本推导过程中使用的公式为化简前的公式，与程序中保持一致。
程序中的公式为：

$$\boldsymbol{a}=\left(\boldsymbol{I}-\boldsymbol{S}_a\right)\boldsymbol{K}_a^{'}\left(\boldsymbol{A}-\boldsymbol{b}_a\right)$$

为了表示方便，将下角标a统一略去：

$$\boldsymbol{a}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\boldsymbol{K}^{'}\left(\boldsymbol{A}-\boldsymbol{b}\right)$$

为了使$\boldsymbol{I}-\boldsymbol{S}$与程序中$\boldsymbol{T}$的意义相同，则$\boldsymbol{S}$的定义如下

$$\boldsymbol{S}=\begin{bmatrix} 1 & 0 & 0 \\ \text{-}S_{yx} & 1 & 0 \\ S_{zx} & \text{-}S_{zy} & 1 \end{bmatrix}$$

$\boldsymbol{a}$展开为矩阵形式：

$$\boldsymbol{a}=\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}=\begin{bmatrix} \left(A_x-b_x\right)K_x^{'} \\ \left(A_x-b_x\right)K_x^{'}S_{yx}+\left(A_y-b_y\right)K_y^{'} \\ -\left(A_x-b_x\right)K_x^{'}S_{zx}+\left(A_y-b_y\right)K_y^{'}S_{zy}+\left(A_z-b_z\right)K_z^{'} \end{bmatrix}$$

残差函数为：

$$\begin{aligned} f&=\|\boldsymbol{g}\|^2-\|\boldsymbol{a}\|^2 \\ &=g_0^2-\left(a_x^2+a_y^2+a_z^2\right) \end{aligned}$$

将矩阵$\boldsymbol{K}^{'}$中的三个非零元素$K_x^{'}$，$K_y^{'}$和$K_z^{'}$作为状态量，解出之后再间接求$\boldsymbol{K}$。

根据链式法则，残差关于九个自变量的偏导数分别为：

$$\frac{\partial f}{\partial S_{yx}}=\frac{\partial f}{\partial a_y}\frac{\partial a_y}{\partial S_{yx}}$$

$$\frac{\partial f}{\partial S_{zx}}=\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial S_{zx}}$$

$$\frac{\partial f}{\partial S_{zy}}=\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial S_{zy}}$$

$$\frac{\partial f}{\partial K_x^{'}}=\frac{\partial f}{\partial a_x}\frac{\partial a_x}{\partial K_x^{'}}+\frac{\partial f}{\partial a_y}\frac{\partial a_y}{\partial K_x^{'}}+\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial K_x^{'}}$$

$$\frac{\partial f}{\partial K_y^{'}}=\frac{\partial f}{\partial a_y}\frac{\partial a_y}{\partial K_y^{'}}+\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial K_y^{'}}$$

$$\frac{\partial f}{\partial K_z^{'}}=\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial K_z^{'}}$$

$$\frac{\partial f}{\partial b_x}=\frac{\partial f}{\partial a_x}\frac{\partial a_x}{\partial b_x}+\frac{\partial f}{\partial a_y}\frac{\partial a_y}{\partial b_x}+\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial b_x}$$

$$\frac{\partial f}{\partial b_y}=\frac{\partial f}{\partial a_y}\frac{\partial a_y}{\partial b_y}+\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial b_y}$$

$$\frac{\partial f}{\partial b_z}=\frac{\partial f}{\partial a_z}\frac{\partial a_z}{\partial b_z}$$

其中涉及到的偏导数为：

$$\frac{\partial f}{\partial a_x}=-2a_x$$

$$\frac{\partial f}{\partial a_y}=-2a_y$$

$$\frac{\partial f}{\partial a_z}=-2a_z$$

$$\frac{\partial a_x}{\partial K_x^{'}}=A_x-b_x$$

$$\frac{\partial a_x}{\partial b_x}=-K_x^{'}$$

$$\frac{\partial a_y}{\partial S_{yx}}=\left(A_x-b_x\right)K_x^{'}$$

$$\frac{\partial a_y}{\partial K_x^{'}}=\left(A_x-b_x\right)S_{yx}$$

$$\frac{\partial a_y}{\partial K_y^{'}}=A_y-b_y$$

$$\frac{\partial a_y}{\partial b_x}=-K_x^{'}S_{yx}$$

$$\frac{\partial a_y}{\partial b_y}=-K_y^{'}$$

$$\frac{\partial a_z}{\partial S_{zx}} = -\left(A_x - b_x\right)K_x^{'}$$

$$\frac{\partial a_z}{\partial S_{zy}} = \left(A_y - b_y\right)K_y^{'}$$

$$\frac{\partial a_z}{\partial K_x^{'}} = -\left(A_x - b_x\right)S_{zx}$$

$$\frac{\partial a_z}{\partial K_y^{'}} = \left(A_y - b_y\right)S_{zy}$$

$$\frac{\partial a_z}{\partial K_z^{'}} = A_z - b_z$$

$$\frac{\partial a_z}{\partial b_x} = K_x^{'}S_{zx}$$

$$\frac{\partial a_z}{\partial b_y} = -K_y^{'}S_{zy}$$

$$\frac{\partial a_z}{\partial b_z} = -K_z^{'}$$