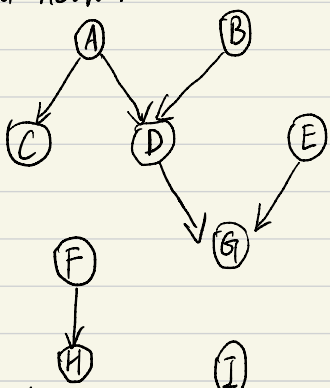
概率图模型 第5次作业

1. 解：

(1) multilated network:



$Q(A)=Q(E)=Q(F)=Q(I)=1$

$W[M]=P(A)\cdot P(E|B,D)$

拓扑排序：$\{A,B,C,D,E,G,F,H,I\}=S$

```
for m in range(M):
    tuplelist = list()
    W[m] = 1
    for x_i in S:
        if x_i not in {A,E,F,I}:
            x_i = sample(P(x_i | Pa_{x_i}))
        else:
            x_i ← {A,E,F,I}<X_i>
            W[m] = W[m] · P(x_i | Pa_{x_i})
    tuplelist.append((x_1, ..., x_n), w))
```

$$P=\frac{\sum_{i=1}^{m} 1\{x[m](D,H)=d,h\}\cdot W[m]}{\sum_{i=1}^{m} W[m]}$$

return P

(2) Gibbs sampling：记 $S=\{A,E,F,I\}$，采样顺序为 $X=\{H,B,D,C,G\}$

初始化 $A=a, E=e, F=f, I=i, B=b_1, C=c_1, D=d_1, G=g_1, H=h_1$.

即初始化 $x[0]=\{B=b_1, C=c_1, D=d_1, G=g_1, H=h_1\}$

```
for Xi in X:
    Ui = x[m](X-Xi) + s
    x[m](Xi) = samplefrom(P(Xi|Ui))
reture x[m]
```

(3) 拓扑排序：$\{A,B,C,D,E,G,F,H,I\}=S$

```
for m in range(M):
    tuplelist = list()
    w[m] = 1
    for Xi in S:
        if Xi not in {A,E,F,I}:
            Xi = sample(P(Xi|Pa_Xi))
            w[m] = w[m] · P[Xi|Pa Xi)
        else:
            Xi ⇐ {A,E,F,I}<Xi>
            w[m] = w[m] · P(Xi|PaXi)
    tuplelist.append((x1,---,xn), w))
```

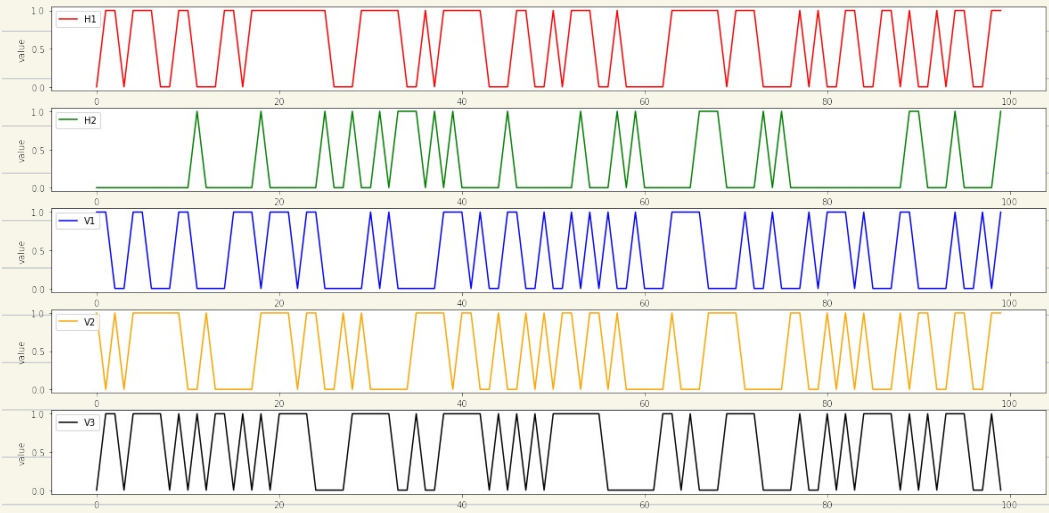
$$p=\frac{\sum_{i=1}^{m} 1\{x[m](D,H)=d,h\}\cdot w[m]/Q(x[m])}{\sum_{i=1}^{m} w[m]/Q(x[m])}$$

return p

2.解:

(1) Gibbs distribution $= \frac{1}{Z}\exp\left(-\left[\sum_{i=1}^{2}\alpha_i h_i + \sum_{i=1}^{3}\beta_i v_i + \sum_{i=1}^{2}\sum_{j=1}^{3} w_{ij} h_i v_j\right]\right)$

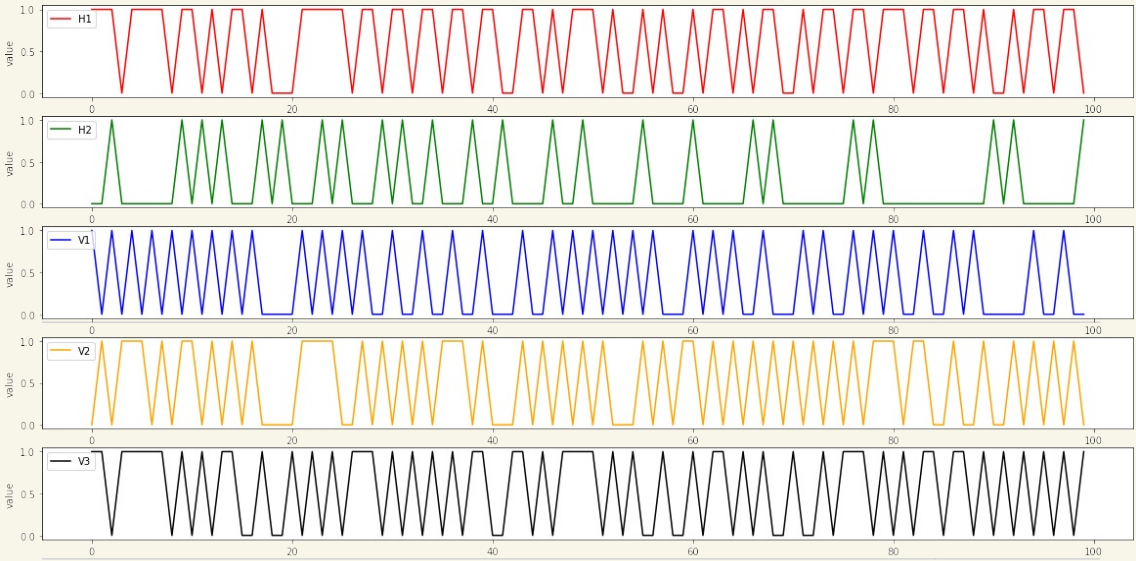
(2) Gibbs 采样 100 次得到的数据点为



(3) 吉布斯采样 10000 次得到的边缘分布为

	节点	取值	采样得到的边缘分布	真实边缘分布
0	H1	0	0.3991	0.396086
1	H1	1	0.6009	0.603914
2	H2	0	0.7759	0.784246
3	H2	1	0.2241	0.215754
4	V1	0	0.5987	0.601142
5	V1	1	0.4013	0.398858
6	V2	0	0.4809	0.484701
7	V2	1	0.5191	0.515299
8	V3	0	0.4209	0.414986
9	V3	1	0.5791	0.585014

(4) MH采样得到的数据点为



(5) MH采样10000次得到的边缘分布为

	节点	取值	采样得到的边缘分布	真实边缘分布
0	H1	0	0.3947	0.396086
1	H1	1	0.6053	0.603914
2	H2	0	0.7824	0.784246
3	H2	1	0.2176	0.215754
4	V1	0	0.5998	0.601142
5	V1	1	0.4002	0.398858
6	V2	0	0.4861	0.484701
7	V2	1	0.5139	0.515299
8	V3	0	0.4039	0.414986
9	V3	1	0.5961	0.585014

(6) 可见，吉布斯采样收敛速度为MH采样快
并且MH采样很难在不同状态之间跳转