1. Derivation of State Graphs and Tables

（1）Designate the circuit states

S_0——a reset state, If a 0 input is received.

S_1——a 1 is received.

S_2——11 is received.

S_3——110 is received, and the output Z=1.

（2）Analyze transition of the states

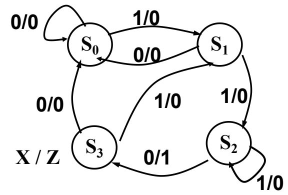
S_0 (0): X=0 ⟶ S_0; X=1 ⟶ S_1

S_1 (1): X=1 ⟶ S_2; X=0 ⟶ S_0

S_2 (11): X=1 ⟶ S_2; X=0 ⟶ S_3

S_3 (110, Z=1): X=0 ⟶ S_0; X=1 ⟶ S_1

（3） State Graphs



（4） State Tables

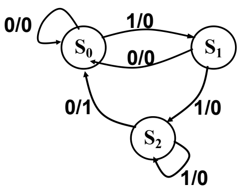
Y_n \ X	Y_{n+1} / Z: 0	1
S_0	S_0 / 0	S_1 / 0
S_1	S_0 / 0	S_2 / 0
S_2	S_3 / 1	S_2 / 0
S_3	S_0 / 0	S_1 / 0

2. Reduction of State Tables.

Y_n \ X	Y_{n+1} / Z: 0	1	
S_0	S_0 / 0	S_1 / 0	√
S_1	S_0 / 0	S_2 / 0	
S_2	S_3 / 1	S_2 / 0	
S_3	S_0 / 0	S_1 / 0	√

⟹

Y_n \ X	Y_{n+1} / Z: 0	1
S_0	S_0 / 0	S_1 / 0
S_1	S_0 / 0	S_2 / 0
S_2	S_0 / 1	S_2 / 0



3. State Assignment

Use 2 flip-flops

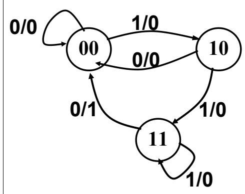
y2y1

S_0 —— 00

S_1 —— 10

S_2 —— 11

4. Form the transition table



X Y_2 Y_1	Y_2^{n+1} Y_1^{n+1}	J_2 K_2 J_1 K_1	Z
0 0 0	0 0	0 × 0 ×	0
0 1 1	0 0	× 1 × 1	1
0 1 0	0 0	× 1 0 ×	0
1 0 0	1 0	1 × 0 ×	0
1 1 1	1 1	× 0 × 0	0
1 1 0	1 1	× 0 1 ×	0
0 0 1	× ×	× × × ×	×
1 0 1	× ×	× × × ×	×

5. Derive the flip-flop input equations and the output functions using K. maps.

J_2:

X \ Y_2Y_1	00	01	11	10
0	0	×	×	×
1	1	×	×	×

$J_2 = X$

K_2:

X \ Y_2Y_1	00	01	11	10
0	×	×	1	1
1	×	×	0	0

$K_2 = \bar{X}$

X Y_2 Y_1	Y_2^{n+1} Y_1^{n+1}	J_2 K_2 J_1 K_1	Z
0 0 0	0 0	0 × 0 ×	0
0 1 1	0 0	× 1 × 1	1
0 1 0	0 0	× 1 0 ×	0
1 0 0	1 0	1 × 0 ×	0
1 1 1	1 1	× 0 × 0	0
1 1 0	1 1	× 0 1 ×	0
0 0 1	× ×	× × × ×	×
1 0 1	× ×	× × × ×	×

J_1:

X \ Y_2Y_1	00	01	11	10
0	0	×	×	0
1	0	×	×	1

$J_1 = XY_2$

K_1:

X \ Y_2Y_1	00	01	11	10
0	×	×	1	×
1	×	×	0	×

$K_1 = \bar{X}$

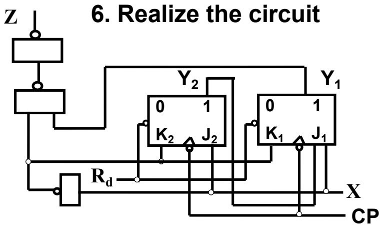
Z:

X \ Y_2Y_1	00	01	11	10
0	0	×	1	0
1	0	×	0	0

$Z = \bar{X}Y_1$

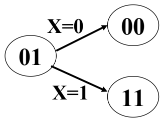
X Y_2 Y_1	Y_2^{n+1} Y_1^{n+1}	J_2 K_2 J_1 K_1	Z
0 0 0	0 0	0 × 0 ×	0
0 1 1	0 0	× 1 × 1	1
0 1 0	0 0	× 1 0 ×	0
1 0 0	1 0	1 × 0 ×	0
1 1 1	1 1	× 0 × 0	0
1 1 0	1 1	× 0 1 ×	0
0 0 1	× ×	× × × ×	×
1 0 1	× ×	× × × ×	×

6. Realize the circuit



7. Check all the don't-care states

$$Y_1^{n+1} = XY_2\bar{Y}_1 + XY_1$$
$$= X(Y_1+Y_2)$$
$$Y_2^{n+1} = X\bar{Y}_2 + XY_2$$
$$= X$$



1. Derivation of State Graphs and Tables

Analyze

1. Reusable:

S_0 (0): X=0 ⟶ S_0; X=1 ⟶ S_1

S_1 (1): X=0 ⟶ S_2; X=1 ⟶ S_1

S_2 (10): X=0 ⟶ S_0; X=1 ⟶ S_3

S_3 (101) (Z=1): X=0 ⟶ S_2; X=1 ⟶ S_1

2. Nonreusable:

S_0 (0): X=0 ⟶ S_0; X=1 ⟶ S_1

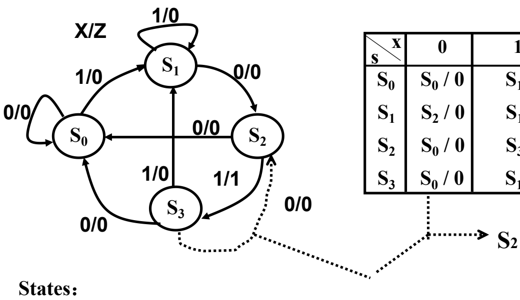
S_1 (1): X=0 ⟶ S_2; X=1 ⟶ S_1

S_2 (10): X=0 ⟶ S_0; X=1 ⟶ S_3

S_3 (101) (Z=1): X=0 ⟶ S_0; X=1 ⟶ S_1

States:
S_0——0 , S_1—— 1
S_2—— 10 , S_3—— 101 , and Z=1

Nonreusable



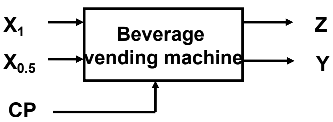
s \ x	0	1
S_0	S_0 / 0	S_1 / 0
S_1	S_2 / 0	S_1 / 0
S_2	S_0 / 0	S_3 / 1
S_3	S_0 / 0	S_1 / 0

States:
S_0——0 , S_1—— 1
S_2—— 10 , S_3—— 101 , and Z=1

Beverage vending machines

- Accept coins only: **0.5￥ , 1￥**
- Accept only one coin each time
- If **1.5** ￥ coins are inserted into the slot , you will get a bottle of drink
- If **2.0** ￥ coins are inserted into the slot , you will get a bottle of drink and 0.5￥ change

Beverage vending machines



X_1 = 1: insert a coin (1￥)
X_1 = 0: don't insert a coin (1￥)
$X_{0.5}$ = 1: insert a coin (0.5￥)
$X_{0.5}$ = 0: don't insert a coin (0.5￥)
Y=1: get a bottle of drink
Y=0: don't get a bottle of drink
Z=1: get a change (0.5 ￥coin)
Z=0: don't get a change (0.5 ￥coin)

Beverage vending machines

1. Derivation of State Graphs and Tables

① Designate the circuit states

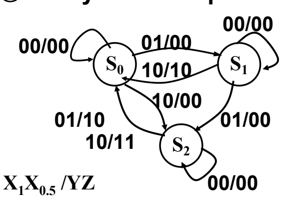
Solution 1:
Mealy circuit

S_0—a reset state, no coin is inserted.
S_1—0.5 ￥ is received.
S_2—1.0 ￥ is received(2 coins of 0.5 ￥, or a coin of 1.0 ￥)
If a 0.5 ￥coin is inserted,
then Y=1, Z=0, state go to S_0
Else If a 1.0 ￥coin is inserted,
then Y=1, Z=1, and state go to S_0

② Analyze

S_0:
- $X_1X_{0.5}=00 \rightarrow S_0$
- $X_1X_{0.5}=01 \rightarrow S_1$:
  - $X_1X_{0.5}=00 \rightarrow S_1$
  - $X_1X_{0.5}=01 \rightarrow S_2$
  - $X_1X_{0.5}=10 \rightarrow S_0, Y=1, Z=0$
- $X_1X_{0.5}=10 \rightarrow S_2$:
  - $X_1X_{0.5}=00 \rightarrow S_2$
  - $X_1X_{0.5}=01 \rightarrow S_0, Y=1, Z=0$
  - $X_1X_{0.5}=10 \rightarrow S_0, Y=1, Z=1$

③ Mealy State Graph



$X_1X_{0.5}$ /YZ

④ State Table

	S_{n+1} / YZ			
S_n \ $X_1X_{0.5}$	0 0	0 1	1 0	1 1
S_0	S_0 / 00	S_1 / 00	S_2 / 00	×/××
S_1	S_1 / 00	S_2 / 00	S_0 / 10	×/××
S_2	S_2 / 00	S_0 / 10	S_0 / 11	×/××

2. Reduction of State Table

3. State Assignment

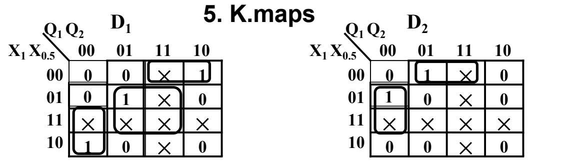
S_0 —— 00
S_1 —— 01
S_2 —— 10

∵ $N=3$, $2^2>3$
∴ $k=2$
Need 2 flip flops

4. transition table

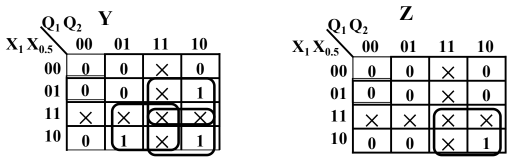
$X_1X_{0.5}$	Q_1Q_2	$Q_1^{n+1}\,Q_2^{n+1}$	Y Z	D_1 D_2
0 0	0 0	0 0	0 0	0 0
0 0	0 1	0 1	0 0	0 1
0 0	1 0	1 0	0 0	1 0
0 0	1 1	× ×	××	××
0 1	0 0	0 1	0 0	0 1
0 1	0 1	1 0	0 0	1 0
0 1	1 0	0 0	1 0	0 0
0 1	1 1	× ×	× ×	××
1 0	0 0	1 0	0 0	1 0
1 0	0 1	0 0	1 0	0 0
1 0	1 0	0 0	1 1	0 0
1 0	1 1	× ×	××	××
1 1	0 0	× ×	××	××
1 1	0 1	× ×	××	××
1 1	1 0	× ×	××	××
1 1	1 1	× ×	××	××

5. K.maps



$D_1=\bar{X}_1\bar{X}_{0.5}Q_1+Q_2X_{0.5}+X_1\bar{Q}_1\bar{Q}_2$

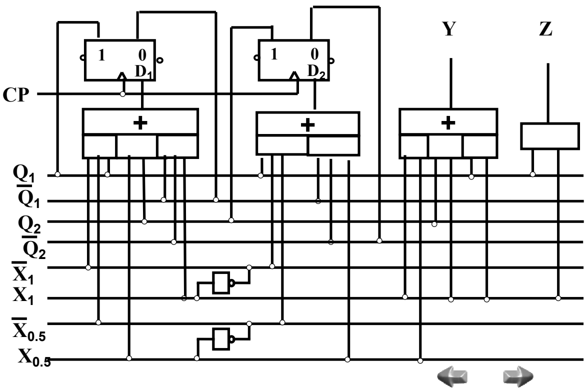
$D_2=\bar{X}_1\bar{X}_{0.5}Q_1+X_{0.5}\bar{Q}_1\bar{Q}_2$



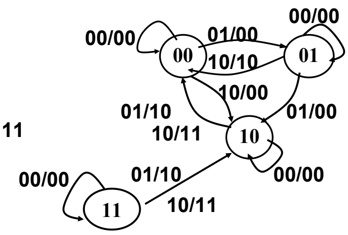
$Y=X_1X_{0.5}+Q_2X_1+X_1Q_1$

$Z=X_1Q_1$

6. Realize



7. Recheck



Don't care state $Q_1Q_2=11$

$X_1X_{0.5}=00$:

$Q_1^{n+1}=D_1=\bar{X}_1\bar{X}_{0.5}Q_1+Q_2X_{0.5}+X_1\bar{Q}_1\bar{Q}_2$

$Q_2^{n+1}=D_2=\bar{X}_1\bar{X}_{0.5}Q_2+X_{0.5}\bar{Q}_1\bar{Q}_2$

$\rightarrow$ 11 Non-Self-Starting

$X_1X_{0.5}=01$ (10): $\rightarrow$ 10 Self-Starting, but error in charging

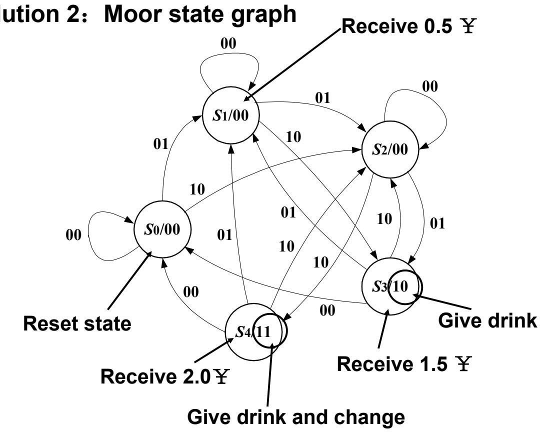
! Reset circuit with R_D at beginning

Moor Circuit

Solution 2：

- **Output:** Z_1 =1→get a bottle of drink, Z_2 =1→change
  all possible output Z_1Z_2 = 00, 10, 11
- **Input：** X_1X_2 = 00—— 0￥;
  X_1X_2 = 01——0.5 ￥;
  X_1X_2 = 10——1.0 ￥。
- **State：remember the money received**
  S_0=0 ￥, S_1 =0.5 ￥, S_2 =1.0￥
  S_3 =1.5 ￥, S_4 =2.0 ￥

Solution 2：Moor state graph



Moor state table

Present state	the next state $X_1X_2=00$	$X_1X_2=01$	$X_1X_2=10$	output Z_1Z_2
S_0	S_0	S_1	S_2	00
S_1	S_1	S_2	S_3	00
S_2	S_2	S_3	S_4	00
S_3	S_0	S_1	S_2	10
S_4	S_0	S_1	S_2	11

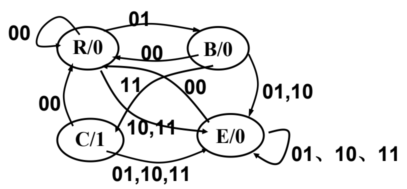
Design a Sequential Lock with JK FF

- Inputs: X_1X_2，Output: Z
  States: R、B、C、E
- Input 00 ⟶ 01 ⟶11 Sequentially from X_1X_2 state of the lock will be changed from **R** to **B** and then to **C**，and Z=1（unlock）
- If the order is not above mentioned, state of the lock will be **E**（error）
- state of the lock will be reset to **R** whenever 00 are inputted from X_1X_2

1. Derivation of State Graphs and Tables

R——receive 00
B——receive 01 after 00,
C——receive 11 after 00 , 01,
Z =1（unlock）
E——error

S \ X_1X_2	00	01	11	10	Z
R	R	B	E	E	0
B	R	E	C	E	0
C	R	E	E	E	1
E	R	E	E	E	0



Design a Sequential Lock with JK FF

2. Reduction of State Table

3. State Assignment

R —— 00 、 B —— 01
C —— 11 、 E —— 10

∵ N=4, 2^2 = 4
∴ k = 2 flip flops

4. transition table

X_1X_2	Y_1Y_2	Y_1^{n+1} Y_2^{n+1}	J_1 K_1	J_2 K_2	Z
0 0	0 0	0 0	0 ×	0 ×	0
0 0	0 1	0 0	0 ×	× 1	0
0 0	1 0	0 0	× 1	0 ×	0
0 0	1 1	0 0	× 1	× 1	1
0 1	0 0	0 1	0 ×	1 ×	0
0 1	0 1	1 0	1×	× 1	0
0 1	1 0	1 0	× 0	0 ×	0
0 1	1 1	1 0	× 0	× 1	1
1 0	0 0	1 0	1 ×	0 ×	0
1 0	0 1	1 0	1 ×	× 1	0
1 0	1 0	1 0	× 0	0 ×	0
1 0	1 1	1 0	× 0	× 1	1
1 1	0 0	1 0	1 ×	0 ×	0
1 1	0 1	1 1	1 ×	× 0	0
1 1	1 0	1 0	× 0	0 ×	0
1 1	1 1	1 0	× 0	× 1	1

X_1X_2 \ Y_1Y_2	00	01	11	10
00	0	0	×	×
01	0	1	×	×
11	1	1	×	×
10	1	1	×	×

$J_1 = X_1 + X_2Y_2$

X_1X_2 \ Y_1Y_2	00	01	11	10
00	×	×	1	1
01	×	×	0	0
11	×	×	0	0
10	×	×	0	0

$K_1 = \bar{X}_1\bar{X}_2$

J_2

X_1X_2 \ Y_1Y_2	00	01	11	10
00	0	×	×	0
01	1	×	×	0
11	0	×	×	0
10	0	×	×	0

$J_2 = \bar{X}_1X_2\bar{Y}_1$

K_2

X_1X_2 \ Y_1Y_2	00	01	11	10
00	×	1	1	×
01	×	1	1	×
11	×	0	1	×
10	×	1	1	×

$K_2 = \bar{X}_2 + \bar{X}_1 + Y_1$

Design a Sequential Lock with JK FF

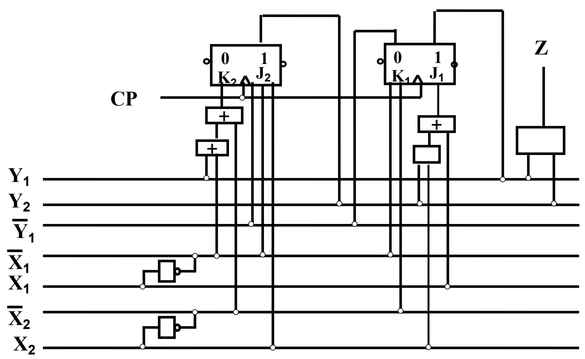
J_1

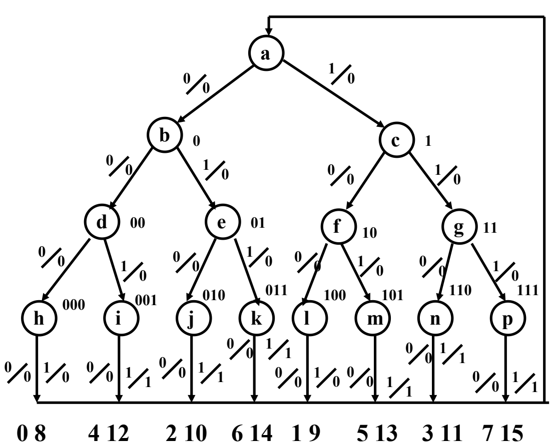
X_1X_2 \ Y_1Y_2	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$Z = Y_1Y_2$

$$\begin{cases} J_1 = X_1 + X_2Y_2 \\ K_1 = \bar{X}_1\bar{X}_2 \\ J_2 = \bar{X}_1X_2\bar{Y}_1 \\ K_2 = \bar{X}_2 + \bar{X}_1 + Y_1 \\ Z = Y_1Y_2 \end{cases}$$

5. Realize





Y_N \ X	Y_{N+1}/Z 0	1	Y_N \ X	Y_{N+1}/Z 0	1
A	B/0	C/0	~~J~~	~~A/0~~	~~A/1~~
B	D/0	E/0	~~K~~	~~A/0~~	~~A/1~~
C	F/0	G/0	~~L~~	~~A/0~~	~~A/0~~
D	H/0	I/0	~~M~~	~~A/0~~	~~A/1~~
E	J/0	K/0	~~N~~	~~A/0~~	~~A/1~~
F	L/0	M/0	~~P~~	~~A/0~~	~~A/1~~
G	N/0	P/0			
~~H~~	~~A/0~~	~~A/0~~			
~~I~~	~~A/0~~	~~A/1~~			

X	0	1
A'	B'/0	B'/0
B'	C'/0	D'/0
C'	E'/0	F'/0
D'	F'/0	F'/0
E'	A'/0	A'/0
F'	A'/0	A'/1

State Assignment

Guideline 1: a given input, same next state, present state code should be adjacent

(C' D') (E'F')

Guideline 1: a given present state, it's next state code should be adjacent

(C'D') (E'F')

Guideline 3: Output same, present state code should be adjacent

(A'B'C'D'E')

Code Converter

② Reduction of State Table

Time	Input Sequence Received (Least Significant Bit First)	Present State	Next State $X=0$	Next State $X=1$	Present Output (Z) $X=0$	Present Output (Z) $X=1$
t_0	reset	A	B	C	1	0
t_1	0	B	D	F	1	0
	1	C	E	G	0	1
t_2	00	D	H	L	0	1
	01	E	I	M	1	0
	10	F	J	N	1	0
	11	G	K	P	1	0
t_3	000	H	A	A	0	1
	001	I	A	A	0	1
	010	J	A	–	0	–
	011	K	A	–	0	–
	100	L	A	–	0	–
	101	M	A	–	1	–
	110	N	A	–	1	–
	111	P	A	–	1	–

Time	Present State	Next State $X=0$	Next State $X=1$	Present Output (Z) $X=0$	Present Output (Z) $X=1$
t_0	A	B	C	1	0
t_1	B	D	E	1	0
	C	E	E	0	1
t_2	D	H	H	0	1
	E	H	M	1	0
t_3	H	A	A	0	1
	M	A	–	1	–

Code Converter

④ Transition table

	$Q_1Q_2Q_3$	$Q_1^+Q_2^+Q_3^+$ $X=0$	$Q_1^+Q_2^+Q_3^+$ $X=1$	Z $X=0$	Z $X=1$
A	0 0 0	1 0 0	1 0 1	1	0
B	1 0 0	1 1 1	1 1 0	1	0
C	1 0 1	1 1 0	1 1 0	0	1
D	1 1 1	0 1 1	0 1 1	0	1
E	1 1 0	0 1 1	0 1 0	1	0
H	0 1 1	0 0 0	0 0 0	0	1
M	0 1 0	0 0 0	x x x	1	x
–	0 0 1	x x x	x x x	x	x