# Homework 1 (Due 3/12/2022)

Advanced Statistical Computing

February 26, 2022

- Univariate optimization
  Chapter 2: PROBLEMS 2.1, 2.2, 2.4

- Multivariate optimization
  Chapter 2: PROBLEMS 2.3, 2.5, 2.6, 2.7

- Consider a multivariate Gaussian distribution constrained on a unit sphere, whose p.d.f. is

$$f(p) \propto \exp\left\{-\frac{1}{2}(p-\mu)'\Sigma^{-1}(p-\mu)\right\} \cdot \mathbb{1}\left(||p||_2 = 1\right).$$

  Such a distribution is called *Fisher-Bingham distribution* in directional statistics. Assuming $||\mu||_2 > 1$, we want to find out the point on the unit sphere that maximizes $f(p)$. Theorem 4.3 in Nocedal and Wright (1999) suggests a numerical optimization method for this problem. To find the maximum point is equivalent to find the $p^*$ satisfying $(\Sigma^{-1} + \lambda I)\, p^* = \Sigma^{-1}\mu$ for some $\lambda$ s.t. $\Sigma^{-1} + \lambda I$ is positive definite. Define

$$p(\lambda) = (\Sigma^{-1} + \lambda I)^{-1}\Sigma^{-1}\mu,$$

  then we only need to seek a value $\lambda$ such that

$$||p(\lambda)|| = 1,$$

  and $\Sigma^{-1} + \lambda I$ is positive definite.

  Generate $\mu$ and $\Sigma$ using the following code in R, and find the maximum point $p^*$ that maximizes $f(p)$.

```
set.seed(2022)
mu = rnorm(10, .5, .1)
Sigma = rWishart(1, 10, diag(runif(10)))[, , 1]
```

  Hint: For a symmetric real matrix, it is positive definite if and only if all its eigenvalues are positive.

**Note (a).** If you are asked to implement an algorithm in a problem, you are expected to code the algorithm in R, python or other languages by your self; otherwise, you can "use" or "apply" a prepackaged software, with a clear statement about which package you used and how you used it.

**Note (b).** Please present your results, codes and necessary mathematical details into one **single pdf or html file**. `R markdown` is recommended and `Jupyter Notebook` is also acceptable.

# References

[1] Nocedal, J. and Wright, S. (1999) *Numerical Optimization. (2nd ed.)* New York, NY: Springer.