Generating Covering Arrays with Pseudo-Boolean Constraint Solving and Balancing Heuristic

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Abstract. Covering arrays (CAs) are interesting objects in combinatorics and they also play an important role in software testing. It is a challenging task to generate small CAs automatically and efficiently. In this paper, we propose a new approach which generates a CA column by column. A kind of balancing heuristic is adopted to guide the searching procedure. At each step (column extension), some pseudo Boolean constraints are generated and solved by a PBO solver. A prototype tool is implemented, which turns out to be able to find smaller CAs than other tools, for some cases.

1 Introduction

In complex software systems, faults usually arise from the interaction of a few components/factors. It is reported that up to 90 % of the faults are caused by interactions of at most 3 factors, among which 70 % are caused by pairwise interactions [6]. Combinatorial testing is a useful black-box testing technique to reveal such faults. It usually uses the concept of Covering Arrays (CAs) [13]. Such arrays are important objects in combinatorics. A covering array of strength t is an array with property that each ordered combination of t values from different columns appears at least once in the rows. Each row corresponds to a test case.

Given parameters and coverage criteria, we would like to obtain CAs with the least number of rows, which correspond to the smallest test suites for the System Under Test (SUT). However, generating such CAs is a challenging task, which has been proved to be NP-complete. Therefore, most works on CA generation are based on heuristic search, so as to obtain a solution in reasonable time. The heuristics adopted in the search algorithms are vital to the optimality of the solutions and the efficiency of the algorithms.

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In general, search algorithms for CAs using problem-specific heuristics fall into two categories: the one-test-at-a-time strategy and its variants, and the In-Parameter-Order (IPO) family [7,8]. The one-test-at-a-time strategy is widely employed in many CA generation algorithms [1]. The basic idea is to generate test cases one by one in a greedy manner until the coverage requirement is met. During the process, each test case covers as many uncovered target combinations as possible, so as to minimize the number of test cases in the test suite. The IPO algorithm expands a CA both horizontally and vertically. It firstly initializes a small sub-array by enumerating all value combinations of the first t parameters, then adds an additional column so that as many target combinations are covered as possible, and then adds rows to cover the remaining uncovered combinations. The horizontal extension stage and the vertical extension stage is repeated alternatively until a CA is completed.

Both of the above strategies are greedy approaches, trying to cover as many target combinations as possible when expanding the array. In this paper, we propose a different heuristic called the balancing heuristic, which only imposes constraints upon each individual column. With this heuristic, our main algorithm generates CAs in the horizontal way: Each time it derives pseudo-Boolean constraints for the next column, and then employs a pseudo-Boolean constraint solver to obtain the solution of the new column. Simple as it is, our algorithm demonstrates much advantage over the state-of-the-art CA generators for instances of strength 2. It can generate smaller CAs, while the execution time is comparable to that of other solvers.

2 Preliminaries

Definition 1. A covering array $CA(N, d_1d_2 \cdots d_k, t)$ of strength t is an $N \times k$ array having the following two properties:

- 1. There are exactly d_i symbols in each column i $(1 \le i \le k)$;
- 2. In every $N \times t$ sub-array, each ordered combination of symbols from the t columns appears at least once.

Each column of the CA corresponds to a factor or parameter p_i $(1 \le i \le k)$, and d_i is called the *level* of p_i .

The parameters in a CA can be combined when their levels are the same. If every parameter has the same level, the array can be denoted by $CA(N, d^k, t)$.

	0 1 1
$0 \ 0 \ 0 \ 0$	0 0 0
0 1 1 1	1 1 0
1 0 1 1	101
1 1 0 1	2 1 1
1 1 1 0	200

Fig. 1. $CA(5, 2^4, 2)$

Fig. 2. $MCA(6, 3^1.2^2, 2)$

Otherwise, it is called a *mixed level covering array* (MCA) [2]. For a quick example, Fig. 1 shows an instance of $CA(5,2^4,2)$. In any two columns, each ordered pair of symbols occurs at least once. Similarly, an instance of $CA(6,3^12^2,2)$ is given in Fig. 2.

3 Encoding Column Restrictions as Pseudo-Boolean Constraints

As mentioned before, our algorithm constructs a CA column by column. As an initial step, the algorithm randomly generates t-1 columns. Assume that we have constructed m columns ($m \ge t-1$), we now discuss how to generate pseudo-Boolean constraints for column m+1.

A pseudo-Boolean (PB) constraint is an equation or inequality between polynomials in 0-1 variables. A linear PB clause has the form: $\sum c_i \cdot L_i \sim d$, where $c_i, d \in \mathcal{Z}, \sim \in \{=, <, \leq, >, \geq\}$, and L_i s are literals.

Let us denote the variable at the *i*th entry of column m+1 by V_i . For each entry i and each value v ($0 \le v < d_{m+1}$), we introduce a Boolean variable $P_{i,v}$ such that $P_{i,v} \equiv (V_i = v)$.

The first class of constraints guarantees that each entry can only take one value. For the *i*th entry of column m + 1, we have:

$$\sum_{0 \le v < d_{m+1}} P_{i,v} = 1$$

Now consider the covering property of the array. Given an array A, suppose we extract t-1 columns (denoted by $C_{i_1}, C_{i_2}, \ldots, C_{i_{t-1}}$) from A and denote the sub-array by A_s . The p-set corresponding to a row vector \boldsymbol{v} is the set of row indices i such that i_{th} row of A_s is \boldsymbol{v} . Apparently, there are $d_{i_1} \times \cdots \times d_{i_{t-1}}$ mutually exclusive p-sets induced by the sub-array A_s .

Example 1. Figure 3 illustrates the p-sets from the $CA(5, 2^4, 2)$ in Fig. 1. Each column induces 2 p-sets. More specifically, the p-set $\{1, 2\}$ is induced by the sub-array formed by column 1 because row 1 and row 2 in the sub-array share the same row vector $\langle 0 \rangle$.

Column	P-set
1	$\{1,2\}\ \{3,4,5\}$
2	$\{1,3\}\ \{2,4,5\}$
3	$\{1,4\}\ \{2,3,5\}$
4	{1,5} {2,3,4}

Fig. 3. p-sets from $CA(5,2^4,2)$

			1	
0	0	0	0	1
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	0
1	1	1	0	1

Fig. 4. Balanced $CA(10, 2^5, 3)$

Theorem 1. An $N \times (m+1)$ array is a $CA(N, d_1 \dots d_{m+1}, t)$ **iff** the array formed by its first m columns is a $CA(N, d_1 \dots d_m, t)$, and for each $N \times (t-1)$ sub-array of the first m columns, the d_{m+1} symbols in column m+1 all appear within the rows indexed by each p-set induced by the sub-array.

Proof. For the **if** way of the implication: Suppose an $N\times (m+1)$ array M satisfies the latter condition and denote the array formed by the first m columns of M by M'. For any t columns extracted from M, we are to show all combinations of symbols from these columns are covered in rows. If column m+1 isn't chosen, the conclusion obviously holds since all the t columns are contained in M' and M' is a $\mathrm{CA}(N,d_1\ldots d_m,t)$. Otherwise, denote the other t-1 columns from M' by $C_{i_1},\ldots,C_{i_{t-1}}$, and label an arbitrary combination of symbols from these columns and column m+1 as $\langle v_{i_1},\ldots,v_{i_{t-1}},v_{m+1}\rangle$. Since M' is also a $\mathrm{CA}(N,d_1\ldots d_m,t-1)$, it covers $\langle v_{i_1},\ldots,v_{i_{t-1}}\rangle$. Among all the p-sets induced by the sub-array formed by the columns $C_{i_1},\ldots,C_{i_{t-1}}$, we denote the one corresponding to the vector $\langle v_{i_1},\ldots,v_{i_{t-1}}\rangle$ by T. From the presumption we know that v_{m+1} of column m+1 appears within the rows indexed by T. Hence the vector $\langle v_{i_1},\ldots,v_{i_{t-1}},v_{m+1}\rangle$ is covered in M. By definition, M is a $\mathrm{CA}(N,d_1\ldots d_{m+1},t)$.

For the **only if** way of implication: Suppose the array M is a CA $(N, d_1 \dots d_{m+1}, t)$. Let the array formed by the first m columns be M'. Obviously M' is a CA $(N, d_1 \dots d_m, t)$. Now suppose there exists a p-set T in the rows of which the symbol from column m+1, namely v_{m+1} does not appear. We denote the symbol combination corresponding to T by $\langle v_{i_1}, \dots, v_{i_{t-1}} \rangle$. Then the symbol combination $\langle v_{i_1}, \dots, v_{i_{t-1}}, v_{m+1} \rangle$ is not covered in M, contradicting the presumption that M is a CA of strength t.

According to Theorem 1, firstly we should calculate all p-sets induced by all $N \times (t-1)$ sub-arrays from the first m columns, then the constraints for the covering property of column m+1 can be obtained directly. In practice, we may add the p-sets incrementally to a stack as the columns expand, so that re-computation can be avoided.

Once all the p-sets are computed, it's easy to translate the constraints to pseudo-Boolean constraints. For an arbitrary p-set T, each of the d_{m+1} symbols from column m+1 should appear at least once in the rows indexed by T. The constraint of a p-set is naturally represented by the following pseudo-Boolean clauses:

$$\bigwedge_{0 \le v < d_{m+1}} \sum_{i \in T} P_{i,v} \ge 1$$

4 The Balancing Heuristic

By observing many CA instances, we find that the CAs of the optimal sizes are likely to have the following property:

Definition 2. A CA is **balanced** if in each column, all symbols occur nearly equally often. Formally, for a $CA(N, d_1d_2 \cdots d_k, t)$, denote the number of occurrences of symbol v in column j by $O_j(v)$. If for any pair $\langle v, j \rangle$, $\left\lfloor \frac{N}{d_j} \right\rfloor \leq O_j(v) \leq \left\lceil \frac{N}{d_j} \right\rceil$, then the $CA(N, d_1d_2 \cdots d_k, t)$ is balanced.

The balancing property indicates that the difference of the numbers of occurrences of any two symbols within the same column is no larger than 1. For example, the $CA(10, 2^5, 3)$ in Fig. 4 is balanced.

Our algorithm employs the balancing heuristic, searching for balanced CAs so as to enhance the probability of finding optimal CAs. Hence in addition to the constraints encoding the CA properties, there are constraints encoding the balancing property. For each value v $(1 \le v < d_{m+1})$, we have:

$$\left\lfloor \frac{N}{d_{m+1}} \right\rfloor \le \sum_{1 \le i \le N} P_{i,v} \le \left\lceil \frac{N}{d_{m+1}} \right\rceil$$

Currently we are unable to prove the rationality of the balancing heuristic, nevertheless we can provide some explanation which may shed light on this issue. Unlike the greedy strategies, which aim to locally optimize the current CA solution, the balancing heuristic is concerned with the expansibility of the current solution. The symbol distribution in column m+1 will influence the column that follows (if any). Take a $CA(N, d^k, 2)$ for example. According to Theorem 1, the d symbols in column m+2 must all appear within each p-set induced by column m+1. Intuitively the smallest p-set is the most restrictive one. Since the sum of the sizes of all these p-sets equals to N, it is better if all these p-sets are nearly of the same size, which means column m+1 is balanced.

5 Implementation and Experimental Evaluation

The column generation approach requires that the size of a CA is specified beforehand. To overcome this limitation, we employ the binary search strategy to determine the optimal size of the CA. Our tool CAB (CA searcher with Balancing heuristic) was implemented in the C++ programming language and integrated with the pseudo-Boolean constraint Solver clasp v3.1.1 [16].

For comparison, we selected three state-of-the-art CA generators, PICT [3], ACTS [9] and CASA [4]. PICT is a widely used test case generator developed by Jacek Czerwonka at Microsoft Corporation. The core generation algorithm of PICT adopts the one-test-at-a-time strategy. ACTS is a powerful test generation tool which implements the IPO algorithms. CASA is a CA generator based on Simulated Annealing. We compared CAB with these tools on various benchmarks. The experiments were conducted on an Intel 1.7 GHZ Core Duo i5-4210U PC with virtual Linux2.6 OS. Timeout (TO) means more than one hour.

The experimental results on pure-level CAs of strength 2 with level d ranging from 2 to 4 are illustrated in Tables 1, 2 and 3. Given the number of parameters (denoted by k), the number of rows (denoted by N) generated by each tool and the running time (denoted by T, measured in milliseconds) are listed. It can be seen that in most cases, CAB produces the best results, covering the pairwise interactions of k parameters with the least number of rows. The running times of CAB are also reasonable. In particular, for strength t=2 and level d=2, CAB is able to obtain the best results in dramatically shorter time. The results

Table 1. $CA(N,2^k,2)$

CAB PICT ACTS К CASA N T Ν N T Ν Т 4 80 4 84 50 6 140 2 10 2 11 2 12 2 14 4 15 22 то то 15 то 37 то crash crach 71 то crash crash то crash crash то crash crash то crashcrash то crash crash то crash crashто crash crash то crash crash то crash crash то 9657700 27

Table 2. $CA(N,3^k,2)$

К	CAB		CA	SA	PIC	PICT		ACTS	
	N	Т	N	Т	N	т	N	Т	
4	9	140	9	29	13	16	9	0	
5	12	90	11	100	12	21	15	4	
6	13	70	12	360	14	16	15	0	
9	14	120	15	340	17	16	15	4	
12	15	150	16	1070	19	16	19	0	
18	17	230	17	12350	22	29	21	0	
24	18	350	19	5080	23	17	24	0	
30	19	1190	21	3600	25	21	25	4	
39	20	1560	21	38540	27	23	26	4	
52	21	2830	22	111140	29	36	28	4	
64	22	4600	23	148610	30	40	29	12	
83	23	10620	25	108110	33	109	31	16	
117	24	20330	26	1376720	34	112	33	32	
137	25	39280	27	855600	35	152	34	48	
170	26	84420	28	2733730	37	268	36	84	
248	27	167000	-	то	39	537	37	184	
289	28	513110	-	то	39	800	39	264	
361	29	996630	-	то	40	1201	40	456	
476	30	919390	-	то	42	2336	42	888	

Table 3. $CA(N,4^k,2)$

K	CA	AB		CASA		PICT		ACTS	
	N	Т	N	Т	N	Т	N	Т	
3	16	160	16	140	17	17	16	0	
5	19	80	16	190	22	41	24	0	
6	21	160	19	1980	25	36	24	0	
7	23	260	22	1660	27	24	32	0	
8	24	380	24	4330	28	27	32	0	
9	25	760	27	820	30	23	32	0	
10	26	6350	26	5290	31	28	33	0	
12	27	5360	28	2720	34	25	33	0	
15	28	13830	29	41270	35	28	36	0	
16	29	10630	29	90910	36	35	38	0	
18	30	151950	30	105930	36	48	41	4	
21	31	169020	31	457320	39	57	41	4	
26	32	313420	33	492560	42	68	43	4	

Table 4. $CA(N,2^k,3)$

K	CA	В	CA	SA	PICT		ACTS	
	N	Т	N	Т	N	Т	N	Т
4	8	40	8	140	8	20	8	0
5	10	50	10	90	13	28	12	0
8	12	90	12	450	16	24	18	0
10	16	120	12	1700	18	28	20	0
12	18	310	16	2410	19	28	22	0
14	22	600	18	6890	22	36	25	4
15	22	540	19	1890	23	29	26	4
17	26	940	20	5640	23	20	26	0
20	26	1210	21	24040	26	32	27	4
24	28	1860	23	150800	29	44	29	8

obtained by CASA are quite close to that of CAB, but it often takes much more running times. ACTS and PICT are the fastest solvers in general. However, the CAs found by them are usually larger.

Model	Description	CAB	;	CASA		PICT		ACTS	
		N	Т	N	Т	N	Т	N	Т
Apache	$CA(2^{158}3^84^45^16^1;2)$	30	3490	33	44230	32	151	33	172
Bugzilla	$CA(2^{49}3^14^2;2)$	16	360	16	2660	17	24	18	4
gcc	$CA(2^{189}3^{10};2)$	15	9780	17	71710	20	133	20	76
SpinS	$CA(2^{13}4^5;2)$	19	320	16	2480	23	13	24	0
SpinV	$CA(2^{42}3^24^{11};2)$	27	6460	28	7010	32	28	33	8
Banking1	$CA(3^44^1;2)$	13	70	12	130	16	17	15	0
Banking2	$CA(2^{14}4^{1};2)$	10	180	10	210	12	12	10	0
CommProtocol	$CA(2^{10}7^1;2)$	14	40	14	320	16	15	14	0
Concurrency	$CA(2^5;2)$	6	0	6	60	7	11	6	0
Healthcare1	$CA(2^63^25^16^1;2)$	30	40	30	220	30	16	30	0
Healthcare2	$CA(2^53^64^1;2)$	15	140	15	210	18	11	15	0
Healthcare3	$CA(2^{16}3^{6}4^{5}5^{1}6^{1};2)$	30	250	32	1590	35	19	32	4
Healthcare4	$CA(2^{13}3^{12}4^{6}5^{2}6^{1}7^{1};2)$	42	320	42	9150	47	27	44	8
Insurance	$CA(2^63^15^16^211^113^117^131^1;2)$	527	26420	527	215930	527	92	527	4
NetworkMgmt	$CA(2^24^15^310^211^1;2)$	110	28610	110	98160	118	28	110	0
ProcessorComm1	$CA(2^33^64^6;2)$	21	270	22	5390	26	16	26	4
ProcessorComm2	$CA(2^33^{12}4^85^2;2)$	28	2690	29	32810	36	21	37	4
Services	$CA(2^33^45^28^210^2;2)$	100	350	102	5810	101	31	102	4
Storage1	$CA(2^13^14^15^1;2)$	20	10	20	100	20	28	20	71
Storage2	$CA(3^46^1;2)$	18	20	18	170	19	32	18	0
Storage3	$CA(2^93^15^36^18^1;2)$	48	80	48	610	52	53	51	0
Storage4	$CA(2^53^74^15^26^27^110^113^1;2)$	130	2330	132	7770	130	68	134	4
Storage5	$CA(2^53^85^36^28^19^110^211^1;2)$	113	521870	113	243140	123	84	119	4
SystemMgmt	$CA(2^53^45^1;2)$	15	50	15	310	19	2	16	0
Telecom	$CA(2^53^14^25^16^1;2)$	30	40	30	4550	30	23	30	4

Table 5. Mixed Covering Array

Table 4 demonstrates the results on CAs of strength 3 and level 2. CASA produces the smallest CAs in all cases, although its execution times are significantly longer than those of the other tools. Compared with PICT and ACTS, CAB usually obtains smaller CAs in longer times.

We also performed experiments on a number of MCAs, as listed in Table 5. These MCAs are derived from some benchmark SUT models in previous papers on combinatorial testing, see [12] for example. CAB outperforms CASA in both the quality of results and the execution times. PICT and ACTS are very fast, and both fail to produce the smallest CAs in most occasions. Interestingly, for many cases in Table 5, the smallest size (in bold type) happens to be the lower bound of the optimal size in theory (the product of t largest levels).

6 Related Work

The computational methods for covering array generation have been extensively studied in literature. Besides the aforementioned one-test-a-time strategy and IPO strategy, there are also some metaheuristic search and evolutionary

algorithms applied to the automatic generation of CAs, including simulated annealing, which is the core algorithm of CASA, genetic algorithms, and particle swarm optimization. For a detailed review, one can refer to [11]. Recently, an efficient local search algorithm has been proposed for generating CAs with constraints [14].

Constraint Solving techniques are also used for automatic generation of CAs. Hinch et al. [5] developed constraint programming models which exploited global constraints and symmetry breaking constraints. They also studied the local search algorithm for a SAT-encoding of the model. Yan and Zhang developed another backtrack search tool for finding CAs [10]. A kind of balancing heuristic is applied to value-tuples in the CA, so as to prune the search space. In particular, pseudo-Boolean constraint solving has also been employed to generate orthogonal arrays (OA), which can be viewed as a special class of CA [15].

7 Conclusion

In this paper, we propose a non-backtracking algorithm which generates covering arrays column by column. It integrates a pseudo-Boolean constraint solver for column generation, and adopts a new heuristic named the balancing heuristic to guide the searching procedure. The principle of balancing heuristic is very different from the greedy strategies such as one-test-at-a-time and IPO, which have been widely employed by CA generators. The balancing heuristic suggests that, rather than locally optimizing the current CA solution, the algorithm should generate balanced columns so that the current solution is more likely to be horizontally expanded. Simple as it is, our algorithm demonstrates advantage over the state-of-the-art CA generators for instances of strength 2. It can generate smaller CAs in reasonable time. In the future, we will study how to improve the performance of our tool CAB on CAs with higher strengths.

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