CAAM 485/586: Stochastic Simulation

Homework 6 (due November 17, 2022)

1. Ross Chapter 10, Problem 7

In addition to the question using the normalized importance sampling, use the resampling method to estimate the mean $E[X]$.

2. Read Example 9u of Ross Chapter 10 on page 203-205.

Consider $S_n = X_1 + \cdots + X_n$, with X_i's being i.i.d. Bernoulli $p = 0.2$. Use the exponential tilting and LDP to simulate $P(S_{40} > 16)$. Find the optimal tilting parameter θ^* using the Legendre transform.

3. Consider a random walk $S_n = X_1 + \cdots + X_n$, with X_i's being i.i.d. normal with mean $\mu = -0.1$ and variance $\sigma^2 = 0.5$. Let $\tau(x) = \inf\{k \geq 1 : S_k > x\}$ be the first passage time. Simulate $P(\tau(x) < \infty)$ for $x = 10$ by Siegmund's algorithm. Also give the 95% confidence interval for the simulated probability.

4. Consider a single server $M/G/1$ queue under first-come first-served discipline. The arrival process is Poisson of rate λ, that is, the interarrival times are i.i.d. exponential with parameter λ. The service times are i.i.d. of an Erlang-2 distribution, that is, $S = \xi_1 + \xi_2$ with ξ_1 and ξ_2 both exponential of rate μ. Assume the arrival and service processes are independent. Let D_n be the waiting time or delay of the n^{th} customer. Recall $D_{n+1} = (D_n + S_n - T_{n+1})^+$ where S_n is the service time of the n^{th} customer and T_{n+1} is the associated interarrival time. Let D be the steady state delay. Given $\lambda = 1, \mu = 3$, simulate the delay tail probability $P(D > 6)$ by Siegmund's algorithm.