Reputational Communication and Private Sponsorships

Cuimin Ba & Beixi Zhou
University of Pittsburgh

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Motivation

Literature review

- Extensive literature on reputation
  - Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989, 1992, 1994), Ely & Välimäki (2003), Cripps, Mailath, and Samuelson (2004), etc
  - **Reputation & Communication**
    Benabou and Laroque (1992), Mathevet et al. (2019), Best and Quigley (2020), Özdoğan and Barlo (2021), Fudenberg, Gao, and Pei (2021)
  - This paper: manipulation on the sender's private incentives
- Recent literature on influencer marketing
  - Mitchell (2021), Fainmesser and Galeotti (2021), Pei and Mayzlin (2021)
  - This paper: crowd-out of unsponsored content

Model

Overview

- Static model of strategic communication with reputation concerns
- A single agent who endorses products or ideas
- A continuum of sponsors
- A continuum of decision makers

Setup

- Agent comes across a product (or an idea) of random quality $\boxed{\theta \in \Theta = \{H, L\}}$.
  - Assume uniform prior for simplicity, $\mu_0 = Pr(H) = 1/2$.
- Agent observes a signal about quality $\boxed{s \in S = \{h, l\}}$:

$$Pr(h|H) = Pr(\ell|L) = \lambda \in (1/2, 1),$$

where λ measures her signal informativeness.

- Agent chooses whether to send an endorsement, $\boxed{m \in M = \{E, \varnothing\}}$.
  - If $m = E$, decision makers decide whether to act on the product.
  - If $m = \varnothing$, decision makers make no decisions and do not observe which product the agent considered.

Agent's incentives (1/2)

- The agent is either strategic or honest, $\boxed{\omega \in \Omega = \{\omega_0, \omega_1\}}$.
  - An honest agent sends E if and only if she receives h.
  - A strategic agent freely chooses her communication strategy.
  - Prior distribution satisfies $Pr(\omega_0) = \pi_0$. $\leftarrow$ initial reputation
- The product may or may not be associated with a sponsorship, $\boxed{z \in Z = \{Y, N\}}$.
  - If $z = Y$ ($z = N$), the product is sponsored (unsponsored/organic).
  - Assume sponsorship is uncorrelated with quality, $\theta \perp z$.
- Write the strategic agent's strategy by $\boxed{\sigma : S \times Z \to \Delta(M)}$:

$$\sigma_{sz} = Pr(m = E|s, z) = 1 - Pr(m = \varnothing|s, z)$$

Agent's incentives (2/2)

The agent earns the following ex post payoff:

$$\underbrace{W(\pi)}_{\text{Reputation payoff}} \quad + \quad \underbrace{\mathbf{1}_{z=Y} \cdot \mathbf{1}_{m=E} \cdot R\left(\frac{\mu}{1-\mu}\right)}_{\text{sponsorship revenue}}$$

end-of-game reputation

induced posterior ratio about product

Assumption: $W : [0,1] \to \mathbb{R}_+$ is continuously differentiable, $W' > 0$.

$R : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, $R' \geq 0$ and $R(\frac{\mu}{1-\mu}) > 0$ if $\mu > 1/2$.

Micro-foundation for sponsorship R (1/2)

- R can be microfounded differently depending on the context.
- **Application 1**: agent = influencer, decision makers = buyers, sponsors = marketers
  - Buyers get 1 from a high-quality product and $-v$ from a low-quality product.
  - A buyer with v buys if $\mu - (1-\mu)v \geq 0 \Rightarrow v \leq \frac{\mu}{1-\mu}$.
  - Suppose $v \sim G \in \Delta[1, \overline{v}]$, then a fraction of $G\left(\frac{\mu}{1-\mu}\right)$ purchase the product.
  - By bargaining with the sponsor, influencer gets commission from sales with rate $c > 0$.
  - In this case, $R\left(\frac{\mu}{1-\mu}\right) = cG\left(\frac{\mu}{1-\mu}\right)$.

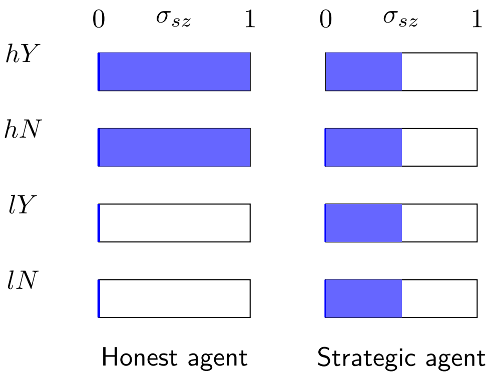
Micro-foundation for sponsorship R (2/2)

- R can be microfounded differently depending on the context.
- **Application 2**: agent $=$ expert, decision makers $=$ policymaker, sponsors $=$ donors
  - Policy maker chooses a level of policy implementation, $a \in [0, 1]$, and their payoff is maximized when $a = \mu$.
  - The donor privately offers the expert funding r if they publicly support a given policy and induce the policymaker to choose $a > 1/2$.
  - In this case, $R\left(\frac{\mu}{1-\mu}\right) = r\mathbf{1}_{\mu>1/2}$.

Timeline

1. Agent receives signal s and sponsorship z.
2. Agent decides whether to endorse by sending m.
3. Agent gets sponsorship revenue $R\left(\frac{\mu}{1-\mu}\right)$, if any.
    - Decision makers update beliefs about the endorsed product and take actions.
4. The true product quality θ is disclosed.
5. Agent's reputation is updated to π, and she gets the reputational payoff $W(\pi)$.

Inferring **quality** from endorsements



Given strategic agent's strategy σ_{sz}:

prob. of endorsing when $\theta = H$

$$q_H = \pi_0\lambda + (1-\pi_0)\sum_{s,z}\sigma_{sz}Pr(s,z|H)$$

prob. of endorsing when $\theta = L$

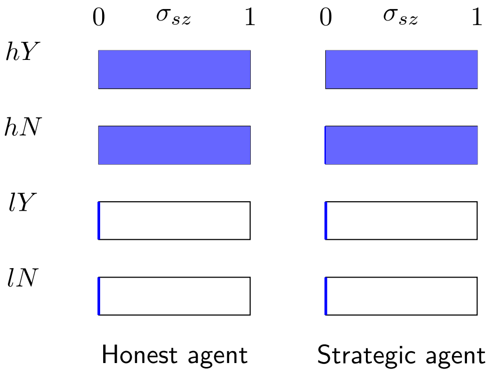
$$q_L = \pi_0(1-\lambda) + (1-\pi_0)\sum_{s,z}\sigma_{sz}Pr(s,z|L)$$

By Bayes' rule,

$$\frac{\mu}{1-\mu} = \frac{q_H}{q_L} \leftarrow \textbf{credibility}$$

Inferring **quality** from endorsements

Truthful endorsing



Suppose strategic agent truthfully reports:

$$q_H = \lambda$$

$$q_L = 1 - \lambda$$

Endorsement has maximum credibility:

$$\frac{\mu}{1-\mu} = \frac{q_H}{q_L} = \frac{\lambda}{1-\lambda}$$

Inferring **honesty** from endorsements and quality

Given strategic agent's strategy σ_{sz}:

reputation following $m = E$ and $\theta = H$

$$\pi_H = \frac{\pi_0 \lambda}{q_H}$$

reputation following $m = E$ and $\theta = L$

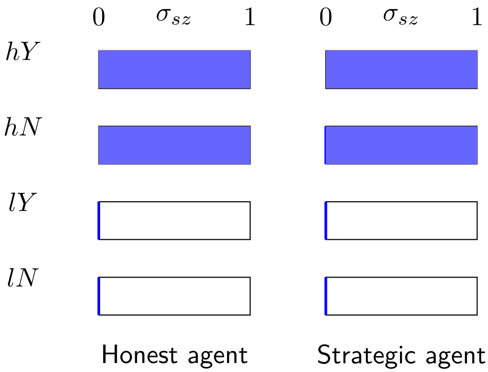
$$\pi_L = \frac{\pi_0 (1 - \lambda)}{q_L}$$

reputation following $m = \varnothing$

$$\pi_\varnothing = \frac{\pi_0}{2 - q_H - q_L}$$

Inferring **honesty** from endorsements and quality

Truthful endorsing



Given strategic agent's strategy σ_{sz}:

$$\pi_H = \pi_L = \pi_\varnothing = \pi_0$$

Reputation remains the same.

Equilibrium concept

Perfect Bayesian equilibrium $(\sigma^*, \mu^*, \pi^*_H, \pi^*_L, \pi^*_\varnothing)$

- Everything is on path.
- Endorsement strategy σ^* maximizes sender's expected payoff given expected sponsorship $R\left(\frac{\mu^*}{1-\mu^*}\right)$ and reputation $(\pi^*_H, \pi^*_L, \pi^*_\varnothing)$.
- Beliefs μ^* and $(\pi^*_H, \pi^*_L, \pi^*_\varnothing)$ are updated by Bayes' rule from q^*_H and q^*_L induced by σ^*.

Equilibrium Analysis

Benchmark Case: $\phi = 0$

Claim 1.

Suppose $\phi = 0$, then the strategic sender truthfully endorses, i.e. $\sigma_h = 1$ and $\sigma_l = 0$.

- An unsponsored agent only cares about reputation.

Benchmark Case: $\phi = 1$

Claim 2.

Suppose $\phi = 1$, the strategic sender truthfully endorses at h but over-endorses at l,

$$\sigma_h = 1 \text{ and } \sigma_l > 0.$$

- Truthful endorsement cannot be an equilibrium because

$$\pi_H = \pi_L = \pi_\varnothing$$

$$\text{yet sponsorship} \quad R\left(\frac{q_H}{q_L}\right) = R\left(\frac{\lambda}{1-\lambda}\right) > 0.$$

- In equilibrium, σ_l may $\in (0, 1)$ or $= 1$, depending on how R compares with W.

Equilibrium characterization for $\phi \in (0, 1)$

Proposition 1.

Suppose $\phi \in (0, 1)$. There exists a unique equilibrium, which satisfies the following:

- The agent truthfully endorses at hY and does not endorses at lN,

$$\sigma_{hY} = 1, \sigma_{lN} = 0.$$

- The agent over-endorses at lY but under-endorses at hN,

$$\sigma_{lY} > 0, \sigma_{hN} < 1.$$

- Two-way distortion: over-endorsement of sponsored, low signal products
  under-endorsement of unsponsored, high signal products

Are more sponsorships bad?

Are more sponsorships bad?

The answer depends on whether this means:

- ***greater*** sponsorship → Yes.
- ***more frequent*** sponsorship → Not necessarily, and sometimes the opposite.

Define a welfare measure

To talk about consumer welfare, we focus on the influencer application:

- Buyers get 1 from a high-quality product and $-v$ from a low-quality product.
- A buyer with v buys if $\frac{\mu}{1-\mu} = \frac{q_H}{q_L} > v$, and $v \sim G \in \Delta[1, \overline{v}]$.
- By bargaining with the sponsor, influencer gets commission from sales with rate $c > 0$.

Consumer surplus is defined by

$$CS := \int_{\underline{v}}^{q_H/q_L} (\mu_0 q_H - (1-\mu_0) v q_L) dG(v)$$

Greater sponsorship

Consider any commission rate $c > 0$ and sponsorship prevalence $\phi \in (0, 1)$.

Let $CS^*(c, \phi)$ denote the consumer surplus in the corresponding equilibrium.

Proposition 2.

Greater sponsorship decreases consumer surplus: given $\phi \in (0, 1)$ and $c < c'$,

$$CS^*(c, \phi) > CS^*(c', \phi).$$

Greater sponsorship

As c increases, the agent has stronger monetary incentive to endorse when sponsored

$\Rightarrow \sigma_{lY}$ increases $\Rightarrow \pi_H, \pi_L$ decreases while $\pi_\varnothing$ increases

$\Rightarrow$ the agent has stronger reputational incentive to send $\varnothing$ when unsponsored

$\Rightarrow \sigma_{hN}$ decreases

$\Rightarrow$ Both more over-endorsement and under-endorsement cause consumer surplus to fall

More frequent sponsorship

Proposition 3.

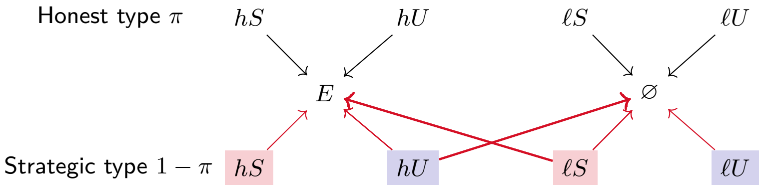
There exist $0 < \underline{\phi} \leq \overline{\phi} < 1$ such that consumer welfare $CS^*(c, \phi)$ **strictly decreases** in ϕ over $[0, \underline{\phi}]$ and **strictly increases** in ϕ over $[\overline{\phi}, 1]$.

More frequent sponsorships decreases welfare but increases later.

Insight. In a market where sponsored endorsement is already prevalent, introducing more sponsorships may be welfare improving.

Idea of Proposition 3

As ϕ increases from 0...



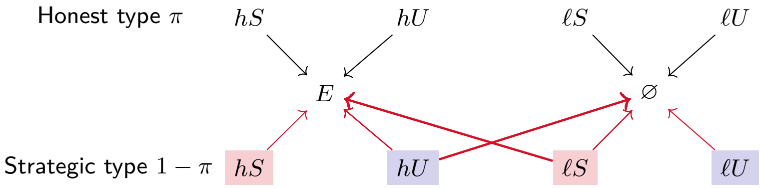
Phase (1): ℓS always endorses while hU randomizes

more sponsorships $\Rightarrow \uparrow$ over-endorsement of ℓ-products $\Rightarrow \uparrow$ reputational effects $\Rightarrow \downarrow \sigma^*_{hU}$

Insight: Matching influencers and sponsors more efficiently can increase consumer welfare

Idea of Proposition 3

As ϕ increases from 0...



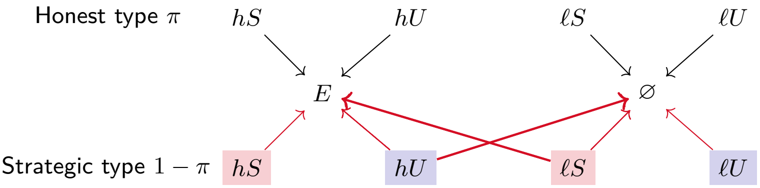
Phase (2): both ℓS and hU randomize

more sponsorships $\Rightarrow \downarrow \sigma^*_{\ell S}$ and $\downarrow \sigma^*_{hU}$ $\Rightarrow q^*_H$ and q^*_L remain exactly the same

Insight: Matching influencers and sponsors more efficiently can increase consumer welfare

Idea of Proposition 3

As ϕ increases from 0...



Phase (3): hU always stays silent while ℓS randomizes

more sponsorships $\Rightarrow \downarrow$ under-endorsement that dominates any $\uparrow$ over-endorsement

Insight: Matching influencers and sponsors more efficiently can increase consumer welfare

Disclosing sponsorships

Suppose influencer can disclose sponsorships and choose between $\{ES, EU, \varnothing\}$

- For example, the influencer may add "$\#ad$" or "$\#sponsored$" to her endorsements
- Assume also honest influencer discloses truthfully

$$\begin{tikzcd}
\text{Honest type } \pi & hS \arrow[dr] & & hU \arrow[dr] & & \ell S \arrow[dr] & & \ell U \arrow[dl] \\
& & ES & & EU & & \varnothing & \\
\text{Strategic type } 1-\pi & hS & & hU & & \ell S & & \ell U
\end{tikzcd}$$

Can transparency help?

Can transparency help?

We compare four transparency regimes:

- Voluntary disclosure
- Regulated disclosure
    - *No-False Disclosure*: U cannot claim to be sponsored
    - *Mandatory Disclosure*: S cannot claim to be unsponsored
    - *Full Disclosure*: Both U and S must disclose sponsorship status truthfully

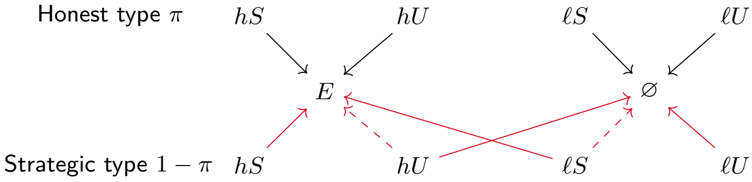
Can transparency help?

Proposition 4

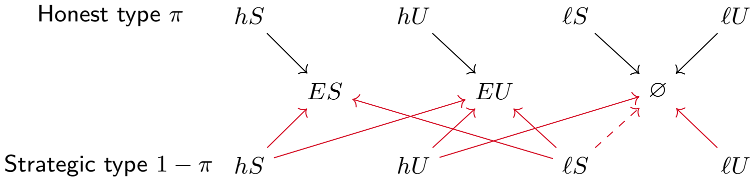
(i) Voluntary Disclosure and No False Disclosure do not meaningfully change equilibrium: same credibility for ES and EU, with consumer welfare unchanged

(ii) Mandatory Disclosure or Full Disclosure both increase consumer welfare

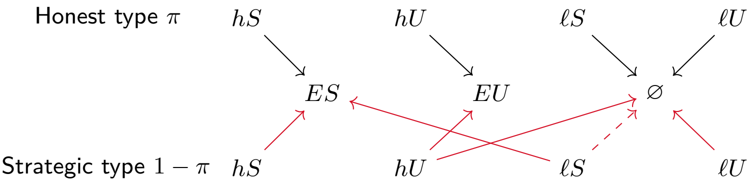
Voluntary and No-False Disclosure



Voluntary and No-False Disclosure



Mandatory and Full Disclosure



Two effects of Mandatory disclosure:

- Better information transmission: EU delivers a highly credible signal of high quality
- Better information provision: $\uparrow$ reputation costs $\Rightarrow$ $\downarrow$ **over-endorsement**

  $\Rightarrow \downarrow$ reputational effects $\Rightarrow$ $\downarrow$ **under-endorsement**

Conclusion

Final Remarks

1. Two-way distortion:
   - over-endorsement of sponsored products and under-endorsement of unsponsored products

2. Non-monotone relationship between amount of sponsorships and consumer welfare

3. Mandatory & Full Disclosure benefit consumers, Voluntary & No-False Disclosure don't