.401433	1.030908	1.941392
500	-4.793152	0.002299	-0.004315	4.793152	1.149354	2.157259
600	-5.123408	0.002132	-0.003919	5.123408	1.278922	2.351472
700	-5.391418	0.002003	-0.003594	5.391418	1.402423	2.515550
800	-5.602517	0.001889	-0.003310	5.602517	1.511004	2.648362
900	-5.764919	0.001780	-0.003058	5.764919	1.601553	2.752584
1000	-5.887537	0.001674	-0.002832	5.887537	1.674308	2.832483
2000	-6.203847	0.000943	-0.001523	6.203847	1.885420	3.045266
3000	-6.213711	0.000631	-0.001018	6.213711	1.892336	3.052948
4000	-6.214465	0.000473	-0.000763	6.214465	1.892413	3.053918
5000	-6.214822	0.000378	-0.000611	6.214822	1.892376	3.054435
6000	-6.215059	0.000315	-0.000509	6.215059	1.892350	3.054778
7000	-6.215229	0.000270	-0.000436	6.215229	1.892332	3.055023
8000	-6.215356	0.000237	-0.000382	6.215356	1.892318	3.055207
9000	-6.215454	0.000210	-0.000339	6.215454	1.892307	3.055350
10000	-6.215533	0.000189	-0.000306	6.215533	1.892299	3.055465
20000	-6.215889	0.000095	-0.000153	6.215889	1.892261	3.055980
30000	-6.216008	0.000063	-0.000102	6.216008	1.892248	3.056152

Table 2: Numerical values of the LDCs $(h'_\ell, l'_\ell, k'_\ell)$ for model REF6371, for some values of harmonic degree ℓ.

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