University of Illinois Fall 2018

ECE 544NA: Pattern Recognition
Lecture 21: Generative Adversarial Nets

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1 Overview

In this scribe, we summarize lecture 12 of ECE 544 offered by Dr. Schwing in Fall 2018. We will first briefly go through a review. Then we are going to talk aboue GANs and WGANs. Note all the images are from the lecture slides [6], except for those who are cited specifically. You may want to check our course at https://courses.engr.illinois.edu/ece544na/fa2018/secure/L21_Slides.pdf.

2 Brief Review

Before we start introducing Generative Adversarial Nets (GAN) [2], let's briefly review some concepts here.

2.1 Discriminative vs. Generative

In previous lectures, we have learned both discriminative model and generative models, which are two common types of models used to model the distribution in machine learning field. In general, given a data sample $(\vec{x}, y)$ such that $\vec{x} \in \Re^{D_X}$ and $y \in \Re$, in which $\vec{x}$ is input data and y is discrete output space, a discriminative model will model distribution $p(y|\vec{x})$ and a generative model will model $p(\vec{x}, y)$, from which we can get $p(\vec{x}|y)$ by Bayesian rule.

Application of discriminative models:

Applications of generate models include:synthesis of objects (images, text),environment simulator (reinforcement learning, planning), item leveraging unlabeled data and etc.

2.2 Minimize loss functions

Loss function is a protocol to evaluate the difference of model output and ground truth for discriminative model, or the how much the clustering criteria is satisfied by the model output for the generative model. The goal of minimizing loss functions is to obtain an optimal set of parameters of a model, which allows the model best reconstruct the distributions of sample data. For example, parameters in a fully connected layer include W and b. Common ways to achieve it includes maximize log likelihood, minimize squared error (MSE) and so on.

2.3 Generative Models: How to model $p(x)$?

Intuitively, we could assume $p(\vec{x})$ can be described well by a simple distribution, e.g.,Guassian distribution, so then we have $p(\vec{x}; \mu, \Sigma)$ where μ and Σ are the mean and covariance matrix respectively. Then we can apply maxmimum likelihood to approach to find the optimal parameter θ (μ and Σ), i.e.,

$$\theta^* = \max_\theta \sum_i \log p(x^i; \theta) \tag{1}$$

Apart from Gaussian distribution, we can also use other sample models like k-means, Gaussian mixture models (GMM) to reconstruct $p(\vec{x})$ by optimizing different parameters. Variational auto-coder [3] is another way to model $p(\vec{x})$, which will be reviewed briefly in next subsection.

2.4 Variational Auto-Coder

The goal of variational auto-coder [3] is to model $p_\theta(\vec{x}|\vec{z})$, i.e., given $\vec{z}$, we can generate, compute or sample data $\vec{x}$. The idea of this method to sample z from a simple Guassian distribution ($\vec{z}$) and transform it through a deep net to more complex distribution $p_\theta(\vec{x}|\vec{z})$. To ensure that we know which $\vec{z}$ maps to which $\vec{x}$, we train $p(\vec{z})$ as an auto-encoder (encode $\vec{x}$ to $\vec{z}$) by a deep net $q_\phi(\vec{z}|\vec{x})$, which approximates the mean and variances $p_\phi(\vec{z}|\vec{x})$ (a Gaussian distribution), as shown in figure 1. In this way, once we have done the training, we could "generate" new data sample by drawing

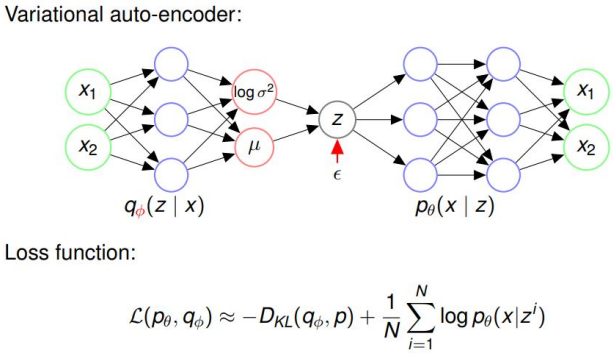

Figure 1: Architecture of Variational Auto-encoder: note that we try to estimate $log\sigma$ ($log\Sigma$) rather than σ because $log\sigma$ ($log\Sigma$) could automatically ensure the range constraints that are hard to encode directly. Moreover, to sample z, we could first sample a $\vec{\epsilon}$ from standard normal $N(\vec{0}, diag(1))$ and calculate $\vec{z} = \Sigma\vec{\epsilon} + \vec{\mu}$.

sample $\vec{z}$ and throwing it to the decoder $p(\vec{x}|\vec{z})$, which is modeled by another deep neural net. This way is innovative in a sense that in the previous, people usually try to generate $\vec{x}$ by finding a complex distribution $p(\vec{x})$ and sampling data from it.

So far, the variational auto-encoder could model the Gaussian distribution well, but what if we do not want to encode with Gaussian? How can we backpropagate some more complex distribution? Generative Adversarial Nets (GAN) is another approach which could address these questions and will be discussed in the coming section.

3 Generative Adversarial Nets (GAN)

Similarily, GAN [2] is another apporach to generate data samples. The main idea of GAN is not to write a formular for $p(\vec{x})$ but to learn to sample directly, so that we do not have to deal with the summation over large probability spaces. Then this problem is formed as a game between two players:

- Generator G - generate examples
- Discriminator D - predict whether the example is artificial or real

In this game, G tries to "trick" D by generating samples $G_\theta(\vec{z})$ that are hard to distinguish from the real data, where is a randomly sampled noise vector. G and D could be deep neural nets. A visualization is shown in figure 2.

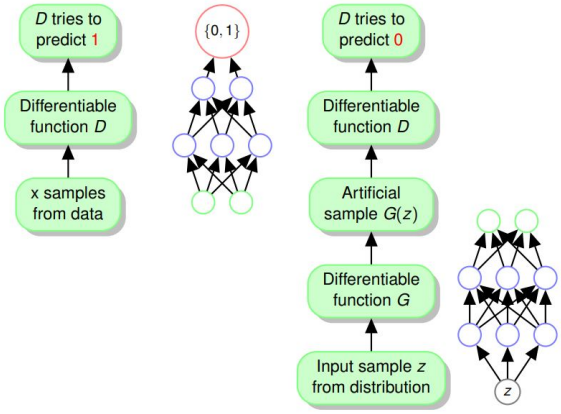

Figure 2: Architecture of GAN

3.1 Loss function

As the game formulated, D is trying to make $D_w(\vec{x}) = p(y = 1|\vec{x})$ and $D_w(G_\theta(\vec{z})) = p(y = 0|G_\theta(\vec{z}))$. G is trying to make $D_w(G_\theta(\vec{z})) = p(y = 1|G_\theta(\vec{z}))$, where y is a binary indicator, if $y = 1$, the data is real and vice versa. To achieve the goal of D, we choose w by:

$$\min_w -\sum_{\vec{x}} \log D_w(\vec{x}) - \sum_{\vec{z}} \log(1 - D_w(G_\theta(\vec{z}))) \tag{2}$$

and we choose θ by:

$$\max_\theta \min_w -\sum_{\vec{x}} \log D_w(\vec{x}) - \sum_{\vec{z}} \log(1 - D_w(G_\theta(\vec{z}))) \tag{3}$$

3.1.1 Why this loss works

Next, we are going to show that by optimizing this loss function, we could obtain a global optimal $p(G(Z)) = p(X)$, under assumptions that we can find a optimal discriminator with arbitrary capability (ability to model distribution) and continuity. Then to train criterion for the discriminator D, given any generator G, we should maximize the quanity $V(G, D)$ (minimizing discrete version of $-V(G, D)$ is equivalent to expression used in (3).)

$$\begin{aligned} V(G, D) &= \int_{\vec{x}} p_{data}(\vec{x}) \log D_w(\vec{x}) dx + \int_{\vec{z}} p_z(\vec{z}) \log(1 - D_w(G_\theta(\vec{z}))) dz \\ &= \int_{\vec{x}} p_{data}(\vec{x}) \log D_w(\vec{x}) dx + \int_{\vec{x}} p_G(\vec{x}) \log(1 - D_w(\vec{x})) dx \\ &= \int_{\vec{x}} p_{data}(\vec{x}) \log D_w(\vec{x}) + p_G(\vec{x}) \log(1 - D_w(\vec{x})) dx \end{aligned} \tag{4}$$

where we can do the integral because of continuity and $p_z(Z)$ is mapped to $p_G(X)$.

By Euler-Lagrange formalism, quantity (4) satisfy the form $S(D) = \int_x L(x, D, D')dx$, so that we have the equality with stationary D:

$$\frac{\partial L(x, D, D')}{\partial D} - \frac{d}{dx} \frac{\partial L(x, D, D')}{\partial D'} = 0 \tag{5}$$

since in our case, $L(x, D, D')$ does not depend on D', the second part in the above equation can be ignored. We then can find the **optimal discriminator** D^* as:

$$\begin{aligned} \frac{\partial L(x, D, D')}{\partial D} &= -\frac{p_{data}}{D} + \frac{p_G}{1-D} = 0 \\ \therefore D^*(\vec{x}) &= \frac{p_{data}}{p_{data}(\vec{x}) + p_G(\vec{x})} \end{aligned} \tag{6}$$

With the optimal discriminator and the consequent criterion for generator $C(G) = C(G|D^*) = \max_w V(G, D)$, we can the optimize the generator G by minimizing $C(G)$ as:

$$\begin{aligned} C(G) &= \min_\theta \int_{\vec{x}} p_{data}(\vec{x}) \log D_w(\vec{x}) + p_G(\vec{x}) \log(1 - D_w(\vec{x}))dx \\ &= \min_\theta \int_{\vec{x}} p_{data}(\vec{x}) \log \frac{p_{data}}{p_{data}(\vec{x}) + p_G(\vec{x})} + p_G(\vec{x}) \log \frac{p_G}{p_{data}(\vec{x}) + p_G(\vec{x})} dx \\ &= \min_\theta \int_{\vec{x}} p_{data}(\vec{x}) (\log \frac{p_{data}}{\frac{p_{data}(\vec{x}) + p_G(\vec{x}))}{2}} - \log 2) + p_G(\vec{x}) (\log \frac{p_G}{\frac{p_{data}(\vec{x}) + p_G(\vec{x}))}{2}} - \log 2) dx \\ &= \min_\theta KL(p_{data} || \frac{p_G + p_{data}}{2}) + KL(p_G || \frac{p_G + p_{data}}{2}) - \log 2 \int_{\vec{x}} p_{data}(\vec{x}) - \log 2 \int_{\vec{x}} p_G(\vec{x}) \\ &= \min_\theta 2 \Big(\frac{1}{2} KL(p_{data} || \frac{p_G + p_{data}}{2}) + \frac{1}{2} KL(p_G || \frac{p_G + p_{data}}{2}) \Big) - 2 \log(2) \\ &= \min_\theta 2 JSD(p_{data}, p_G) - \log(4) \end{aligned} \tag{7}$$

where KD is Kullback-Leibler divergence and JSD is Jensen-Shannon divergence:

$$KL(P||Q) = \int_{-\infty}^{\infty} p(x) \log \Big(\frac{p(x)}{q(x)} \Big) dx \tag{8}$$

$$JSD(P||Q) = \frac{1}{2} KL(P||M) + \frac{1}{2} KL(Q||M), \text{ where } M = \frac{1}{2}(P + Q) \tag{9}$$

Since the Jensen-Shannon divergence between two distribution is always non-negative and zero only when the two distribution is equal, we prove that $C* = -\log 4$ when and only if $p_G = pdata$, which is to say, the generator perfectly models the real data's distribution. Hence, we show that the loss function mathematically work to achieve the goal of GAN.

3.2 Optimize loss function

The next question will be how we can optimize the loss function (3). A intuitive way to do it would be stochastic gradient descent:

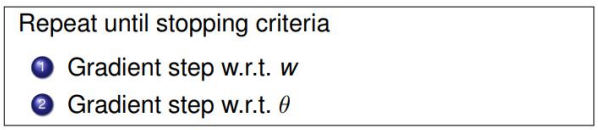

Figure 3: SGD of GAN

However, in practice, there are some problems with SGD with GAN directly and heuristics would work better. For example, if we get discriminator D trained pretty well but generator G is very

bad, we will nearly get no gradient, if we try to improve optimization of:

$$\begin{aligned} C(G) &= \max_{\theta} V(D,G) \\ &= \boxed{\max_{\theta}} \min_{w} -\sum_{\vec{x}} \log D_w(\vec{x}) - \boxed{\sum_{\vec{z}} \log(1 - D_w(G_\theta(\vec{z})))} \end{aligned} \tag{10}$$

because the value of $V(D,G)$ is already small enough and thus we could not get large change in y even with a large x on a negative log function as shown in figure 4. To avoid it, we can play a little

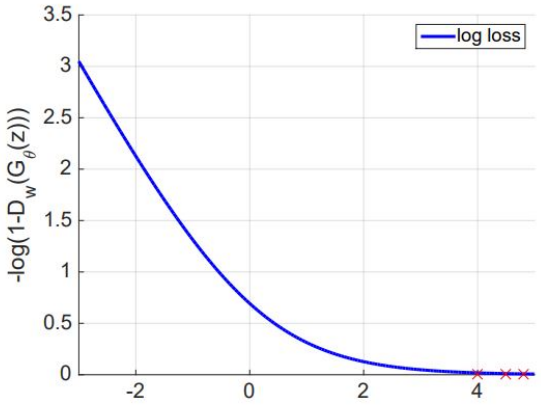

Figure 4: Example when GAN not working well

trick that instead of try to optimize $V(D,G)$ as (10) jointly, to optimize G, we do:

$$\min_{\theta} -_w(G_\theta(\vec{z})) \tag{11}$$

In this way, we flip the log curve horizontally, so that we could get enough gradient change every time to approach the optimal as shown in figure 5. Though in this way, the proof we just showed in the previous is no longer held, it works well in practice.

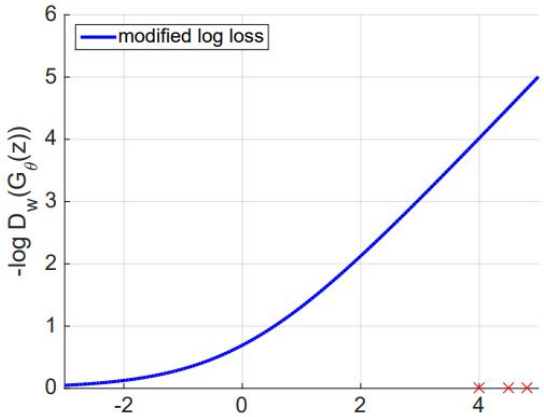

Figure 5: The modified loss to address the above problem

4 Evaluation of GAN

Here we are going to show some generated images output out of GAN model and its variance model. GAN is currently a very hot topic in Computer Vision field and many interesting research show up every day. There seemingly has not been a general way to quantitatively evaluate the generation solutions, so only the quality results are showing here in figures 6. 7, 8 and 9.

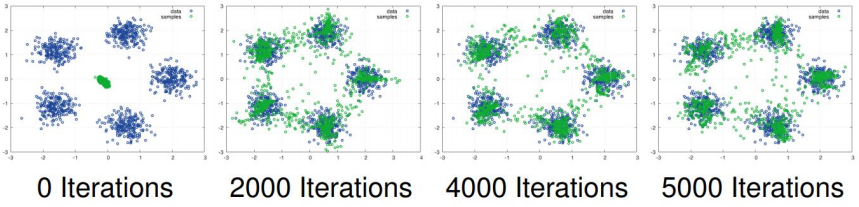

Figure 6: This is simulation of distribution modeling. The blue dots are from real set and green dots are generated. As shown, the more iterations were trained, the learned distribution is closer to the real distribution.

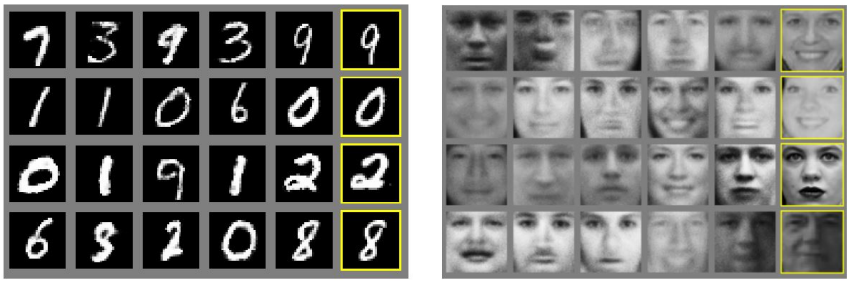

Figure 7: Outputs from GAN [2]. The rightmost column shows the nearest training sample, which can demonstrate the model is not memorizing the training set.

5 Wasserstein GAN

a variation of GAN called Wasserstein GAN [1] was introduced in 2017.

5.1 Wasserstein distance

Wasserstein distance between two distributions μ and υ is measured as:

$$W(\mu, \upsilon) = \inf_{\gamma \in \prod(\mu, \upsilon)} \mathbb{E}_{X,G}\Big[||X - G||_1\Big] \tag{12}$$

Wasserstein distance has the following properties:

- Given $G \sim \upsilon$ and $X \sim \mu$, if $W(\upsilon, \mu) = 0$, then $\upsilon = \mu$.

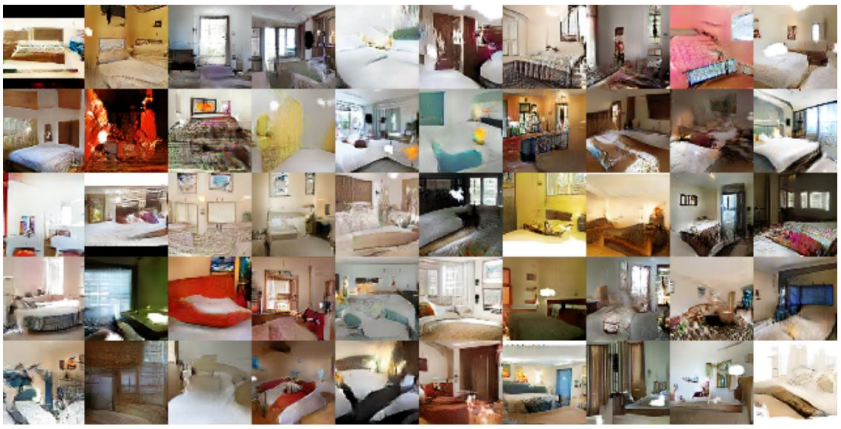

Figure 8: Outputs of a variation of GAN, DCGAN [4].

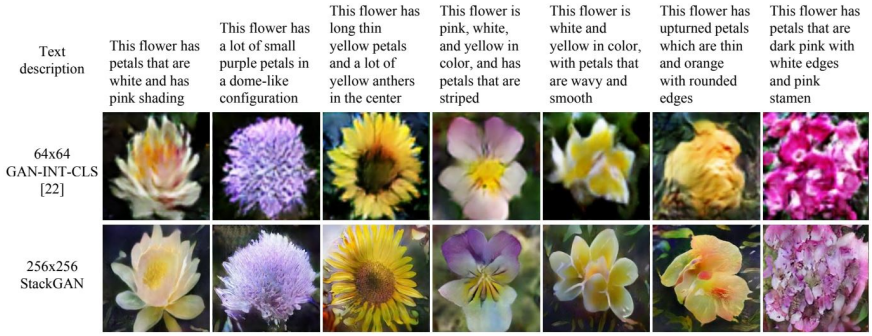

Figure 9: Example results StackGAN [8] and GAN-INT-CLS [5], which is conditioned on text description from Oxford-102 test set.

- Let $G^N \sim \upsilon^N$ and $X \sim \mu$. If $W(\mu, \upsilon^N) \to 0$ as $N \to \infty$, then $G^N \xrightarrow{d} X$, where $G^N \xrightarrow{d} X$ denotes "convergence in distribution, i.e. $\mathbb{E}\Big[f(G^N)\Big] \to \mathbb{E}\Big[f(X)\Big]$ for any continuous, bounded function f.

By the Kantorovich-Rubinstein duality,

$$W(\mu, \upsilon) = \sup_{||f||_L \leq 1} \big[\mathbb{E}_{X\sim\mu}[f(X)] - \mathbb{E}_{G\sim\upsilon}[f(G)]\big], \tag{13}$$

where $\{f : ||f||_L \leq 1\}$ is the set of functions f such that

$$||f(x) - f(x')||_1 \leq ||x - x'||_1 \tag{14}$$

That is, function f which are Lipschitz continuous with Lipschitz constant $K = 1$.

5.2 Wasserstein GAN

Wasserstein GAN uses a different metrics to measure distance between the two distribution p_{data} and p_G:

$$W(p_{data}, p_G) = \min_{p_J(x,x') \in \prod(p_{data}, p_G)} \mathbb{E}_{p_G}\Big[||x - x'||_1\Big] \tag{15}$$

where $\prod(p_{data}, p_G)$ is any probability measure with marginal p_{data} and p_G. Thus, we can formulate

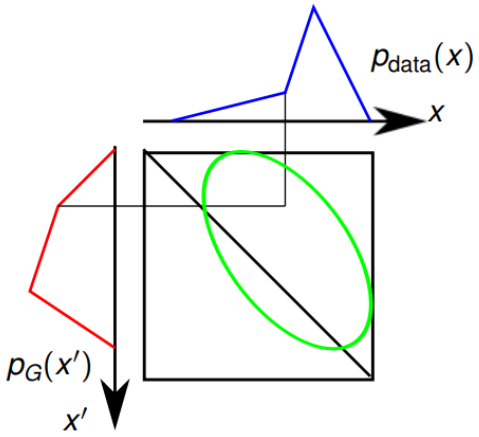

Figure 10: Wasserstein distance: the more similar the two distributions are, the closer the joint distribution is to the diagonal so that the smaller distance is.

GAN using Wasserstein distance as:

$$\min_{P_G} W(p_{data}, p_G) = \min_{P_G} \min_{p_J(x,x') \in \prod(p_{data}, p_G)} \mathbb{E}_{p_G}\Big[||X - G||_1\Big] \tag{16}$$

However, we cannot directly calculate it since p_data is not available. Instead, by Kantorovich-Rubinstein duality, we know:

$$W(p_{data}, p_G) = \max_{||f||_L \leq 1} \mathbb{E}_{p_{data}}[f(x)] - \mathbb{E}_{p_G}[f(x')] \tag{17}$$

Then, the loss becomes:

$$\min_{P_G} W(p_{data}, p_G) = \min_{P_G} \max_{||f||_L \leq 1} \mathbb{E}_{p_{data}}[f(x)] - \mathbb{E}_{p_G}[f(x')]. \tag{18}$$

Here is an example result comparing with GAN.

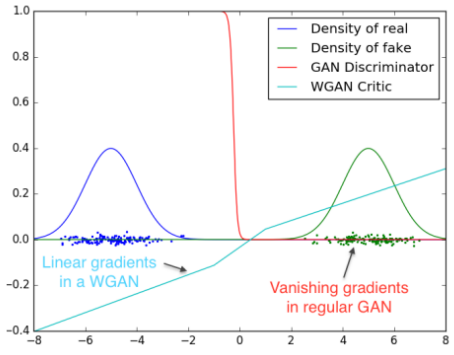

Figure 11: As shown, GAn suffers from gradient vanishing problem, but not for WGAN.

6 Quiz

- What generative modeling techniques do you know about?
  - Variational auto-encoder, GAN and RNN and so on.
- What are the short-comings of those techniques?
  - Variational Auto-encoder (VAEs): the generate images is slightly blurry. [7]
  - RNN: inefficient sampling and no low-dimensional code. [7]
- What differentiates GANs from other generative models?
  - rather than learn a given distribution, GAN tries to model the data directly.
- What are the short-comings of GANs?
  - difficult to optimize (unstable) [7]

References

[1] M. Arjovsky, S. Chintala, and L. Bottou. Wasserstein gan. *arXiv preprint arXiv:1701.07875*, 2017.

[2] I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial nets. In *Advances in neural information processing systems*, pages 2672–2680, 2014.

[3] D. P. Kingma and M. Welling. Auto-encoding variational bayes. *arXiv preprint arXiv:1312.6114*, 2013.

[4] A. Radford, L. Metz, and S. Chintala. Unsupervised representation learning with deep convolutional generative adversarial networks. *arXiv preprint arXiv:1511.06434*, 2015.

[5] S. Reed, Z. Akata, X. Yan, L. Logeswaran, B. Schiele, and H. Lee. Generative adversarial text to image synthesis. *arXiv preprint arXiv:1605.05396*, 2016.

[6] A. G. Schwing. L21: Generative adversarial nets. *ECE 544, University of Illinois at Urbana-Champaign*, 2018.

[7] A. G. Schwing. L22: Autoregressive methods (rnns/lstms/grus). *ECE 544, University of Illinois at Urbana-Champaign*, 2018.

[8] H. Zhang, T. Xu, H. Li, S. Zhang, X. Huang, X. Wang, and D. Metaxas. Stackgan: Text to photo-realistic image synthesis with stacked generative adversarial networks. *arXiv preprint*, 2017.