

# Modelling Physical Systems

an introduction at many levels

**Hans Vangheluwe**

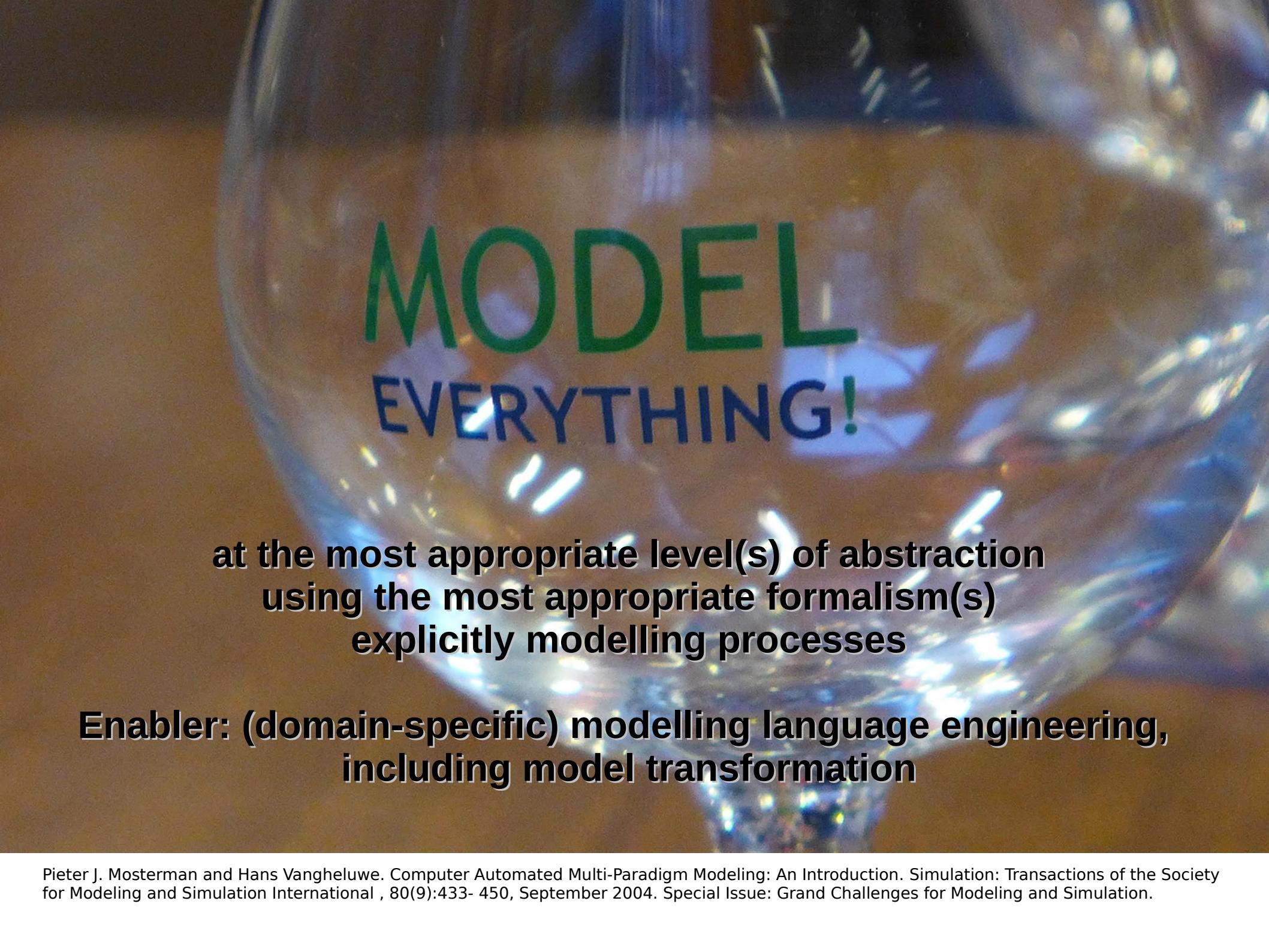
(with thanks to Joachim Denil for some CBD slides)

<http://msdl.cs.mcgill.ca/>

CUSO Winter School in Computer Science

Modelling of knowledge and the cyber-physical systems

5 – 9 February 2018  
Champéry, Switzerland



# **MODEL EVERYTHING!**

**at the most appropriate level(s) of abstraction  
using the most appropriate formalism(s)  
explicitly modelling processes**

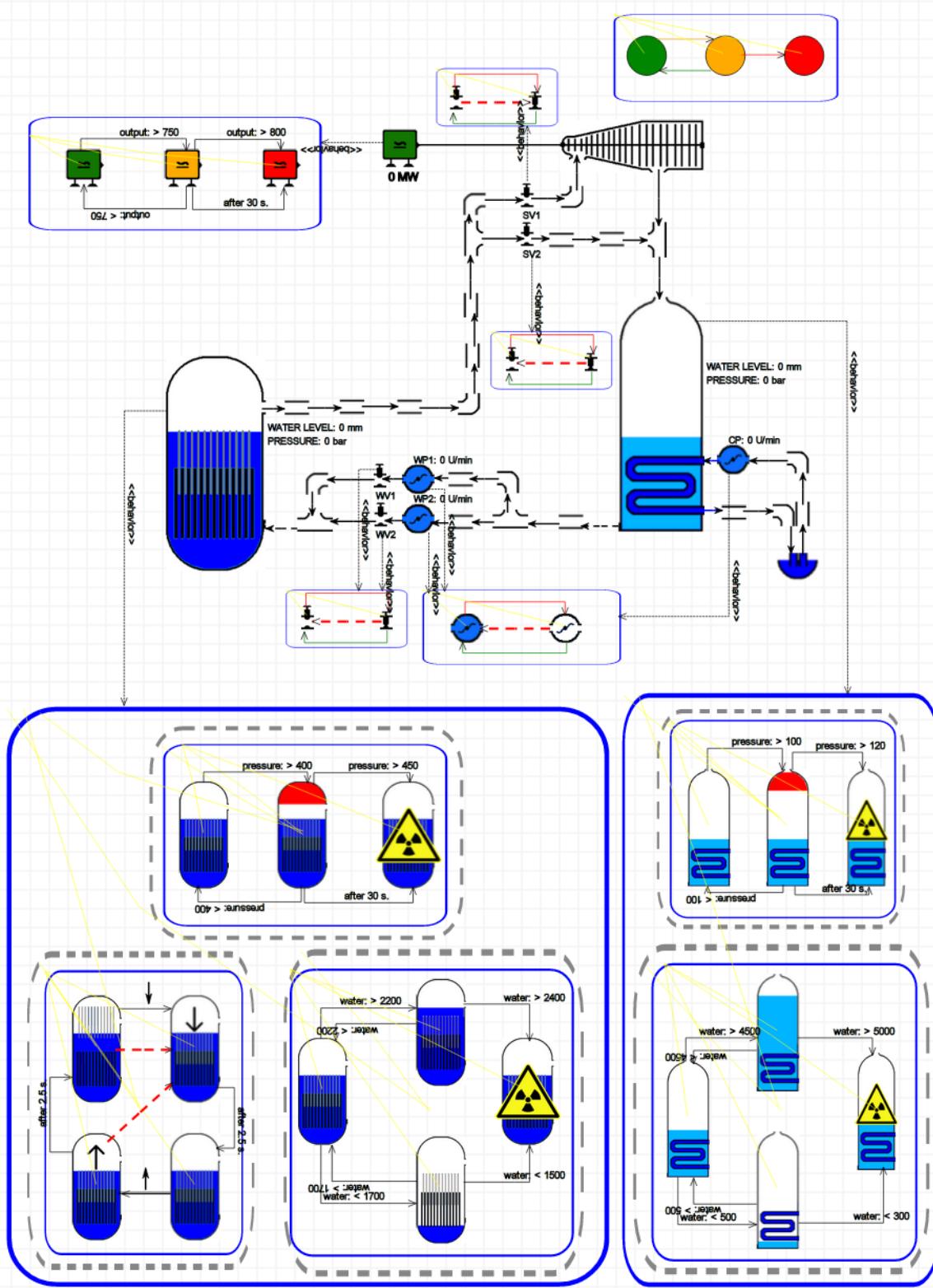
**Enabler: (domain-specific) modelling language engineering,  
including model transformation**

# Modelling Physical Systems

- Domain/Problem-Specific
- Laws of Physics
- Power Flow
- a-causal (Mathematical) ← **Modelica**
- Causal Block Diagrams
- Numerical (Discrete) Approximations
- Computer Numerical (Floating Point, Fixed Point)
- As-Fast-As-Possible vs. Real-time
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)

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### Boric Acid Transportation Pump

#### Product parameters

Design standards : RCC-M

Flow : 16.6m<sup>3</sup>/h

Head : 85m

Temperature : ~80°C

Pressure : 1.6MPa

Used in 600MWe、900MWe、1000MWe PWR nuclear power plant boric acid transportation system.



lithium-ion battery



$$R = V * i$$

$$R = R_{\text{ref}} [1 + \alpha(T - T_{\text{ref}})]$$

Where,

$R$  = Conductor resistance at temperature "T"

$R_{\text{ref}}$  = Conductor resistance at reference temperature  
 $T_{\text{ref}}$ , usually  $20^{\circ}\text{C}$ , but sometimes  $0^{\circ}\text{C}$ .

$\alpha$  = Temperature coefficient of resistance for the conductor material.

$T$  = Conductor temperature in degrees Celcius.

$T_{\text{ref}}$  = Reference temperature that  $\alpha$  is specified at for the conductor material.

# SimHydraulics

File Edit View Display Diagram Simulation Analysis Code Tools Help

sh\_actuator\_with\_2\_chamber\_snubbers\_model/Double-Acting Hydraulic Cylinder with Snubbers

Double-Acting Hydraulic Cylinder with Snubbers

sh\_actuator\_with\_2\_chamber\_snubbers\_model > Double-Acting Hydraulic Cylinder with Snubbers

1 C  
2 A  
3 R  
4 B

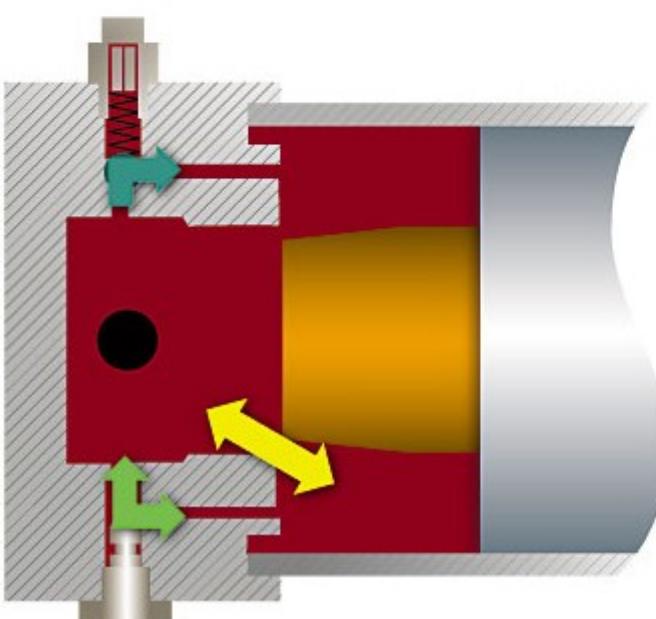
Cushioning Orifice A  
Variable Orifice A  
Check Valve A  
Main Piston A  
Motion Sensor  
Hard Stop  
Cushioning Orifice B  
Variable Orifice B  
Check Valve B  
Main Piston B  
Cushion Piston A  
Cushion Piston B

Velocity /Position

Pressure

Time offset: 0

Diagram showing a double-acting hydraulic cylinder model with two chambers and snubbers. The cylinder has four ports: C (top inlet), A (bottom inlet), R (top exhaust), and B (bottom exhaust). The circuit includes two main pistons (A and B) connected to cushion pistons (A and B) via check valves (A and B). Motion sensors and variable orifices are used for damping and cushioning. A hard stop is also present. The Velocity /Position plot shows position (magenta) and velocity (yellow) over time (0 to 10 seconds). The Pressure plot shows pressure (x10^6 Pa) over time (0 to 10 seconds) for four different ports.



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## \* **Newton's Three Laws of Motion:**

Fundamental relationship between the acceleration of an object and the total forces acting upon it.

- First Law states that in order for the motion of an object to change, a force must act upon it, a concept generally called inertia.
- Second Law defines the relationship between acceleration, force, and mass.
- Third Law states that any time a force acts from one object to another, there is an equal force acting back on the original object.

## \* **Newton's Law of Gravity:**

Explains the attractive force between a pair of masses. In the twentieth century, it became clear that this is not the whole story, as Einstein's theory of general relativity has provided a more comprehensive explanation for the phenomenon of gravity.

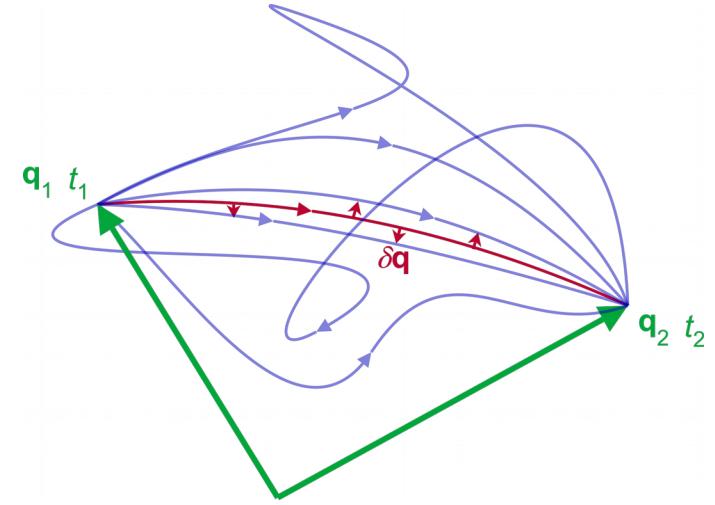
## \* **Conservation of Mass-Energy:**

The total energy in a closed or isolated system is constant, no matter what happens.

## \* **Conservation of Momentum:**

The total momentum in a closed or isolated system remains constant. An alternative of this is the law of conservation of angular momentum.

\* ...



In non-relativistic physics, the **principle of least action** – or, more accurately, the **principle of stationary action** – is a variational principle that, when applied to the action of a mechanical system, can be used to obtain the equations of motion for that system by stating a system follows the path where the average difference between the kinetic energy and potential energy is minimized or maximized over any time period. It is called stable if minimized. In relativity, a different average must be minimized or maximized. The principle can be used to derive Newtonian, Lagrangian, and Hamiltonian equations of motion.

The starting point is the *action*, denoted  $\mathcal{S}$  (calligraphic S), of a physical system. It is defined as the integral of the *Lagrangian*  $L$  between two instants of *time*  $t_1$  and  $t_2$  - technically a *functional* of the  $N$  *generalized coordinates*  $\mathbf{q} = (q_1, q_2 \dots q_N)$  which define the *configuration* of the system:

$$\mathcal{S}[\mathbf{q}(t)] = \int_{t_1}^{t_2} L(\mathbf{q}(t), \dot{\mathbf{q}}(t), t) dt$$

where the dot denotes the *time derivative*, and  $t$  is time.

Mathematically the principle is<sup>[11][12][13]</sup>

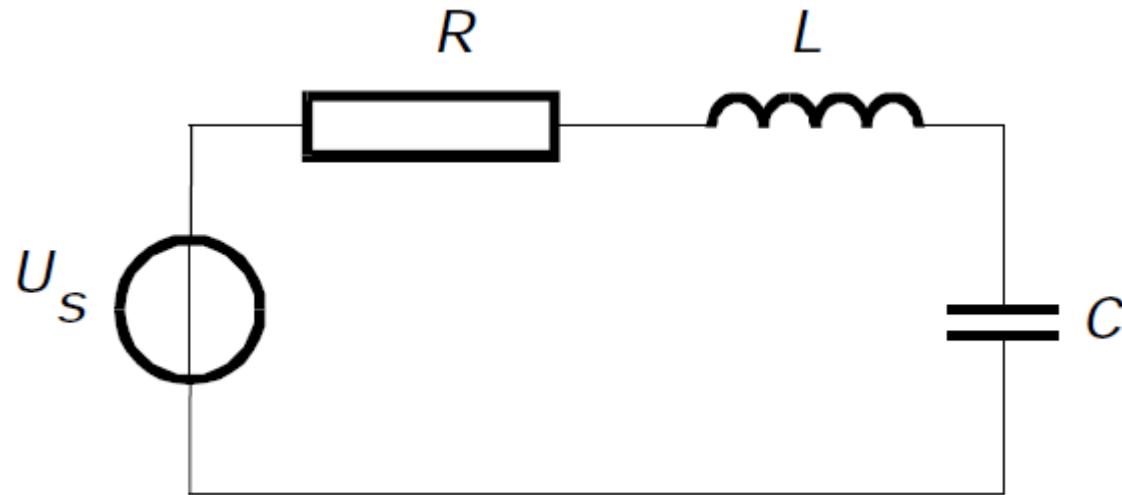
$$\delta\mathcal{S} = 0$$

where  $\delta$  (Greek lowercase *delta*) means a *small* change. In words this reads:<sup>[10]</sup>

*The path taken by the system between times  $t_1$  and  $t_2$  is the one for which the action is stationary (no change) to first order.*

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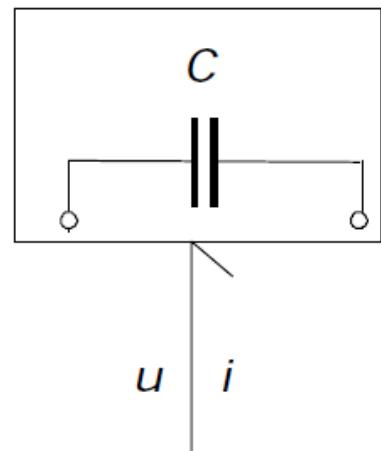
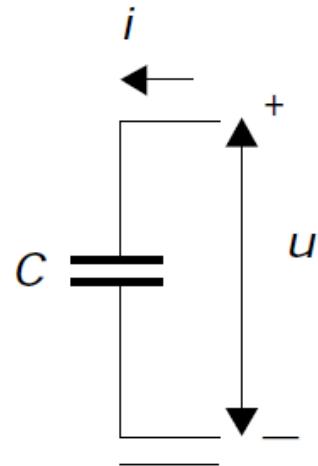
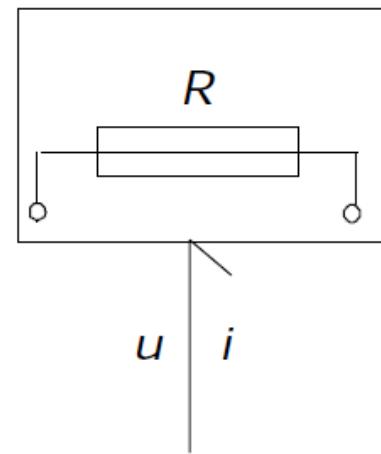
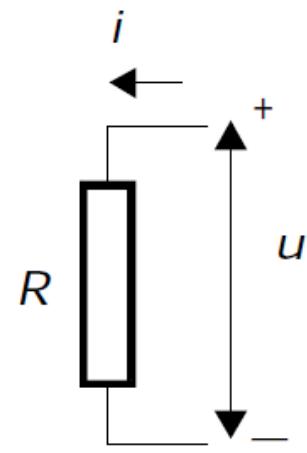
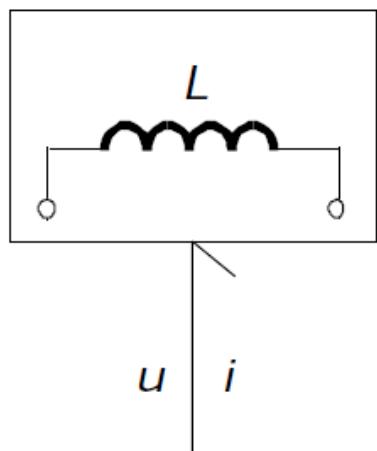
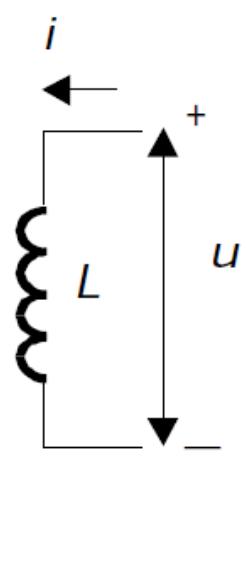
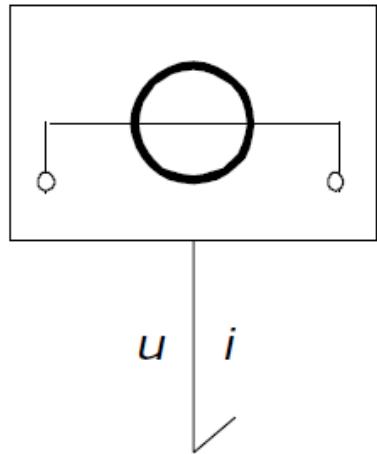
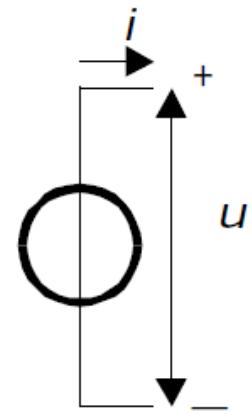


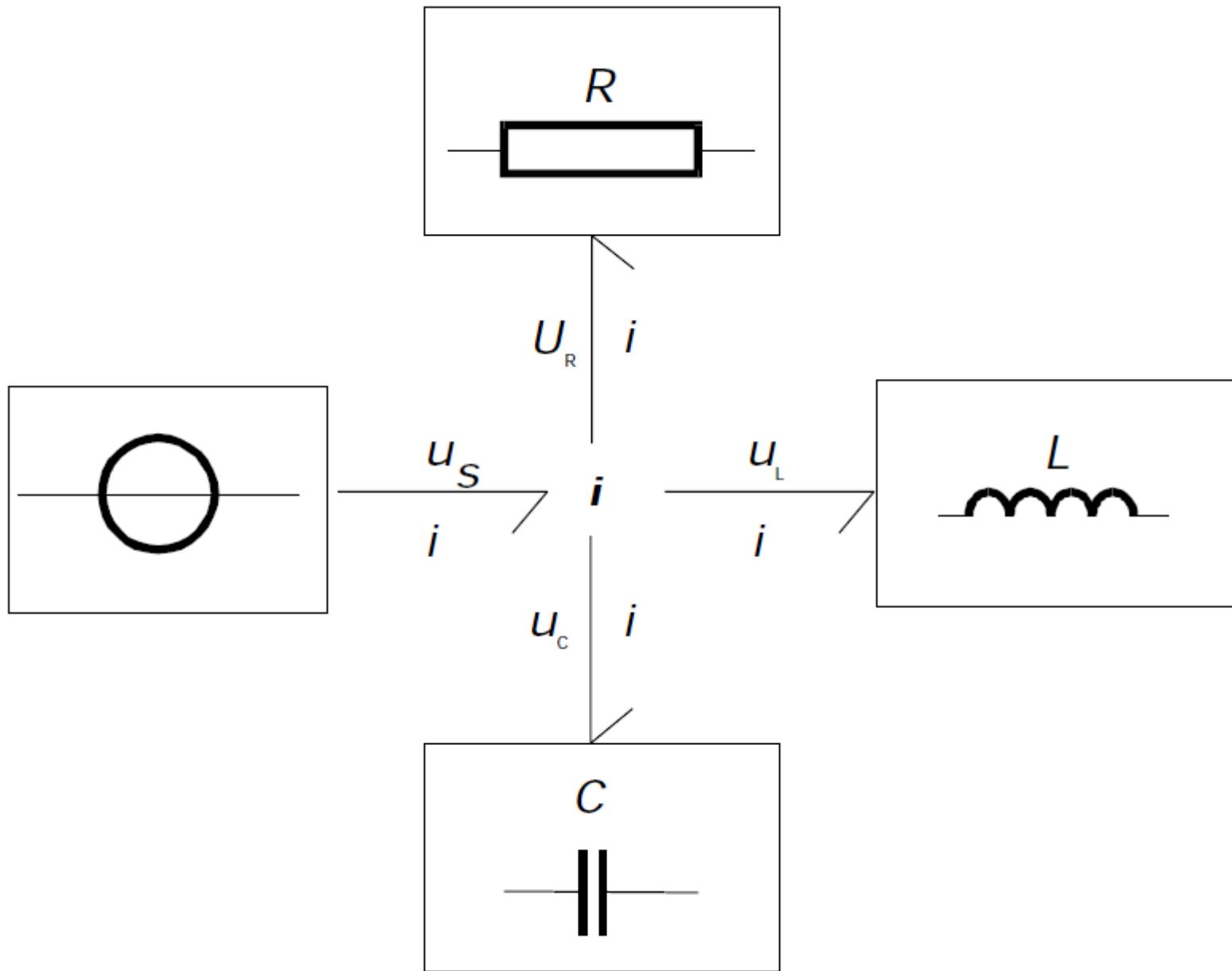
From Jan Broenink's [Introduction to physical systems modelling with bond graphs](#)

$$u_R = iR$$

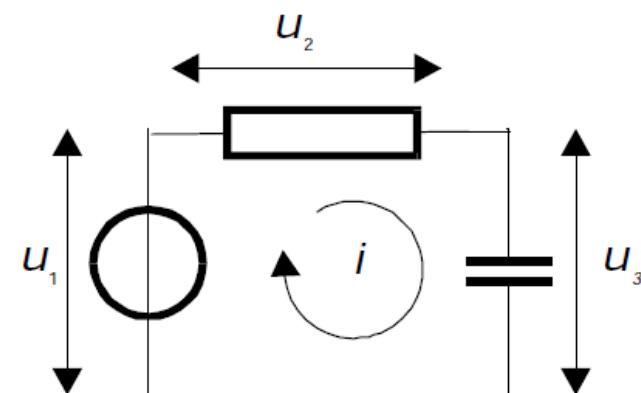
$$u_C=\frac{1}{C}\int idt$$

$$u_L=L\frac{di}{dt} \text{ or } i_L=\frac{1}{L}\int u dt$$

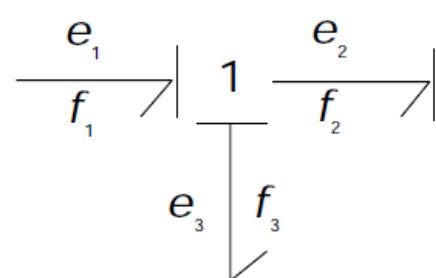




Domain-specific  
symbols



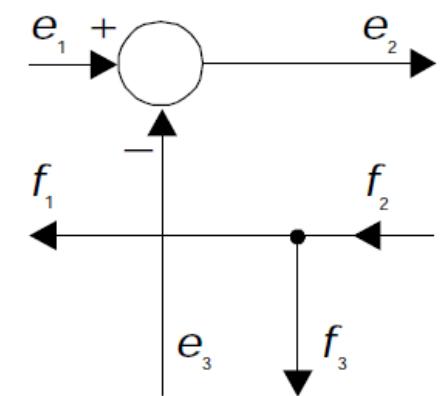
Bond-graph element



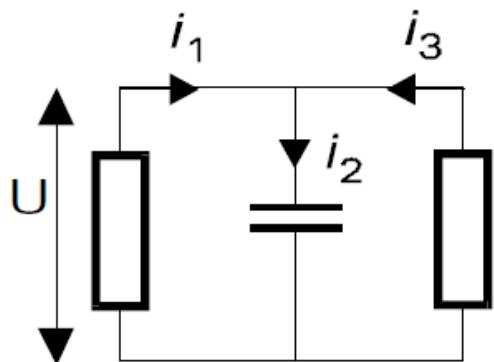
Equations

$$\begin{aligned}f_1 &= f_2 \\f_3 &= f_2 \\e_2 &= e_1 - e_3\end{aligned}$$

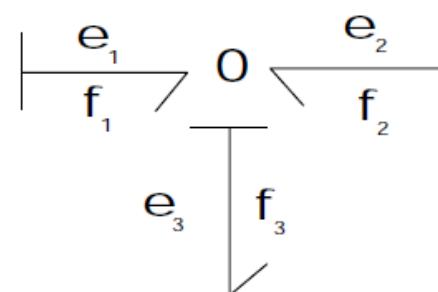
Block diagram  
expansion



Domain-specific  
symbols



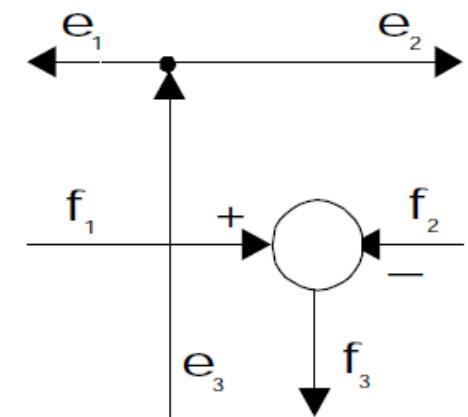
Bond-graph  
element

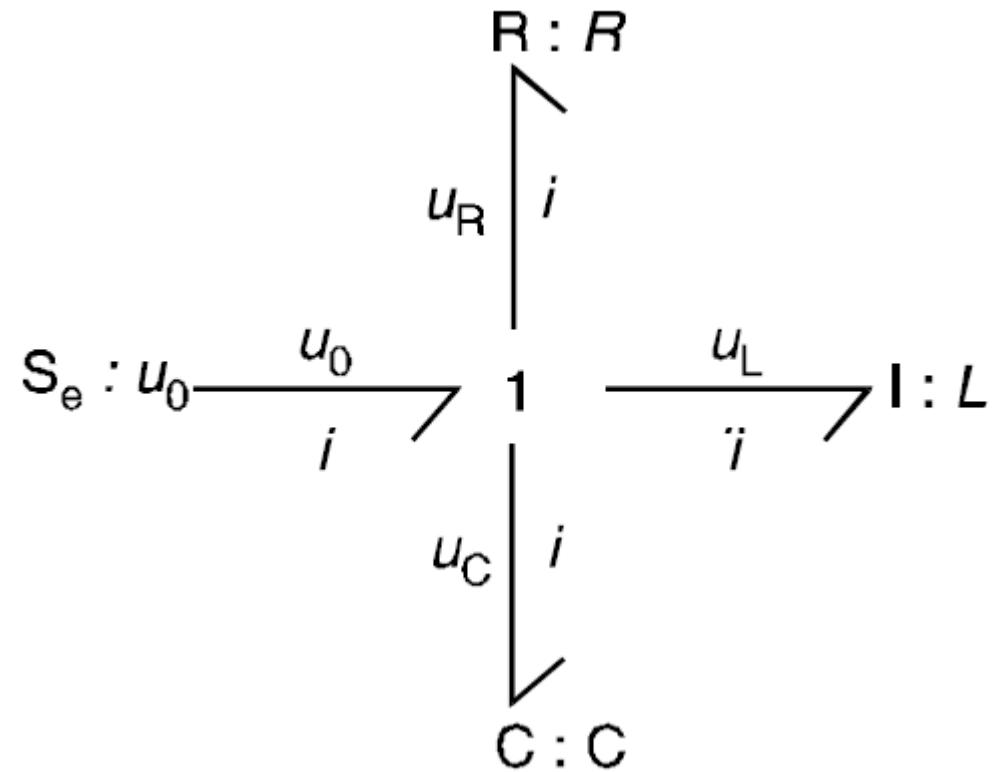


Equations

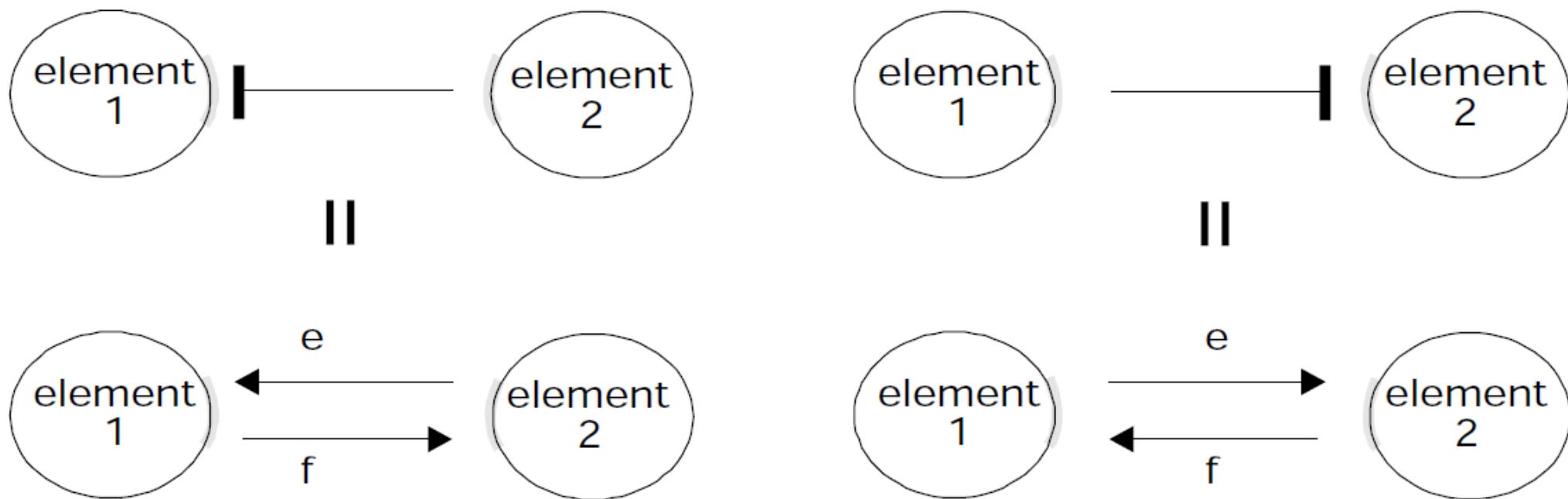
$$\begin{aligned}e_1 &= e_3 \\e_2 &= e_3 \\f_3 &= f_1 - f_2\end{aligned}$$

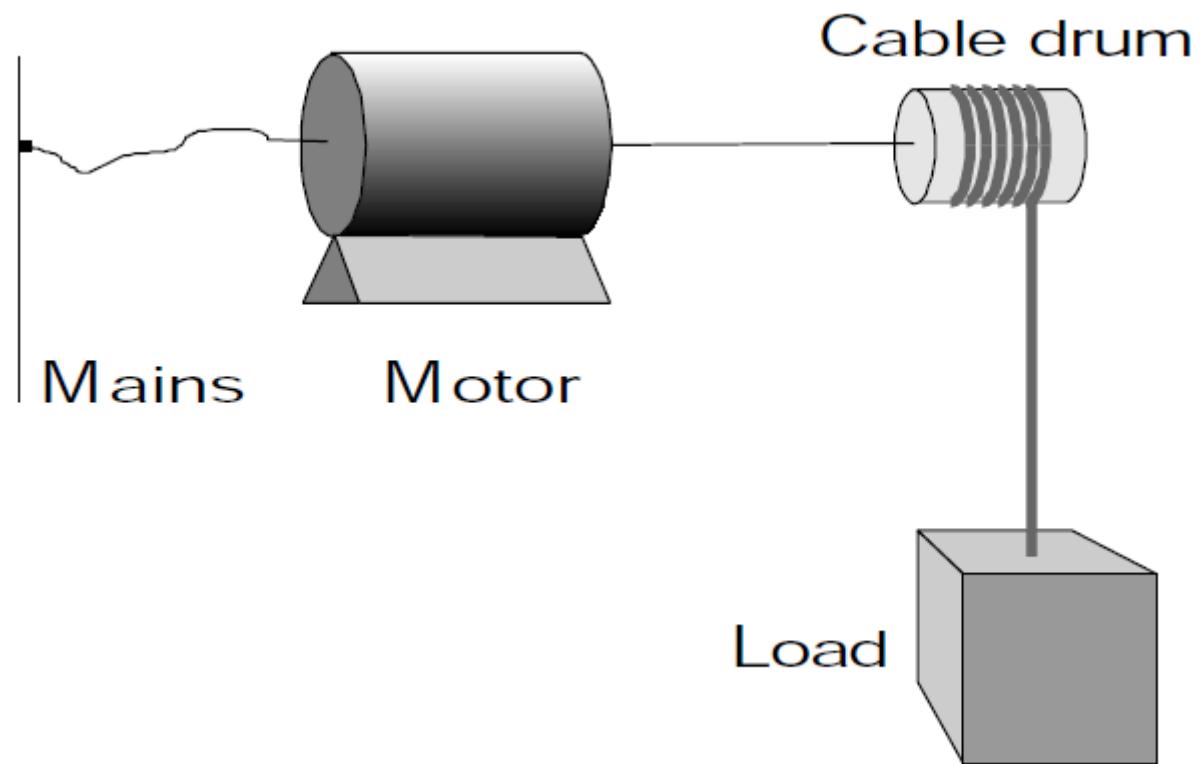
Block-diagram  
expansion



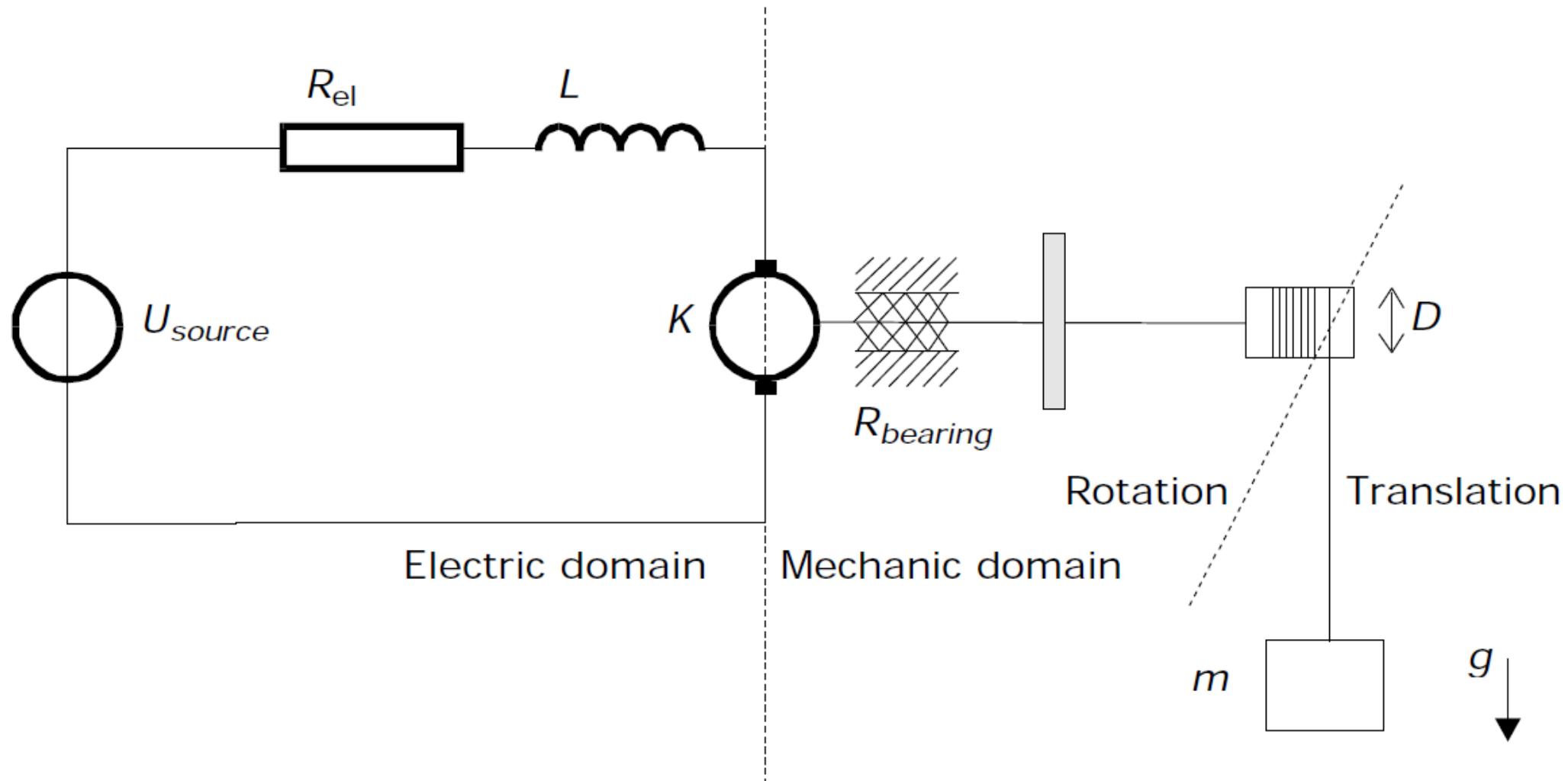


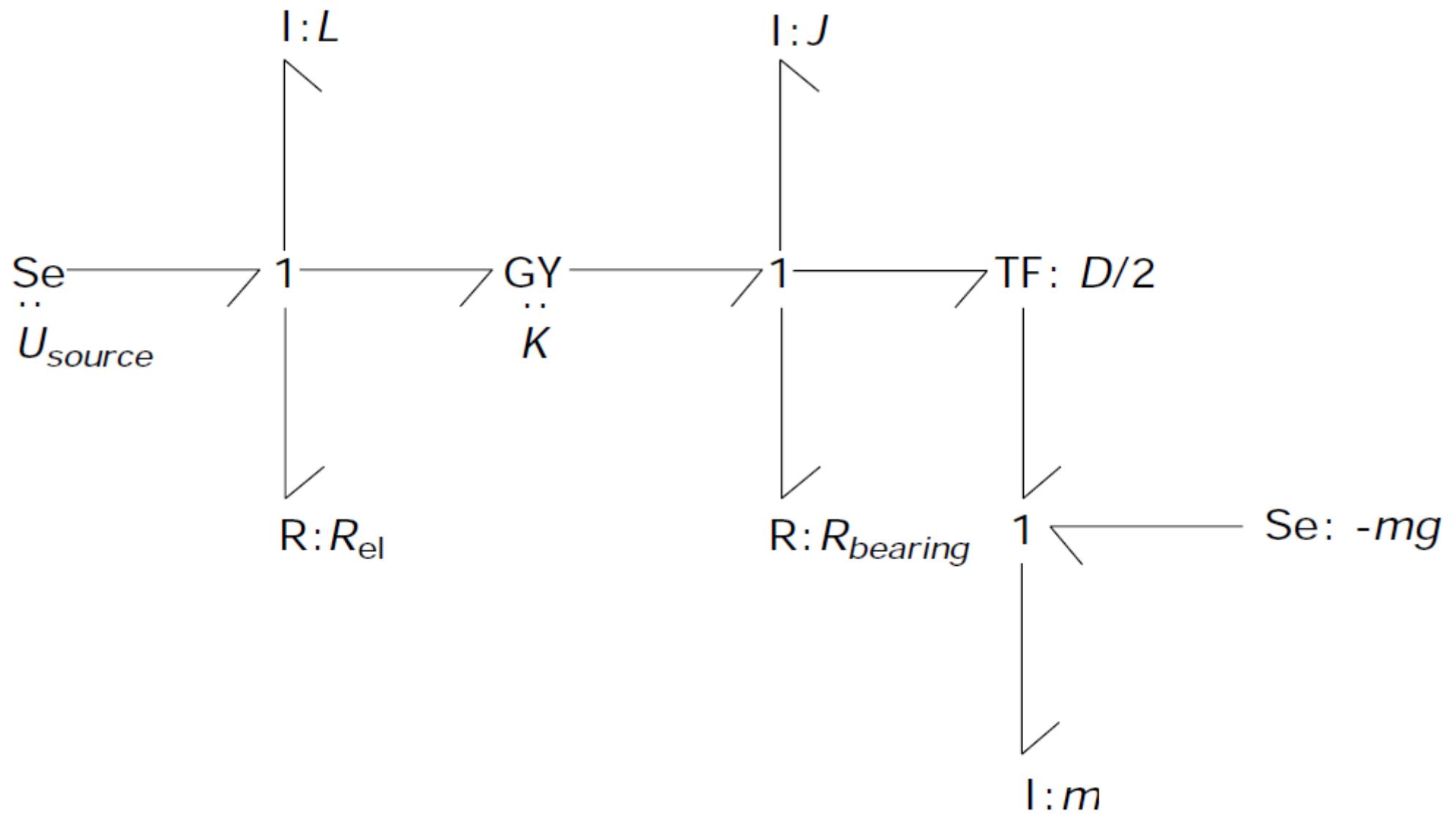
(computational) causality

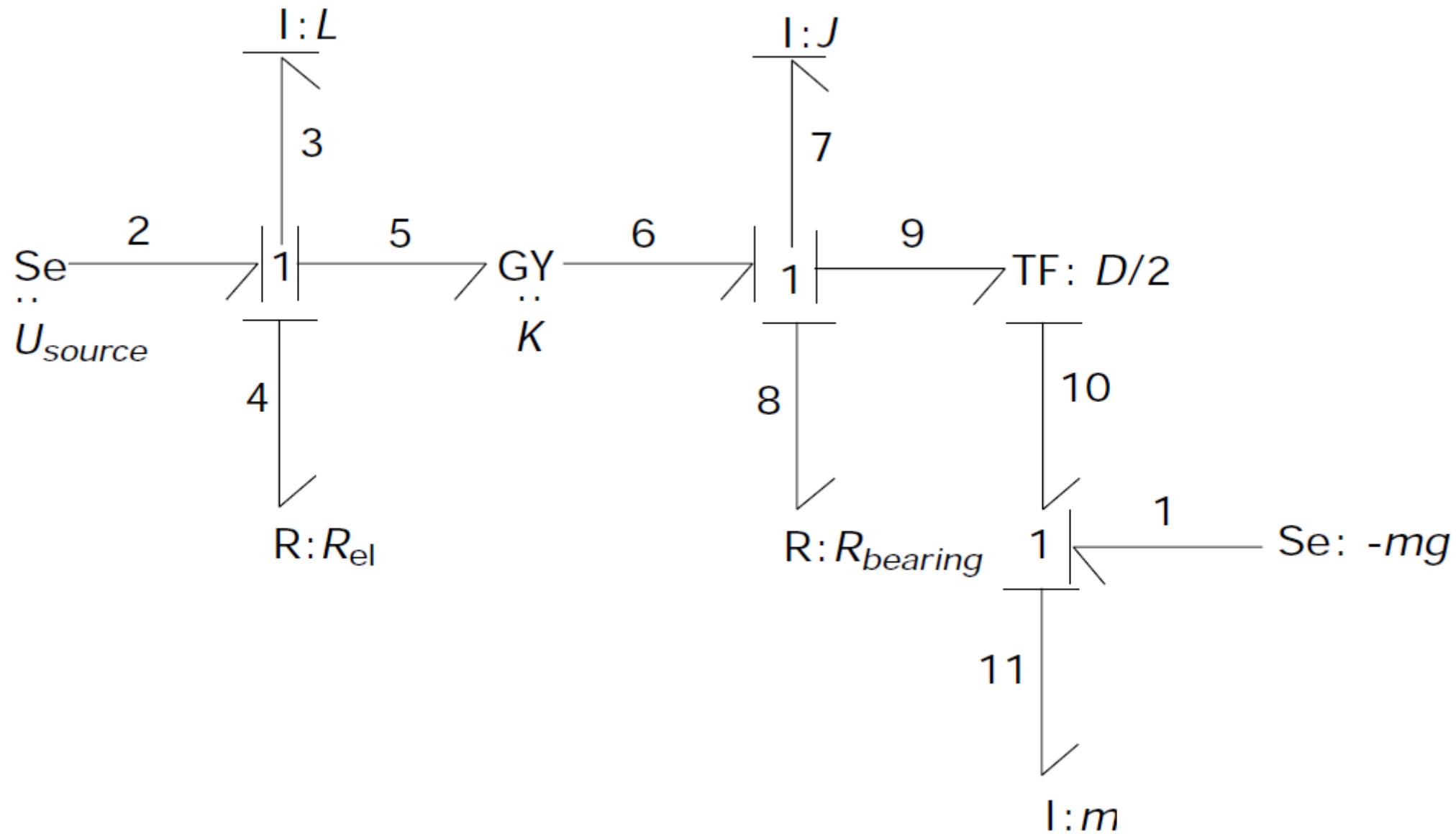


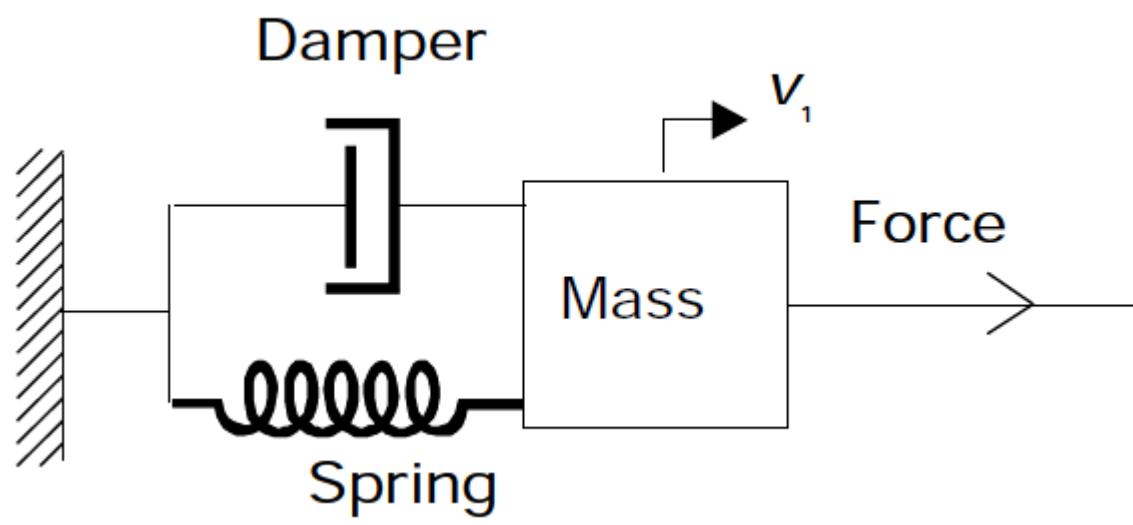


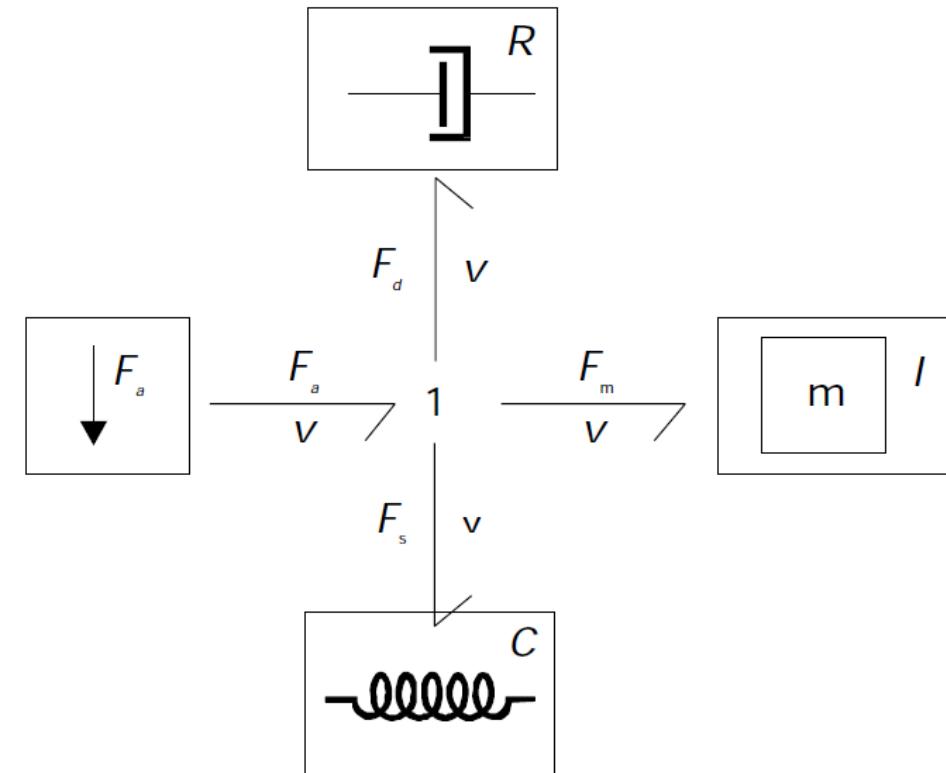
## Idealized Physical Model







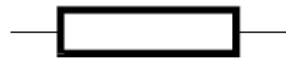




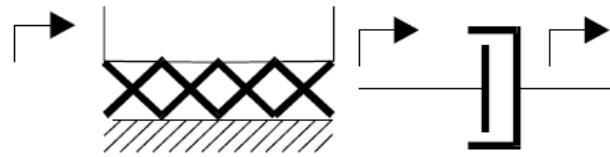
$$S_e : F_a \xrightarrow[F_a/v]{1} \xrightarrow[F_m/v]{I:m} R : R$$

$$C : C = 1/K$$

Domain-specific  
symbols

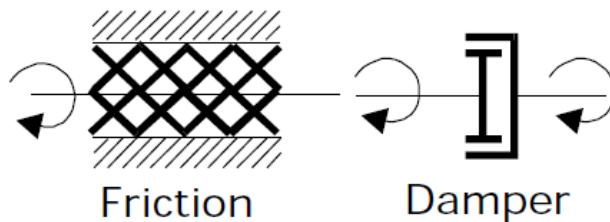


Resistor



Friction

Damper



Friction

Damper

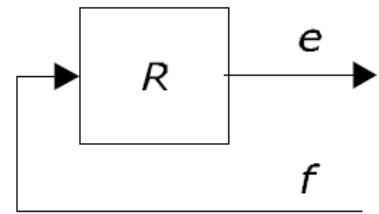
Bond-graph element



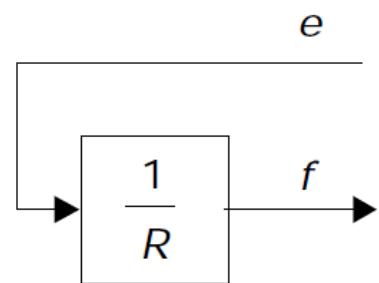
Equations

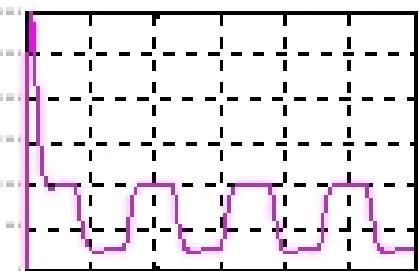
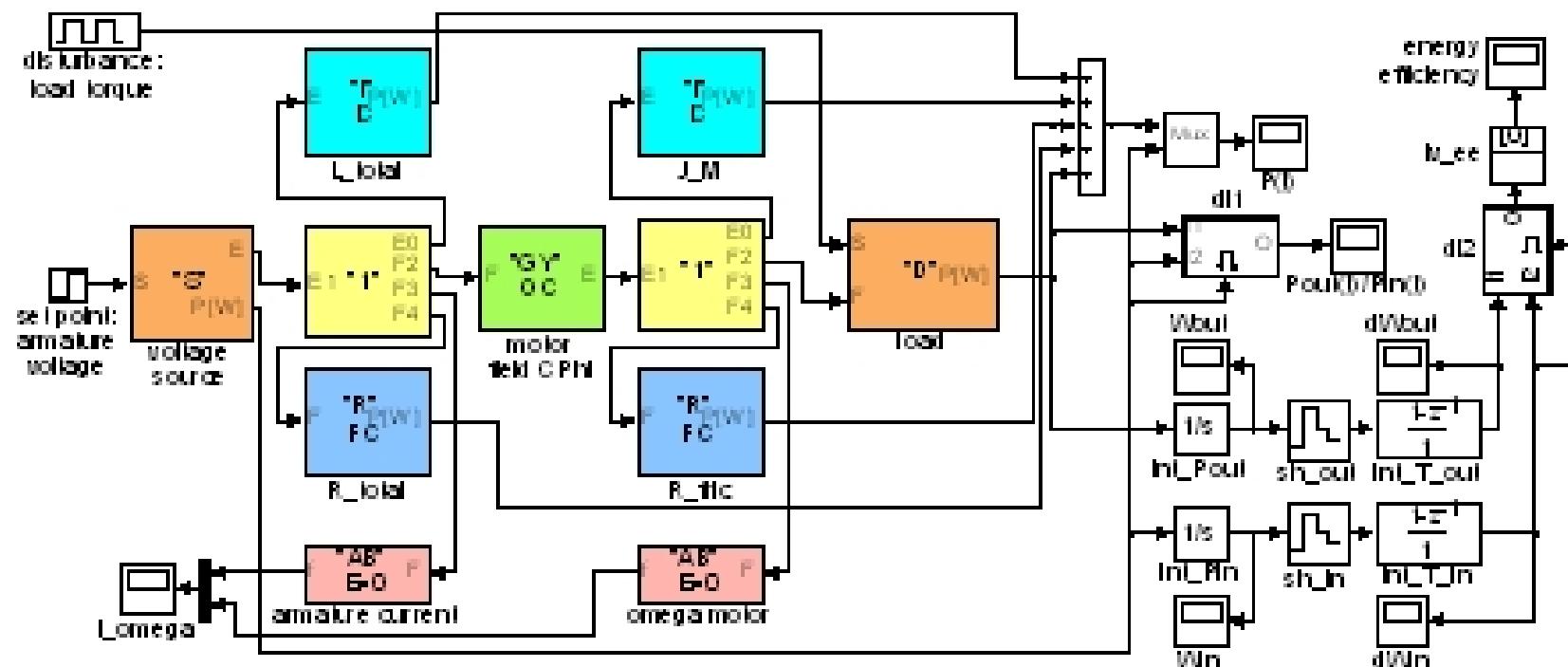
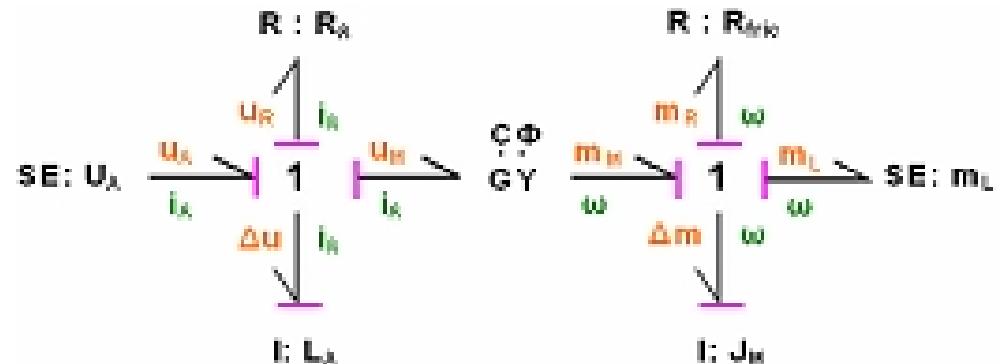
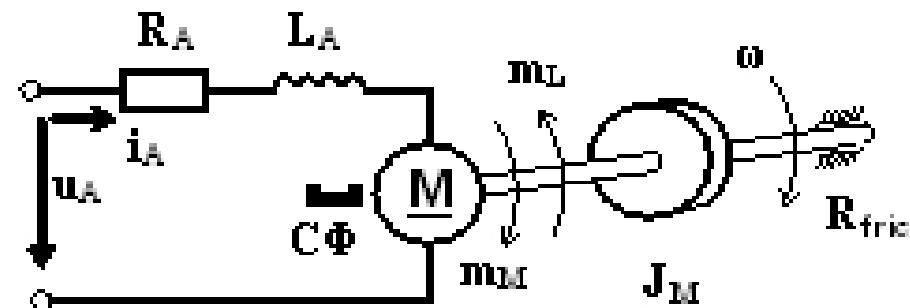
$$e = Rf$$

Block diagram  
expansion

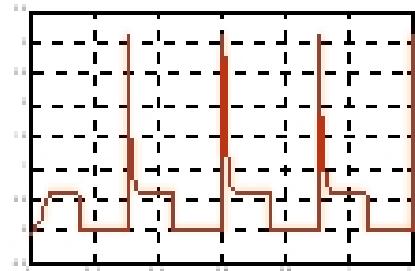


$$f = \frac{1}{R} e$$

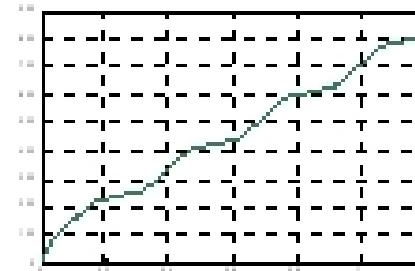




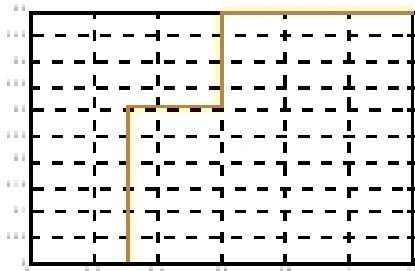
Power  $P(t)$



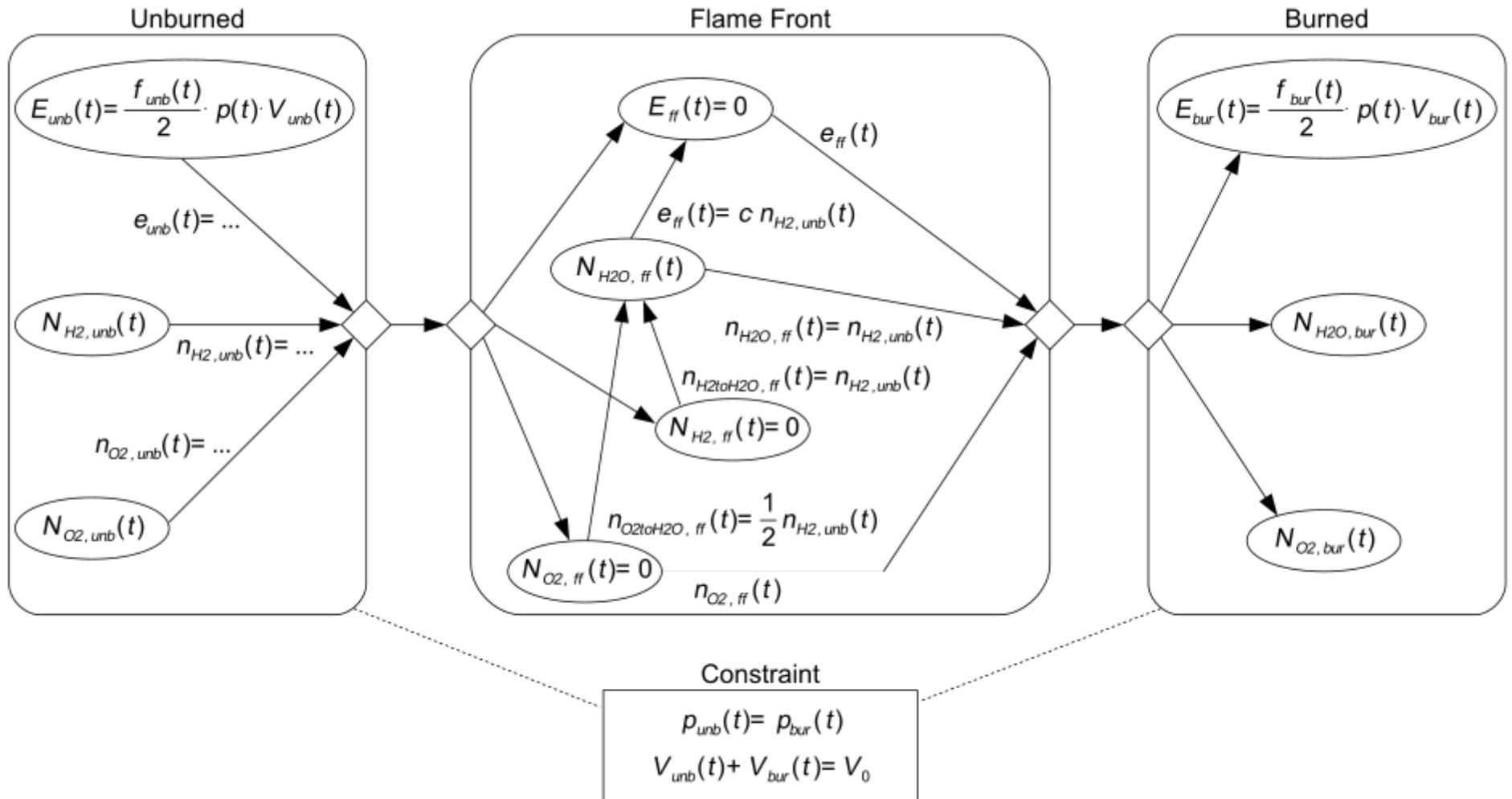
$P_{out}(t)/P_{in}(t)$



Energy  $W_{in}$

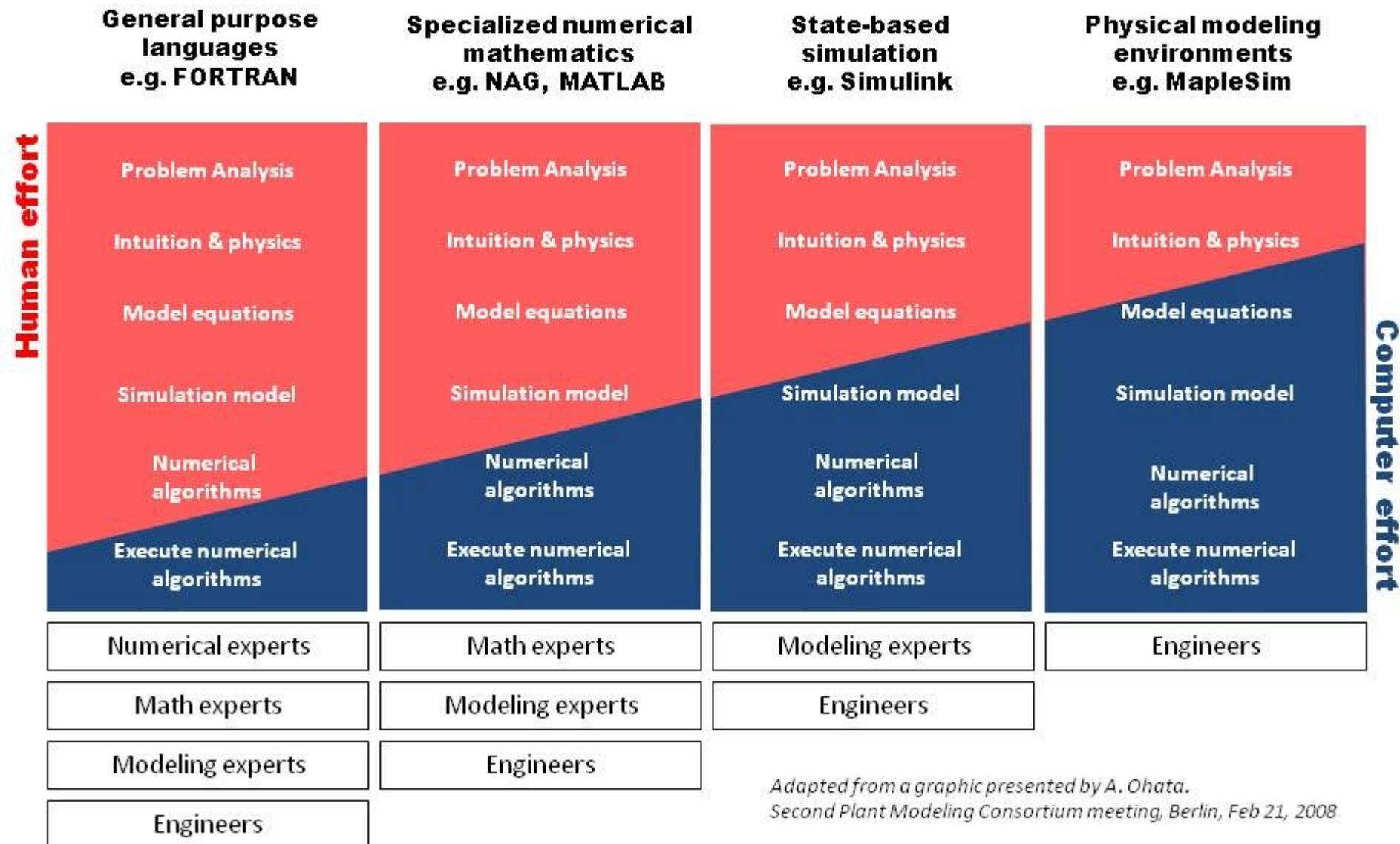


Energy efficiency



**Figure 2.** High Level Model Description (HLMD) example - hydrogen-oxygen combustion in a closed chamber.

Akira Ohata @ Toyota



*Adapted from a graphic presented by A. Ohata.  
Second Plant Modeling Consortium meeting, Berlin, Feb 21, 2008*

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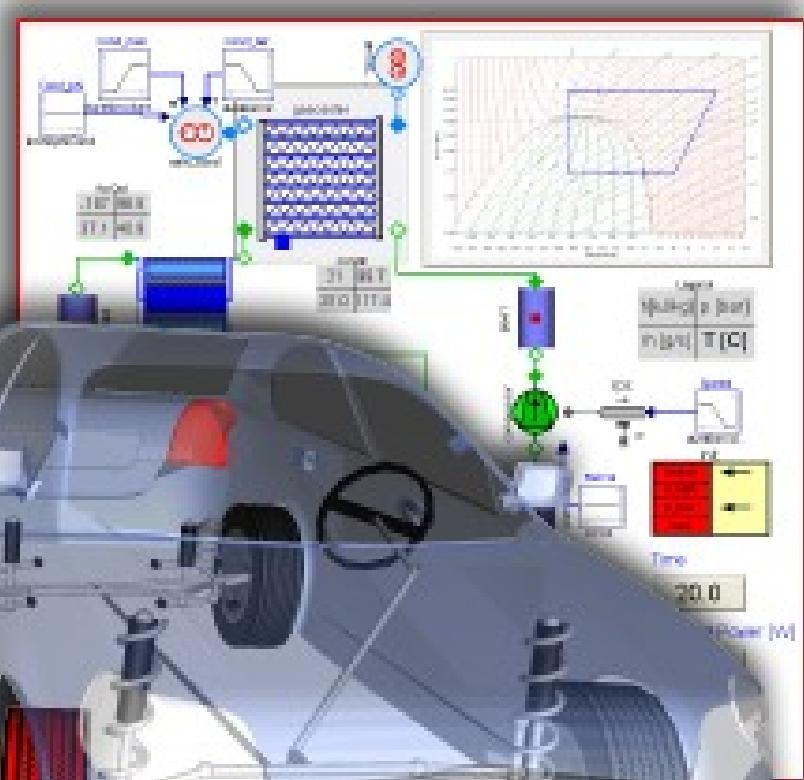
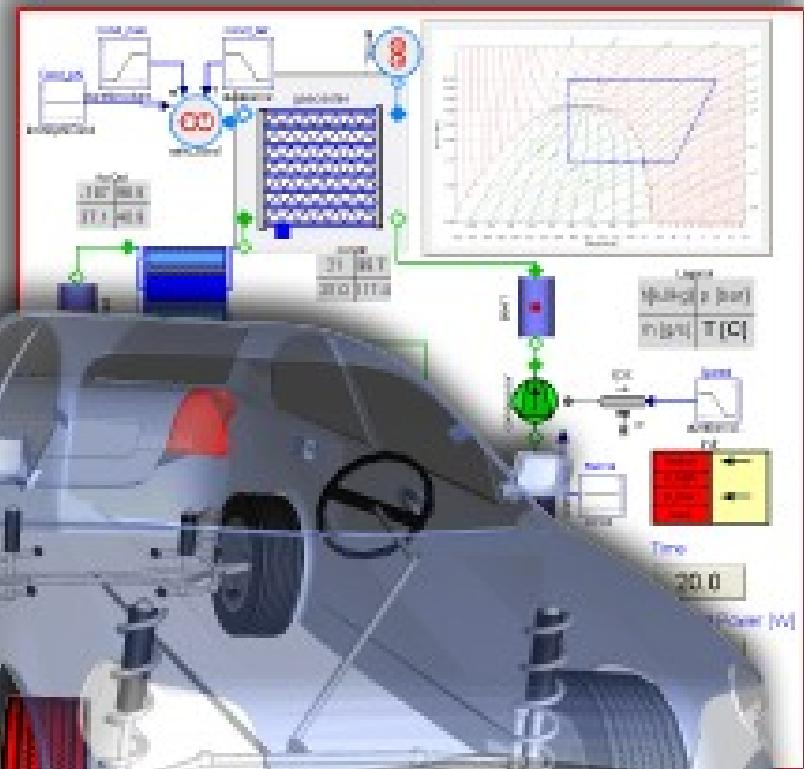
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- Modelica
- User's Guide
- Blocks
- Mechanics
- Fluid
- Electrical
- Analog
- Examples
- Basic
  - Ground
  - Resistor
  - Conductor
  - Capacitor
  - Inductor
  - SaturatingInductor
  - DCTransformer
  - M\_Transformer
  - Gyrator

```

model Capacitor "Ideal linear electrical capacitor"
  parameter SI.Capacitance C "Capacitance";
  Interfaces.PositivePin p;
  Interfaces.NegativePin n;
  SI.Voltage v "Voltage drop between pins";
equation
  0 = p.i + n.i;
  v = p.v - n.v;
  C*der(v) = p.i;
end Capacitor;

```

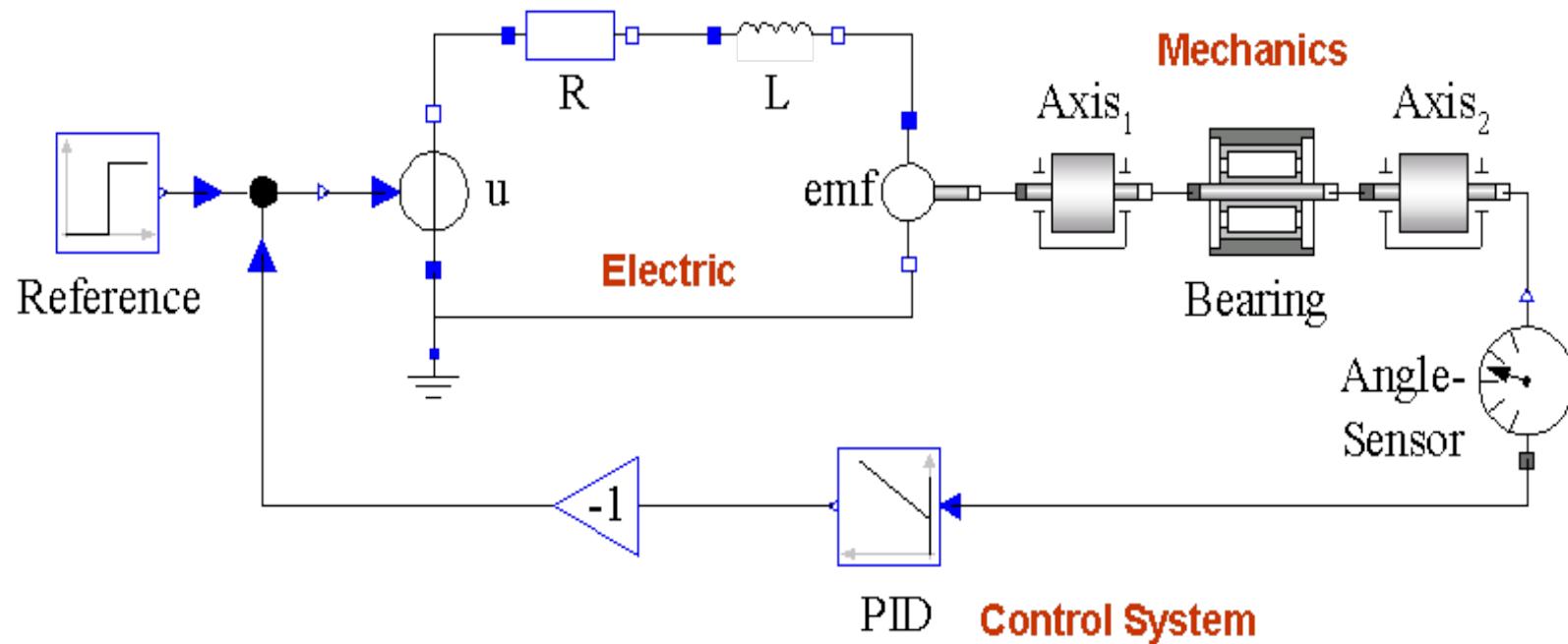


# M O D E L I C A

# Multi-Domain Modeling



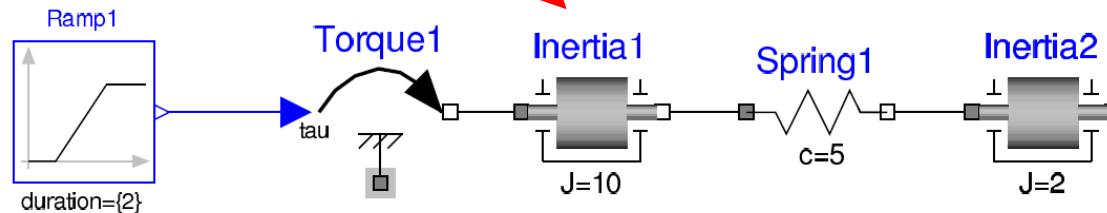
<http://www.modelica.org>



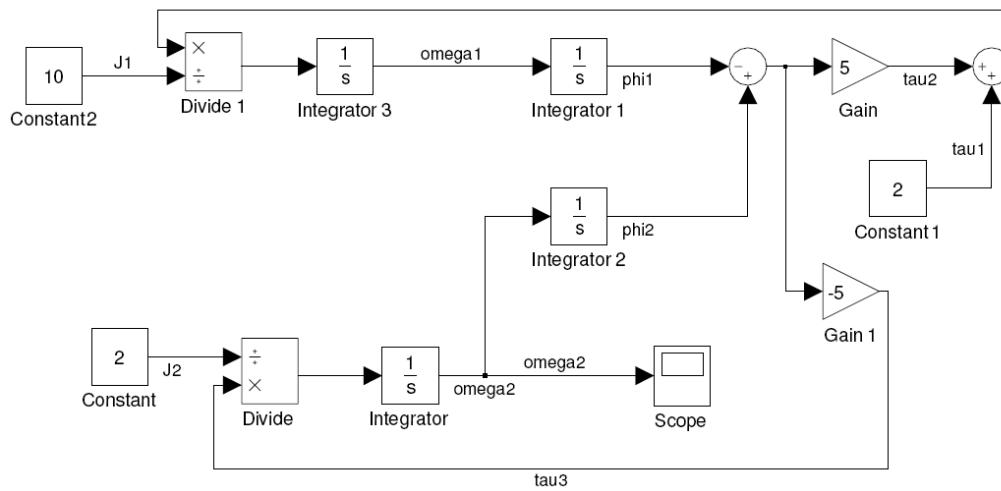
this slide from Peter Fritzson's Modelica tutorial

Keeps the physical structure

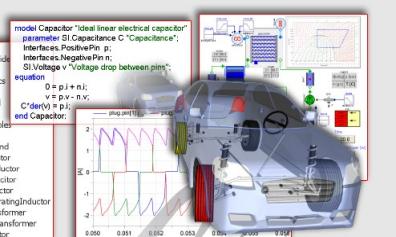
## Acausal model (Modelica)



## Causal block-based model (Simulink)

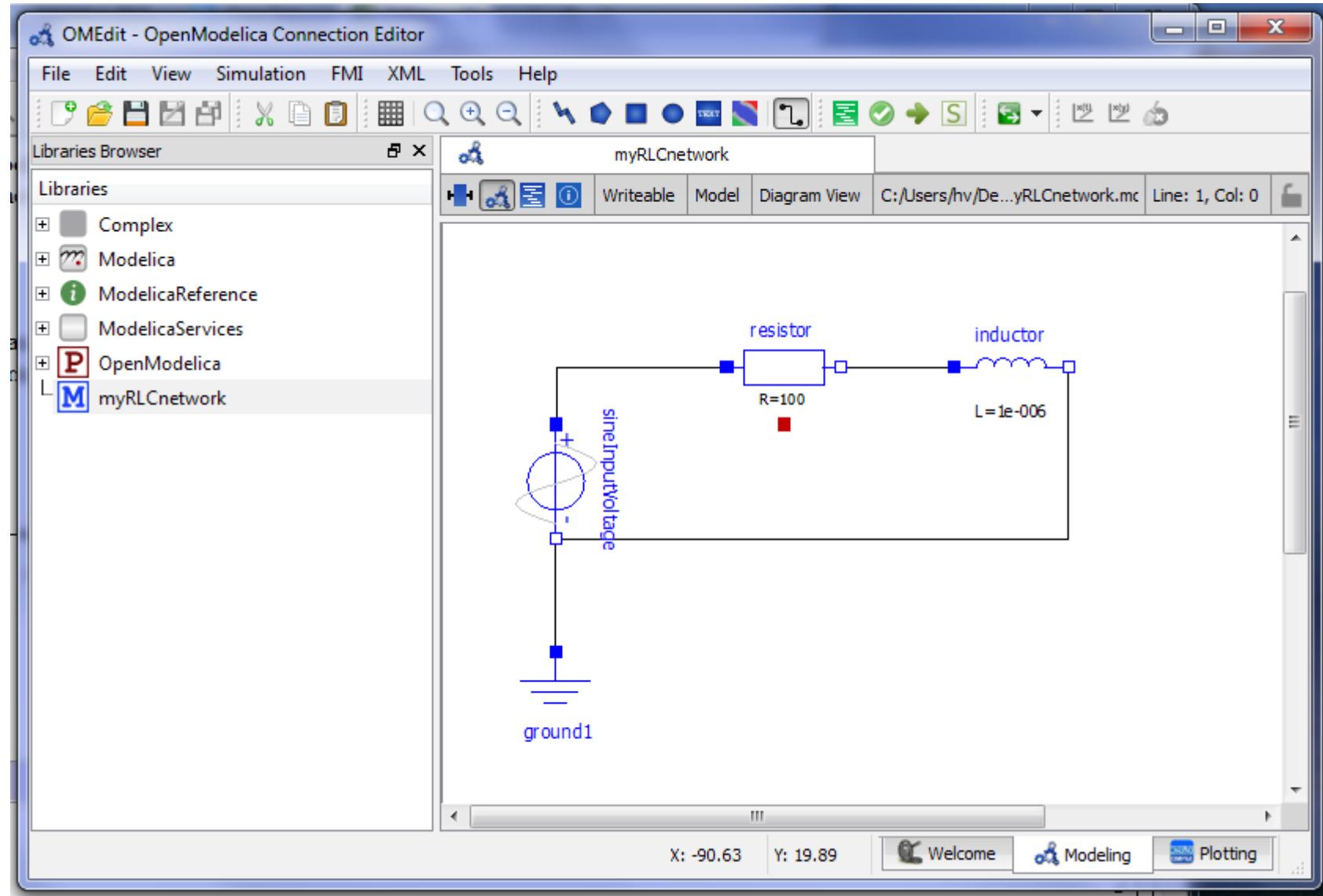


- Model exchange/re-use standard (Modelica Association)
- Modelica Standard Library (MSL)
- Object-oriented, hierarchical; semantics based on flattening
- Computationally a-causal modelling; semantics based on DAEs
- Originated in Hilding Elmquist's 1978 PhD thesis @ Lund
- Early 1990's: Modelica Design Team (started in SiE)



MODELICA

- hybrid (discrete-time/discrete-event) constructs  
(e.g., used to model network protocols based  
on TrueTime <http://www.control.lth.se/trutime/>)
- Limited support for Dynamic Structure models (i.e., no “agents”)
- Separate model from its (numerical) solution ...
- Generate Functional Mockup Interface (FMI) compliant simulation units
- Currently: many commercial and open (OpenModelica) tools
- Related: Mathworks Simscape, EcosimPro, NMF, gProms, ...



OMEdit - OpenModelica Connection Editor

File Edit View Simulation FMI XML Tools Help

Libraries Browser

Libraries

- + Complex
- + Modelica
- + ModelicaReference
- + ModelicaServices
- + OpenModelica
- L myRLCnetwork**

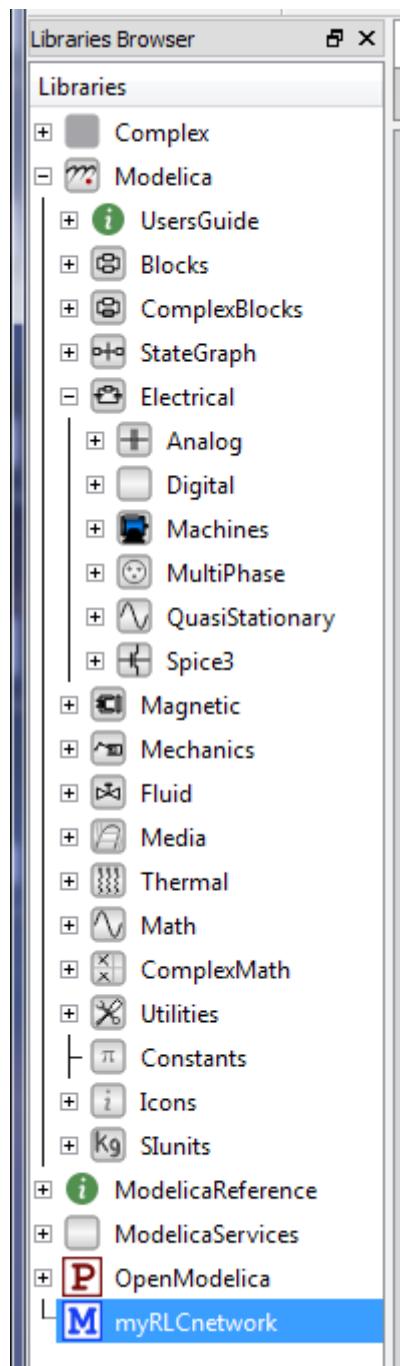
myRLCnetwork

Writeable Model Modelica Text View C:/Users/hv/Desktop/scribbles/myRLCnetwork.mo Line: 1, Col: 11

```
1 model myRLCnetwork
2   Modelica.Electrical.Analog.Basic.Resistor resistor(R = 100) annotation(Placement(visible
= true, transformation(origin = {-40,60}, extent = {{-10,-10},{10,10}}, rotation = 0)));
3   Modelica.Electrical.Analog.Basic.Inductor inductor(L = 1e-006)
annotation(Placement(visible = true, transformation(origin = {0,60}, extent = {{-10,-10},
{10,10}}, rotation = 0)));
4   Modelica.Electrical.Analog.Sources.SineVoltage sineInputVoltage(V = 10, freqHz = 50)
annotation(Placement(visible = true, transformation(origin = {-80,40}, extent =
{{-10,-10},{10,10}}, rotation = -90)));
5   Modelica.Electrical.Analog.Basic.Ground ground1 annotation(Placement(visible = true,
transformation(origin = {-80,0}, extent = {{-10,-10},{10,10}}, rotation = 0)));
6 equation
7   connect(resistor.p,sineInputVoltage.p) annotation(Line(points = {{-50,60}, {-79.8913,60},
{-79.8913,49.4565}, {-79.8913,49.4565}}));
8   connect(resistor.n,inductor.p) annotation(Line(points = {{-30,60}, {-9.78261,60},
{-9.78261,59.2391}, {-9.78261,59.2391}}));
9   connect(sineInputVoltage.n,inductor.n) annotation(Line(points = {{-80,30}, {-80,29.8913},
{10.0543,29.8913}, {10.0543,59.7826}, {10.0543,59.7826}}));
10  connect(sineInputVoltage.n,ground1.p) annotation(Line(points = {{-80,30}, {-80,9.78261},
{-79.61960000000001,9.78261}, {-79.61960000000001,9.78261}}));
11  annotation(Icon(coordinateSystem(extent = {{-100,-100},{100,100}}, preserveAspectRatio =
true, initialScale = 0.1, grid = {2,2})), Diagram(coordinateSystem(extent = {{-100,-100},
{100,100}}, preserveAspectRatio = true, initialScale = 0.1, grid = {2,2})));
12 end myRLCnetwork;
```

X: -90.63 Y: 19.89

Welcome Modeling Plotting



OMEdit - OpenModelica Connection Editor

File Edit View Simulation FMI XML Tools Help

Libraries Browser

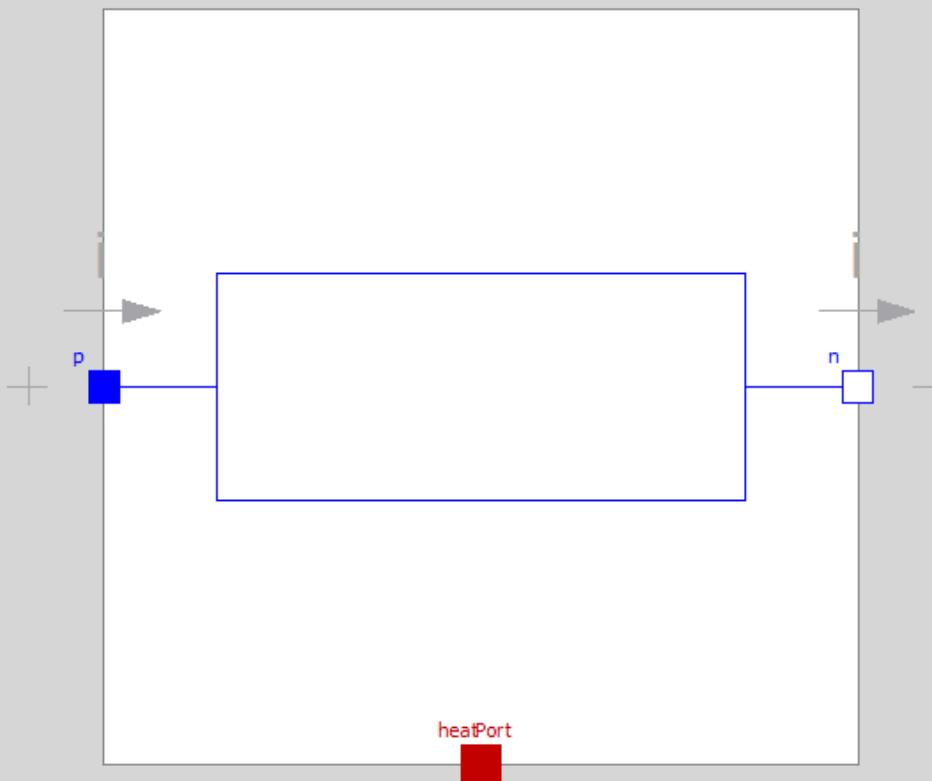
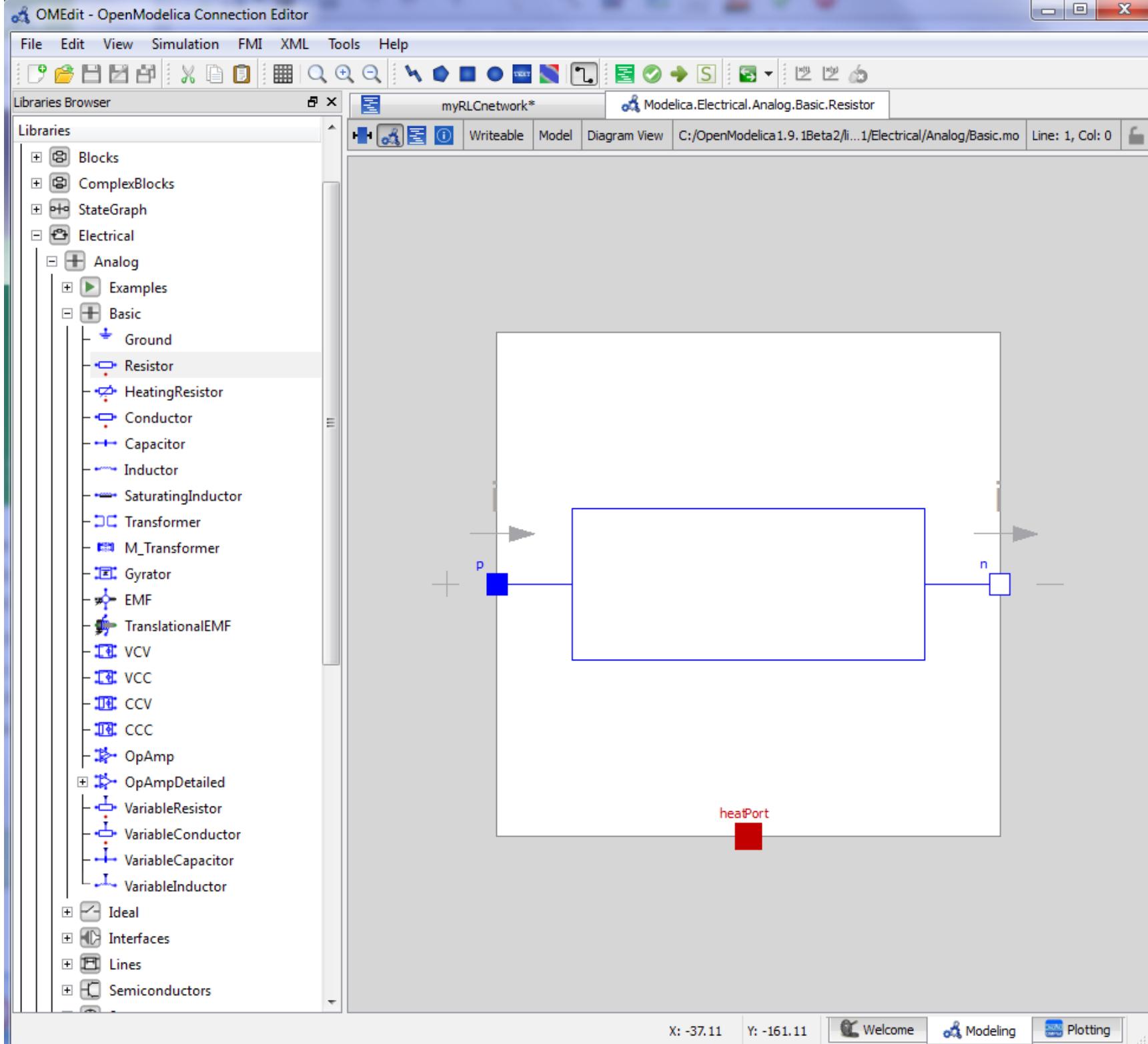
myRLCnetwork\* Modelica.Electrical.Analog.Basic.Resistor

C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Basic.mo Line: 1, Col: 0

```

1 model Resistor "Ideal linear electrical resistor"
2   parameter Modelica.SIunits.Resistance R(start = 1) "Resistance at temperature T_ref";
3   parameter Modelica.SIunits.Temperature T_ref = 300.15 "Reference temperature";
4   parameter Modelica.SIunits.LinearTemperatureCoefficient alpha = 0 "Temperature coefficient of resistance
(R_actual = R*(1 + alpha*(T_heatPort - T_ref)));
5   extends Modelica.Electrical.Analog.Interfaces.OnePort;
6   extends Modelica.Electrical.Analog.Interfaces.ConditionalHeatPort(T = T_ref);
7   Modelica.SIunits.Resistance R_actual "Actual resistance = R*(1 + alpha*(T_heatPort - T_ref))";
8 equation
9   assert(1 + alpha * (T_heatPort - T_ref) >= Modelica.Constants.eps, "Temperature outside scope of model!");
10  R_actual = R * (1 + alpha * (T_heatPort - T_ref));
11  v = R_actual * i;
12  LossPower = v * i;
13  annotation(Documentation(info = "<html>
14 <p>The linear resistor connects the branch voltage <i>v</i> with the branch current <i>i</i> by <i>i*R = v</i>.
The Resistance <i>R</i> is allowed to be positive, zero, or negative.</p>
15 </html>", revisions = "<html>
16 <ul>
17 <li><i> August 07, 2009 </i>
18     by Anton Haumer<br> temperature dependency of resistance added<br>
19 </li>
20 <li><i> March 11, 2009 </i>
21     by Christoph Clauss<br> conditional heat port added<br>
22 </li>
23 <li><i> 1998 </i>
24     by Christoph Clauss<br> initially implemented<br>
25 </li>
26 </ul>
27 </html>"), Icon(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}), graphics =
{Rectangle(extent = {{-70,30},{70,-30}}, lineColor = {0,0,255}, fillColor = {255,255,255}, fillPattern =
FillPattern.Solid), Line(points = {{-90,0},{-70,0}}, color = {0,0,255}), Line(points = {{70,0},{90,0}}, color =
{0,0,255}), Text(extent = {{-144,-40},{142,-72}}, lineColor = {0,0,0}, textString = "R=%R"), Line(visible =
useHeatPort, points = {{0,-100},{0,-30}}, color = {127,0,0}, smooth = Smooth.None, pattern =
LinePattern.Dot), Text(extent = {{-152,87},{148,47}}, textString = "%name", lineColor = {0,0,255})), Diagram(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}), graphics =
{Rectangle(extent = {{-70,30},{70,-30}}, lineColor = {0,0,255}), Line(points = {{-96,0},{-70,0}}, color =
{0,0,255}), Line(points = {{70,0},{96,0}}, color = {0,0,255}))});
28 end Resistor;
```

X: -15.03 Y: 154.06 Welcome Modeling Plotting



X: -37.11

Y: -161.11

Welcome

Modeling

Plotting

Libraries

- VolumeDensityOfCharge
- SurfaceDensityOfCharge
- ElectricFieldStrength
- ElectricPotential
- Voltage

Writeable Type Modelica Text View C:/Open...nts.mo Line: 1, Col: 0

```
1 type Voltage = ElectricPotential;
```

Libraries

- VolumeDensityOfCharge
- SurfaceDensityOfCharge
- ElectricFieldStrength
- ElectricPotential

Writeable Type Modelica Text View C:/OpenModelica 1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/SIunits.mo Line: 1, Col: 0

```
1 type ElectricPotential = Real(final quantity = "ElectricPotential", final unit = "V");
```

Libraries

- CCC
- OpAmp
- + OpAmpDetailed
- VariableResistor
- VariableConductor
- VariableCapacitor
- VariableInductor
- + Ideal
- Interfaces
- Pin
- PositivePin
- NegativePin
- TwoPin
- OnePort
- TwoPort
- ConditionalHeatPort
- AbsoluteSensor
- RelativeSensor
- VoltageSource
- CurrentSource
- + Lines
- + Semiconductors
- + Sensors
- + Sources
- + Digital
- + Machines
- + MultiPhase

File: C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Interfaces.mo | Line: 1, Col: 0

```

1 partial model OnePort "Component with two electrical pins p and n and current i from p to n"
2   SI.Voltage v "Voltage drop between the two pins (= p.v - n.v)";
3   SI.Current i "Current flowing from pin p to pin n";
4   PositivePin p "Positive pin (potential p.v > n.v for positive voltage drop v)"
5     annotation(Placement(transformation(extent = {{-110,-10},{-90,10}}), rotation = 0));
6   NegativePin n "Negative pin" annotation(Placement(transformation(extent = {{110,-10},{90,10}}),
7     rotation = 0));
8   equation
9     v = p.v - n.v;
10    0 = p.i + n.i;
11    i = p.i;
12    annotation(Documentation(info = "<html>
13      <p>Superclass of elements which have <b>two</b> electrical pins: the positive pin connector
14      <i>p</i>, and the negative pin connector <i>n</i>. It is assumed that the current flowing into pin p
15      is identical to the current flowing out of pin n. This current is provided explicitly as current
16      i.</p>
17    </html>"), revisions = "<html>
18      <ul>
19        <li><i> 1998 </i>
20          by Christoph Clauss<br> initially implemented<br>
21        </li>
22      </ul>
23    </html>"), Diagram(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}),
24      graphics = {Line(points = {{-110,20}, {-85,20}}, color = {160,160,164}), Polygon(points = {{-95,23},
25        {-85,20}, {-95,17}, {-95,23}}, lineColor = {160,160,164}, fillColor = {160,160,164}, fillPattern =
26        FillPattern.Solid), Line(points = {{90,20}, {115,20}}, color = {160,160,164}), Line(points = {{-125,0},
27        {-115,0}}, color = {160,160,164}), Line(points = {{-120,-5}, {-120,5}}, color =
28        {160,160,164}), Text(extent = {{-110,25}, {-90,45}}, lineColor = {160,160,164}, textString =
29        "i"), Polygon(points = {{105,23}, {115,20}, {105,17}, {105,23}}, lineColor = {160,160,164}, fillColor =
30        {160,160,164}, fillPattern = FillPattern.Solid), Line(points = {{115,0}, {125,0}}, color =
31        {160,160,164}), Text(extent = {{90,45}, {110,25}}, lineColor = {160,160,164}, textString = "i")});
32  end OnePort;

```

Libraries

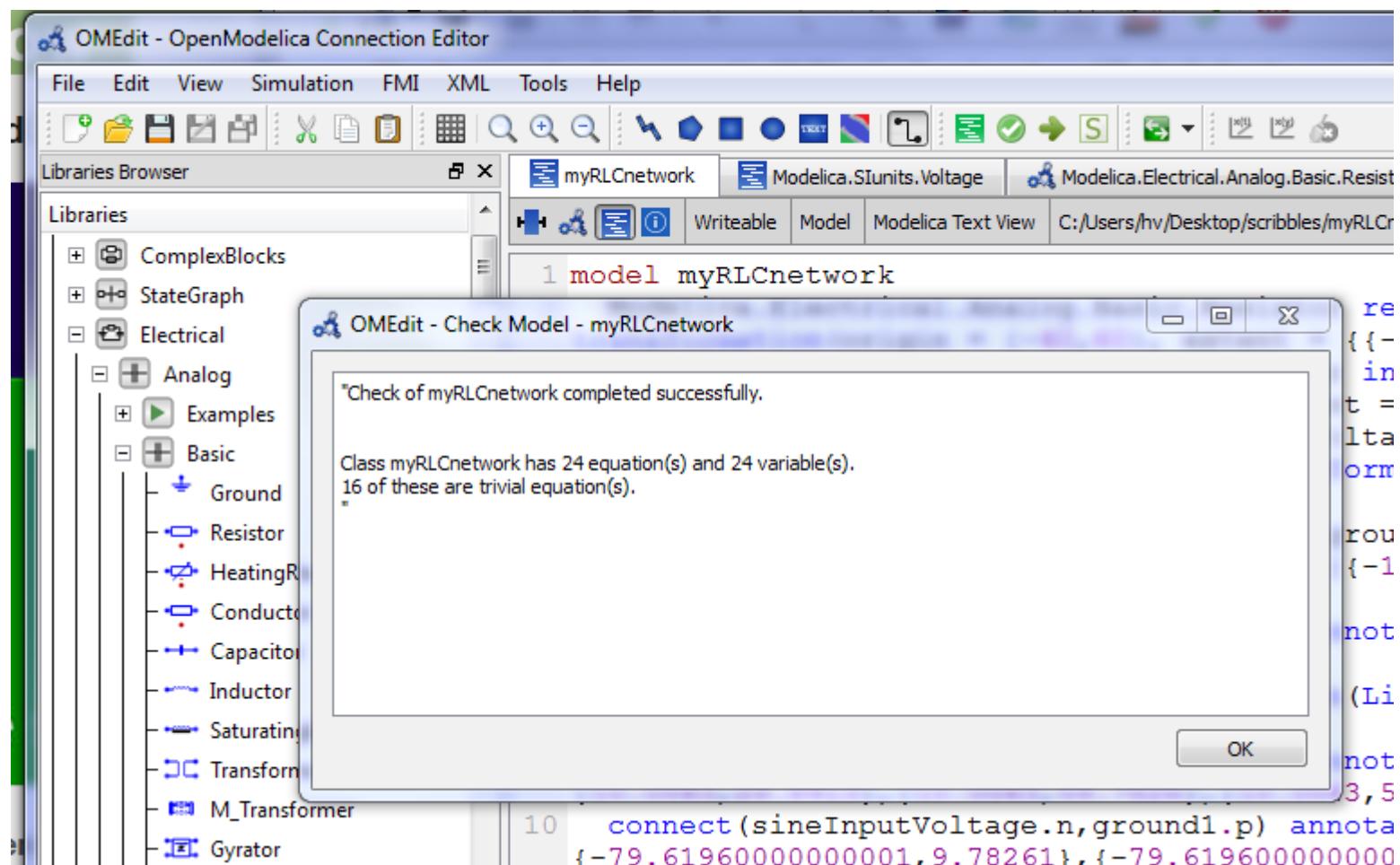
- CCC
- OpAmp
- OpAmpDetailed
- VariableResistor
- VariableConductor
- VariableCapacitor
- VariableInductor
- Ideal
- Interfaces
  - Pin
  - PositivePin
  - NegativePin
  - TwoPin
  - OnePort
  - TwoPort
  - ConditionalHeatPort
  - AbsoluteSensor
  - RelativeSensor
  - VoltageSource
  - CurrentSource
- Lines
- Semiconductors
- Sensors
- Sources
- Digital
- Machines

Writable Connector Modelica Text View C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Interfaces.mo Line: 1, Col: 0

```

1 connector PositivePin "Positive pin of an electric component"
2   Modelica.SIunits.Voltage v "Potential at the pin" annotation(unassignedMessage = "An electrical
3 potential cannot be uniquely calculated.
4 The reason could be that
5 - a ground object is missing (Modelica.Electrical.Analog.Basic.Ground)
6 to define the zero potential of the electrical circuit, or
7 - a connector of an electrical component is not connected.");
8   flow Modelica.SIunits.Current i "Current flowing into the pin" annotation(unassignedMessage = "An
9 electrical current cannot be uniquely calculated.
10 The reason could be that
11 - a ground object is missing (Modelica.Electrical.Analog.Basic.Ground)
12 to define the zero potential of the electrical circuit, or
13 - a connector of an electrical component is not connected.");
14 annotation(defaultComponentName = "pin_p", Documentation(info = "<html>
15 <p>Connectors PositivePin and NegativePin are nearly identical. The only difference is that the
16 icons are different in order to identify more easily the pins of a component. Usually, connector
17 PositivePin is used for the positive and connector NegativePin for the negative pin of an electrical
18 component.</p>
19 </html>"), revisions = "<html>
20 <ul>
21 <li><i> 1998 </i>
22 by Christoph Clauss<br> initially implemented<br>
23 </li>
24 </ul>
25 </html>"), Icon(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}),
26 graphics = {Rectangle(extent = {{-100,100},{100,-100}}, lineColor = {0,0,255}, fillColor =
27 {0,0,255}, fillPattern = FillPattern.Solid)}), Diagram(coordinateSystem(preserveAspectRatio = true,
28 extent = {{-100,-100},{100,100}}), graphics = {Rectangle(extent = {{-40,40},{40,-40}}, lineColor =
29 {0,0,255}, fillColor = {0,0,255}, fillPattern = FillPattern.Solid),Text(extent = {{-160,110},
30 {40,50}}, lineColor = {0,0,255}, textString = "%name")});
31 end PositivePin;

```



```

class myRLCnetwork
Real resistor.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real resistor.i(quantity = "ElectricCurrent", unit = "A") "Current flowing from pin p to pin n";
Real resistor.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real resistor.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real resistor.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real resistor.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Boolean resistor.useHeatPort = false "=true, if HeatPort is enabled";
Real resistor.LossPower(quantity = "Power", unit = "W") "Loss power leaving component via HeatPort";
Real resistor.T_heatPort(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) "Temperature of HeatPort";
parameter Real resistor.R(quantity = "Resistance", unit = "Ohm", start = 1.0) = 100.0 "Resistance at temperature T_ref";
parameter Real resistor.T_ref(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) = 300.15 "Reference temperature";
parameter Real resistor.alpha(quantity = "LinearTemperatureCoefficient", unit = "1/K") = 0.0 "Temperature coefficient of resistance (R_actual = R*(1 + alpha*(T_heatPort - T_ref)))";
Real resistor.R_actual(quantity = "Resistance", unit = "Ohm") "Actual resistance = R*(1 + alpha*(T_heatPort - T_ref))";
parameter Real resistor.T(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) = resistor.T_ref "Fixed device temperature if useHeatPort = false";
Real inductor.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real inductor.i(quantity = "ElectricCurrent", unit = "A", start = 0.0) "Current flowing from pin p to pin n";
Real inductor.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real inductor.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real inductor.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real inductor.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Real inductor.L(quantity = "Inductance", unit = "H", start = 1.0) = 1e-006 "Inductance";
Real sineInputVoltage.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real sineInputVoltage.i(quantity = "ElectricCurrent", unit = "A") "Current flowing from pin p to pin n";
Real sineInputVoltage.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real sineInputVoltage.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real sineInputVoltage.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real sineInputVoltage.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Real sineInputVoltage.offset(quantity = "ElectricPotential", unit = "V") = 0.0 "Voltage offset";
parameter Real sineInputVoltage.startTime(quantity = "Time", unit = "s") = 0.0 "Time offset";
parameter Real sineInputVoltage.V(quantity = "ElectricPotential", unit = "V", start = 1.0) = 10.0 "Amplitude of sine wave";
parameter Real sineInputVoltage.phase(quantity = "Angle", unit = "rad", displayUnit = "deg") = 0.0 "Phase of sine wave";
parameter Real sineInputVoltage.freqHz(quantity = "Frequency", unit = "Hz", start = 1.0) = 50.0 "Frequency of sine wave";
output Real sineInputVoltage.signalSource.y "Connector of Real output signal";
parameter Real sineInputVoltage.signalSource.amplitude = sineInputVoltage.V "Amplitude of sine wave";
parameter Real sineInputVoltage.signalSource.freqHz(quantity = "Frequency", unit = "Hz", start = 1.0) = sineInputVoltage.freqHz "Frequency of sine wave";
parameter Real sineInputVoltage.signalSource.phase(quantity = "Angle", unit = "rad", displayUnit = "deg") = sineInputVoltage.phase "Phase of sine wave";
parameter Real sineInputVoltage.signalSource.offset = sineInputVoltage.offset "Offset of output signal";
parameter Real sineInputVoltage.signalSource.startTime(quantity = "Time", unit = "s") = sineInputVoltage.startTime "Output = offset for time < startTime";
protected constant Real sineInputVoltage.signalSource.pi = 3.141592653589793;
Real ground1.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real ground1.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";

```

```

equation
  assert(1.0 + resistor.alpha * (resistor.T_heatPort - resistor.T_ref) >= 1e-015,"Temperature outside scope of model!");
  resistor.R_actual = resistor.R * (1.0 + resistor.alpha * (resistor.T_heatPort - resistor.T_ref));
  resistor.v = resistor.R_actual * resistor.i;
  resistor.LossPower = resistor.v * resistor.i;
  resistor.v = resistor.p.v - resistor.n.v;
  0.0 = resistor.p.i + resistor.n.i;
  resistor.i = resistor.p.i;
  resistor.T_heatPort = resistor.T;
  inductor.L * der(inductor.i) = inductor.v;
  inductor.v = inductor.p.v - inductor.n.v;
  0.0 = inductor.p.i + inductor.n.i;
  inductor.i = inductor.p.i;
  sineInputVoltage.signalSource.y = sineInputVoltage.signalSource.offset + (if time <
sineInputVoltage.signalSource.startTime then 0.0 else sineInputVoltage.signalSource.amplitude * sin(6.283185307179586 *
sineInputVoltage.signalSource.freqHz * (time - sineInputVoltage.signalSource.startTime) +
sineInputVoltage.signalSource.phase));
  sineInputVoltage.v = sineInputVoltage.signalSource.y;
  sineInputVoltage.v = sineInputVoltage.p.v - sineInputVoltage.n.v;
  0.0 = sineInputVoltage.p.i + sineInputVoltage.n.i;
  sineInputVoltage.i = sineInputVoltage.p.i;
  ground1.p.v = 0.0;
  resistor.p.i + sineInputVoltage.p.i = 0.0;
  resistor.n.i + inductor.p.i = 0.0;
  inductor.n.i + sineInputVoltage.n.i + ground1.p.i = 0.0;
  resistor.p.v = sineInputVoltage.p.v;
  inductor.p.v = resistor.n.v;
  ground1.p.v = inductor.n.v;
  ground1.p.v = sineInputVoltage.n.v;
end myRLCnetwork;

```

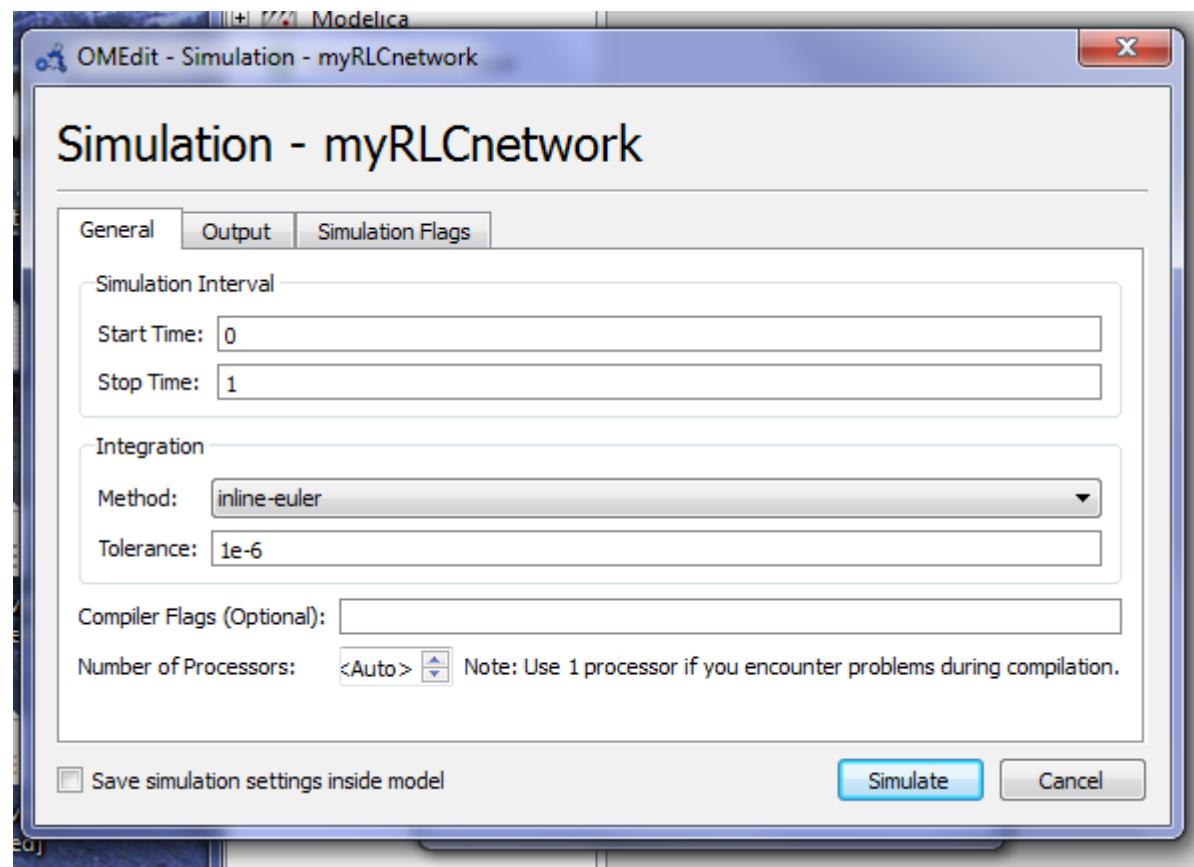
	29/09/20...	C File	myRLCnetwork	14 KB
	29/09/20...	Application	myRLCnetwork	9,960 KB
	29/09/20...	LIBS File	myRLCnetwork.lib	0 KB
	29/09/20...	Text Document	myRLCnetwork	0 KB
	29/09/20...	MAKEFILE File	myRLCnetwork.makefile	2 KB
	29/09/20...	O File	myRLCnetwork.o	17 KB
	29/09/20...	C File	myRLCnetwork_01exo	2 KB
	29/09/20...	O File	myRLCnetwork_01exo.o	2 KB
	29/09/20...	C File	myRLCnetwork_02nls	2 KB
	29/09/20...	O File	myRLCnetwork_02nls.o	1 KB
	29/09/20...	C File	myRLCnetwork_03isy	2 KB
	29/09/20...	O File	myRLCnetwork_03isy.o	1 KB
	29/09/20...	C File	myRLCnetwork_04set	2 KB
	29/09/20...	O File	myRLCnetwork_04set.o	1 KB
	29/09/20...	C File	myRLCnetwork_05evt	3 KB
	29/09/20...	O File	myRLCnetwork_05evt.o	2 KB
	29/09/20...	C File	myRLCnetwork_06inz	7 KB
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	29/09/20...	C File	myRLCnetwork_07dly	2 KB
	29/09/20...	O File	myRLCnetwork_07dly.o	1 KB
	29/09/20...	C File	myRLCnetwork_08bnd	7 KB
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	29/09/20...	O File	myRLCnetwork_10asr.o	1 KB
	29/09/20...	C File	myRLCnetwork_11mix	2 KB
	29/09/20...	H File	myRLCnetwork_11mix.h	0 KB
	29/09/20...	O File	myRLCnetwork_11mix.o	1 KB
	29/09/20...	C File	myRLCnetwork_12jac	4 KB
	29/09/20...	H File	myRLCnetwork_12jac.h	2 KB

## OMEdit - myRLCnetwork Simulation Output

## Output

## Compilation

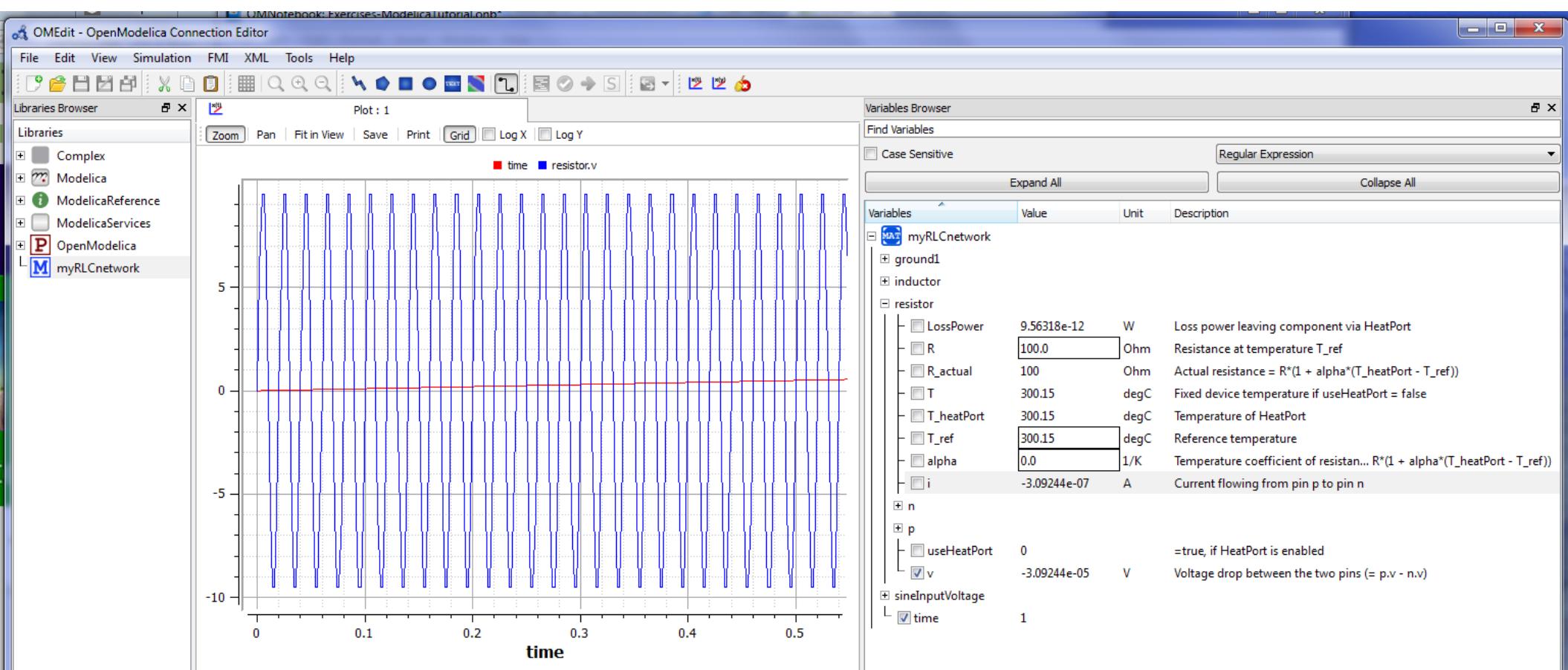
```
"C:\OpenModelica1.9.1Beta2\MinGW\bin\mingw32-make.exe" -j4 -f myRLCnetwork.makefile
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork.o myRLCnetwork.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_functions.o myRLCnetwork_functions.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_records.o myRLCnetwork_records.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_01exo.o myRLCnetwork_01exo.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_02nls.o myRLCnetwork_02nls.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_03lsy.o myRLCnetwork_03lsy.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_04set.o myRLCnetwork_04set.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_05evt.o myRLCnetwork_05evt.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_06inz.o myRLCnetwork_06inz.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_07dly.o myRLCnetwork_07dly.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_08bnd.o myRLCnetwork_08bnd.c
myRLCnetwork_05evt.c: In function 'myRLCnetwork_zeroCrossingDescription':
myRLCnetwork_05evt.c:51: warning: assignment discards qualifiers from pointer target type
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_09alg.o myRLCnetwork_09alg.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_10asr.o myRLCnetwork_10asr.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_11mix.o myRLCnetwork_11mix.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_12jac.o myRLCnetwork_12jac.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_13opt.o myRLCnetwork_13opt.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_14lnz.o myRLCnetwork_14lnz.c
gcc -I. -o myRLCnetwork.exe myRLCnetwork.o myRLCnetwork_functions.o myRLCnetwork_records.o myRLCnetwork_01exo.o myRLCnetwork_02nls.o
myRLCnetwork_03lsy.o myRLCnetwork_04set.o myRLCnetwork_05evt.o myRLCnetwork_06inz.o myRLCnetwork_07dly.o myRLCnetwork_08bnd.o myRLCnetwork_09alg.o
myRLCnetwork_10asr.o myRLCnetwork_11mix.o myRLCnetwork_12jac.o myRLCnetwork_13opt.o myRLCnetwork_14lnz.o -
I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -falign-functions -msse2 -mfpmath=sse -
L"C:/OpenModelica1.9.1Beta2/lib/omc" -L"C:/OpenModelica1.9.1Beta2/lib" -Wl,--stack,0x2000000,-rpath,"C:/OpenModelica1.9.1Beta2/lib/omc" -Wl,-
rpath,"C:/OpenModelica1.9.1Beta2/lib" -lregex -lexpat -lgc -lpthread -fopenmp -loleaut32 -lSimulationRuntimeC -lgc -lexpat -lregex -static-
libgcc -luuid -loleaut32 -lole32 -lws2_32 -lsundials_kinsol -lsundials_nvecserial -lipopt -lcoinmumps -lcoinmetis -lpthread -lm -lgfortranbegin -
lgfortran -lmingw32 -lgcc_eh -lmoldname -lmingwex -lmsvcrt -luser32 -lkernel32 -ladvapi32 -lshell32 -llapack-mingw -ltmglib-mingw -lblas-mingw -
lf2c -linteractive -lwsock32 -llis -lstdc++
```



OMEdit - myRLCnetwork Simulation Output

Output    Compilation

```
C:/Users/hv/AppData/Local/Temp/OpenModelica/OMEdit/myRLCnetwork.exe -port=49502 -logFormat=xml -w -lv=LOG_STATS
LOG_STATS           | info    | ### STATISTICS ####
LOG_STATS           | info    | timer
|                   | |         | | 0.0150538s [ 46.9%] pre-initialization
|                   | |         | | 4.18139e-005s [ 0.1%] initialization
|                   | |         | | 2.0907e-005s [ 0.1%] steps
|                   | |         | | 0.0157118s [ 49.0%] creating output-file
|                   | |         | | 0.000115558s [ 0.4%] event-handling
|                   | |         | | 0.000295738s [ 0.9%] overhead
|                   | |         | | 0.000824114s [ 2.6%] simulation
|                   | |         | | 0.0320637s [100.0%] total
LOG_STATS           | info    | events
|                   | |         | | 0 state events
|                   | |         | | 0 time events
LOG_STATS           | info    | solver: DASSL
|                   | |         | | 2431 steps taken
|                   | |         | | 3266 calls of functionODE
|                   | |         | | 165 evaluations of jacobian
|                   | |         | | 73 error test failures
|                   | |         | | 0 convergence test failures
LOG_STATS           | info    | ### END STATISTICS ###
```



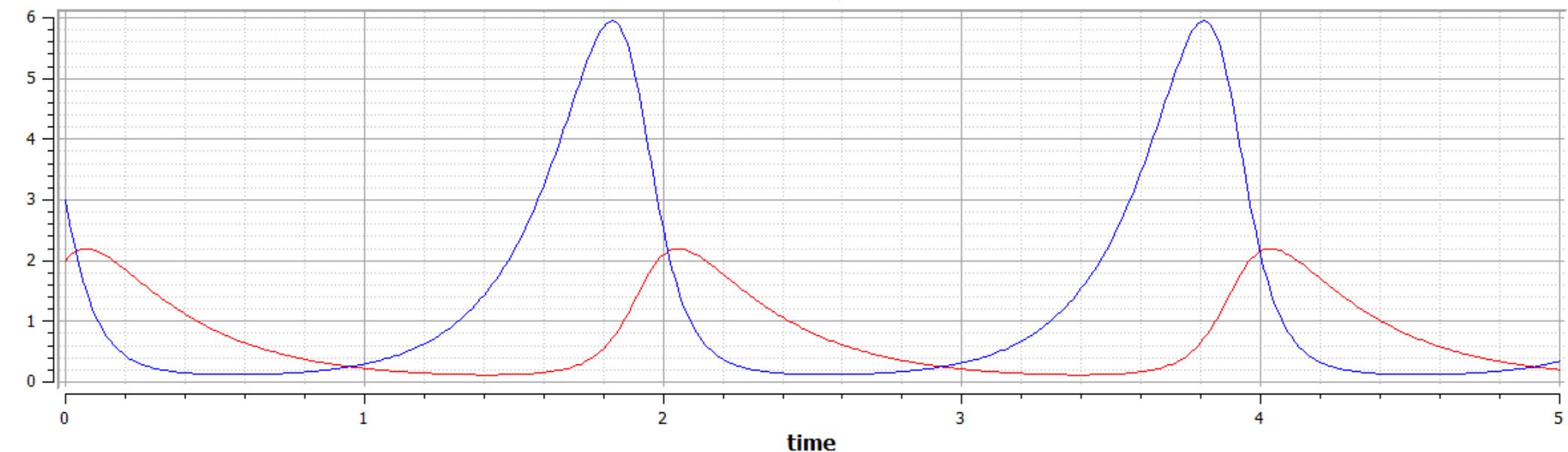
```
model mySimpleEqnSet "simple equation set"
    Real x(start=2, fixed=true);
    Real y(start=3, fixed=true);
equation
    der(x) = 2*x*y-3*x;
    der(y) = 5*y-7*x*y;
end mySimpleEqnSet;
```

```
plot({x,y})
```

[done]

Zoom Pan Fit in View Save Print Grid Log X Log Y

x y

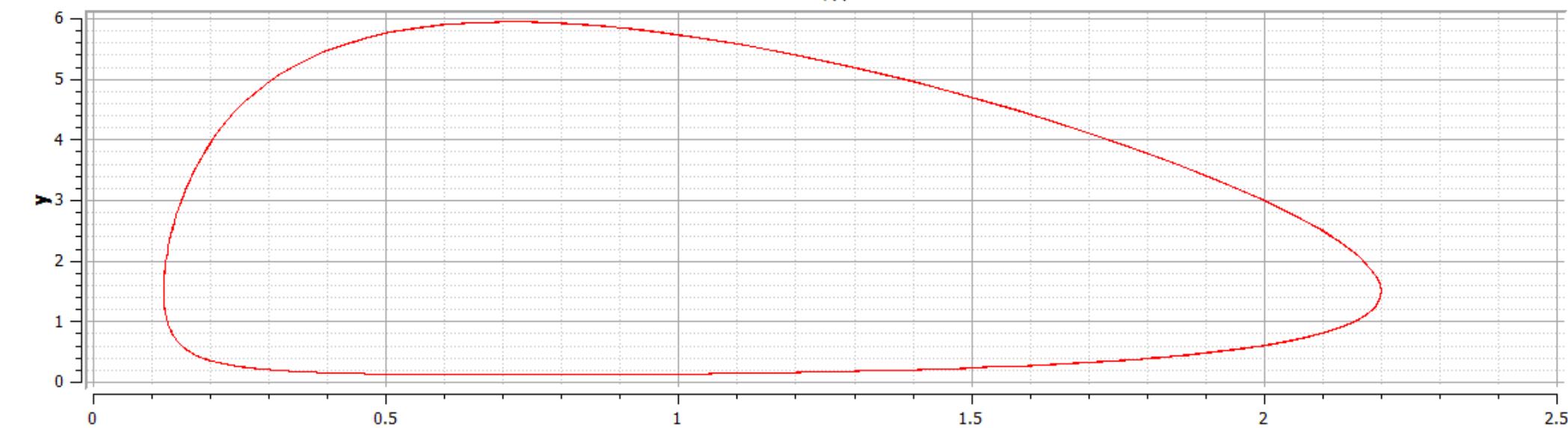


```
plotParametric(x,y)
```

[done]

Zoom Pan Fit in View Save Print Grid Log X Log Y

y(x)



## Electrical Types

```
type Time = Real (final quantity="Time", final unit="s");
type ElectricPotential = Real (final quantity="ElectricPotential",
                                final unit="V");
type Voltage = ElectricPotential;
type ElectricCurrent = Real (final quantity="ElectricCurrent",
                             final unit="A");
type Current = ElectricCurrent;
```

Beware: variables are **signals** (functions of **time**)!

## Electrical Pin Interface

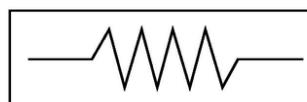
```
connector PositivePin "Positive pin of an electric component"
    Voltage v "Potential at the pin";
    flow Current i "Current flowing into the pin";
end PositivePin;
```

## Electrical Port

```
partial model OnePort
  "Component with two electrical pins p and n
   and current i from p to n"
  Voltage v "Voltage drop between the two pins (= p.v - n.v)";
  Current i "Current flowing from pin p to pin n";
  PositivePin p;
  NegativePin n;
equation
  v = p.v - n.v;
  0 = p.i + n.i;
  i = p.i;
end OnePort;
```

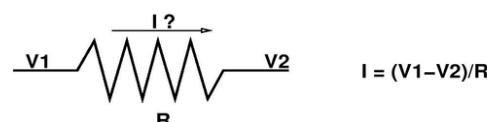
## Object-oriented re-use and causality

## Electrical Resistor

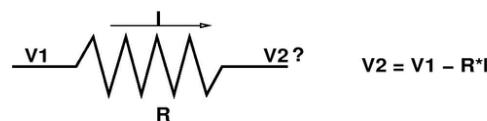


$$V_1 - V_2 = R \cdot I$$

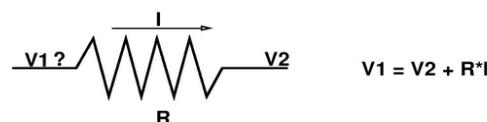
Object "resistor"



$$I = (V_1 - V_2)/R$$



$$V_2 = V_1 - R \cdot I$$



$$V_1 = V_2 + R \cdot I$$

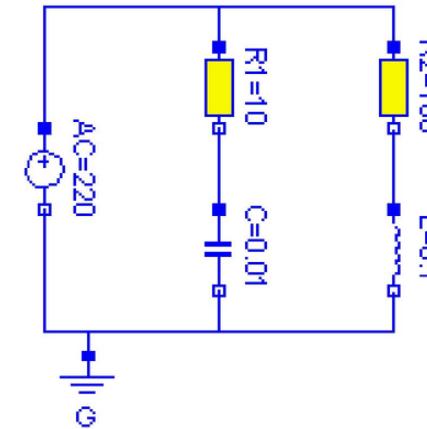
```
model Resistor "Ideal linear electrical resistor"
  extends OnePort;
  parameter Resistance R=1 "Resistance";
  equation
    R*i = v;
end Resistor;
```

## The circuit

```

model circuit
  Resistor R1(R=10);
  Capacitor C(C=0.01);
  Resistor R2(R=100);
  Inductor L(L=0.1);
  VsourceAC AC;
  Ground G;
equation
  connect(AC.p, R1.p);
  connect(R1.n, C.p);
  connect(C.n, AC.n);
  connect(R1.p, R2.p);
  connect(R2.n, L.p);
  connect(L.n, C.n);
  connect(AC.n, G.p);
end circuit;

```



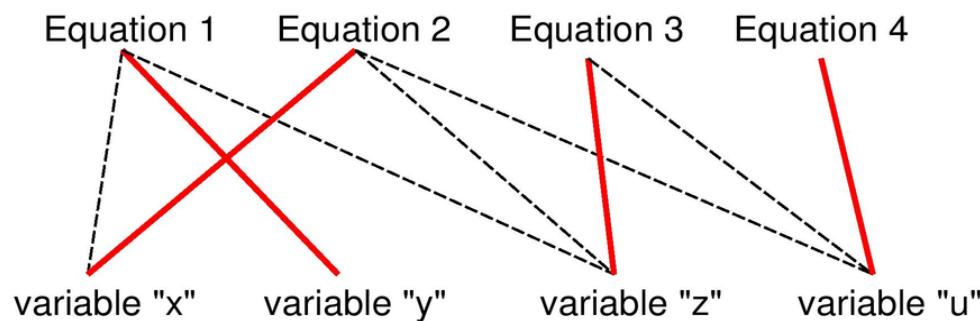
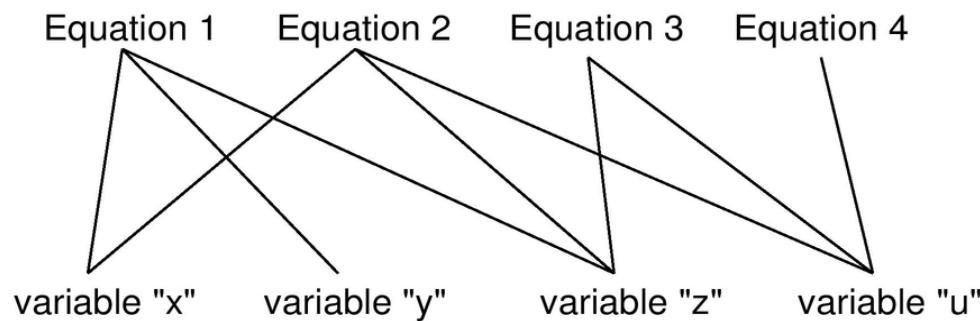
Meaning: set of Differential Algebraic Equations (DAEs) obtained by

1. expanding inheritance/instantiation
2. flattening hierarchy, unique names
3. expanding connect() into equations (across vs. flow)

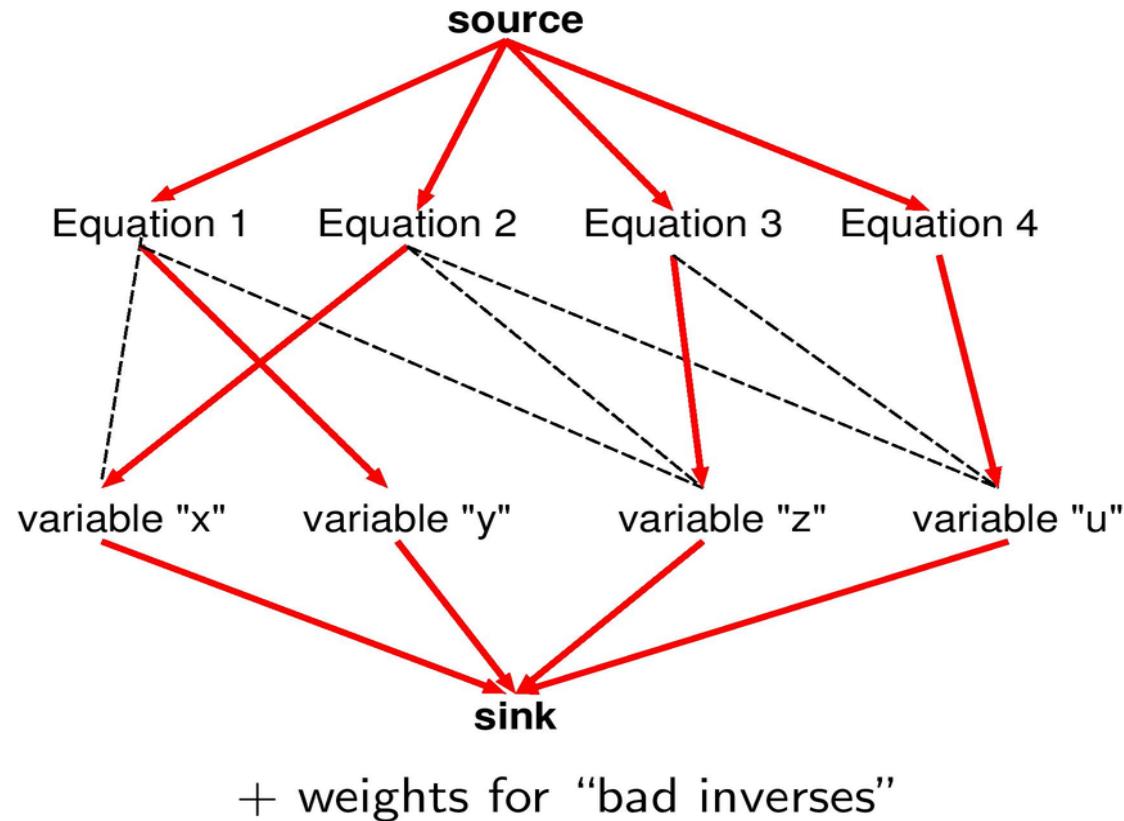
Non-causal model  
(e.g., from physical conservation laws)

$$\left\{ \begin{array}{l} x + y + z = 0 \quad \text{Equation 1} \\ x + 3z + u^2 = 0 \quad \text{Equation 2} \\ z - u - 16 = 0 \quad \text{Equation 3} \\ u - 5 = 0 \quad \text{Equation 4} \end{array} \right.$$

## Causality assignment: bipartite graph, maximum cardinality matching



## Causality assignment: network flow



## Causality assigned

$$\begin{cases} x + \underline{y} + z = 0 & \text{Equation 1} \\ \underline{x} + 3z + u^2 = 0 & \text{Equation 2} \\ \underline{z} - u - 16 = 0 & \text{Equation 3} \\ \underline{u} - 5 = 0 & \text{Equation 4} \end{cases}$$

re-write in causal form

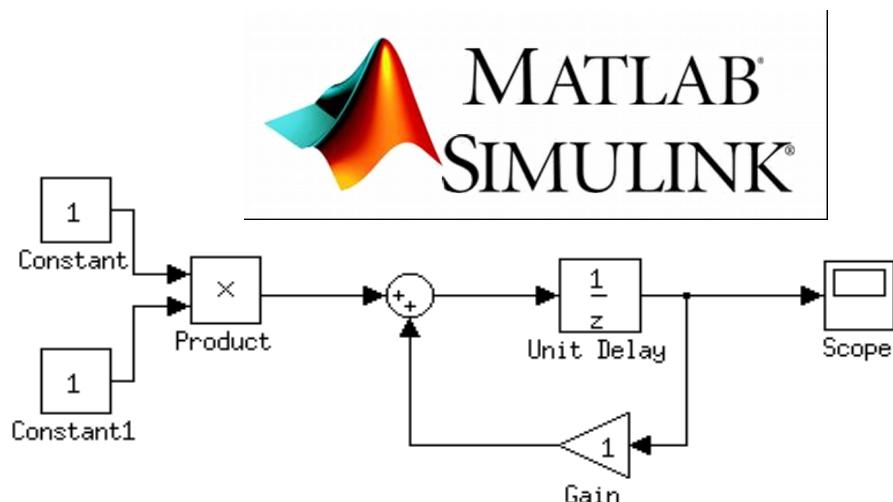
$$\begin{cases} \underline{y} = -x - z \\ \underline{x} = -3z - u^2 \\ \underline{z} = u + 16 \\ \underline{u} = 5 \end{cases}$$

# Set of Algebraic Eqns (no cyclic dependencies)

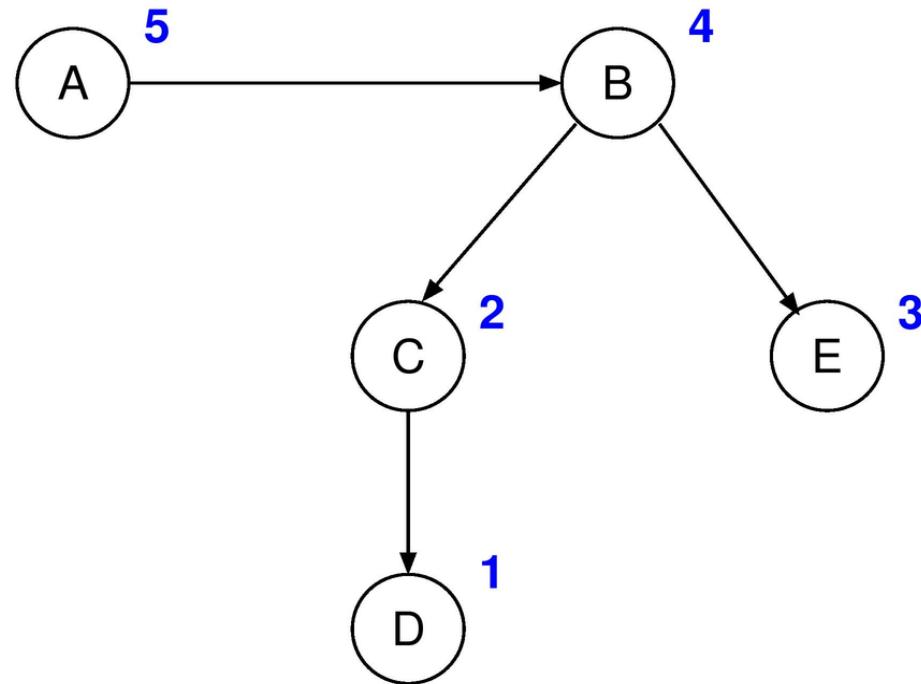
**WRONG:**

$$\begin{cases} a = b^2 + 3 \\ b = \sin(c \times e) \\ c = \sqrt{d - 4.5} \\ d = \pi/2 \\ e = u() \end{cases}$$

$$\begin{bmatrix} a & = & b^2 + 3 = 3 \\ b & = & \sin(c \times e) = 0 \\ c & = & \sqrt{d - 4.5} = \text{error} \\ d & = & \pi/2 \\ e & = & u() \end{bmatrix}$$



Sorting (no cyclic dependencies)  
DFS, postorder numbering of dependency graph



## Dependency Cycle (aka Algebraic Loop)

$$\begin{cases} x = y + 16 \\ y = -x - z \\ z = 5 \end{cases}$$

Can *never* be sorted

due to a dependency *cycle* aka *strong component*

(every vertex in the component is reachable from every other)

$$x \rightarrow y \rightarrow x$$

May be solved implicitly

$$\left[ \begin{array}{l} z = 5 \\ \left\{ \begin{array}{rcl} x - y & = & -6 \\ x + y & = & -z \end{array} \right. \end{array} \right]$$

Implicit set of  $n$  equations in  $n$  unknowns.

- non-linear  $\rightarrow$  non-linear solver.
- linear  $\rightarrow$  numerical or symbolic solution.

Linear: may be solved symbolically (Cramer)

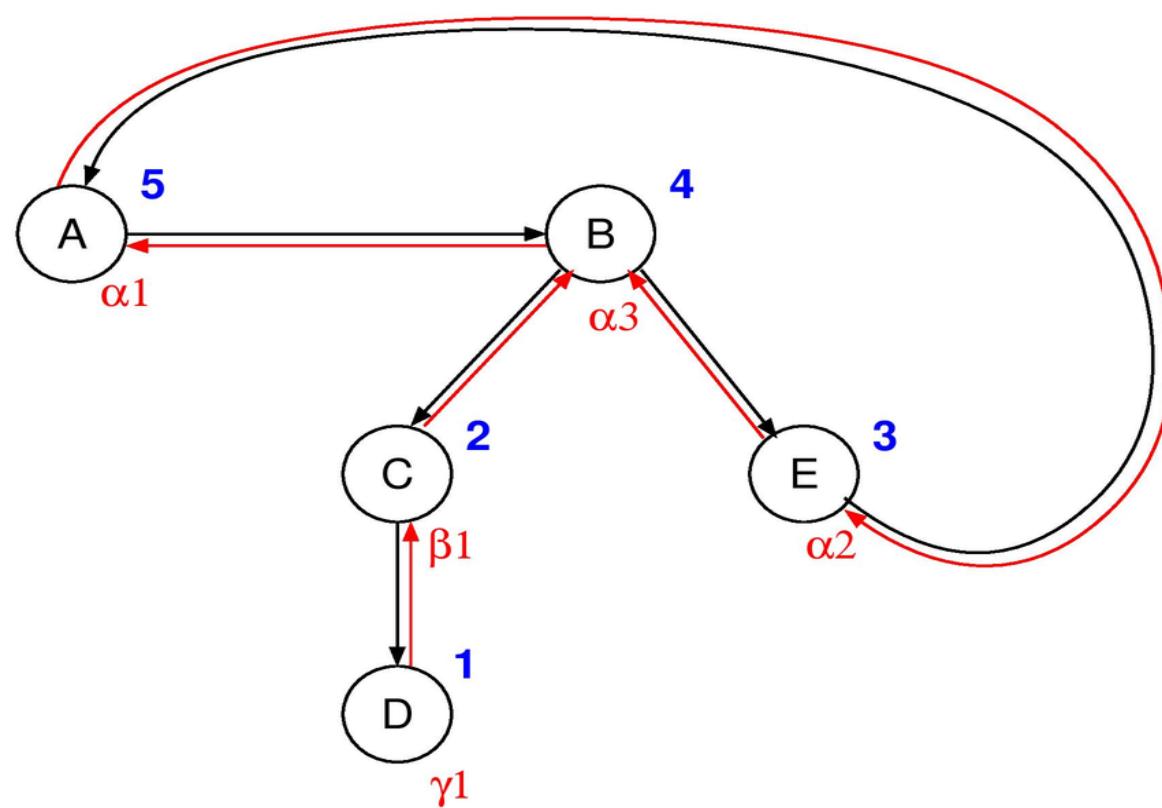
$$x = \frac{\begin{vmatrix} -6 & -1 \\ -z & 1 \\ 1 & -1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -6 \\ 1 & -z \\ 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{-6-z}{2}; \quad y = \frac{\begin{vmatrix} 1 & -6 \\ 1 & -z \\ 1 & -1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -6 \\ 1 & -z \\ 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{6-z}{2}$$

$$\begin{bmatrix} z & = & 5 \\ x & = & \frac{-6-z}{2} \\ y & = & \frac{6-z}{2} \end{bmatrix}$$

## Tarjan's algorithm for Cycle Detection

$$\begin{cases} a &= b^2 + 3 \\ b &= \sin(c \times e) \\ c &= \sqrt{d - 4.5} \\ d &= \pi/2 \\ e &= a^2 + u() \end{cases}$$

# Algebraic Loop (Cycle) Detection



## Algebraic Loop (Cycle) Detection Result

$$\left[ \begin{array}{l} d = \pi/2 \\ c = \sqrt{d - 4.5} \\ \left\{ \begin{array}{l} b = \sin(c \times e) \\ a = b^2 + 3 \\ e = a^2 + u() \end{array} \right. \end{array} \right] ; \left[ \begin{array}{l} d = \pi/2 \\ c = \sqrt{d - 4.5} \\ \left\{ \begin{array}{l} b = -\sin(c \times e) = 0 \\ a = -b^2 = -3 = 0 \\ a^2 = -e = +u() = 0 \end{array} \right. \end{array} \right]$$

# Modelling Physical Systems

- Domain/Problem-Specific
- Laws of Physics
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- a-causal (Mathematical) ← **Modelica**
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- Computer Numerical (Floating Point, Fixed Point)
- As-Fast-As-Possible vs. Real-time
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)

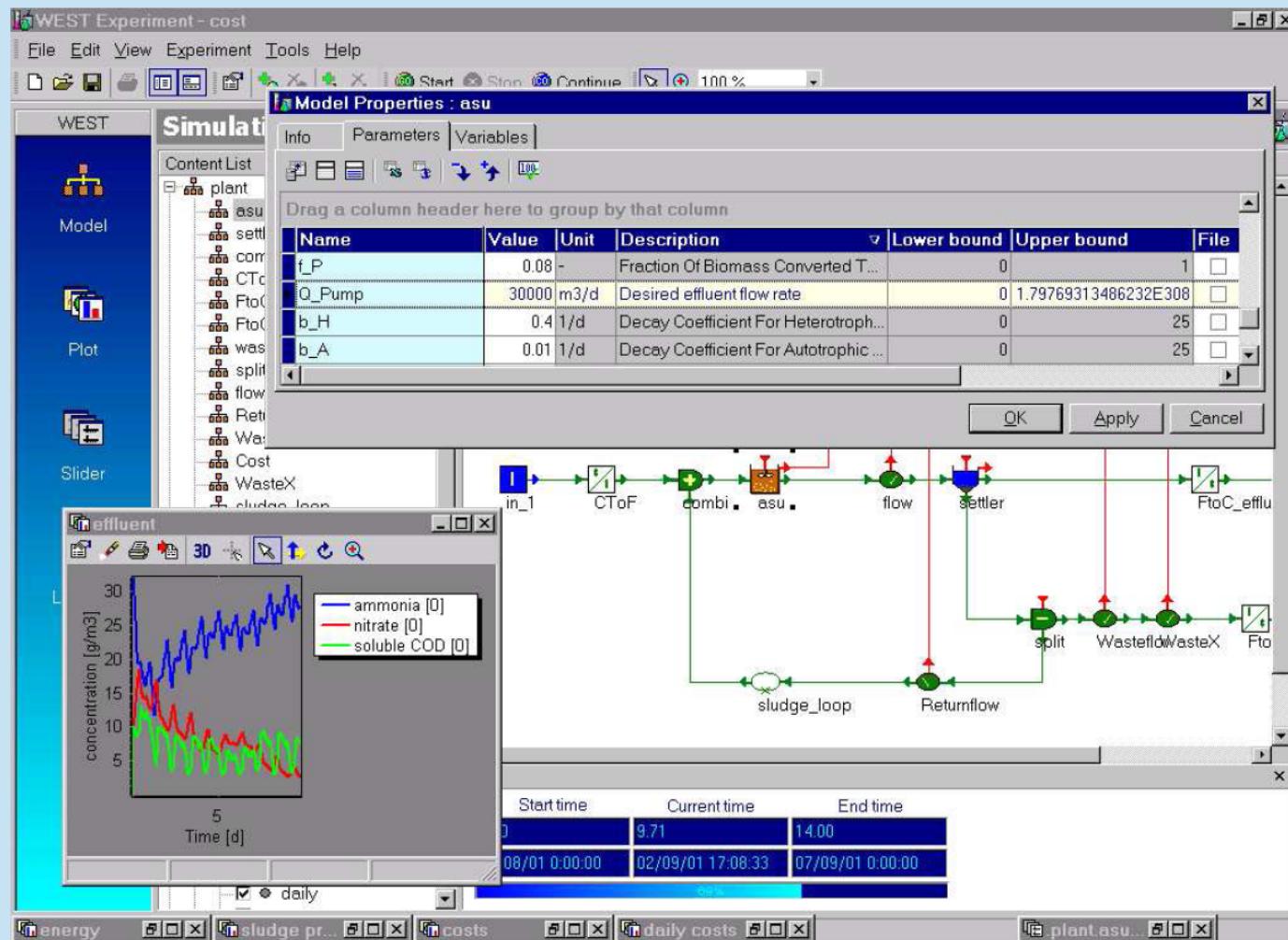
## Operational vs. Denotational (Translational) semantics



NATO's Sarajevo Waste Water Treatment Plant

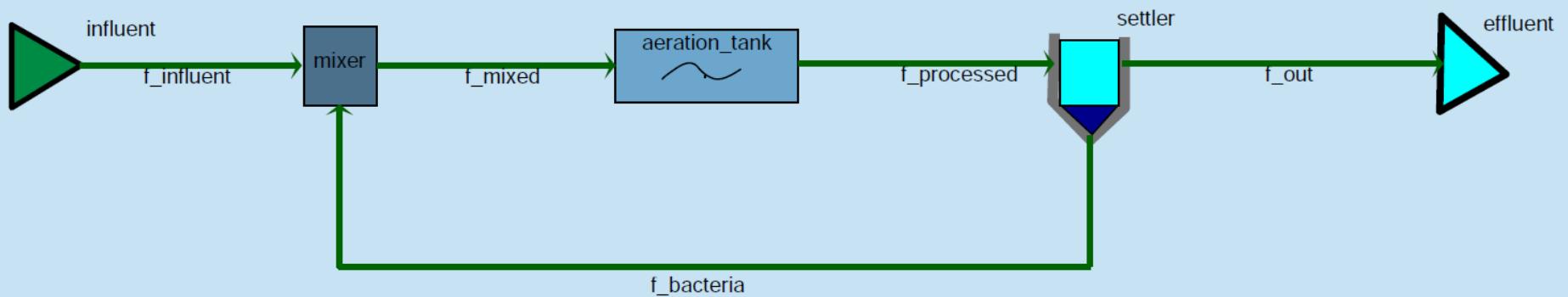
[www.nato.int/sfor/cimic/env-pro/waterpla.htm](http://www.nato.int/sfor/cimic/env-pro/waterpla.htm)

# DS(V)M Environment

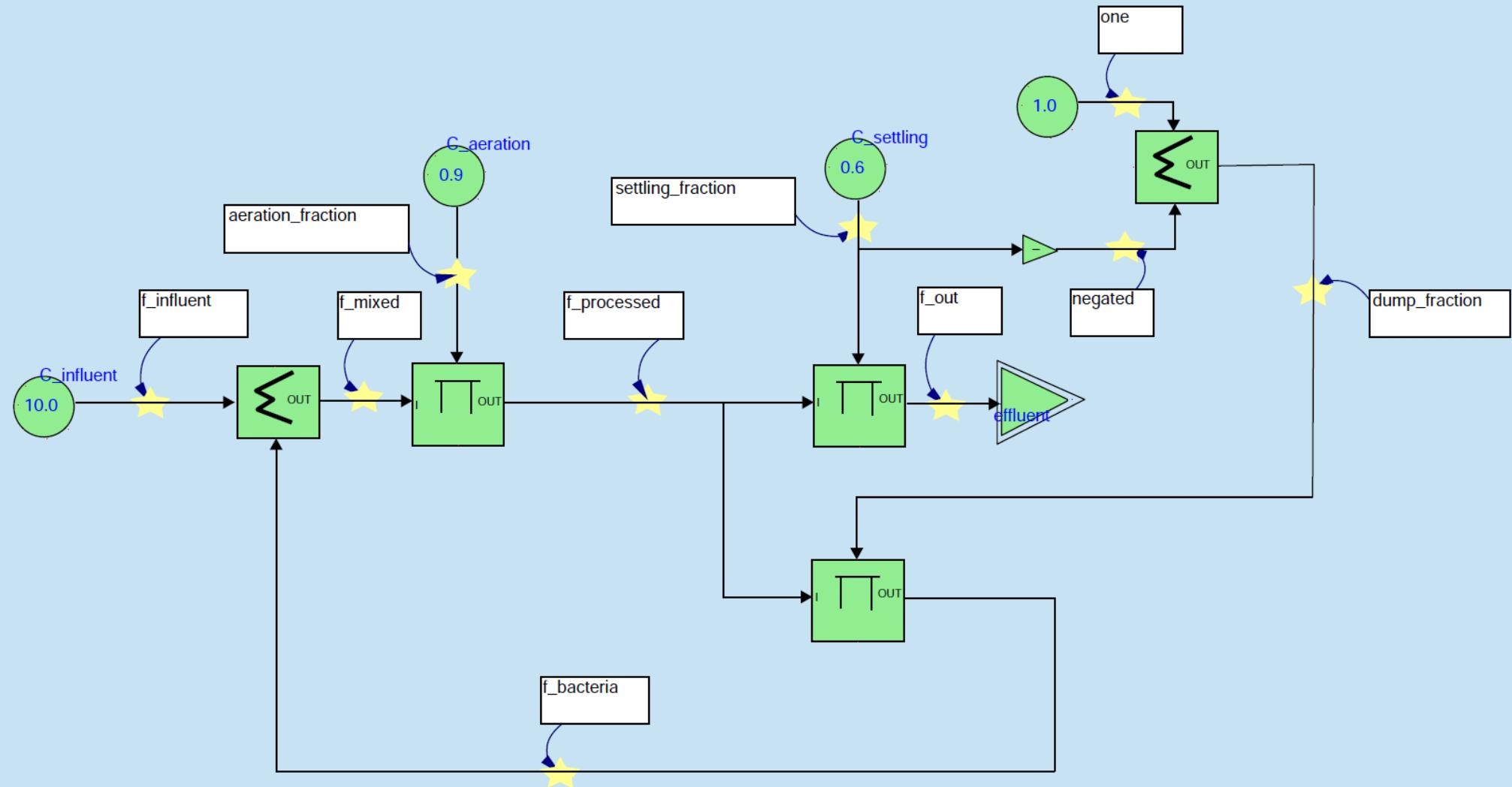


[www.hemmis.com/products/west/](http://www.hemmis.com/products/west/)

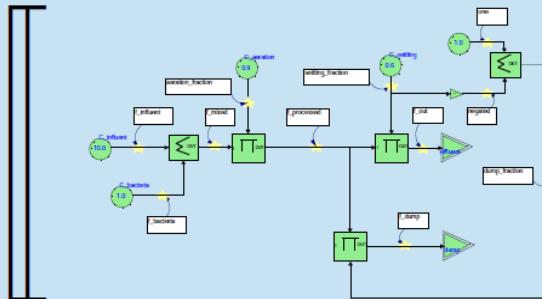
## What does this WWTP model mean?



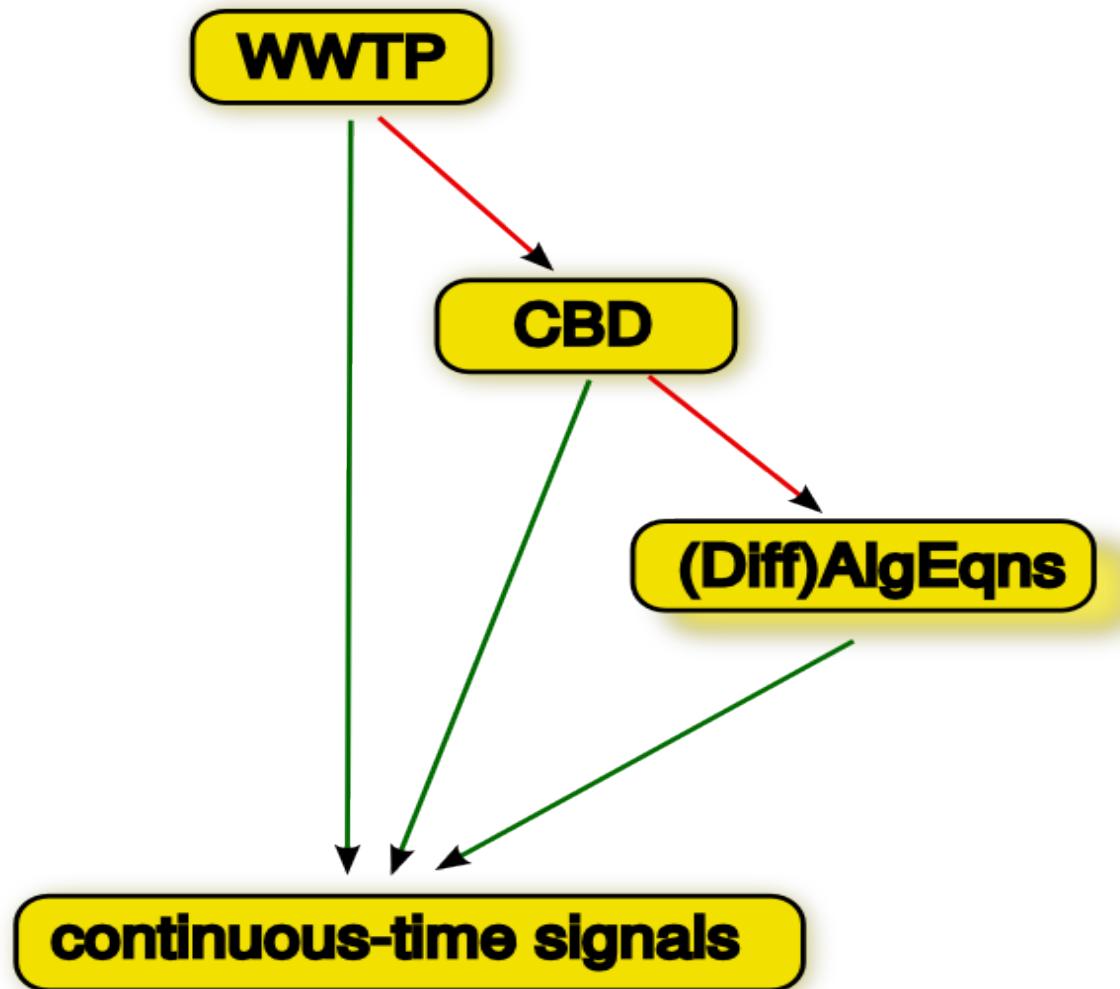
# ... its meaning (steady-state abstraction): Causal Block Diagram (CBD)



## Meaning of the CBD . . . semantic mapping onto algEqns



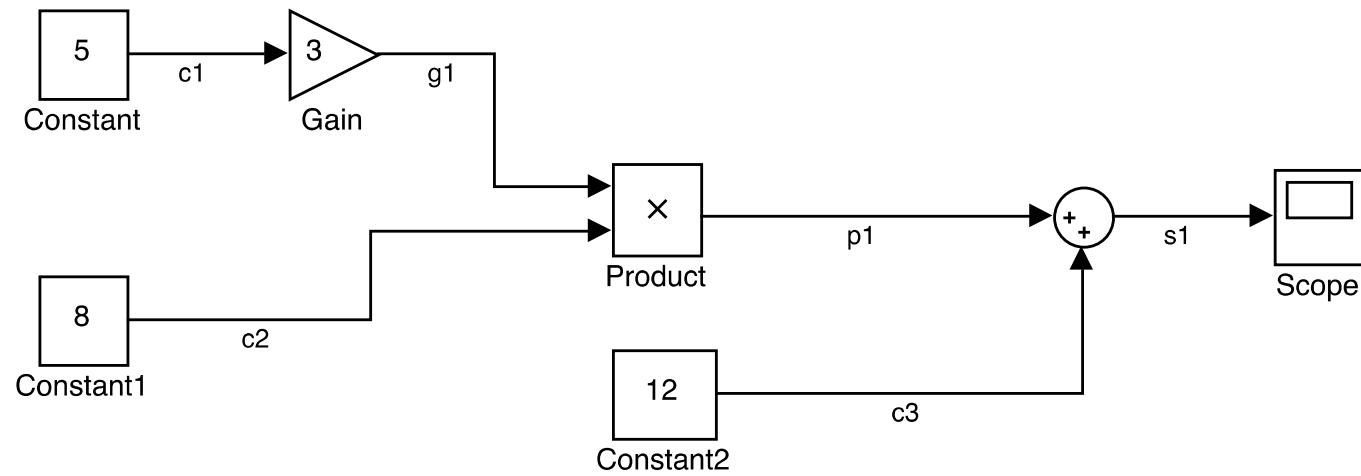
$$= \left\{ \begin{array}{lcl} f_{influent} & = & C_{influent} \\ f_{bacteria} & = & C_{bacteria} \\ f_{mixed} & = & f_{influent} + f_{bacteria} \\ aeration\_fraction & = & C_{aeration} \\ f_{processed} & = & aeration\_fraction * f_{mixed} \\ settling\_fraction & = & C_{settling} \\ negated & = & -settling\_fraction \\ one & = & 1 \\ dump\_fraction & = & one + negated \\ f_{dump} & = & f_{processed} * dump\_fraction \\ f_{out} & = & settling\_fraction * f_{processed} \end{array} \right.$$



# Causal-Block Diagrams

- CBD vs. ABD
- Blocks connected with Links via Ports to each other  
(aka “plex”, as opposed to “graph”)
- Types:
  - Algebraic
  - Discrete-time
  - Continuous-time
- Translational/Operational semantics

# Algebraic CBD



# Transformational Semantics

$$c1 = 5$$

$$c2 = 8$$

$$c3 = 12$$

$$g1 = c1 * 3$$

$$p1 = g1 * c2$$

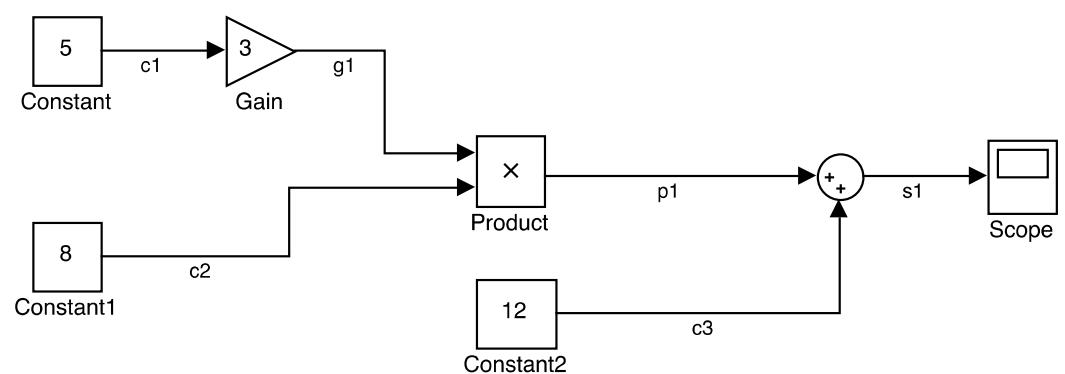
$$s1 = c3 + p1$$

$$= 12 + (g1 * c2)$$

$$= 12 + ((c1 * 3) * 8)$$

$$= 12 + ((5 * 3) * 8)$$

$$= 132$$



# Topological SORT

Kahn 1962

$L \leftarrow$  Empty list that will contain the sorted elements

$S \leftarrow$  Set of all nodes with no incoming edges

**while**  $S$  is non-empty **do**

    remove a node  $n$  from  $S$

    add  $n$  to *tail* of  $L$

**for each** node  $m$  with an edge  $e$  from  $n$  to  $m$  **do**

        remove edge  $e$  from the graph

**if**  $m$  has no other incoming edges **then**

            insert  $m$  into  $S$

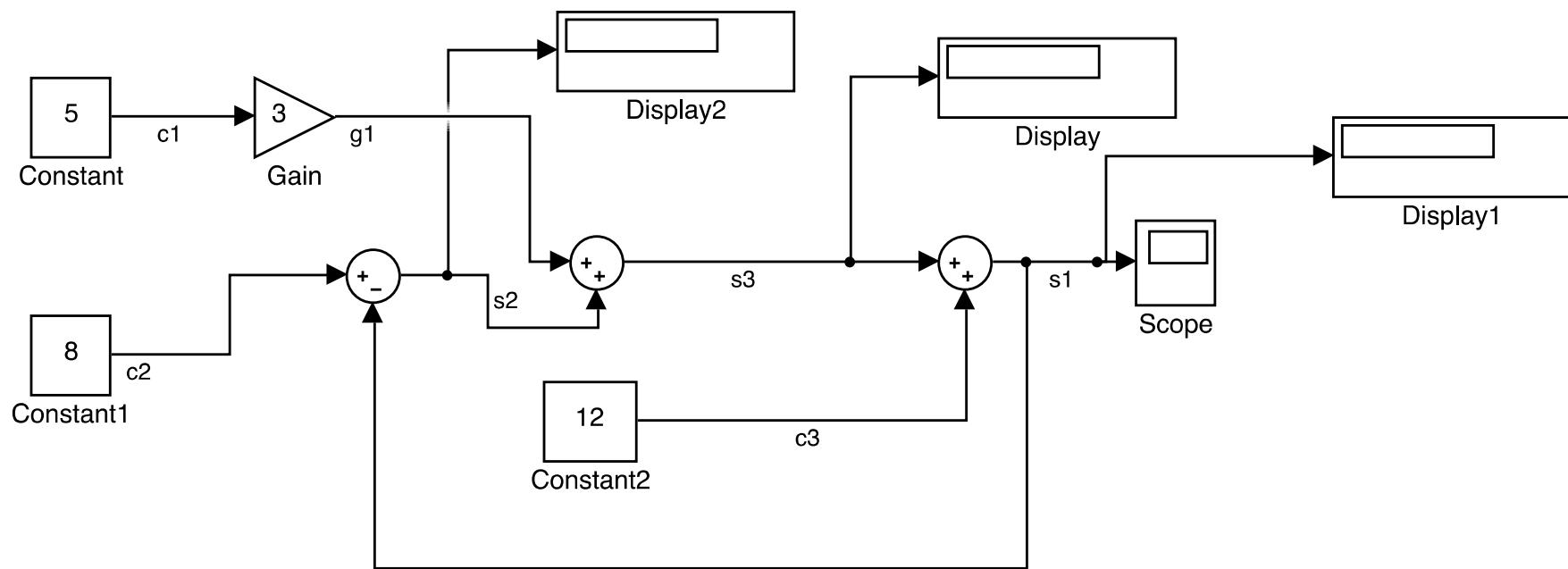
**if** graph has edges **then**

        return error (graph has at least one cycle)

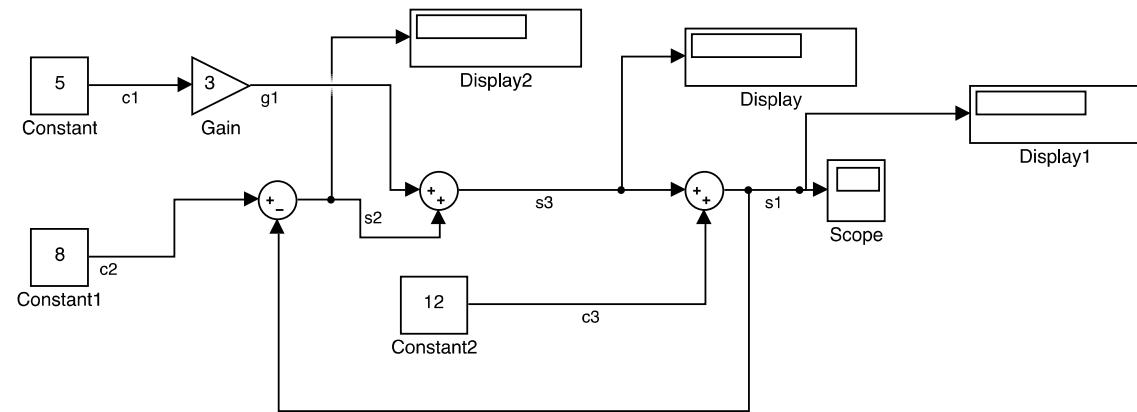
**else**

        return  $L$  (a topologically sorted order)

# (Algebraic) Loops



# Translational Semantics



$$c1 = 5 ; c2 = 8; c3 = 12$$

$$g1 = 3*c1$$

$$s1 = c3 + s3$$

$$s2 = c2 - s1$$

$$s3 = g1 + s2$$

=> Linear Algebraic Loop

# Gaussian Elimination

<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
-1	0	1	-c3
-1	-1	0	-c2
0	1	-1	-g1

<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
-1	0	1	-12
-1	-1	0	-8
0	1	-1	-15

<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
1	0	-1	12
0	1	1	-4
0	1	-1	-15

<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
1	0	-1	12
0	1	1	-4
0	0	-2	-11

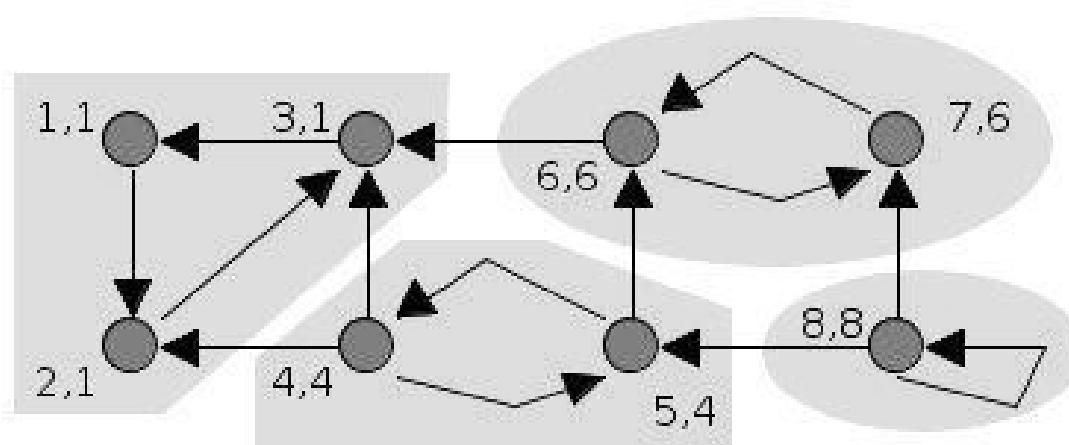
<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
1	0	-1	12
0	1	1	-4
0	0	1	5.5

<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>v</b>
1	0	0	17.5
0	1	0	-9.5
0	0	1	5.5

# Operational Semantics

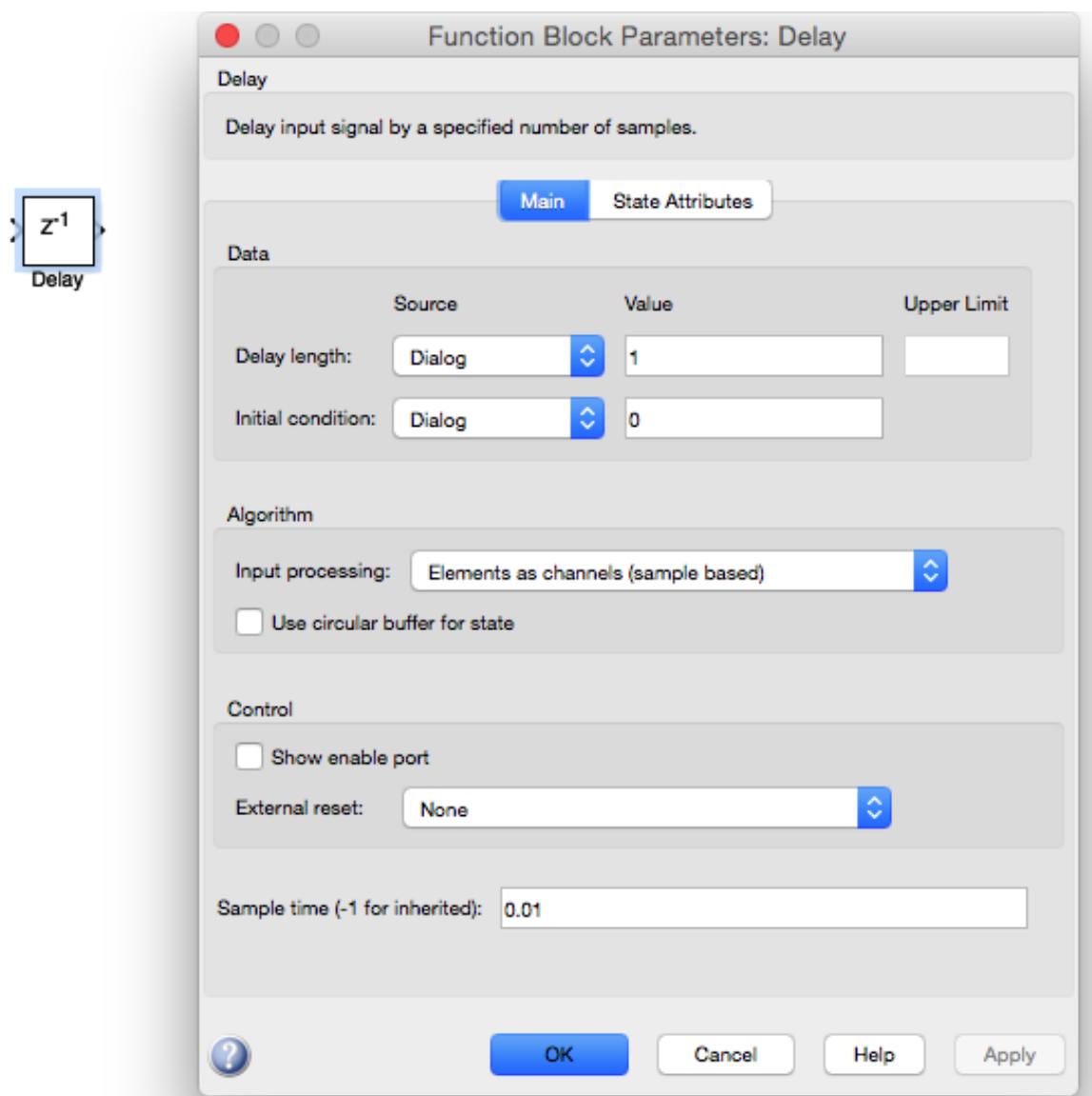
```
schedule ← LOOPDETECT(DEPGRAPH(cbd))
for gblock in schedule do
    COMPUTE(gblock)
end for
```

# Tarjan's Algorithm

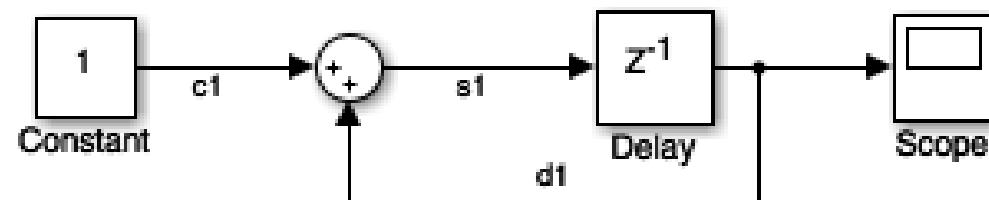


Source: [www.wikipedia.org](http://www.wikipedia.org)

# Discrete Time CBD's



# Example



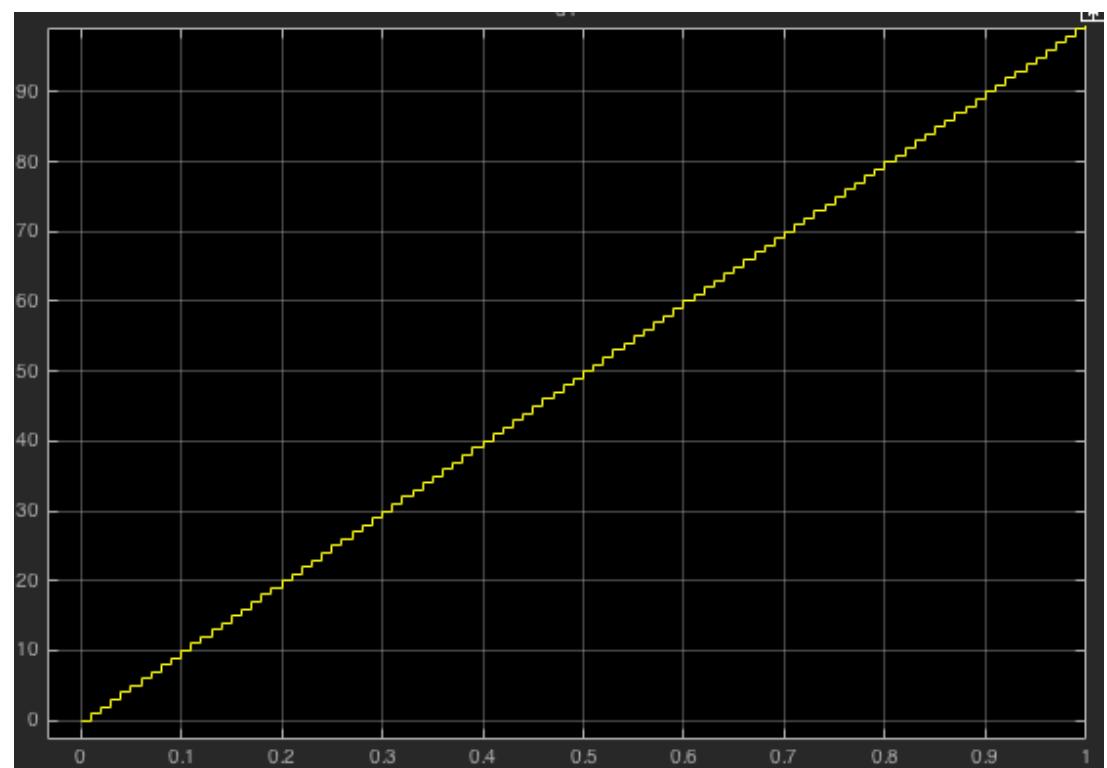
$$c_1(n) = 1$$

$$s_1(n) = d_1(n) + c_1(n)$$

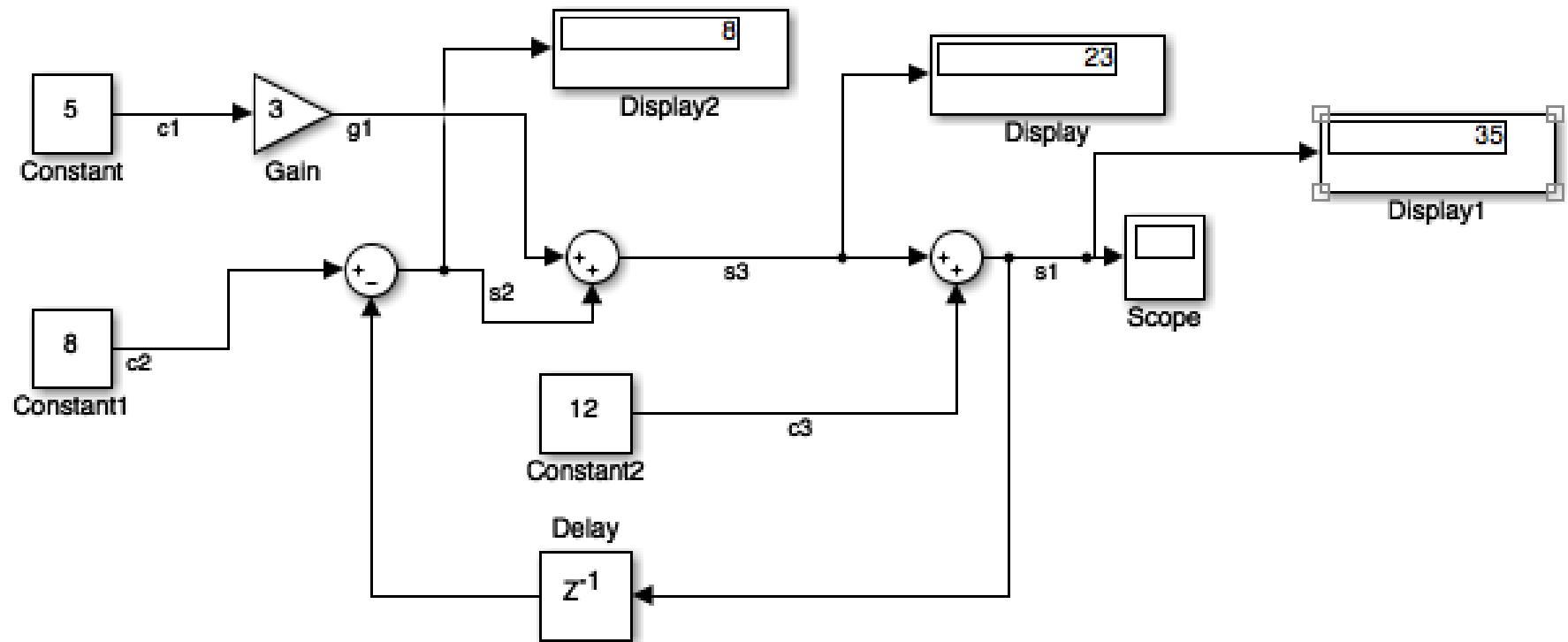
$$d_1(n) = s_1(n-1)$$

$$\text{with } \mathbf{d_1(0) = 0}$$

=> Difference Equation



# Delay and Loops



# Translational Semantics

$$c1(\mathbf{n}) = 5$$

$$c2(n) = 8$$

$$c3(n) = 12$$

$$g1(n) = 3*c1(n)$$

$$s2(n) = c2(n) - d1(n)$$

$$s3(n) = s2(n) + g1(n)$$

$$s1(n) = c3(n) + s3(n)$$

$$d1(n) = d1(n-1) \text{ with } \mathbf{d(0) = 0}$$

# Operational Semantics

---

```
1: time_step  $\leftarrow 0$ 
2: while not end_condition do
3:   schedule  $\leftarrow \text{LOOPDETECT}(\text{DEPGRAPH}(cbd))$ 
4:   for gblock in schedule do
5:     COMPUTE(gblock)
6:   end for
7:   time_step  $\leftarrow \text{time\_step} + 1$ 
8: end while
```

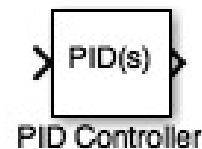
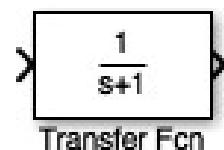
---

# Topological Sort!

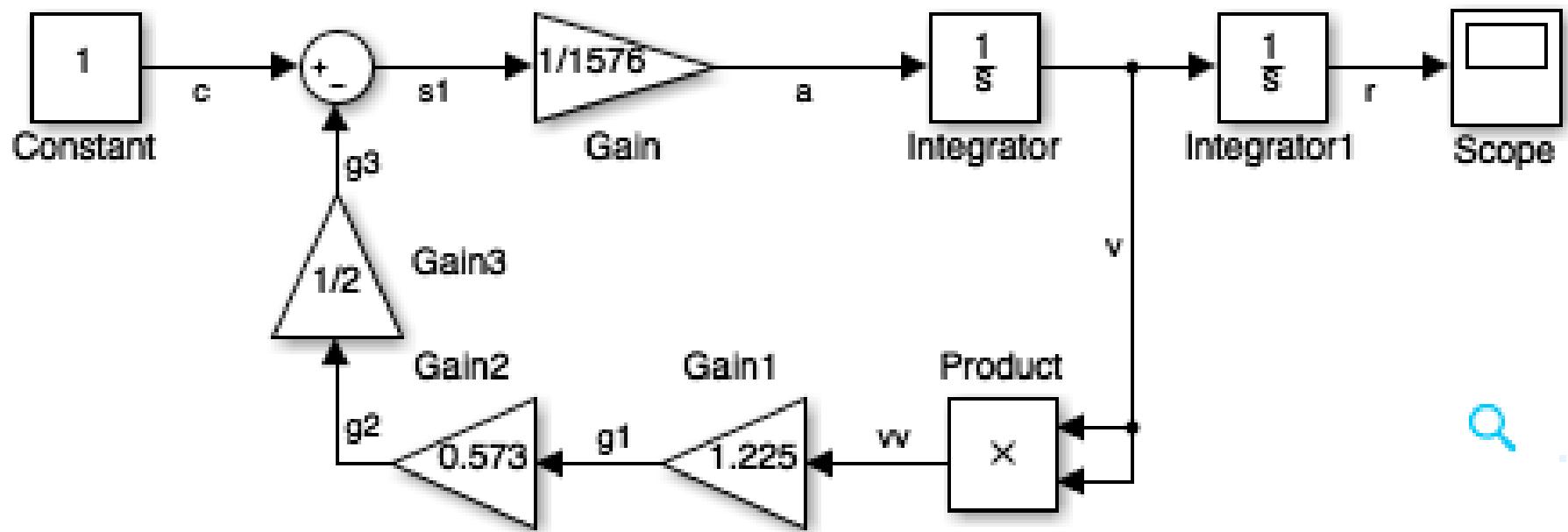
Delay “breaks” the loop!!!

- Value of  $d_1(n)$  is known at beginning
- Value of  $d_1(n+1)$  is calculated in this step
- Evaluation order changes
- Loop is broken at delay block

# Continuous Time



# Example



.

# Translational Semantics

$$a(t) = \mathbf{dv/dt} = 1/1576 * s1(t)$$

$$v(t) = dr/dt$$

$$vv(t) = v(t)*v(t)$$

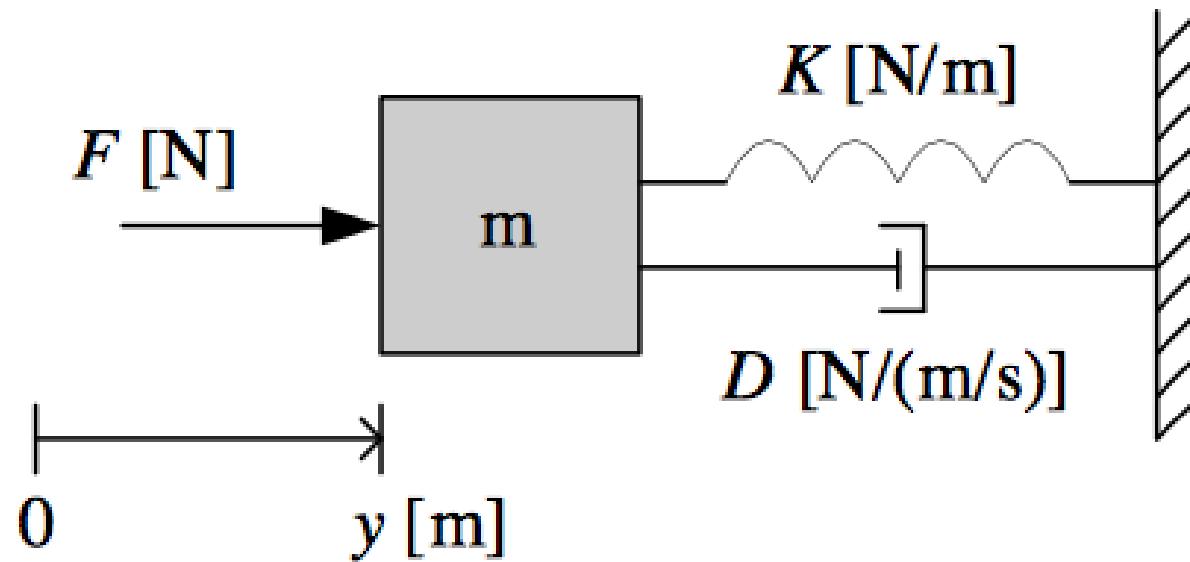
$$g1(t) = 1.225*vv(t)$$

$$g2(t) = 0.573*g1(t)$$

$$g3(t) = 1/2 * g2(t)$$

$$s1(t) = c(t) + g3(t)$$

# Example

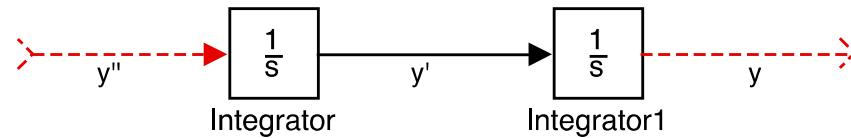


$$\ddot{y}(t) = \frac{1}{m} [F(t) - D\dot{y}(t) - Ky(t)]$$

# ODE to Block Diagram

For each differentiation, draw integrator

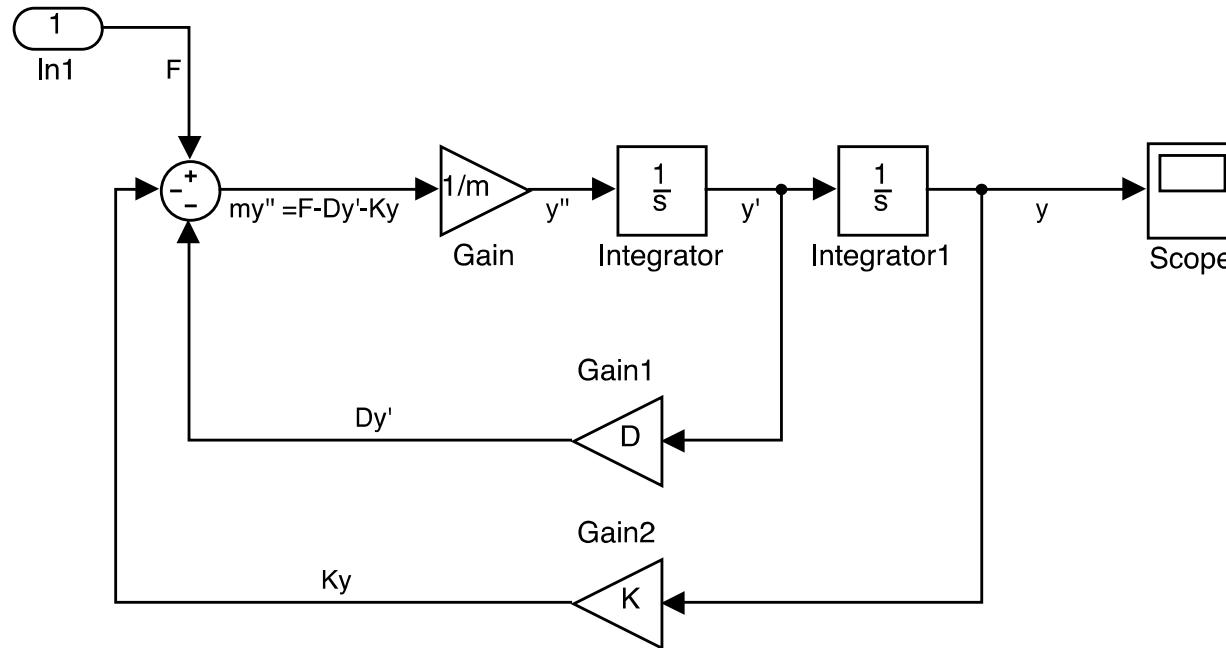
$$\ddot{y}(t) = \frac{1}{m} [F(t) - D\dot{y}(t) - Ky(t)]$$



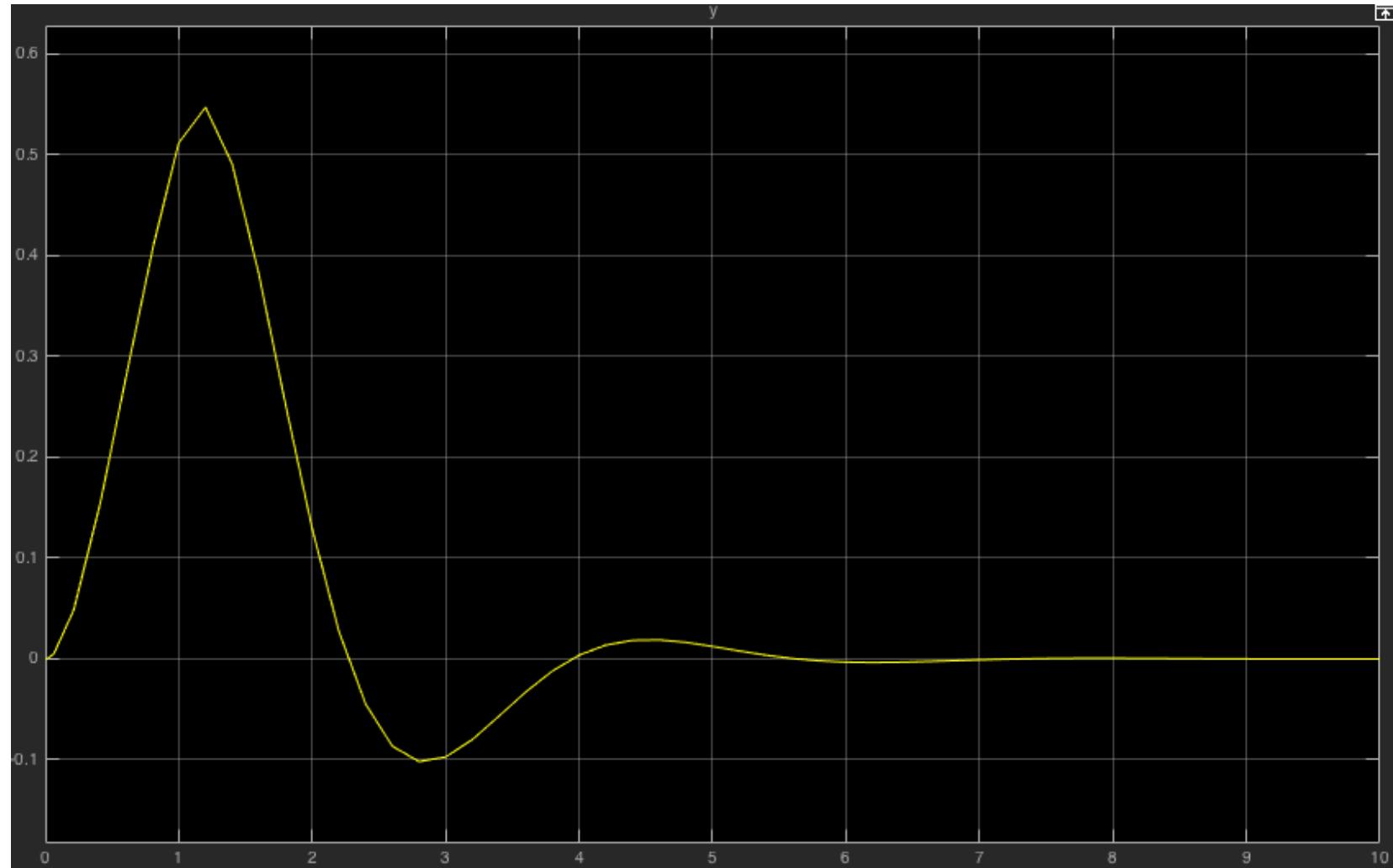
(integrator vs. differentiator)

# ODE to Block Diagram

Connect together using proper blocks

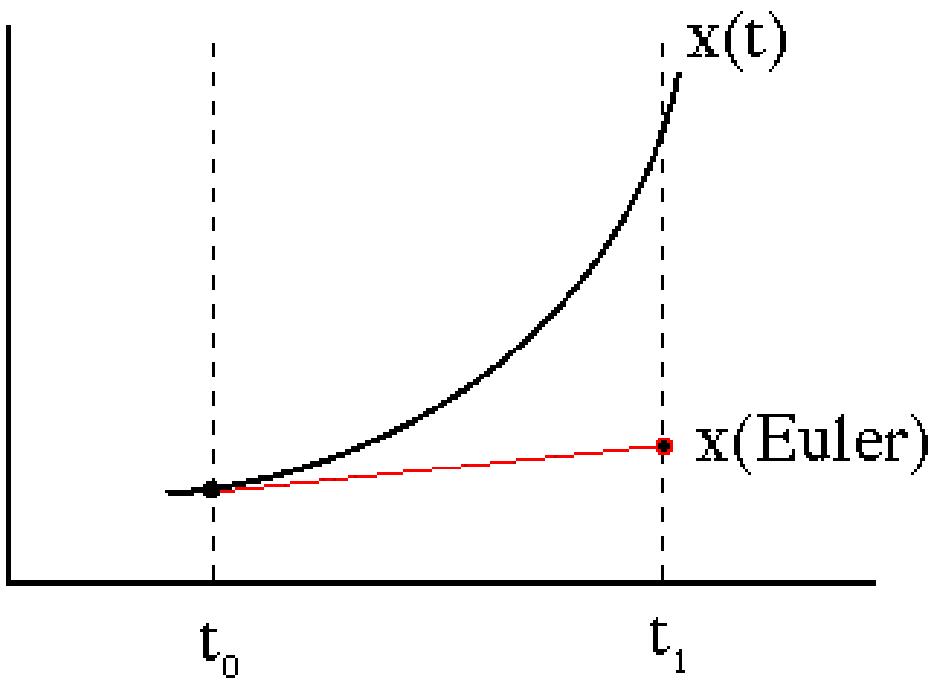


# ODE to Block Diagram



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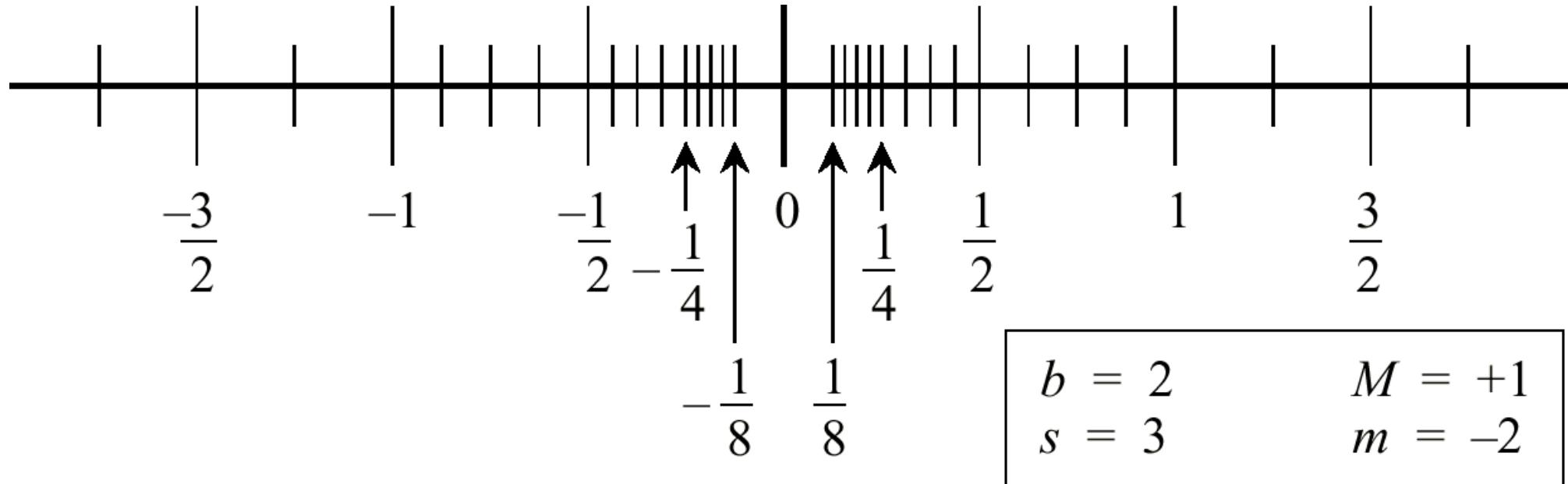


$$\begin{aligned}v_{x,n+1} &= v_{x,n} + \Delta t \, a_x(x_n, y_n, t) \\x_{n+1} &= x_n + \Delta t \, v_{x,n}\end{aligned}$$

# Modelling Physical Systems

- Domain/Problem-Specific
- Laws of Physics
- Power Flow
- a-causal (Mathematical) ← **Modelica**
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# Example Floating Point Format

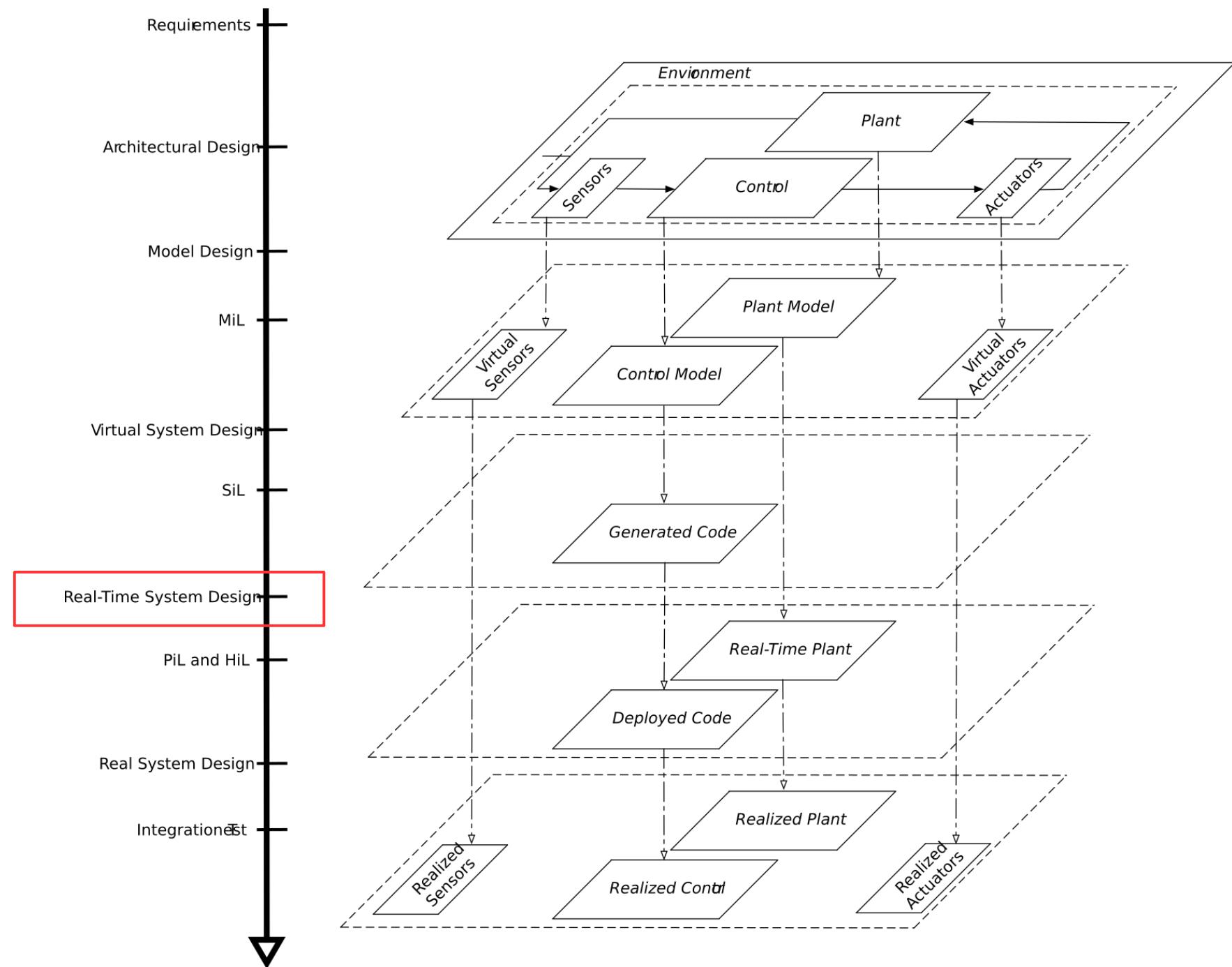


- Smallest non-zero positive number =  $b^m \times b^{-1} = 1/8$
- Largest non-zero positive number =  $b^M \times (1 - b^{-s}) = 7/4$
- Smallest gap =  $b^m \times b^{-s} = 1/32$
- Largest gap =  $b^M \times b^{-s} = 1/4$
- Number of representable numbers =  $2 \times ((M-m)+1) \times (b-1) \times b^{s-1} + 1 = 33$   
... fits into available bits? Optimal number of bits?
- Note: fill the gap around 0: **de-normalized**

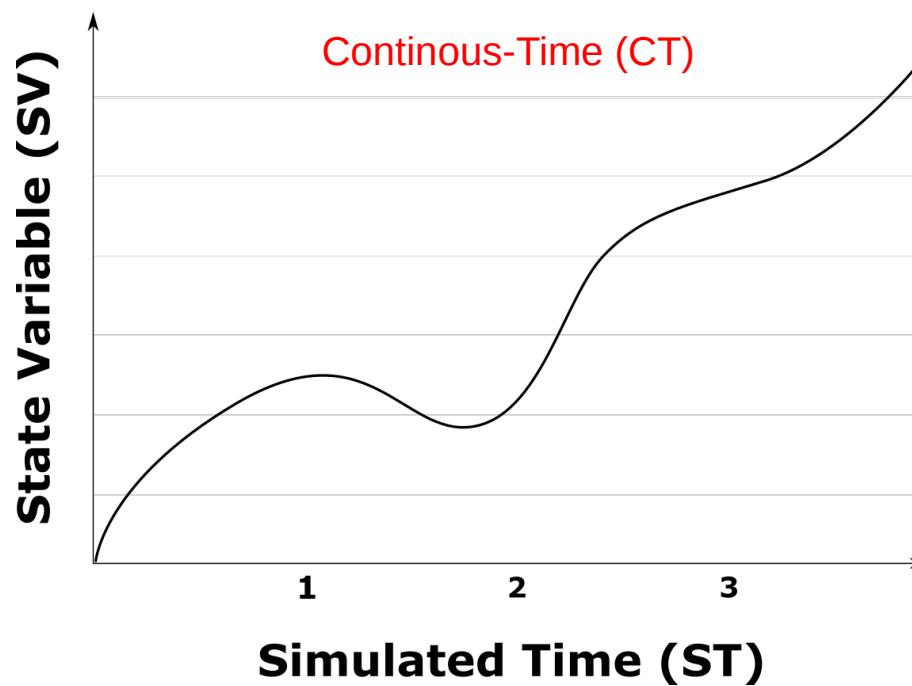
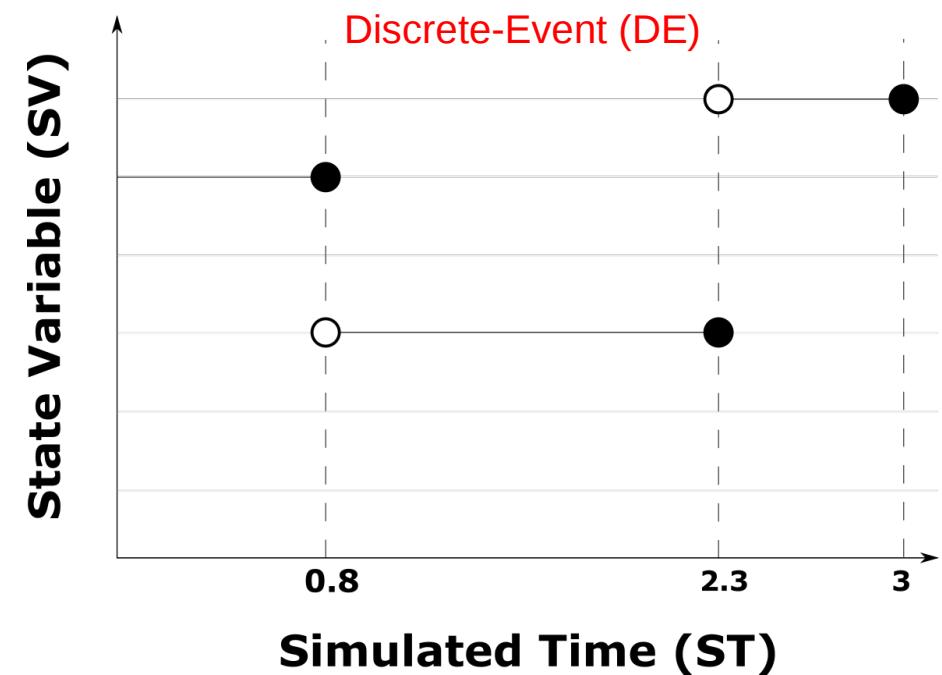
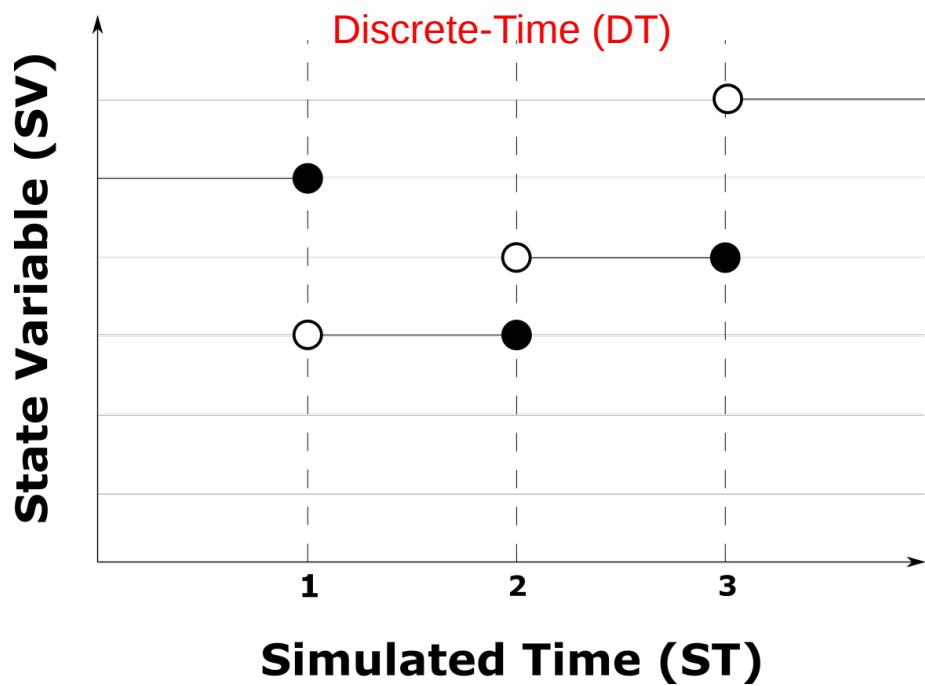
# Modelling Physical Systems

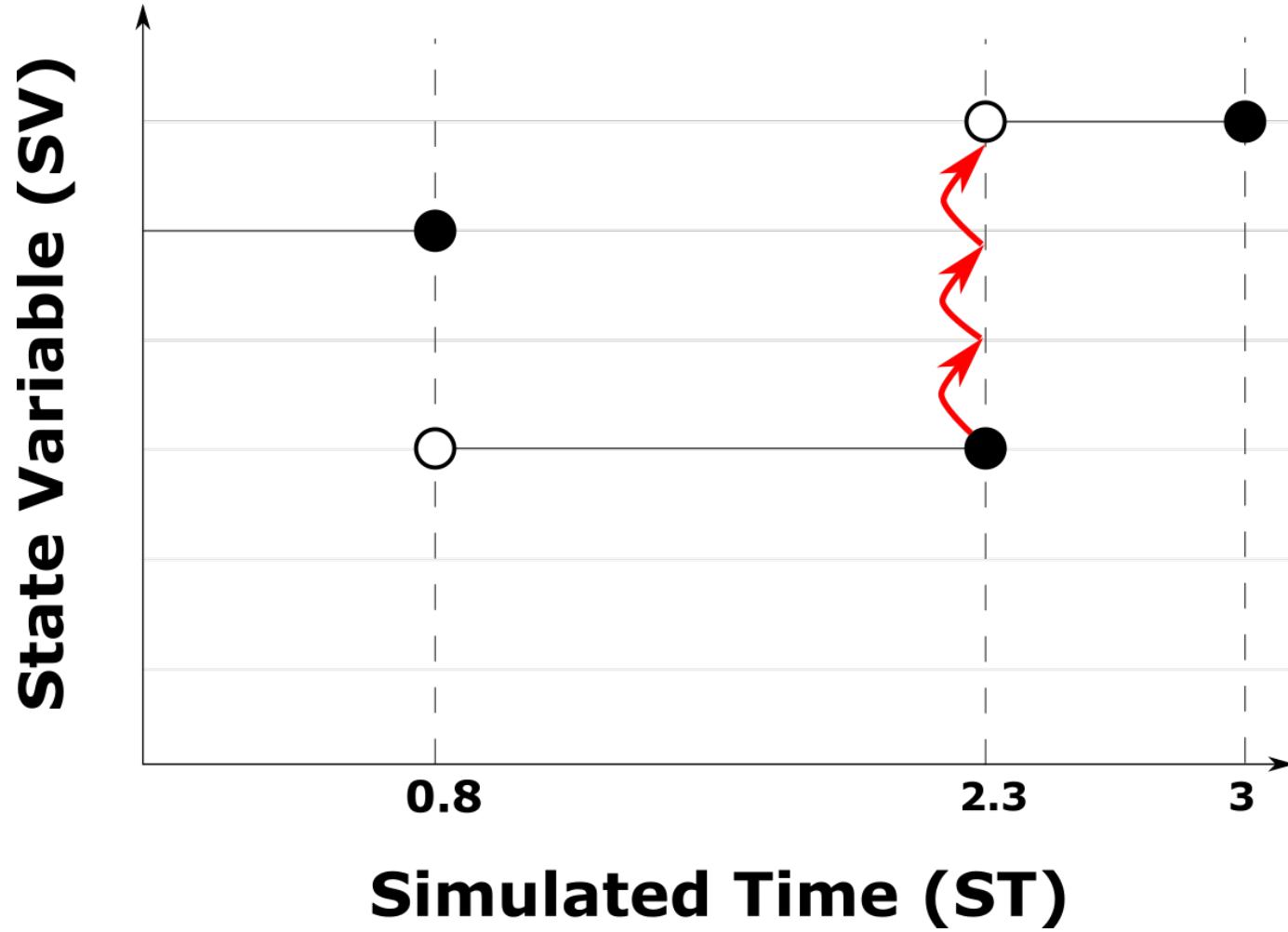
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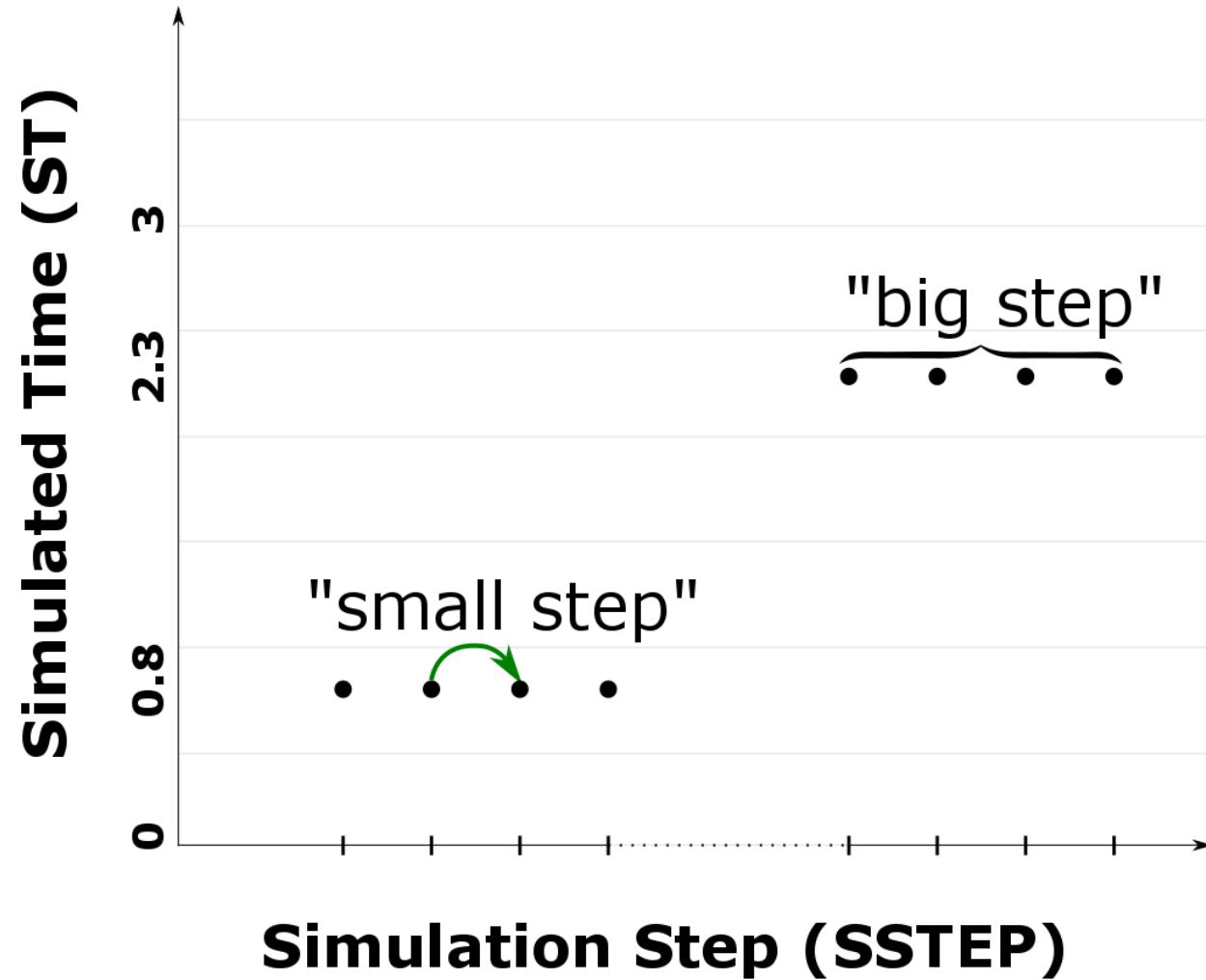
# XiL: X = Model, Software, Processor, Hardware



**simulation** of a **model** of the **dynamics** of a system produces **behaviour traces**

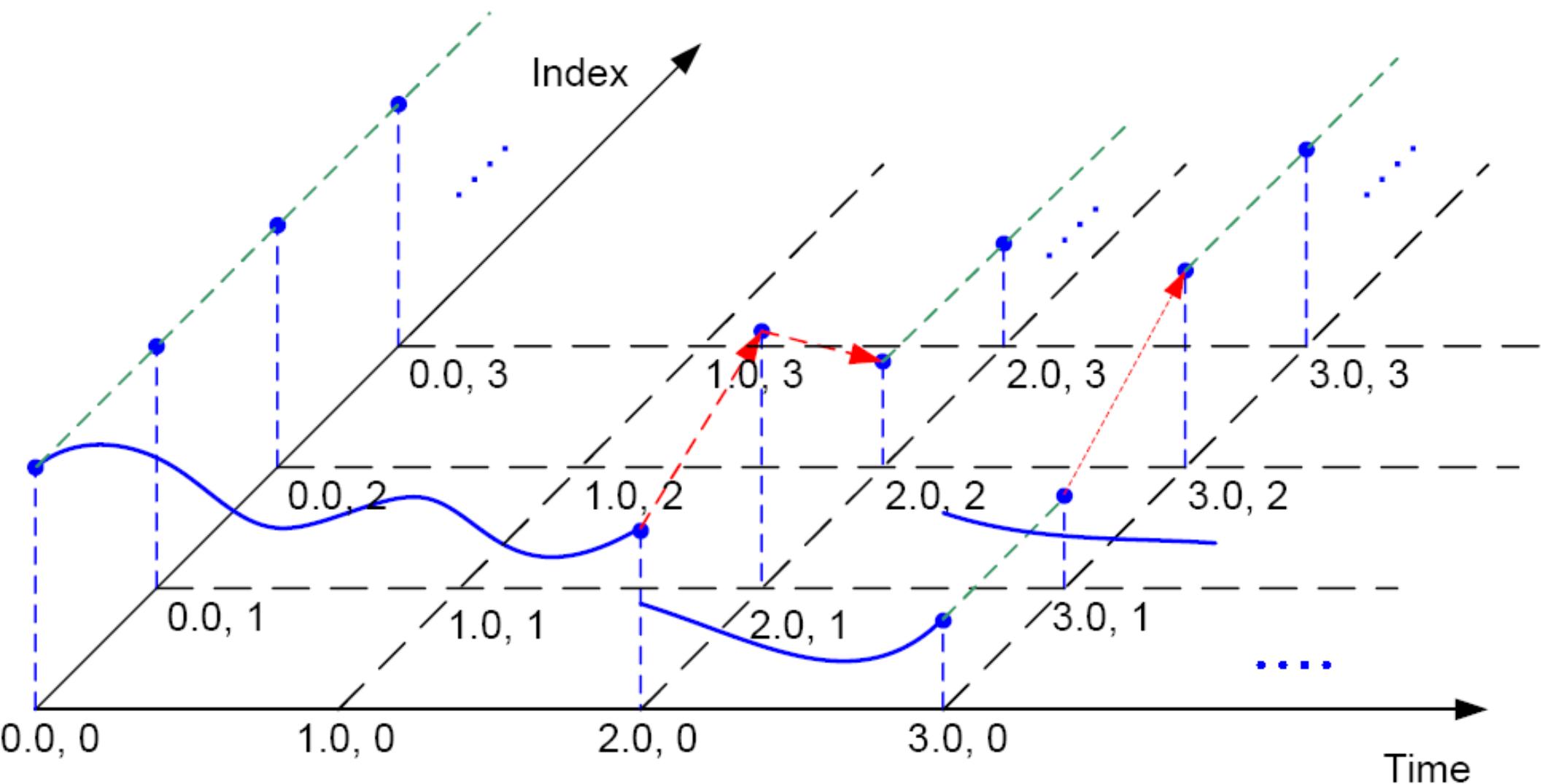


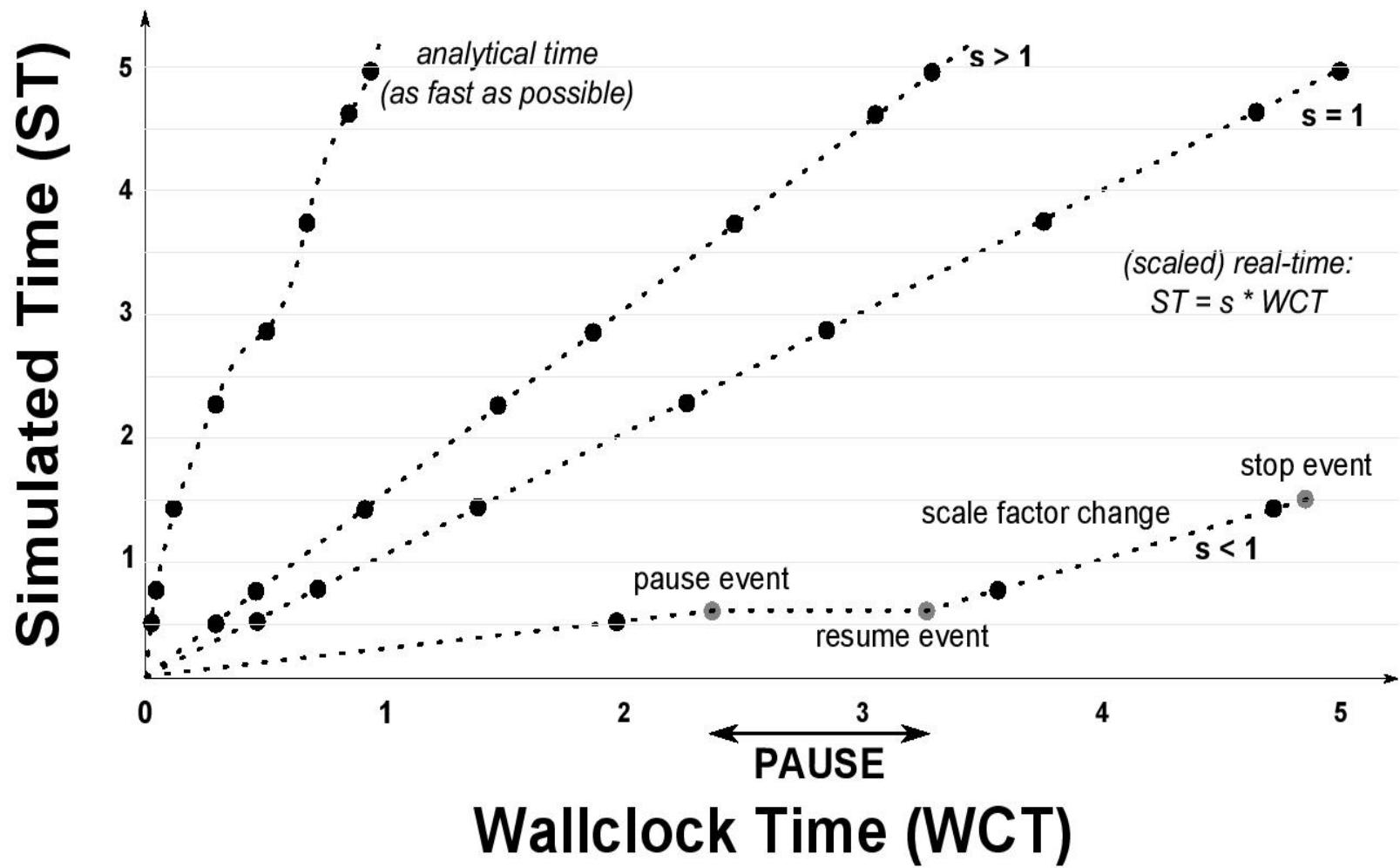




Superdense Time (Edward Lee)

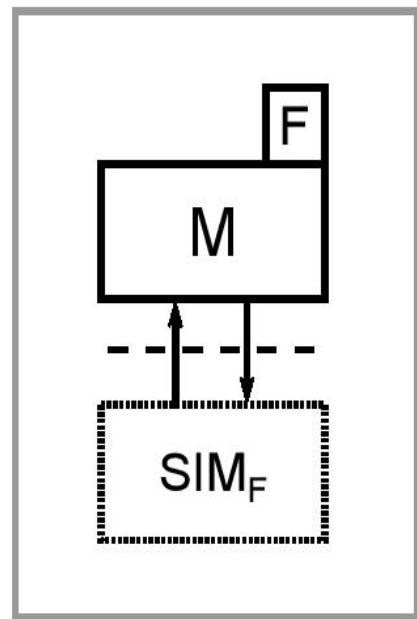
$$T = \mathbb{R}_+ \times \mathbb{N}$$



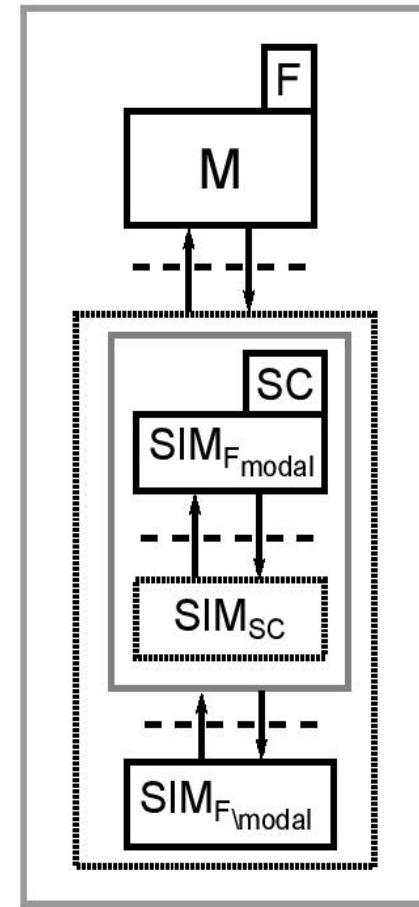


# De/Reconstructing the Simulator

Extracting the Modal Part



(a) A model  $M$  in formalism  $F$  and a simulation kernel  $SIM_F$  for  $F$ .

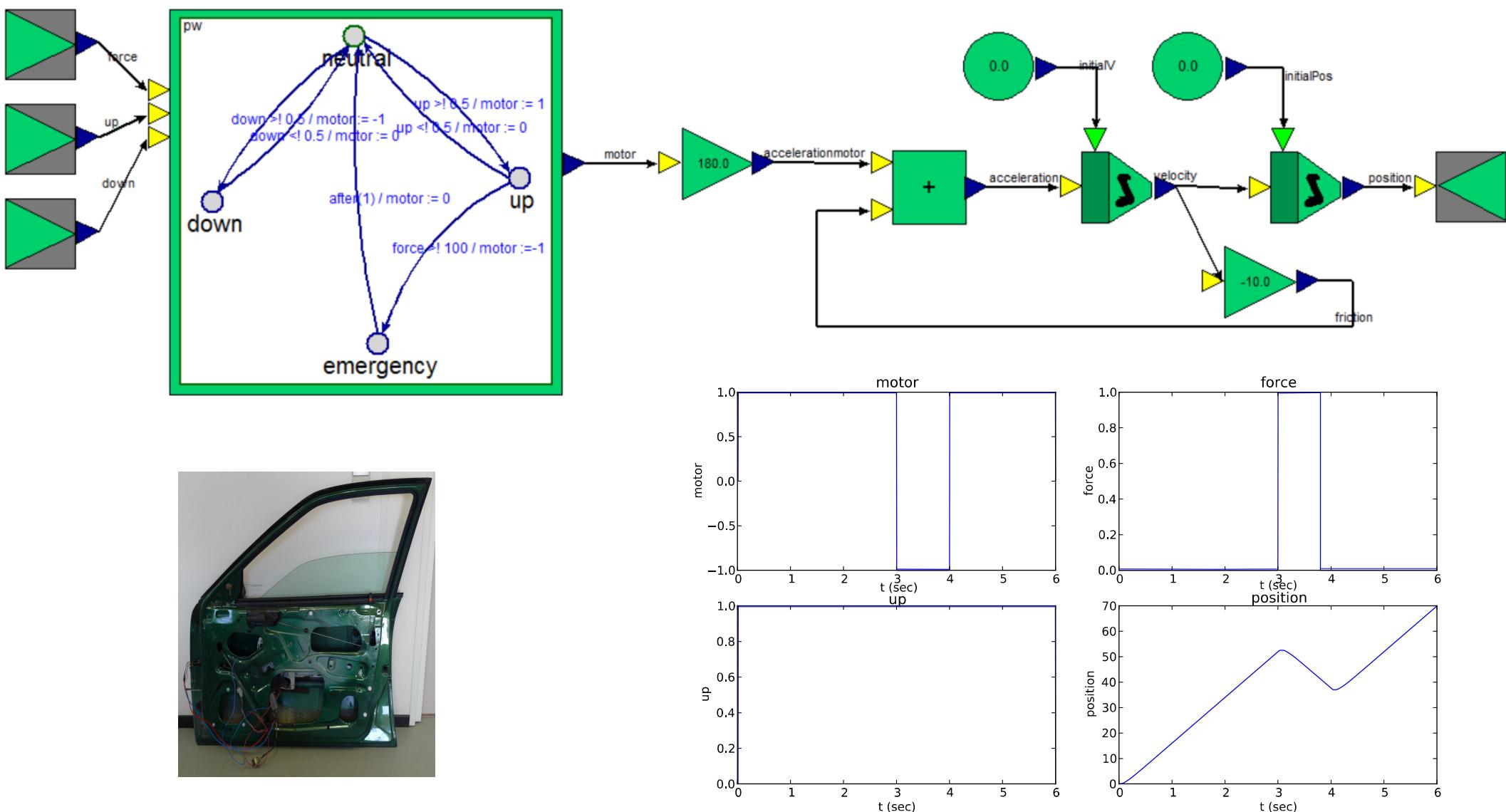


(b) De-/Re-constructing the simulator.

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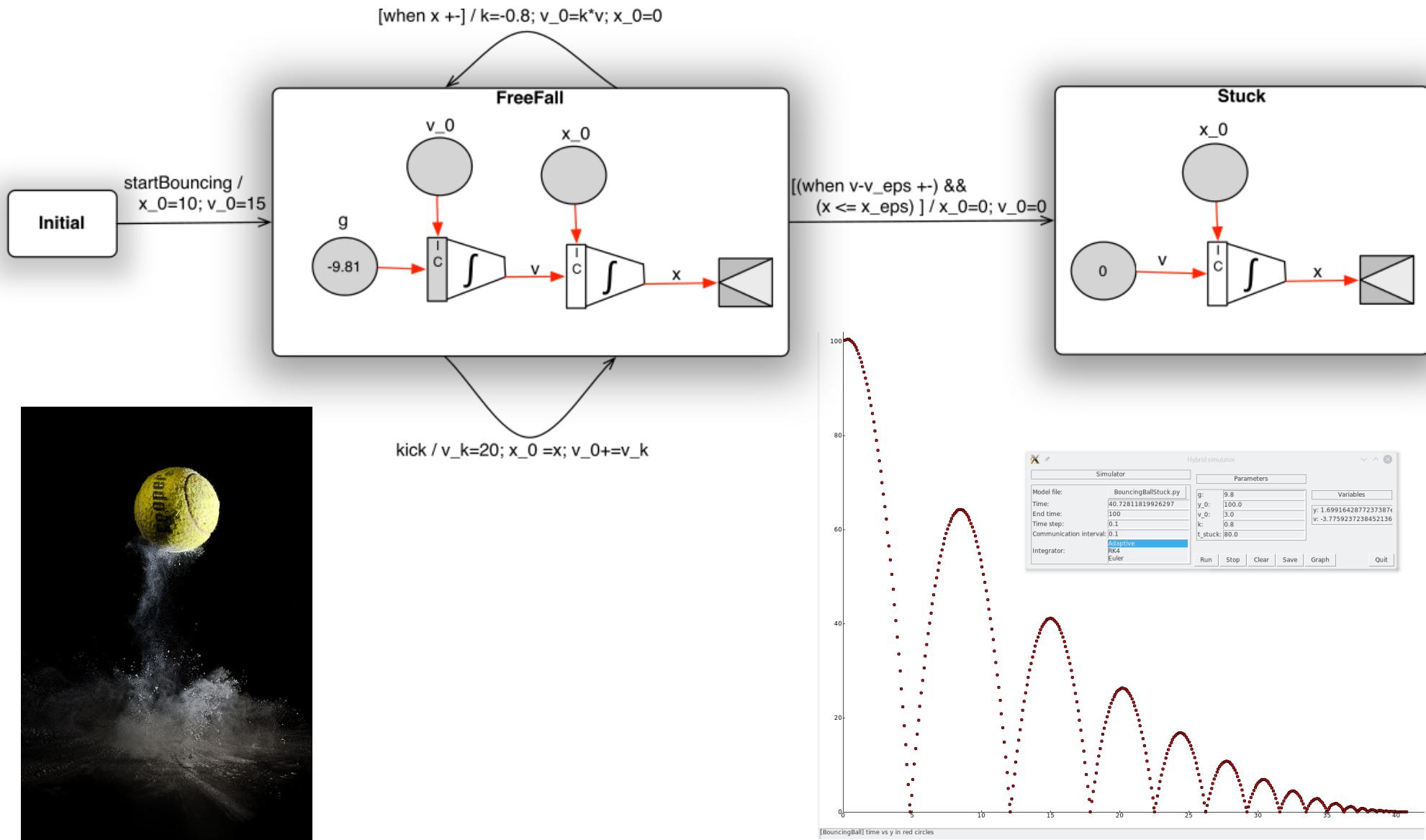
# Example 1: CBD (host) + TFSA (embedded)



Bart Meyers, Joachim Denil, Frederic Boulanger, Cecile Hardebolle, Christophe Jacquet, Hans Vangheluwe. A DSL for Explicit Semantic Adaptation. MPM@MoDELS 2013:47-56.

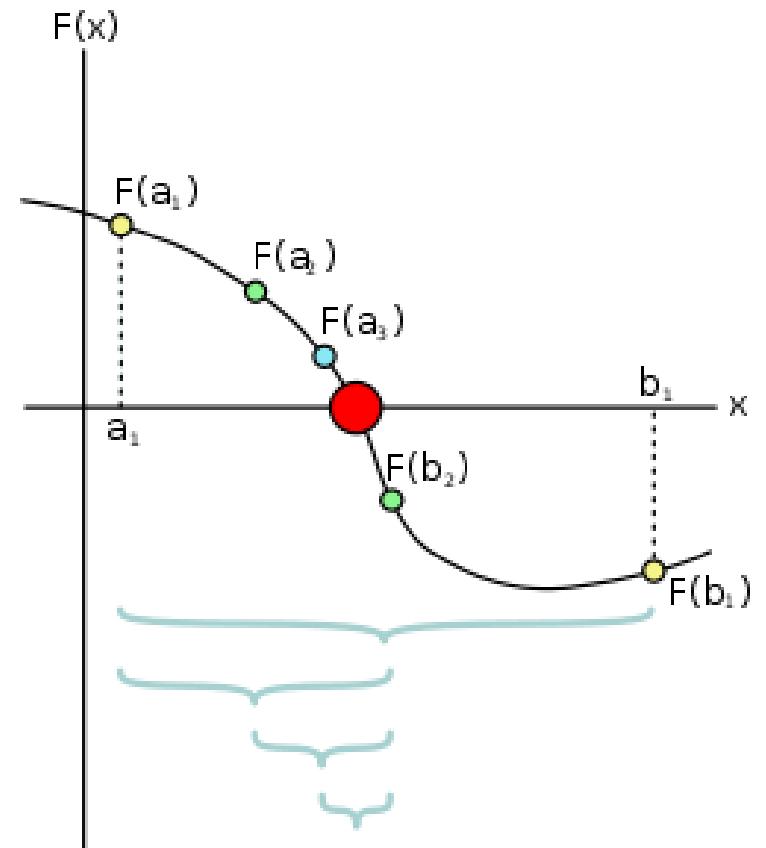
Joachim Denil, Bart Meyers, Paul De Meulenaere, and Hans Vangheluwe. Explicit semantic adaptation of hybrid formalisms for FMI co-simulation. In Proceedings of the 2015 Spring Simulation Multi-Conference, pages 852 - 859. SCS, April 2015.

# Example 2: TFSA (host) + CBD (embedded)



# “when”: Zero Crossing Detection

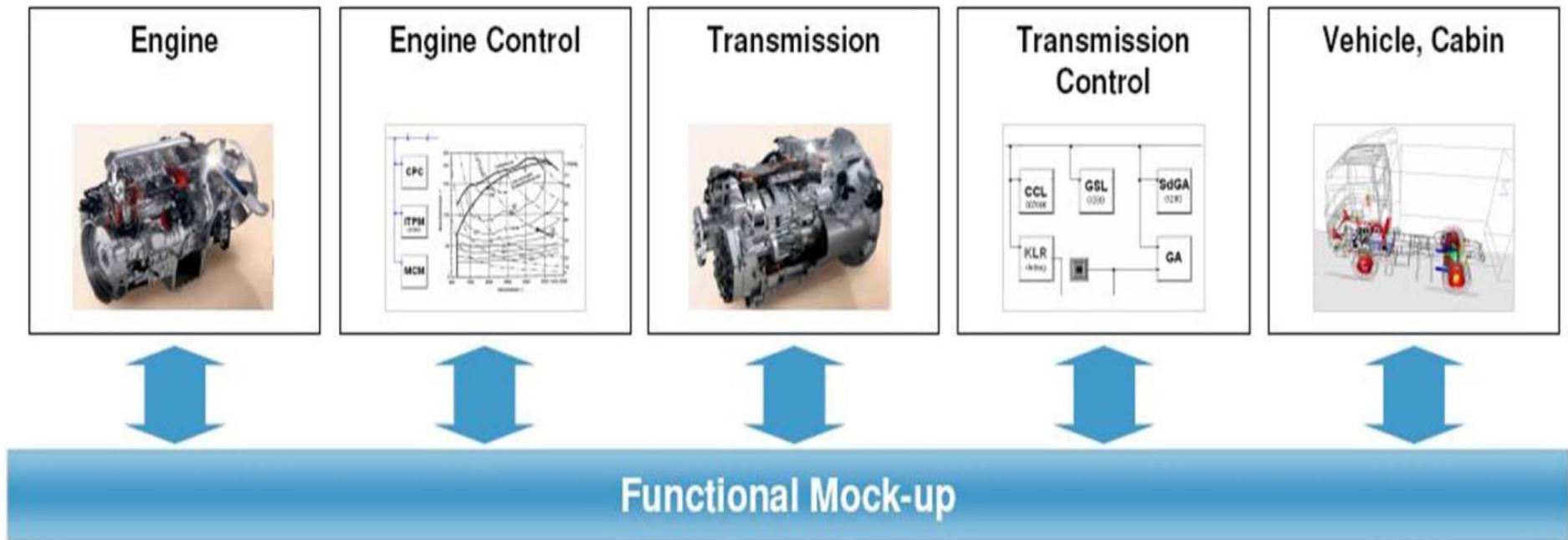
- Need to detect crossing of 0 in bouncing ball model (for mode switch)
  - Certain resolution is required...
  - Fixed step can end up “below ground”. Impossible!
- Use Root-finding algorithms:
  - Bisection method (see figure)
  - Secant method
  - Regula Falsi
  - Etc.



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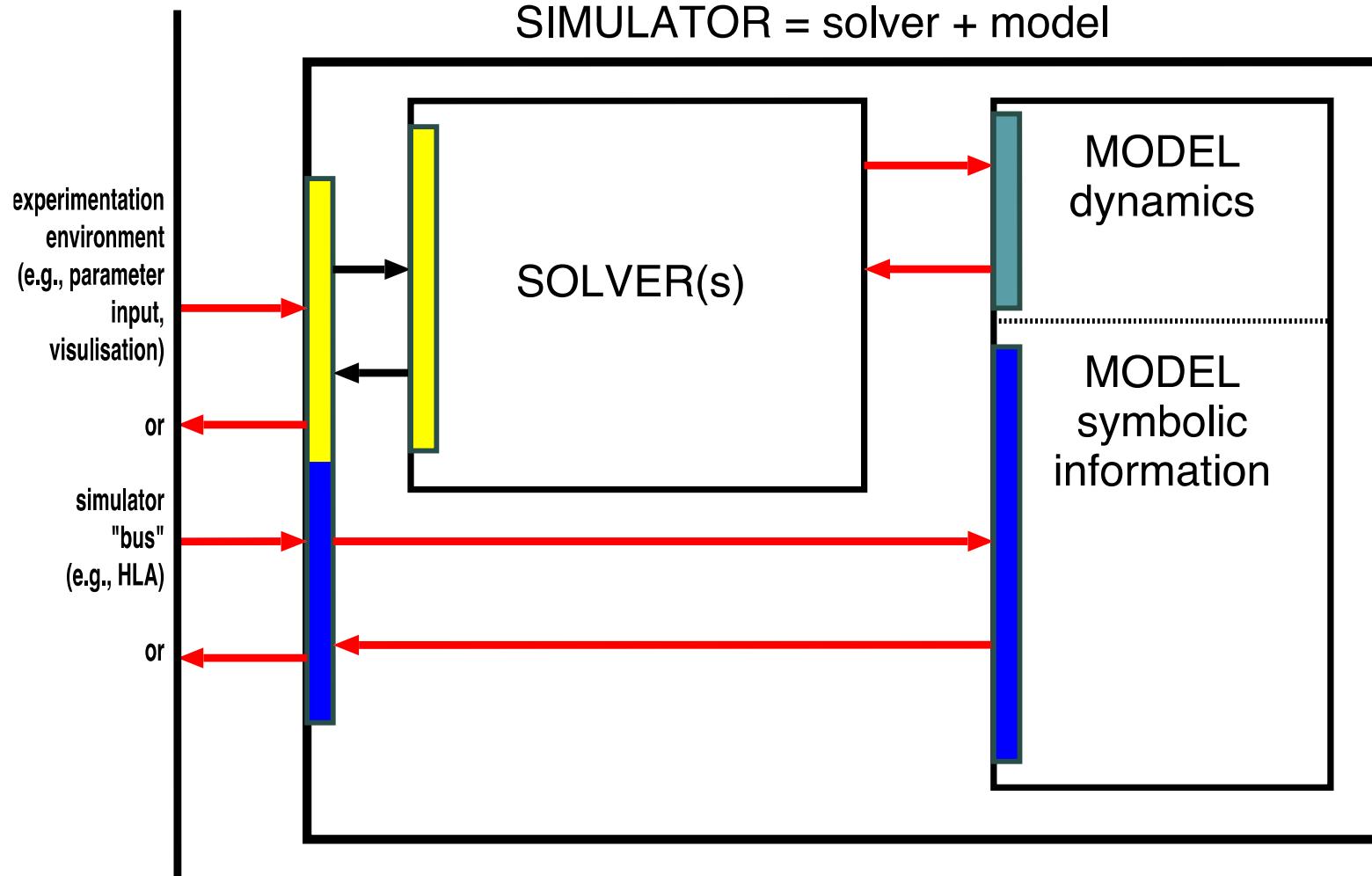
# Functional Mockup Units (FMUs)



[www.fmi-standard.org](http://www.fmi-standard.org)

Bart Pussig, Bert Van Acker, Claudio Gomes

## SIMULATOR = solver + model



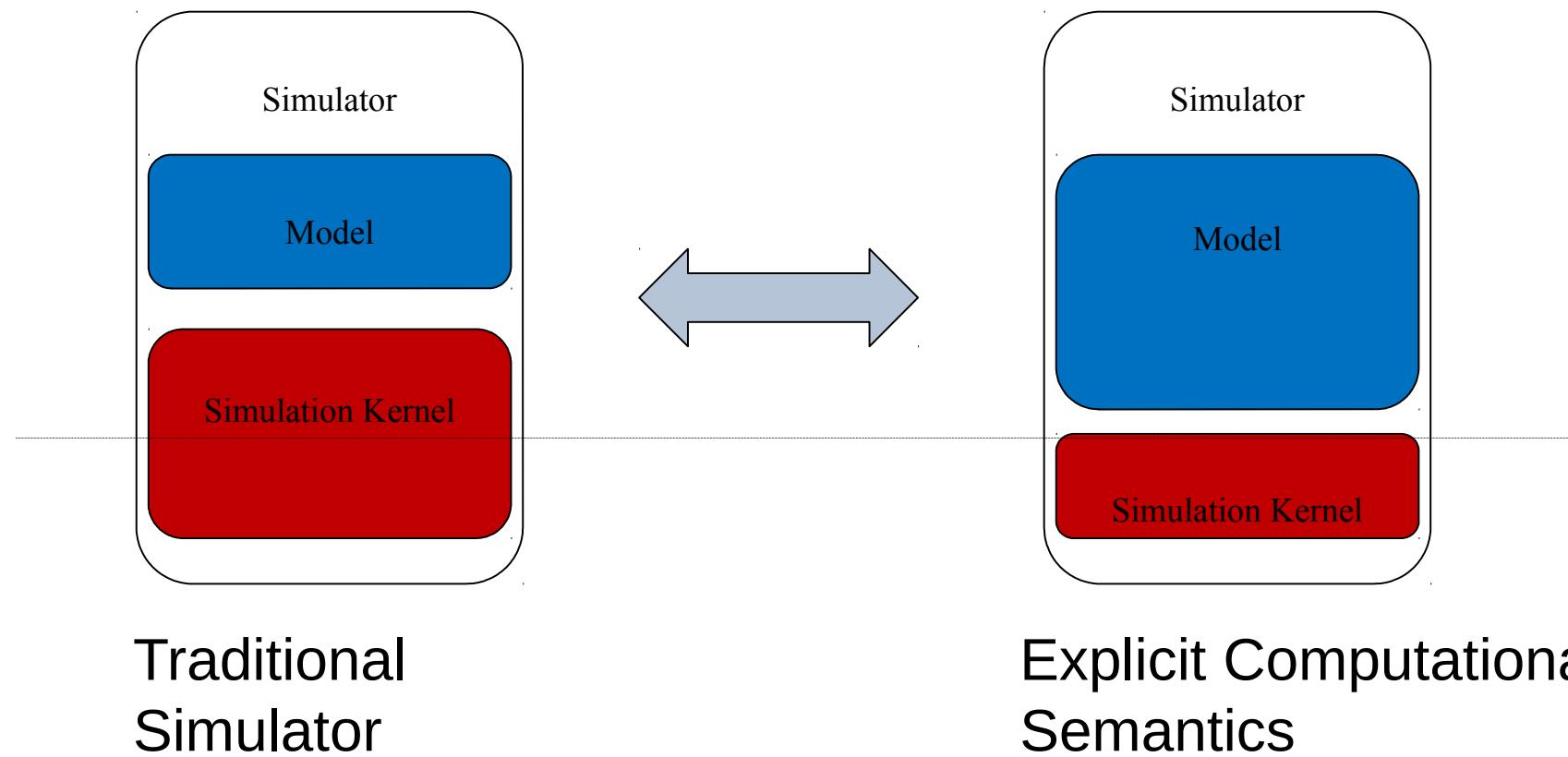
DSblock

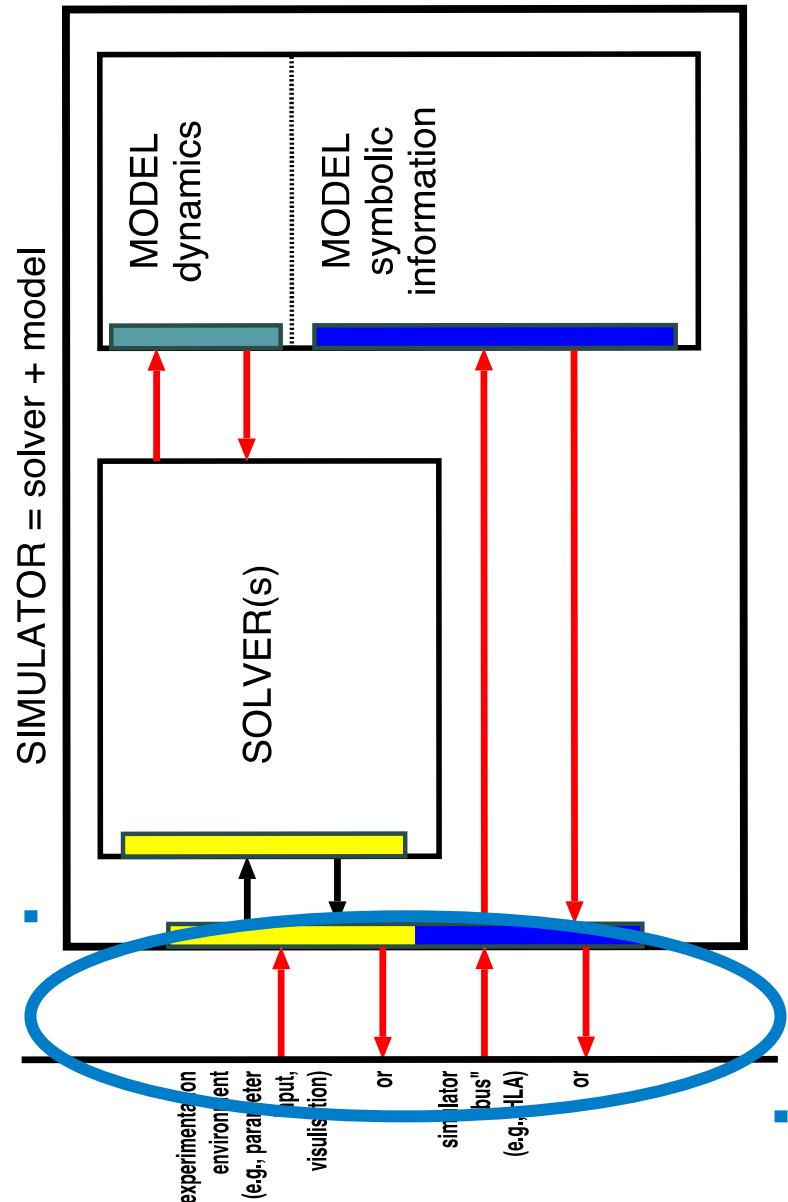
Martin Otter and Hilding Elmquist.  
The DSblock interface for exchanging model components.  
Eurosim '95 Simulation Congress. pp. 505- 510. 1995.

MSL-EXEC

Henk Vanhooren, Jurgen Meirlaen, Youri Amerlinck, Filip Claeys, Hans Vangheluwe,  
and Peter A. Vanrolleghem.  
WEST: Modelling biological wastewater treatment. Journal of Hydroinformatics ,  
5(1):27--50, 2003.

# meaningful operational semantics (Models of Computation)





Function	start, end	instantiated	Initialization Mode	stepComplete	stepInProgress	stepFailed	stepCancelled	terminated	error	fatal
fmi2GetTypesPlatform	x	x	x	x	x	x	x	x	x	
fmi2GetVersion	x	x	x	x	x	x	x	x	x	
fmi2SetDebugLogging		x	x	x	x	x	x	x	x	
fmi2Instantiate	x									
fmi2FreeInstance		x	x	x		x	x	x	x	
fmi2SetupExperiment		x								
fmi2EnterInitializationMode	x									
fmi2ExitInitializationMode		x								
fmi2Terminate			x			x				
fmi2Reset	x	x	x			x	x	x	x	
fmi2GetReal		2	x		8	7	x	7		
fmi2GetInteger		2	x		8	7	x	7		
fmi2GetBoolean		2	x		8	7	x	7		
fmi2GetString		2	x		8	7	x	7		
fmi2SetReal	1	3	6							
fmi2SetInteger	1	3	6							
fmi2SetBoolean	1	3	6							
fmi2SetString	1	3	6							
fmi2GetFMIState	x	x	x		8	7	x	7		
fmi2SetFMIState	x	x	x		x	x	x	x		
fmi2FreeFMIState	x	x	x		x	x	x	x		
fmi2SerializedFMIStateSize	x	x	x		x	x	x	x		
fmi2SerializeFMIState	x	x	x		x	x	x	x		
fmi2DeSerializeFMIState	x	x	x		x	x	x	x		
fmi2GetDirectionalDerivative	x	x			8	7	x	7		
fmi2SetRealInputDerivatives	x	x	x							
fmi2GetRealOutputDerivatives			x		8	x	x	7		
fmi2DoStep		x								
fmi2CancelStep			x							
fmi2GetStatus		x	x	x				x		
fmi2GetRealStatus		x	x	x				x		
fmi2GetIntegerStatus		x	x	x				x		
fmi2GetBooleanStatus		x	x	x				x		
fmi2GetStringStatus		x	x	x				x		

$t_0, p$ , initial values (a subset of  $\{\dot{x}_0, x_0, y_0, v_0, m_0\}$ )

## Enclosing Model

- $t$  time
- $m$  discrete states (constant between events)
- $p$  parameters of type Real, Integer, Boolean, String
- $u$  inputs of type Real, Integer, Boolean, String
- $v$  all exposed variables
- $x$  continuous states (continuous between events)
- $y$  outputs of type Real, Integer, Boolean, String
- $z$  event indicators

## External Model (FMU instance)

$t$

$x$

$\dot{x}, m, z$

Solver

# Behaviour information: FMU - C

```
fmi2Status fmi2DoStep(fmi2Component fc , fmi2Real currentCommPoint, fmi2Real commStepSize, fmi2Boolean
noPrevFMUState)
{
    FMUInstance* fi = (FMUInstance *)fc;
    fmi2Status simStatus = fmi2OK;
    printf("%s in fmi2DoStep()\n", fi->instanceName);
    fi->currentTime = currentCommPoint + commStepSize;
    printf("Motor_in: %f\n", fi->r[_motor_in]);
    printf("slave CBD_PART2 now at time: %f\n", fi->currentTime);

    fi->r[_position] = fi->r[_position] + fi->r[_velocity] * commStepSize;
    fi->r[_velocity] = fi->r[_velocity] + fi->r[_acceleration_after_friction] * commStepSize;
    fi->r[_friction] = fi->r[_velocity] * 5.81;
    fi->r[_motor_acceleration] = fi->r[_motor_in] * 40;
    fi->r[_acceleration_after_friction] = fi->r[_motor_acceleration] - fi->r[_friction];

    return simStatus;
}

fmi2Status fmi2GetReal(fmi2Component fc, const fmi2ValueReference vr[], size_t nvr, fmi2Real value[])
{
    FMUInstance* comp = (FMUInstance *)fc;
    int i;
    for (i = 0; i < nvr; i++)
    {
        value[i] = comp->r[(vr[i])];
    }
    return fmi2OK;
}
```

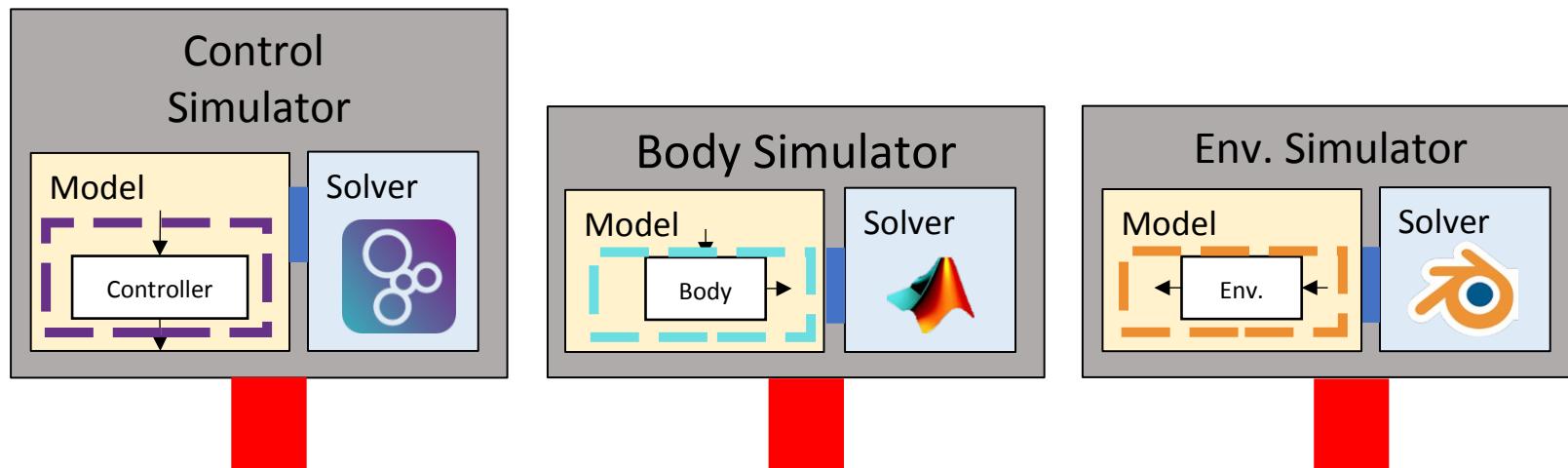
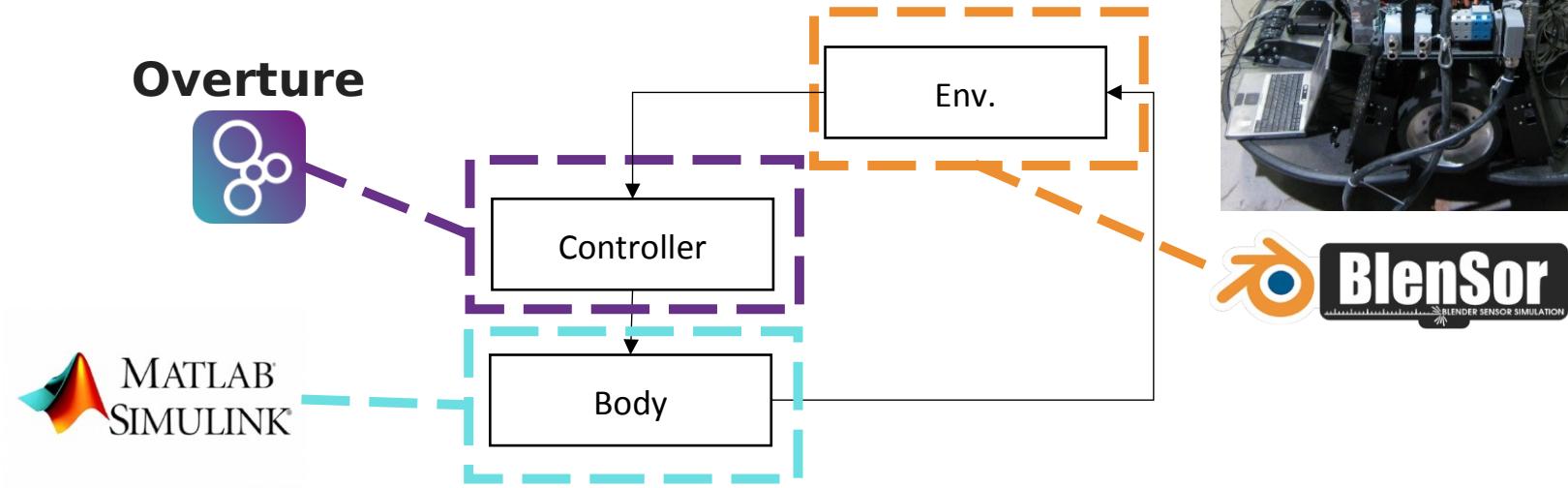


FUNCTIONAL  
MOCK-UP  
INTERFACE

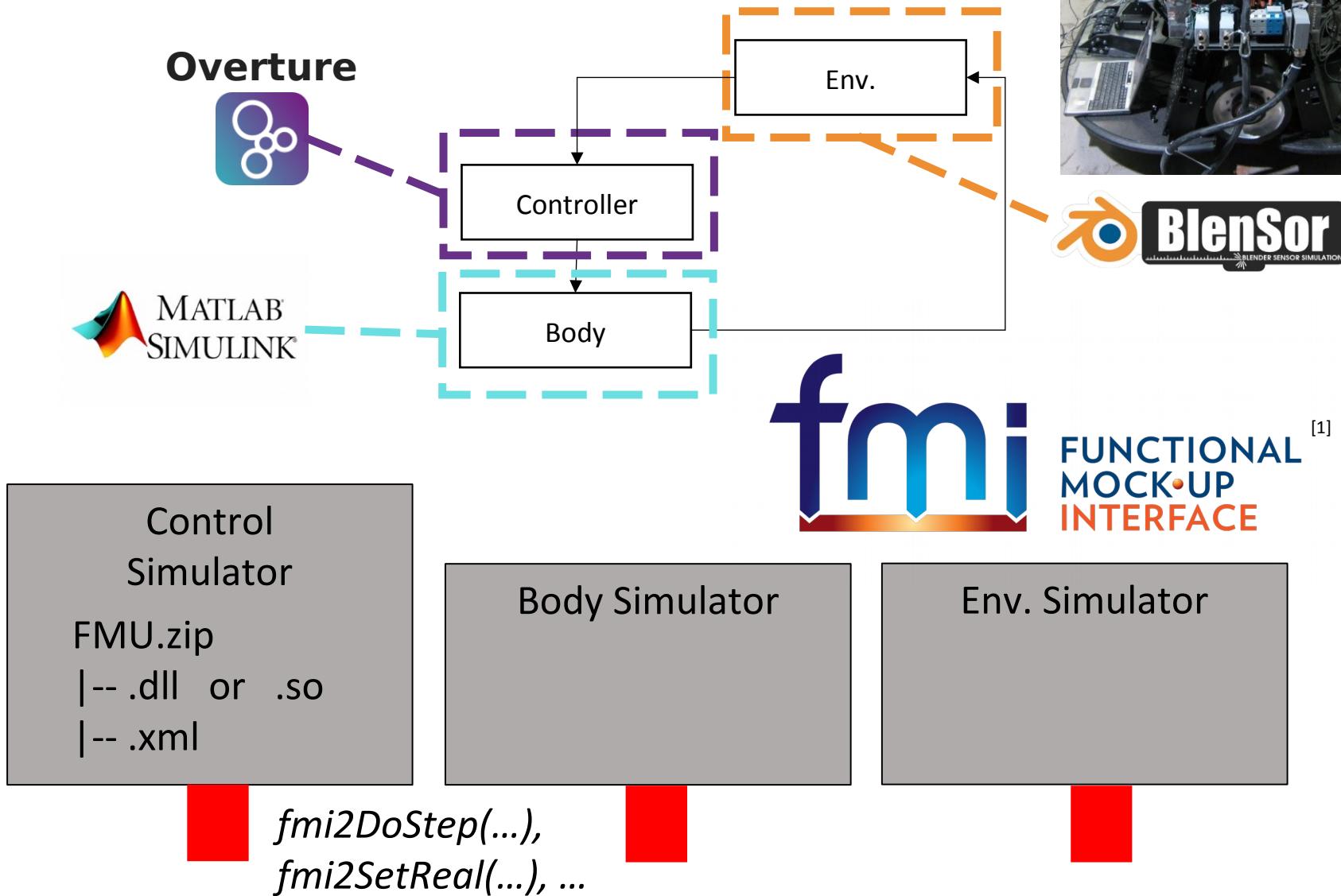
# Symbolic information: FMU - XML

```
<?xml version="1.0" encoding="iso-8859-1"?>
<fmiModelDescription fmiVersion="1.0"
    modelName="BackEulerExmpl"
    modelIdentifier="BackEulerExmpl"
    guid="{7c4e810f-3da3-4a00-8276-176fa3c9f000}"
    numberOfContinuousStates="1"
    numberOfEventIndicators="0">
    <ModelVariables>
        <ScalarVariable
            name="Delay" valueReference="0">
            <Real start="0.0" fixed="true" />
        </ScalarVariable>
        <ScalarVariable
            name="stepSize" valueReference="1">
            <Real start="0.1" fixed="true" />
        </ScalarVariable>
        <ScalarVariable
            name="Product" valueReference="2">
            <Real start="0.0" fixed="true" />
        </ScalarVariable>
        <ScalarVariable
            name="Negator" valueReference="3">
            <Real start="0.0" fixed="true" />
        </ScalarVariable>
        <ScalarVariable
            name="Adder" valueReference="4">
            <Real start="0.0" fixed="true" />
        </ScalarVariable>
    </ModelVariables>
</fmiModelDescription>
```

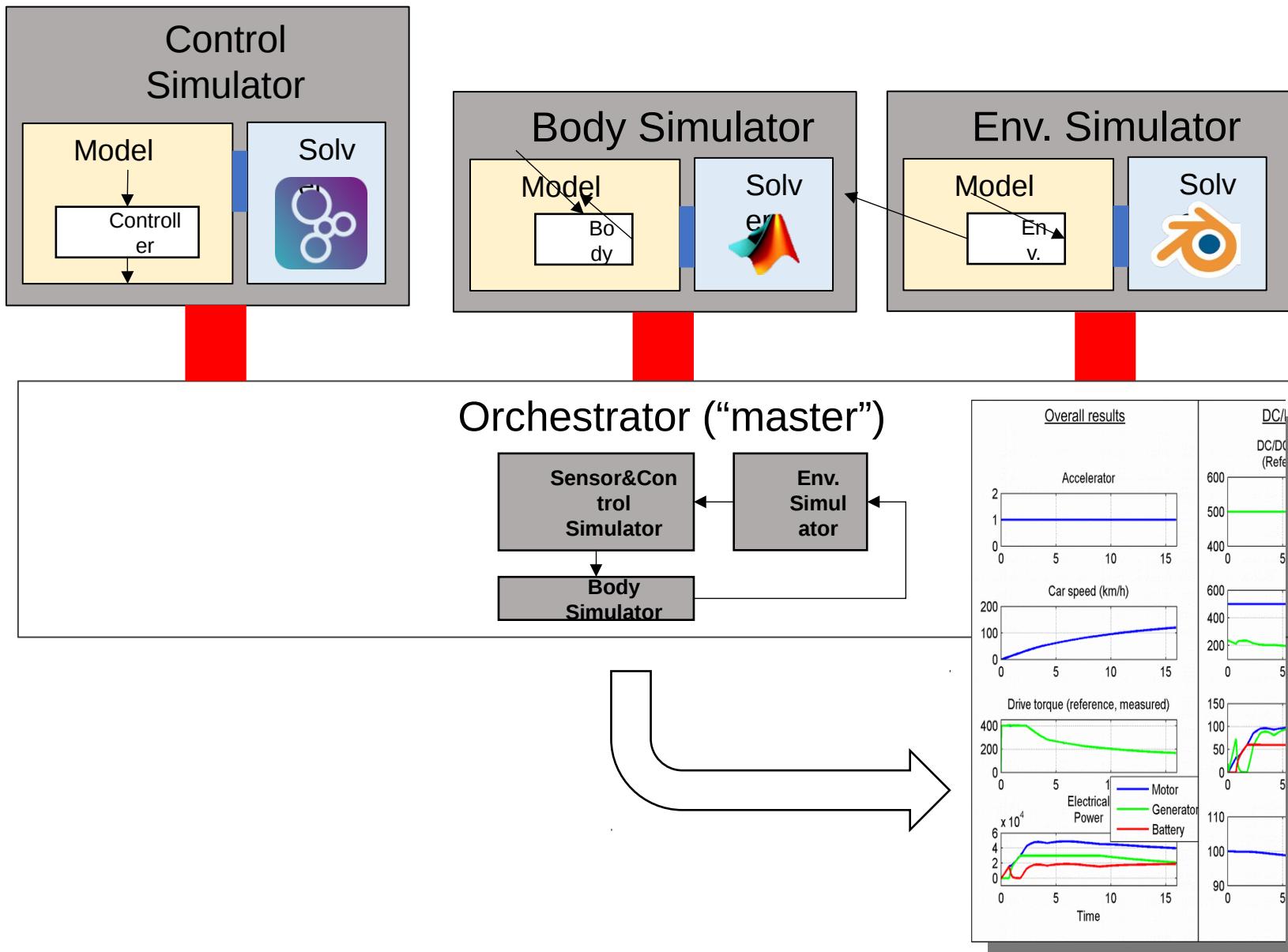
# Co-simulation: how?



# Co-simulation: how?



# Co-simulation: how?

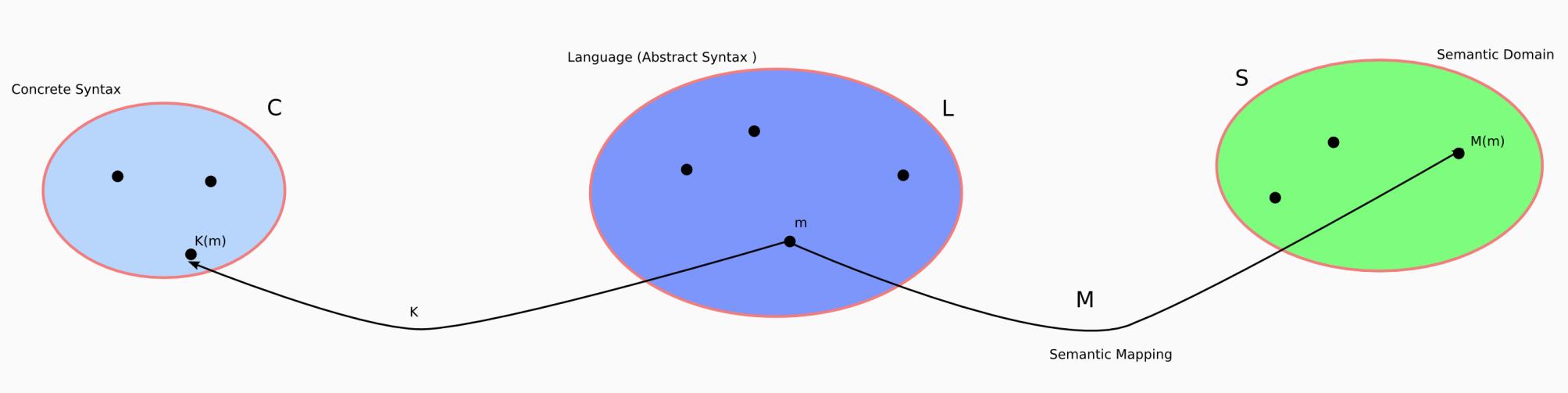


# Modelling Physical Systems

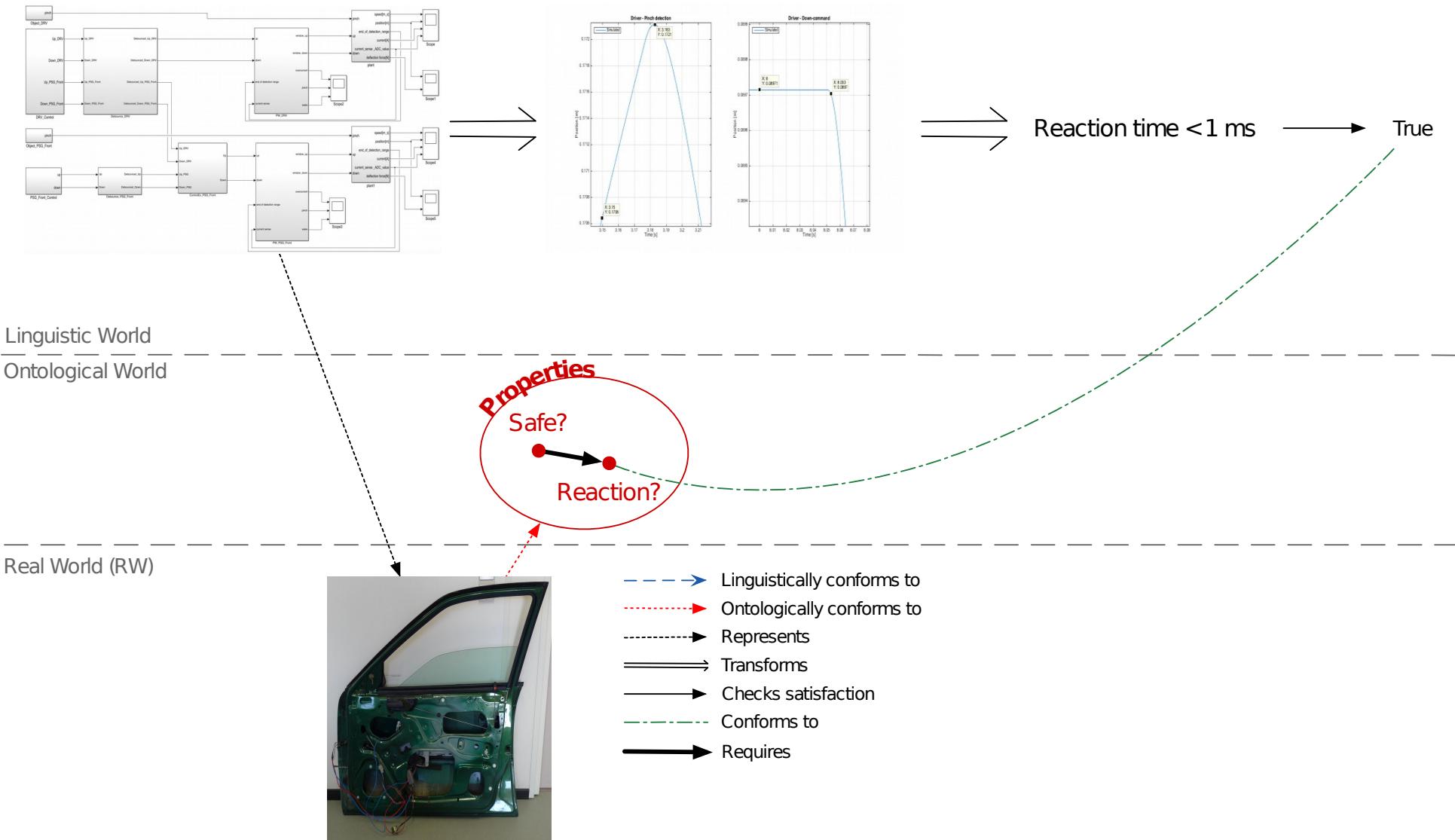
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  - As-Fast-As-Possible vs. Real-time; XIL
  - Hybrid (discrete-continuous) modelling/simulation
  - Hiding IP: Composition of Functional Mockup Units (FMI)
- Inter-disciplinary teams/multiple views:  
“linguistic” vs. “ontological”

# "linguistic" view on Modelling Languages/Formalisms: Syntax and Semantics

Concrete Formalism F



# Linguistic vs. Ontological Reasoning in MBSE



Based on: [4] B. Barroca, T. Kühne, and H. Vangheluwe. Integrating language and ontology engineering. In MPM '14, volume 1237 of CEUR, pages 77–86, September 2014.

# Linguistic vs. Ontological Reasoning in MBSE

