THU-70250403, Convex Optimization (Fall 2021) **Homework: 5**

Lagrange Functions and Dual Problems

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Problem 1

Please find the dual problem of the following primary problem (lasso problem)

$$\min_{\boldsymbol{x}} \ |A\boldsymbol{x} - \boldsymbol{b}|_2^2 + |\boldsymbol{x}|_1 \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{b} \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times n}$, and $\text{rank}(A) = n$.

Problem 2

Considering the following optimization problem

$$\min_{\boldsymbol{x},\boldsymbol{z}\in\mathbb{R}^n} \quad \frac{1}{2}|\boldsymbol{x}|_2^2 + \frac{1}{2}|A\boldsymbol{z} - \boldsymbol{b}|_2^2 \tag{2}$$

$$\text{s.t.} \quad \boldsymbol{x} - \boldsymbol{z} = \boldsymbol{c} \tag{3}$$

where $A \in \mathbb{R}^{m\times n}$ are known constant matrix with $\text{rank}(A) = n$, $\boldsymbol{b} \in \mathbb{R}^m$, $\boldsymbol{c} \in \mathbb{R}^n$ are known constant vectors.

Please derive the corresponding dual problem.

References