建模

估计罐子里的液体温度

一、估计

假设在稳定状态下液体的温度是以每秒 0. 1C 的速度加热。但是真实液体温度可能会出现一些波动，用下面的公式来描述系统**状态方程**：

$$\hat{T_t^-} = \hat{T_{t-1}} + v * \Delta t + w$$

> $w_n $是随机过程噪声，假设我们的模型是很准确的，方差为Q设为0.15
>
> 测量误差（标准差）为 0.1摄氏度,R=0.1
>
> 采样时间：每 5 秒进行一次测量
>
> ^表示估计值
>
> -表示先验值

预估噪声协方差方程有

$$P_t^- = P_{t-1} + Q$$

二、更新

观测方程泰勒展开

$$z_t = T_t + v$$

> T_t就是传感器的温度，v是更新误差

卡尔曼增益更新

$$K = P_t^- * (P_t^- + R)^{-1}$$

状态更新（修正）方程

$$\hat{T_t} = \hat{T_t^-} + K * (z_t - \hat{T_t^-})$$

更新后的后验估计协方差

$$P_t = (I - K)P_t^-$$

在测量点的真实温度是：50.479℃, 51.025℃, 51.5℃, 52.003℃, 52.494℃, 53.002℃, 53.499℃, 54.006℃, 54.498℃, and 54.991℃.

误差0.1内

温度计测量数据：z=

{50.45℃, 50.967℃, 51.6℃, 52.106℃, 52.492℃, 52.819℃, 53.433℃, 54.007℃, 54.523℃, and 54.99℃}

三、迭代

第零次迭代

在第一次迭代之前，需要初始化卡尔曼滤波器，然后预测下一个状态（也就是第一个状态）。

1.初始化

用手摸，盲猜初始化温度是

$$\hat{T}_{(0,\ 0)} = 10$$

由于没有温度计，这个猜测值是非常不精准的，于是把初始估计标准差（σ）设为 $\pm100 $。初始化的估计不确定性(Estimate Uncertainty)是预测误差的方差（自己设置的）

$$P_{0,\ 0} = \sigma^2 = 10000$$

这里方差非常高。如果我们的初始值更合理，卡尔曼滤波器会收敛更快。

2.估计

$$\hat{T}_{(1,\ 0)} = 10$$

预估噪声协方差

$$P_{1,\ 0} = P_{0,\ 0} + Q = \sigma^2 + Q = 10000.15$$

第一次迭代

1.更新

卡尔曼增益更新

$$K = P_{1,\ 0} * (P_{1,\ 0} + R)^{-1} = 0.999999$$

状态更新（修正）

$$\hat{T}_{(1,\ 1)} = \hat{T}_{(1,\ 0)} + K * (z_0 - \hat{T}_{(1,\ 0)}) = 50.45$$

更新后的后验估计协方差

$$P_{1,\ 1} = (I - K)P_t * P_{1,\ 0} = 0.01$$

2.估计

$$\hat{T}_{(2,\ 1)} = \hat{T}_{(1,\ 1)} = 50.45$$

预估噪声协方差

$$P_{2,\ 1} = P_{1,\ 1} + Q = \sigma^2 + Q = 0.16$$

第二次迭代

1.更新

卡尔曼增益更新

$$K = P_{2,\ 1} * (P_{2,\ 1} + R)^{-1} = 0.9412$$

状态更新（修正）

$$\hat{T}_{(2,\ 2)} = \hat{T}_{(2,\ 1)} + K * (z_1 - \hat{T}_{(2,\ 1)}) = 50.94$$

...

n	y_n	Current state estimates (K_n , $\hat{x}_{n,n}$, $p_{n,n}$)	Prediction ($\hat{x}_{n+1,n}$, $p_{n+1,n}$)
1	50.45^oC	$K_1 = \frac{10000.15}{10000.15 + 0.01} = 0.999999$ $\hat{x}_{1,1} = 10 + 0.999999\,(50.45 - 10) = 50.45^oC$ $p_{1,1} = (1 - 0.999999)\,0.10000.15 = 0.01$	$\hat{x}_{2,1} = \hat{x}_{3,3} = 50.45^oC$ $p_{2,1} = 0.01 + 0.15 = 0.16$
2	50.967^oC	$K_2 = \frac{0.16}{0.16 + 0.01} = 0.9412$ $\hat{x}_{2,2} = 50.45 + 0.9412\,(50.967 - 50.45) = 50.94^oC$ $p_{2,2} = (1 - 0.9412)\,0.16 = 0.0094$	$\hat{x}_{3,2} = \hat{x}_{3,3} = 50.94^oC$ $p_{3,2} = 0.0094 + 0.15 = 0.1594$
3	51.6^oC	$K_3 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{3,3} = 50.94 + 0.941\,(51.6 - 50.94) = 51.56^oC$ $p_{3,3} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{4,3} = \hat{x}_{3,3} = 51.56^oC$ $p_{4,3} = 0.0094 + 0.15 = 0.1594$
4	52.106^oC	$K_4 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{4,4} = 51.56 + 0.941\,(52.106 - 51.56) = 52.07^oC$ $p_{4,4} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{5,4} = \hat{x}_{4,4} = 52.07^oC$ $p_{5,4} = 0.0094 + 0.15 = 0.1594$
5	52.492^oC	$K_5 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{5,5} = 52.07 + 0.941\,(52.492 - 52.07) = 52.47^oC$ $p_{5,5} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{6,5} = \hat{x}_{5,5} = 52.47^oC$ $p_{6,5} = 0.0094 + 0.15 = 0.1594$
6	52.819^oC	$K_6 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{6,6} = 52.47 + 0.941\,(52.819 - 52.47) = 52.8^oC$ $p_{6,6} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{7,6} = \hat{x}_{6,6} = 52.8^oC$ $p_{7,6} = 0.0094 + 0.15 = 0.1594$
7	53.433^oC	$K_7 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{7,7} = 52.8 + 0.941\,(53.433 - 52.8) = 53.4^oC$ $p_{7,7} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{8,7} = \hat{x}_{7,7} = 53.4^oC$ $p_{8,7} = 0.0094 + 0.15 = 0.1594$
8	54.007^oC	$K_8 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{8,8} = 53.4 + 0.941\,(54.007 - 53.4) = 53.97^oC$ $p_{8,8} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{9,8} = \hat{x}_{8,8} = 53.97^oC$ $p_{9,8} = 0.0094 + 0.15 = 0.1594$

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9	54.523^oC	$K_9 = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{9,9} = 53.97 + 0.941\,(54.523 - 53.97) = 54.49^oC$ $p_{9,9} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{10,9} = \hat{x}_{9,9} = 54.49^oC$ $p_{10,9} = 0.0094 + 0.15 = 0.1594$
10	54.99^oC	$K_{10} = \frac{0.1594}{0.1594 + 0.01} = 0.941$ $\hat{x}_{10,10} = 54.49 + 0.941\,(54.99 - 54.49) = 54.96^oC$ $p_{10,10} = (1 - 0.941)\,0.1594 = 0.0094$	$\hat{x}_{11,10} = \hat{x}_{10,10} = 54.96^oC$ $p_{11,10} = 0.0094 + 0.15 = 0.1594$

下图比较了真实值、测量值及估计值：