Extended Kalman Filter

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Linearization for non-linear system

Nonlinear system

$x_t = f(x_{t-1}, u_{t-1}, \omega_{t-1}), \omega_{t-1} \sim \mathcal{N}(0, Q)$
$z_t = h(x_t, v_t), v_t \sim \mathcal{N}(0, R)$
f and h are both nonlinear function

Note

The nonlinear mapping of a Gaussian distribution is not Gaussian

Linearization

Taylor Series

$$f(x) = f(x_0) + \frac{\partial f}{\partial x}(x - x_0)$$

The best way to choose initial point is the real-value. However, we don't have this.
We linearize the system from the posterior estimation $\hat{x}_{t-1}$ of the last timestep.

$x_t = f(\hat{x}_{t-1}, u_{t-1}, \omega_{t-1}) + A_t(x_t - \hat{x}_{t-1}) + W_t\omega_{t-1}$

Assuming $\omega_{t-1} = 0$, we get $\tilde{x}_t = f(\hat{x}_{t-1}, u_{t-1}, 0)$

$$A_t = \left(\frac{\partial f}{\partial x}\right)_{\hat{x}_{t-1}, u_{t-1}}$$

$$W_t = \left(\frac{\partial f}{\partial \omega}\right)_{\hat{x}_{t-1}, u_{t-1}}$$

$z_t = h(\tilde{x}, v_t) + H_t(x_t - \tilde{x}_t) + V_t v_t$

Linearization at $\tilde{x}_t$

Assuming $v_t = 0$, we get $\tilde{z}_t = h(\tilde{x}_t, 0)$

$$H_t = \left(\frac{\partial h}{\partial x}\right)_{\tilde{x}_t}$$

$$V_t = \left(\frac{\partial h}{\partial v}\right)_{\tilde{x}_t}$$

$\omega_t \sim \mathcal{N}(0, Q)$
$W_t\omega_t \sim \mathcal{N}(0, W_t Q W_t^\top)$
$V_t v_t \sim \mathcal{N}(0, V_t R V_t^\top)$

E.g.
$f_1 \coloneqq x_1 \leftarrow x_1 + \sin x_2$
$f_2 \coloneqq x_2 \leftarrow x_1^2$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \cos x_2 \\ 2x_1 & 0 \end{bmatrix}$$

Summary

propagation		**correction**	
prior	$\hat{x}_t^- = f(\tilde{x}_{t-1} + u_{t-1}, 0)$	Kalman gain	$K_t = (P_t^- H^\top)(HP_t^- H^\top + V_{t-1}RV_{t-1}^\top)^{-1}$
prior error covariance	$P_t^- = AP_{t-1}A^\top + W_{t-1}QW_{t-1}^\top$	posterior estimation	$\hat{x}_t = \hat{x}_t^- + K_t\big(z_t - h(\hat{x}_t^-)\big)$
		update error covariance	$P_t = (I - K_t H)P_t^-$