THU-70250403, Convex Optimization (Fall 2021) **Homework: 12**

Unconstrained Minimization (continue)

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Student:

Problem 1

Please prove or disporve that the set of all interior points of a convex set C is a convex set.

Problem 2

In the book [1], we use the composition rules for convex functions to prove that the sum of log-convex functions is log-convex function. Now, we need you to prove this conclusion by definition of log-concave functions.

That is, if $f : \Omega \to \mathbb{R}$ and $g : \Omega \to \mathbb{R}$ are two log-convex functions, for any $\boldsymbol{x}, \boldsymbol{y} \in \Omega$, $\alpha \in [0, 1]$, we have

$$(f+g)\left(\alpha \boldsymbol{x} + (1-\alpha)\boldsymbol{y}\right) \leq (f+g)^{\alpha}(\boldsymbol{x}) \cdot (f+g)^{(1-\alpha)}(\boldsymbol{y}) \tag{1}$$

(Hints: first prove Hölder inequality and thus use Hölder inequality to prove the above conclusion.)

Problem 3

Consider following unconstrained optimization problems:

$$\min f(x) = -\sum_{i=1}^{m} \log\left(1 - a_i^T x\right) - \sum_{i=1}^{n} \log(1 - x_i^2) \tag{2}$$

where $x \in R^n$, and dom $f = \left\{ x \middle| \, a_i^T x < 1, i = 1, \cdots, m; |x_i| < 1, i = 1, \cdots, n \right\}$.

Combine *Newton Algorithm* and *Backtracking Line Search* to find the optimal solutions x^* and optimal value p^* of function $f(x)$ in two scales (m=50,n=50) and (m=100,n=100). The stopping error of the algorithm is $\|\nabla f(x)\|_2 \leq 10^{-8}$. You need to plot the logarithmic error $\log\left(f\left(x^k\right) - p^*\right)$ and iteration step size t^k with respect to the number of iterations k separately. We have given two sets of data and the corresponding coefficient matrix $A \in R^{n \times m}$ in the attachment, where $A = [a_1, a_2, \cdots, a_m]$. Note that you need to submit program and analysis documents.

References

[1] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004. http://www.stanford.edu/~boyd/cvxbook/