THU-70250403, Convex Optimization (Fall 2021) **Homework: 13**

Constrained Minimization

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Student:

Problem 1

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \quad \sum_{i=1}^{n} |x_i|^{\frac{3}{2}} \tag{1}$$

$$\text{s.t.} \quad A\boldsymbol{x} \leq \boldsymbol{b} \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $m < n$.

1) Please prove that the above problem is a convex optimization problem.

2) Please derive the Lagrange dual problem of the above problem.

3) Please derive the necessary and sufficient conditions when the strong Lagrange duality holds for the above problem.

Problem 2

Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is a quadratic differentiable quasi-convex function. Can the following conclusion be derived from the second-order condition of the proposed quasi-convex function: there exists a $\sigma \in \mathbb{R}$ satisfying

$$\nabla^2 f(\boldsymbol{x}) + \sigma \nabla f(\boldsymbol{x}) \nabla f(\boldsymbol{x})^T \in \mathbb{S}_+^n, \forall \boldsymbol{x} \in \mathrm{dom} f \tag{3}$$

Problem 3

Consider the equation constrained optimization problem

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \quad \sum_{i=1}^{n} x_i \log x_i \tag{4}$$

$$\text{s.t.} \quad A\boldsymbol{x} = \boldsymbol{b} \tag{5}$$

where $\mathrm{dom} f = \mathbb{R}_{++}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $m < n$.

1) Solve the above problem using the standard Newton method for the optimal solution and the associated optimal value at $m = 30$, $n = 100$ and when the feasible initial point is given in the data file. Use backtracking linear search, choose the backtracking parameters reasonably, require the error $\eta = 10^{-10}$, and draw the relationship between $\log(f(x^{(k)}) - p^*)$ and the number of iterations k.

2) Use the Newton method with infeasible initial points to solve the above problem with infeasible initial points given in the data file. The method uses a backward linear search, and the backward parameters are chosen reasonably with an error $\eta = 10^{-10}$, and draw the relationship between $\log(f(x^{(k)}) - p^*)$ and the number of iterations k.

You need to submit the program, the calculation results and the analysis documents electronically, and submit them through the "Course Assignment" section of the Online Learning Center, using the data provided in the folder. log means logarithm with natural logarithm as the base. Please make sure that the program can be run directly in the folder when submitting the assignment.

References