Homework1

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September 22, 2018

1 Prove $2n + \Theta(n^2) = \Theta(n^2)$

Let $f(n) \in \Theta(n^2)$, we can find c_1, c_2, n_0 such that $\forall n \geq n_0, 0 \leq c_1 n^2 \leq f(n) \leq c_2 n^2$, that is $c_1 n^2 + 2n \leq f(n) + 2n \leq c_2 n^2 + 2n$.
So we only need to prove $f(n) + 2n \in \Theta(n^2)$, that means we need to find c_3, c_4 so that $\forall n \geq n_0, 0 \leq c_3 n^2 \leq f(n) + 2n \leq c_4 n^2$.
Solve inequations: $\begin{cases} c_3 n^2 \leq c_1 n^2 + 2n \\ c_4 n^2 \geq c_2 n^2 + 2n \end{cases}$, We can find $\begin{cases} c_3 < c_1 \\ c_4 > 2/n_0 + c_2 \end{cases}$ such that $\forall n \geq n_0, 0 \leq c_3 n^2 \leq f(n) + 2n \leq c_4 n^2$.
Thus $f(n) + 2n \in \Theta(n^2)$, and $2n + \Theta(n^2) = \Theta(n^2)$.

2 Prove $\Theta(g(n)) \cap o(g(n)) = \emptyset$

Assume $\Theta(g(n)) \cap o(g(n)) \neq \emptyset$, then exists $f(n) \in \Theta(g(n))$ and $f(n) \in o(g(n))$, so $\exists c_1 > 0, c_2 > 0, n_0 > 0$ for that $\forall n \geq n_0, \forall c > 0, \begin{cases} 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \\ 0 \leq f(n) < cg(n) \end{cases}$,
let c $= \frac{c_1}{2}$, so $c_1 g(n) \leq f(n) < cg(n) = \frac{c_1}{2} g(n), c_1 < 0$.
This is a contradiction, thus $\Theta(g(n)) \cap o(g(n)) = \emptyset$.

3 Prove $\Theta(g(n)) \cup o(g(n)) \neq O(g(n))$

Let $f(n) = \begin{cases} g(n) \ when \ n \ is \ even \\ 0 \qquad otherwise \end{cases}$, $\exists c = 1, n_0 > 0, s.t. \forall n \geq n_0, 0 \leq f(n) \leq cg(n)$,so $f(n) \in O(g(n))$.
If $0 < c < 1$ and n is even number, $f(n) > cg(n)$, so $f(n) \notin o(g(n))$.
If n is not even number, f(n)=0, there is not a $c_1 > 0$ so that $0 \leq c_1 g(n) \leq f(n)$, so $f(n) \notin \Theta(g(n))$.
Thus $f(n) \notin \Theta(g(n)) \cup o(g(n))$. Thus $\Theta(g(n)) \cup o(g(n)) \neq O(g(n))$.