Integrating ILP and SMT for Shortwave Radio Broadcast Resource Allocation and Frequency Assignment

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Abstract. Shortwave radio broadcasting is the principal way for broadcasting of voice in many countries. The broadcasting quality of a radio program is determined not only by the parameters of the transmission device, but also by the radio frequency. In order to optimize the overall broadcasting quality, it is desirable to designate both devices and frequencies to radio programs, subject to various constraints including the non-interference of radio programs. In this paper, we propose a two-phase approach to this constrained optimization problem. It integrates ILP and SMT solving, as well as a local search algorithm. These methods are evaluated using real data, and the results are promising.

Keywords: Integer Linear Programming · Satisfiability modulo theories · Shortwave radio broadcast

1 Introduction

Shortwave radio is a significant medium in long distance broadcasting transmission, which uses shortwave frequencies ranging from 2 to 30 megahertz(MHz). [5] introduced the history of shortwave radio broadcasting. Nowadays, it remains the principal way for broadcasting of voice in many countries. There are various factors affecting the broadcasting quality of shortwave radio programs. How to arrange these factors properly is critical to shortwave broadcasting.

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In the past, the staff with the Division of Radio Frequency Assignment of State Administration of Press,Publication,Radio,Film and Television (SAPPRFT) of the People's Republic of China have been managing the allocation of broadcast resource manually. In [10], Ma et al. studied the shortwave radio broadcast resource allocation problem (SRBRA), which concerns how to allocate proper transmission devices to radio programs so that the overall broadcasting quality is optimized. They proved the NP-hardness of the SRBRA problem, and proposed a Pseudo-Boolean formulation and a local search algorithm.

In the SRBRA problem [10], the frequencies for the programs were assigned in advance. In real applications, it may be necessary to find a suitable frequency for each program, without introducing any interference among the programs. The frequency assignment problem (FAP) is another important problem in broadcasting transmission [1]. In the literature, FAP has been solved via several kinds of techniques, such as CSP and Local Search [8,9,11]. But in our application, FAP interleaves with broadcast resource allocation, hence cannot be solved separately. Thus we extend the SRBRA problem further to embody frequency assignment. In [10], only 87 programs were used in the empirical evaluation. (There are 87 programs in a single region.) But in total, there are 948 programs, if all regions are considered. This paper tries to deal with such challenges and investigates new approaches to the extended SRBRA problem.

The contributions of this paper include (1) extending the SRBRA problem to the Shortwave Radio Broadcast Resource Allocation and Frequency Assignment Problem (SRBRAFA), which involves both device allocation and frequency assignment, and (2) developing a two-phase approach to the SRBRAFA problem which integrates Integer Linear Programming (or local search) with Satisfiability Modulo Theories (SMT) [4]. Our methods are evaluated using real data from SAPPRFT, and the results are promising.

2 Problem Description

Given a set of programs $\mathcal{P} = \{P_1, P_2, ..., P_n\}$, a set of transmission devices $\mathcal{D} = \{D_1, D_2, ..., D_m\}$, and a set of frequencies $\mathcal{F} = \{F_1, F_2, ..., F_l\}$, the SRBRAFA problem involves allocating devices and assigning frequencies to programs, and maximizing the broadcasting quality. It is an extension of the SRBRA problem which only allocates devices to programs [10]. We use $A_i = <P_i, D_j, F_k>$ to represent the allocation of device D_j and frequency F_k to program P_i. The allocations should not conflict with each other or interfere with each other.

Conflicting Allocations. A program, which has predetermined target area and time span, can only be transmitted with one device and one frequency. If two programs P_i and P_j overlap by the broadcasting time span, then they are called overlapping programs, denoted by $overlap(P_i, P_j)$. A transmission device is assembled by a transmitter and an antenna. If two devices D_i and D_j share the same transmitter or antenna, then they are called conflicting devices, denoted by $conflict(D_i, D_j)$. For any two allocations $A_i = <P_i, D_j, F_k>$ and

$A_{i'} = <P_{i'}, D_{j'}, F_{k'}>$, if $overlap(P_i, P_{i'})$ and $conflict(D_j, D_{j'})$ hold, then they are called conflicting allocations.

Interfering Allocations. The broadcasting quality of an allocation at a monitoring site is measured by field strength and circuit reliability. Their values can be calculated through dedicated programs such as REC533 [2] or VOACAP [3]. The broadcasting quality at a monitoring site is considered to be acceptable by the SAPPRFT if the field strength is above 38 dB. The site is qualified if the field strength is above 55 dB and the circuit reliability is above 70%. Suppose that there are two allocations A_i and $A_{i'}$, they have a monitoring site in common and the field strengths at the site are both acceptable. If the absolute difference between the field strengths of these two allocations is less than 18 dB, and the absolute difference of their frequencies is no larger than 5 kHz, then the two allocations will interfere each other and weaken the broadcasting quality.

Bands and Frequencies. A band is a frequency interval, denoted by B with subscript. According to the requirement of SAPPRFT, frequencies in the same band have equivalent quality in broadcasting.

Diplomatic Programs. Besides the domestic program broadcasted by the SAPPRFT of China, there are also diplomatic programs broadcasted by other countries and regions with the fixed devices and frequencies. Unless otherwise specified, the term program is referred to as domestic program in this paper.

Optimization Goal. For an allocation $<P_i, D_j, F_k>$, if at least 60% of the sites in the target area of P_i (R_i) are acceptable, then the allocation is admissible. We use $N_{<i,j,k>}$ to represent the number of qualified sites in R_i. The optimization goal of the SRBRAFA problem is to maximize the total coverage rate $(\sum_{P_i \in \mathcal{P}} N_{<i,j,k>}/|R_i|)$ in the target areas of all programs. In [10], Ma et al. use the total number of qualified sites as the optimization goal. One drawback of this objective function is that the programs with large target areas will dominate those with small target areas. However, according to the Division of Radio Frequency Assignment of SAPPRFT, all programs are equally important. So we use the total coverage rate as the optimization goal in this paper.

In summary, the SRBRAFA problem can be defined in the following way.

Definition 1. *(The Shortwave Radio Broadcast Resource Allocation and Frequency Assignment Problem (SRBRAFA)). Given n radio programs, m transmission devices and l frequencies, for each program P_i select a device D_j and a frequency f_k such that:*

- *The allocation $<P_i, D_j, F_k>$ is admissible.*
- *For any two allocations A_i and A_j, A_i and A_j don't conflict with each other.*
- *For any two allocations A_i and A_j, A_i and A_j don't interfere with each other.*
- *The total coverage rate $(\sum_{P_i \in \mathcal{P}} N_{<i,j,k>}/|R_i|)$ is maximized.*

Since the decision version of the SRBRA problem, which is proved to be NP-complete [10], is a special case of the decision version of the SRBRAFA problem, the SRBRAFA problem is NP-hard.

3 The Two-Phase Approach

3.1 The Framework

In the previous section, we know that the broadcasting quality, i.e. field strength and circuit reliability is determined by device and band, not by frequency. This observation motivates us to use band instead of frequency for assignment in the first phase. After the allocations of devices and the assignments of bands, we will assign frequencies with consideration for interference. There are two advantages to the two-phase approach. Firstly, since it uses band instead of frequency in the first phase, the number of variables and constraints in the model is reduced. Secondly, it only takes consideration of interference in the second phase which means the number of allocation pairs which are potentially interfering is reduced.

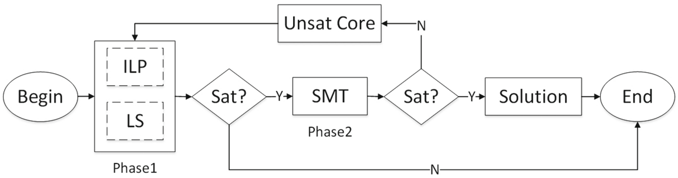

Fig. 1. The framework of the two-phase approach

The framework of the two-phase approach is shown in Fig. 1. In phase 1, an ILP-based method and a local search method are designed to solve problems under different scopes. They allocate one device and assign one band to each program, assuring no conflicting allocation exists and the total coverage rate is maximized. In phase 2, the algorithm constructs an SMT model based on the results of phase 1 to assign frequencies to programs with the condition that interference is not admitted. If a solution is found, it will be returned as the final solution. Otherwise, a new constraint representing that the previous results is not allowed will be added to phase 1 to avoid being stuck on these false allocations. The process repeats until the algorithm gets a final solution or no solution if the model of phase 1 is unsatisfiable.

3.2 Phase 1: Device Allocation and Band Assignment

The ILP Model. We first introduce two sets of 0–1 integer variables $\{Y_{i,j}\}$ and $\{Z_{i,k}\}$ to indicate whether device D_j and band B_k is allocated to program P_i respectively. For clarity, we also introduce two sets $\mathcal{QY}_i$ and $\mathcal{QZ}_i$ to represent available devices and bands for program P_i respectively:

$$\mathcal{QY}_i = \{j \mid \exists k, allocation <P_i, D_j, B_k> \textit{ is admissible}\}$$
$$\mathcal{QZ}_i = \{k \mid \exists j, allocation <P_i, D_j, B_k> \textit{ is admissible}\}$$

Recall that $N_{<i,j,k>}$ is the number of qualified sites, the objective function is as follows (**quadratic**):

$$Maximize \sum_{P_i \in \mathcal{P}} \sum_{j \in \mathcal{QY}_i} \sum_{k \in \mathcal{QZ}_i} (N_{<i,j,k>}/|R_i| \times Y_{i,j} \times Z_{i,k}) \tag{1}$$

There are two kinds of linear integer constraints. One represents that one program is allocated only one device and is assigned only one band:

$$\sum_{j \in \mathcal{QY}_i} Y_{i,j} = 1, \ \forall P_i \in \mathcal{P} \tag{2}$$

$$\sum_{k \in \mathcal{QZ}_i} Z_{i,k} = 1, \ \forall P_i \in \mathcal{P} \tag{3}$$

The other represents that no conflicting allocation is allowed.

$$Y_{i,u} + Y_{j,v} \leq 1, \ \forall overlap(P_i, P_j), \ conflict(D_u, D_v), u \in \mathcal{QY}_i, v \in \mathcal{QY}_j \tag{4}$$

The objective function (1) is a quadratic objective. In order to improve the performance of the approach, we rewrite it to a linear objective by introducing a set of variables $\{X_{i,j,k}\}$ to indicate whether device D_j and frequency F_k are allocated to P_i. We introduce $\mathcal{Q}_i$ to represent all pairs $<j,k>$ which make allocation $<P_i, D_j, B_k>$ admissible for P_i:

$$\mathcal{Q}_i = \{<j,k>| allocation <P_i, D_j, B_k> is \ admissible\}$$

The linear objective is as follows (**linear**):

$$Maximize \sum_{P_i \in \mathcal{P}} \sum_{<j,k> \in \mathcal{Q}_i} (N_{<i,j,k>}/|R_i| \times X_{i,j,k}) \tag{5}$$

In order to build connection between the two groups of variables, we need the following constraints to represent $X_{i,j,k} \leftrightarrow Y_{i,j} \wedge Z_{i,k}$:

$$Y_{i,j} + Z_{i,k} - X_{i,j,k} \leq 1, \ \forall <j,k> \in \mathcal{Q}_i \tag{6}$$

$$Z_{i,k} - X_{i,j,k} \geq 0, \ \forall <j,k> \in \mathcal{Q}_i \tag{7}$$

$$Y_{i,j} - X_{i,j,k} \geq 0, \ \forall <j,k> \in \mathcal{Q}_i \tag{8}$$

The Local Search Method. The local search method is an extension to the one in [10], which only allocates devices to programs. We modify the search procedure for device allocation and band assignment.

The local search method introduced in [10] consists of three steps, i.e. Construct, Swap and Substitute. Band assignment is completed in the process of Construct. For each unassigned program P_i, if $<P_i, D_j, B_k>$ is admissible, then we assign the band B_k and D_j to P_i so that $N_{<i,j,k>}$ is maximized and $<P_i, D_j, B_k>$ is consistent with the solution $\mathcal{S}$. That is to say, we apply a greedy strategy in band assignment. Similarly, in the process of Swap and Substitute, if we allocate a new device to a program, then we choose the band which can maximize the coverage rate of the program.

3.3 Phase 2: Frequency Assignment

In Phase 2, we determine the frequencies of the programs on the basis of the band assignment in Phase 1. The potential interfering program pairs (denoted by $IP(P_i, P_j)$) can be derived from the result of Phase 1. Suppose P_i and P_j are a pair of such programs, whose frequencies are denoted by F_{P_i} and F_{P_j} respectively. Generally, the domain of frequency is limited to the multiples of five, such as 6015 kHz and 7200 kHz. In order to avoid interference between P_i and P_j, the difference between F_{P_i} and F_{P_j} should be larger than 5, or formally:

$$F_{P_i} - F_{P_j} > 1 \vee F_{P_j} - F_{P_i} > 1,\ \forall IP(P_i, P_j),\ P_i \in \mathcal{P},\ P_j \in \mathcal{P} \tag{9}$$

Note that the difference of frequency between interfering programs is larger than 1 instead of 5 since we divide the value of frequency by 5 in calculation.

Recall that there are diplomatic programs with fixed devices and frequencies. In order to deal with the interference of such programs, we divide the band of a program into several domains. Suppose that the band assigned to program P_i is $[7000, 7040]$. In order to prevent interference from a diplomatic program with frequency 7020 kHz, F_{P_i} should be greater than 7025 kHz or less than 7015 kHz. As a result, the domain of F_{P_i} is divided into two intervals, $[7000, 7010]$ and $[7030, 7040]$. We denote the collection of the domains for F_{P_i} by $\mathcal{C}_i$. Each $C \in \mathcal{C}_i$ is an interval $[f, f']$ of frequencies. The following constraint ensures that F_{P_i} should fall in one of these intervals.

$$\bigvee_{C \in \mathcal{C}_i} f \leq F_{p_i} \leq f',\ \forall P_i \in \mathcal{P} \tag{10}$$

The above constraints naturally form an SMT formula on difference logic (SMT(DL)). If the SMT formula is unsatisfiable, it suggests that the allocations in Phase 1 would inevitably lead to interference. By extracting the unsatisfiable core of the SMT formula, we can identify the allocations responsible for this inconsistency. Suppose $\mathcal{F}^{UC}$ is the set of frequency variables involved in the unsatisfiable core, then $\mathcal{X}^{IC} = \{X_{i,j,k} | F_{P_i} \in \mathcal{F}^{UC}, X_{i,j,k} = 1\}$ is the set of allocations with interference. In order to avoid the same inconsistency, we add the following constraint to the ILP model:

$$\sum_{X_{i,j,k} \in \mathcal{X}^{IC}} X_{i,j,k} \leq |\mathcal{X}^{IC}| - 1 \tag{11}$$

This trick also applies to the local search procedure by adding the unsat core to a taboo list.

4 Experimental Results and Analysis

This section evaluates the proposed approach on the entire data set of the Division of Radio Frequency Assignment of SAPPRFT. There are 948 programs in total. The total number of transmission devices is 7061, on the premise that the transmitters and antennas located in the same shortwave radio station can be fully connected. However, due to the limitation of the current circuits, only 873 devices are available in practice. So our experiments were conducted on two sets of devices: the practical devices, and the fully connected devices. We employ CPLEX [6] as the ILP solver, and Z3 [7] for SMT solving. All instances are available on the website[1]. The experiments were performed in windows 7 on 2.8 GHz Intel processor with 16 GB RAM.

Tables 1 and 2 show the comparison of ILP against local search (LS) in Phase 1, on the instances with practical devices, and the instances with fully connected

Table 1. Experimental results on the practical devices

$\|\mathcal{P}\|$	$\|\mathcal{D}\|$	ILP		Local search	
		Obj	Time(s)	Max(avg)	Time(s)
100	100	0.706	1.201	0.651(0.632)	0.06
100	200	0.763	1.716	0.699(0.682)	0.01
200	100	-	2.59	-	-
200	200	0.716	7.769	-	-
200	300	0.802	7.191	0.707(0.697)	0.01
300	200	-	9.797	-	-
300	300	0.795	21.403	-	-
300	400	0.855	24.944	0.741(0.730)	0.03
400	300	0.753	39.281	-	-
400	400	0.836	57.346	-	-
400	500	0.851	50.576	0.711(0.702)	0.05
500	400	0.810	111.478	-	-
500	500	0.835	88.499	0.692(0.676)	5.84
500	600	0.850	82.977	0.715(0.702)	0.12
600	500	0.823	153.661	-	-
600	600	0.844	135.736	-	-
600	700	0.856	126.439	0.713(0.70)	0.2
700	600	0.841	240.039	-	-
700	700	0.855	186.952	0.707(0.701)	0.19
700	800	0.870	250.569	0.717(0.711)	0.28
800	700	0.849	248.26	0.708(0.693)	0.24
800	800	0.863	333.359	0.719(0.709)	0.27
800	873	0.869	385.182	0.726(0.715)	0.36
948	800	0.847	447.145	0.703(0.690)	0.7
948	873	0.854	599.059	0.707(0.696)	0.86

[1] http://lcs.ios.ac.cn/~maff/.

Table 2. Experimental results on the fully connected devices

$\|\mathcal{P}\|$	$\|\mathcal{D}\|$	ILP		Local search	
		Obj	Time(s)	Max(avg)	Time(s)
100	1000	0.663	63.679	-	-
200	2000	0.790	387.148	0.624(0.611)	0.87
300	3000	0.788	1567.12	0.606(0.590)	0.43
400	4000	OM	OM	0.596(0.587)	0.71
500	5000	OM	OM	0.702(0.693)	1.19
600	6000	OM	OM	0.701(0.693)	2.07
700	7000	OM	OM	0.713(0.703)	3.23
948	7061	OM	OM	0.694(0.684)	4.6

devices respectively. Since all instances are solved in a single iteration of the two phases, and Phase 2 only took less than one second, we only perform the comparison for Phase 1. The instances in Table 1 are randomly taken from 948 programs and 873 practical devices. In Table 2, the instances are randomly taken from 948 programs and 7061 fully connected devices. For clarity, the `obj` shown in the table is the average of coverage rate rather than the sum of coverage rate. The time limit for LS is 10 s and for `CPLEX` is 3600 s. The LS method is executed 10 times for each instance, and both the maximum and average rates are listed. The average time for LS to reach a locally optimal solution is also listed. The symbol `-` indicates the instance has no solution, or LS failed to find a solution, and `OM` indicates that `CPLEX` ran out of memory.

We can observe from Table 1 that `CPLEX` can solve all these instances within 10 min. By constrast, LS failed on nearly half of the instances. For the rest, the coverage rates provided by LS is less optimal than `CPLEX`, but the time for LS to find the locally optimal solution within the time limit is always less than 1 second. The reason for the unsatisfactory performance of LS on these instances is that the ratios of the numbers of programs to the numbers of devices are much larger than those in [10], making it very hard for a stochastic algorithm to find a legal solution. In Table 2, there are much more devices in each instance. `CPLEX` ran out of memory for the larger instances, while LS can always provide a solution very quickly for most of the instances besides the first one.

5 Conclusions

In this paper, we studied the SRBRAFA problem. We proposed a two-phase approach integrating ILP (or local search) with SMT to solve the problem. The approach is evaluated using real data from the Division of Radio Frequency Assignment of SAPPRFT, and the results are promising. In the current artificial plan, the average coverage rate is only 0.534, and there are as many as 40 pairs of interfering programs. By contrast, we can achieve the optimal coverage rate 0.854 with `CPLEX`, and 0.696 with local search. Moreover, no interference exists.

Overall, the ILP method can solve the current problem of SAPPRFT completely. It achieves optimal solution for real-world instances with practical devices (up to 873 devices). But on instances with fully connected devices (up to 7061 devices), ILP doesn't scale and we use local search as an alternative. In the future, our aim is to solve larger-scale instances with better approach.

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