# Assignment #1

[***Seven*** problem sets in this assignment]

1. Please prove the following statements:

(1) $\mathrm{P}(A, B|C) = \mathrm{P}(A|C)\mathrm{P}(B|C) \Leftrightarrow \mathrm{P}(A|B, C) = \mathrm{P}(A|C)$

(2) $\mathrm{P}(A = \mathrm{a}, B|C) \neq \mathrm{P}(B|C)$

(3) $\mathrm{P}(Z|X) \neq \Sigma_{\mathrm{Y}}\mathrm{P}(Z|X, Y)$

(4) $P(Y|X) = \mathrm{P}(X|Y)\mathrm{P}(Y)/\sum_{Y}\mathrm{P}(X|Y)\mathrm{P}(Y)$

2. In an entertainment TV program, you were asked to open one out of ***K*** **(*K*⩾3)** doors, but only one of the doors had bonus behind. After you chose a door, the TV host opened another door and it was empty. Then, the host told you that he knew the location of the bonus and asked you "Would you like to change your choice?" What is your answer under the principle of ***Bayesian probability***? If the host does not know the location of the bonus, will be your answer changed? Please explain the reasons under the principle of ***Probability***.

3. According to your common sense, it should be nearly equal probability to get upward or downward of a coin if you throw it up arbitrarily. It means that the prior probability for "upward" is 0.5. Now, there is a special coin, after 100 runs of experiments, you get 55 "upward" and 45 "downward". Then, what is the probability that you will get "upward" for the 101-th experiment?

4. For a common effect model, two factors *X* and *Y* will independently cause *Z*. We can model this case as:

$$\mathrm{P}(XYZ) = \mathrm{P}(X)\mathrm{P}(Y)\mathrm{P}(Z|X, Y).$$

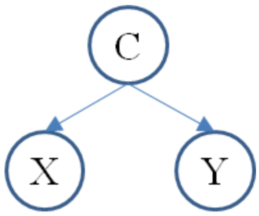
Assume all the variables are binary.

1) If *Z* is ***observed*** (for example, we know *Z*=1), please prove that *X* and *Y* are dependent with each other given *Z* (Hint: check the correctness of the equation of the conditional distribution $\mathrm{P}(XY|Z) = \mathrm{P}(X|Z)\mathrm{P}(Y|Z)$ ).

2) If $Z$ is ***<u>not observed</u>***, please prove that $X$ and $Y$ are independent. (Hint: check the correctness of the equation of the marginal distribution $\mathrm{P}(XY) = \mathrm{P}(X)\mathrm{P}(Y)$ )

5. What are clique, maximal clique and upward closure? Please explain these terms by giving a few concrete examples (*please draw a graph with at least 10 nodes by yourself*).

6. For a simple *Naïve* Bayesian model as below (all variables are binary):




$$\mathrm{P}(C = 1) = 0.6$$

$$\mathrm{P}(X = 1|C = 1) = 0.5 \qquad \mathrm{P}(X = 1|C = 0) = 0.6$$

$$\mathrm{P}(Y = 1|C = 1) = 0.3 \qquad \mathrm{P}(Y = 1|C = 0) = 0.5$$

If $Y$=1, please calculate P($X$=0|$Y$=1).

7. Talk about the applications of Probabilistic Graphical Models in your research fields or why you choose this course.