首先说明，公式推导过程中矩阵相乘时的维度是自洽的，不再显示书写“矩阵转置”。表达式的具体实现体现在程序中。

先推导面特征残差。
课件中的面特征残差为：

$$d_H = \left| \left(\tilde{p}_i - p_j \right) \bullet \frac{\left(p_l - p_j \right) \times \left(p_m - p_j \right)}{\left| \left(p_l - p_j \right) \times \left(p_m - p_j \right) \right|} \right|$$

面特征雅克比为：

$$J_H = \frac{\partial d_H}{\partial T} = \frac{\partial d_H}{\partial \tilde{p}_i} \frac{\partial \tilde{p}_i}{\partial T}$$

令：

$$X = \left(\tilde{p}_i - p_j \right) \bullet \frac{\left(p_l - p_j \right) \times \left(p_m - p_j \right)}{\left| \left(p_l - p_j \right) \times \left(p_m - p_j \right) \right|}$$

则：

$$\frac{\partial d_H}{\partial \tilde{p}_i} = \frac{\partial |X|}{\partial \tilde{p}_i} = \frac{\partial |X|}{\partial X} \frac{\partial X}{\partial \tilde{p}_i} = \frac{X}{|X|} \frac{\partial X}{\partial \tilde{p}_i}$$

其中：

$$\frac{\partial X}{\partial \tilde{p}_i} = \frac{\left(p_l - p_j \right) \times \left(p_m - p_j \right)}{\left| \left(p_l - p_j \right) \times \left(p_m - p_j \right) \right|}$$

根据《视觉 SLAM 十四讲》有：

$$\frac{\partial \tilde{p}_i}{\partial T} = \left[-\left(R p_i + t \right)^{\wedge} \quad I \right]$$

联合上面的公式可得：

$$\begin{aligned} J_H &= \frac{\partial d_H}{\partial T} = \frac{\partial d_H}{\partial \tilde{p}_i} \frac{\partial \tilde{p}_i}{\partial T} \\ &= \frac{X}{|X|} \frac{\partial X}{\partial \tilde{p}_i} \frac{\partial \tilde{p}_i}{\partial T} \\ &= \frac{X}{|X|} \frac{\left(p_l - p_j \right) \times \left(p_m - p_j \right)}{\left| \left(p_l - p_j \right) \times \left(p_m - p_j \right) \right|} \left[-\left(R p_i + t \right)^{\wedge} \quad I \right] \end{aligned}$$

作业中的面特征残差为：

$$d_H = \left(\tilde{p}_i - p_j \right) \bullet \frac{\left(p_l - p_j \right) \times \left(p_m - p_j \right)}{\left| \left(p_l - p_j \right) \times \left(p_m - p_j \right) \right|}$$

通过对比可知，只需要去掉链式法则中的第一项即可，结果为：

$$J_H = \frac{(p_l - p_j)\times(p_m - p_j)}{\left|(p_l - p_j)\times(p_m - p_j)\right|}\left[-(Rp_i + t)^\wedge \quad I \right]$$

下面推导线特征。
作业中的线特征残差为：

$$d_\varepsilon = \frac{\left|(\tilde{p}_i - p_b)\times(\tilde{p}_i - p_b)\right|}{\left|p_a - p_b\right|}$$

参考面特征的推导过程，只需要在课件的结果中加上与“取模”相关的项即可，令：

$$Y = \frac{(\tilde{p}_i - p_b)\times(\tilde{p}_i - p_b)}{\left|p_a - p_b\right|}$$

结果为：

$$J_\varepsilon = \frac{Y}{|Y|}\frac{(p_a - p_b)^\wedge}{\left|p_a - p_b\right|}\left[-(Rp_i + t)^\wedge \quad I \right]$$