THU-70250403, Convex Optimization (Fall 2021) **Homework: 9**

Convex Relaxation and Geometric Problems

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Student:

Problem 1

A problem appeared in the 2012 Final Examination of EE364 Convex Optimization I. Many thanks to Prof. Boyd for recommending this problem!

Consider random variables X_1, X_2, X_3, X_4 that take values in $\{0, 1\}$. We are given the following marginal and conditional probabilities:

$$\begin{aligned} \mathrm{Prob}(X_1 = 1) &= 0.9 \\ \mathrm{Prob}(X_2 = 1) &= 0.9 \\ \mathrm{Prob}(X_3 = 1) &= 0.1 \\ \mathrm{Prob}(X_1 = 1, X_4 = 0 | X_3 = 1) &= 0.7 \\ \mathrm{Prob}(X_4 = 1 | X_2 = 1, X_3 = 0) &= 0.6 \end{aligned} \tag{1}$$

Explain how to find the minimum and maximum possible values of $\mathrm{Prob}(X_4 = 1)$, over all (joint) probability distributions consistent with the given data. Find these values by using CVX toolbox [1] and report them.

Hints. (You may feel free to ignore these hints. These hints may also be useful to the following two questions.)

1. CVX supports multidimensional arrays; for example, variable $p(2, 2, 2, 2)$ declares a 4-dimensional array of variables, with each of the four indices taking the values 1 or 2.

2. The function $sum(p, i)$ sums a multidimensional array p along the ith index.

3. The expression $sum(a(:))$ gives the sum of all entries of a multidimensional array a. You might want to use the function definition $sum_all = @(A)sum(A(:));$, so $sum_all(a)$ gives the sum of all entries in the multidimensional array a.

Problem 2

Please explain the geometric meaning of the following optimization problems and derive its dual problem.

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \quad \boldsymbol{x}^T Q \boldsymbol{x} \tag{2}$$

$$\text{s.t.} \quad \left(\boldsymbol{a}_i^T \boldsymbol{x}\right)^2 = b_i \tag{3}$$

where $Q \in \mathbb{S}^n_{++}$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m_+$.

References

[1] CVX: Matlab Software for Disciplined Convex Programming http://cvxr.com/cvx/