## 1 Nonlinear VCO response dynamics, 3rd generation prototype

Here we study a network of two mutually coupled 3rd generation PLLs taking into account the nonlinearity of the VCO's frequency response to the input control signal. The funtion we use is

$$f(x^{c}(t)) = 2\pi (a^{\text{VCO}} + b^{\text{VCO}} k \sqrt{V_{bias} + x^{c}(t)})$$

$$\tag{1}$$

where  $x^{c}(t)$  the control signal,  $a^{\text{VCO}}$  and  $b^{\text{VCO}}$  are constant parameters which depend on the architecture and hence the nonlinearities of the VCO. Note, that here  $a^{\text{VCO}}$  and  $b^{\text{VCO}}$  are not the parameters of the LF.

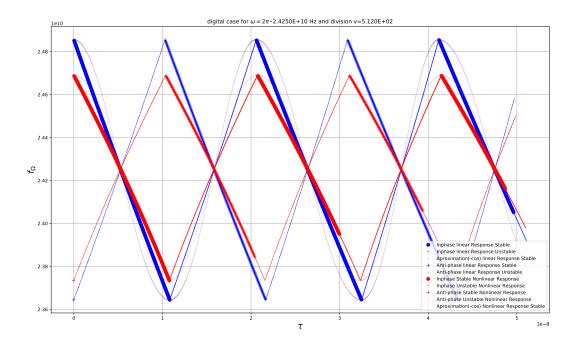


Figure 1:

In Fig. (1) we plot the frequency of synchronized states, i.e., inphase and antiphase as a function of the delay. The blue marks refer to the model with the linear function of frequency response and the red to the model with the nonlinear function. The inphase synchronized states are presented with dots while the antiphase synchronized states with crosses. Larger sized markers denote stable synchronized states.

In Fig. (2) we plot the perturbation decay rate,  $\sigma$ , for both models as a function of the delay. We find that both models have the same  $\sigma$  for frequencies close to the intrinsic frequency of the PLL. The perturbation decay rate for the nonlinear VCO response is different from that for the linear response approach. In this particular case the nonlinearity of the VCO response leads to an asymmetric perturbation decay-rate about the intrinsic frequency of the PLL. This means that the nonlinearity of the VCO affects the stability of the synchronized states. So the perturbation decay of a state with a frequency higher than the intrinsic frequency of the PLL is faster than that with a frequency that is smaller then the intrinsic PLL frequency. This means that synchronized states with frequencies smaller than the intrinsic PLL frequency decay perturbations more slowly then those obtained assuming a linear VCO response.

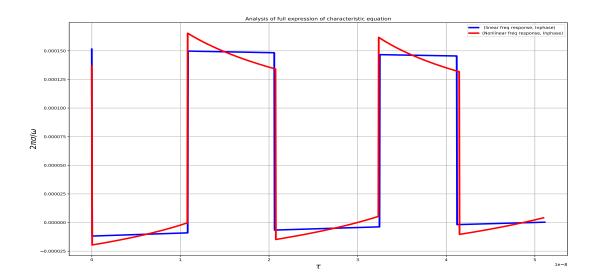


Figure 2: