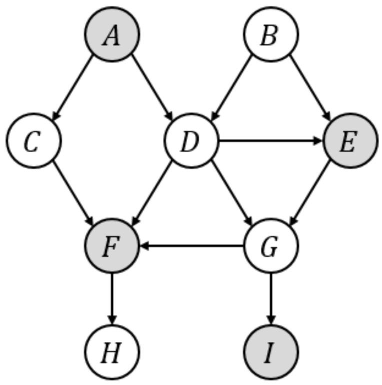
Assignment #5: *Monte Carlo* Inference

[**Requirement**: You need to <u>derive the formula for the sampling algorithm</u>. Try to use ***pseudo codes*** to present the sampling procedure.]

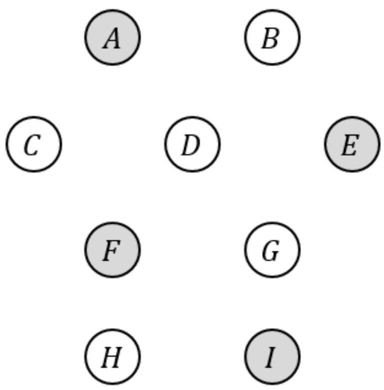
1. Below is a Bayesian network (all parameters are known). If <u>variables AEFI are always observed</u>, we aim at **inferring** $\mathbf{P(D, H|A, E, F, I)}$.

1) Please draw the corresponding *multilated network* and write down the algorithm of *multilated network* based **importance sampling** to do the inference.

2) Please write down the algorithm if you are required to use **Gibbs sampling**.

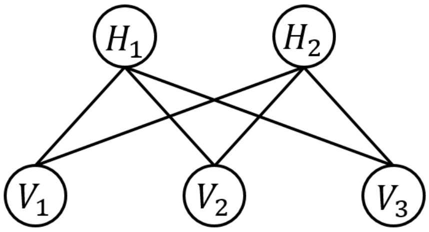


3) If you are required to generate samples from the network as below (proposal distribution Q, assuming that all variables are complete independent, and the parameters of $Q(\mathrm{A}), \dots, Q(\mathrm{I})$ are known), please write down the algorithm by **importance sampling**.



2. For a 5-node **Markov random field** of 0-1 binary variables, the weights for pairwise interactions are $\boldsymbol{w}$, and the weights for single nodes are $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

1) Please write down its Gibbs distribution.



$$\alpha_1 = 1, \alpha_2 = 1$$
$$\beta_1 = 1, \beta_2 = 0.5, \beta_3 = 1$$

$$w_{11} = w_{12} = w_{13} = -1$$
$$w_{21} = w_{22} = w_{23} = 0.2$$

2) Please write down **Gibbs sampling** inference algorithm for this model.

Practice: write your own codes to do Gibbs sampling. Please:

a) Plot the sample generation process of the first 100 samples;

b) Calculate the marginal distribution of the five nodes, respectively.

3) Please write down the **Metropolis-Hastings algorithm** if the proposal distribution Q is chosen as a completely independent random multinomial distribution:

$$Q(H_1, H_2, V_1, V_2, V_3) = \mathrm{P}(H_1|\theta_1)\mathrm{P}(H_2|\theta_2)\mathrm{P}(V_1|\theta_3)\mathrm{P}(V_2|\theta_4)\mathrm{P}(V_3|\theta_5),$$

$$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0.5.$$

Also, **write your own codes for inference:**

a) Plot the sample generation process of the first 100 samples;

b) Calculate the marginal distribution of the five nodes, respectively.

Please compare the convergence for inferring the marginal distributions of the two different approaches. Give your own comments.