On estimation of nonparametric regression models with autoregressive and moving average errors

Qi Zheng *
Department of Bioinformatics & Biostatistics, University of Louisville
and
Yunwei Cui
Department of Mathematics, Towson University
and
Rongning Wu
Zicklin School of Business at Baruch College, The City University of New York

January 22, 2024

The `stats::arima()` function of R is capable of fitting the regression coefficients and ARMA parameter simultaneously. We performed a study to compare the performance of our R code and `stats::arima()` for estimating the smooth functions. Especially, we compared $\rho(\cdot)$ and $\rho_{19}(\cdot)$ for three smooth functions as used in Zheng, Cui, and Wu (2023)

$$\begin{cases} f_1(X_t) = 1 - 6X_t + 36X_t^2 - 53X_t^3 + 22X_t^5; \\ f_2(X_t) = \sin(2\pi X_t) + 2X_t^2; \\ f_3(X_t) = \arctan(5X_t - 5/2) - X_t^2/3. \end{cases}$$

In this simulation, the B-spline basis matrix is generated by the `splines::bs()` function, for which the knots are automatically generated by its default method as a sequence of equally spaced quantile points. For sample path with length $n = 1000$, 9 inner knots are used.

After examining various model specifications, we discovered that the two functions produce nearly identical results in terms of smooth function accuracy. However, our R code appears to generate slightly better results than stats::arima() does.

*Corresponding author. University of Louisville, Louisville, KY 40202, USA (Phone: 502-852-8780; Fax: 502-852-3294; E-mail: qi.zheng@louisville.edu)

Table 1: Comparing $\hat{g}(\cdot)$ and $g_0(\cdot)$, when ϵ_t's follow an ARMA(1, 1), or an AR(2), or an MA(2) process, X_t's are serially correlated and satisfy the conditions of Theorem 2, and innovations ζ_t's have a t distribution with degrees of freedom ν. All sample paths have $n = 1000$. In the table, `one step` denotes the R routine developed by Zheng, Cui, and Wu (2023), whereras `arima()` denotes the R function provided by `stats` package.

	ARMA(1,1), $(\phi, \theta) = (0.2, -0.5)$					
	$f_1(X_t)$		$f_2(X_t)$		$f_3(X_t)$	
	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$
`one step`	0.1986	0.0568	0.1680	0.0444	0.1556	0.0406
`arima()`	0.1987	0.0568	0.1684	0.0444	0.1559	0.0407
	AR(2), $(\phi_1, \phi_2) = (0.5, 0.1)$					
	$f_1(X_t)$		$f_2(X_t)$		$f_3(X_t)$	
	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$
`one step`	0.1939	0.0609	0.1783	0.0682	0.1828	0.0581
`arima()`	0.1957	0.0612	0.1797	0.0686	0.1832	0.0583
	MA(2), $(\theta_1, \theta_2) = (0.4, 0.2)$					
	$f_1(X_t)$		$f_2(X_t)$		$f_3(X_t)$	
	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$	$\rho(g_0, \hat{g})$	$\rho_{19}(g_0, \hat{g})$
`one step`	0.2068	0.0542	0.1937	0.0646	0.1950	0.0542
`arima()`	0.2068	0.0542	0.1937	0.0646	0.1950	0.0542