Convex Optimization Theory and Applications

Topic 0 - Introduction

Li Li

Department of Automation
Tsinghua University

Fall, 2009-2021.

0.0. Outline

0.1. Course Information

Co-Instructor

Dr. Li Li

☯ Office: #806, Main Building

☎ Telephone: +86 (10) 62782071

Email: li-li@tsinghua.edu.cn

Lectures

Location: Room 6A416, 6th Teaching Building

Time: Tuesday, 13:30am - 16:05am

1st-16th week

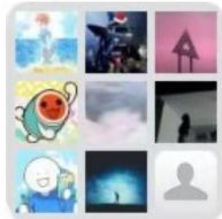
No Office Hours. Please contact me via email if really needed!

Our TAs can handle all the problems that you met, ☺

0.1. Course Information

新型肺炎病毒给我们的教与学带来的新的挑战，我们将充分利用各种形式互动

1. 日常上课以课堂为主，所有资料和作业在网络学堂
2. 备用上课方式是腾讯会议，您可用微信小程序登入
3. 课堂也有微信群，有事直接群里联系



2021秋季学期凸优化课程群

该二维码7天内(9月15日前)有效，重新进入将更新

0.1. Course Information

作业，共 60 分。12 次，每次满分 5 分。

简短读书报告，共 10 分，采取团队完成模式，往年记录得分在 8-10 分

~~小题目天梯竞赛，共 10 分，采取团队完成模式，只要基本完成就能拿 7 分基础分，后面 3 分打榜，按成绩划档，分成 4 档：0, 1, 2, 3 分~~

期末考试，共 30 分

The grading policy MAY be alternated. Any change will be announced in advance.

0.1. Course Information

考虑如下基本形式的优化问题

$$\begin{aligned} &\min_x \ f(x) \\ &\text{s.t.} \ g(x) \geq 0, \ h(x) = 0 \end{aligned} \tag{0.1}$$

如果目标函数 $f(x)$ 是**凸函数**，约束条件 $g(x) \geq 0, \ h(x) = 0$ 构成的可行域为**凸集**，则该问题为凸优化问题。

为什么研究凸优化问题：

- 良好的理论分析特性
- 高效的实际可计算性
- 强大的建模能力

0.1. Course Information

优化问题的四大要素

1. 目标函数 Objective Function
   - 很多时候我们要面临多目标的情况

2. 决策变量 Decision Variables
   - 实数变量、整数变量等

3. 约束条件 Constraints
   - 常常需要我们仔细挖掘

4. 求解方法 Solving Method
   - 很多时候我们将求解视为一种搜寻的过程

0.1. Course Information

对于凸优化问题

1. 目标函数 Convex Function
2. 决策变量 Real Variables
3. 约束条件 Constraints Formulate Convex Sets
4. 求解方法 Generally no analytical solution, but often mature, reliable and efficient algorithms

我们系统深入的课程和灵活的联系实践让学生对凸优化领域有一个整体的认识，了解如何解决问题，并专注于建模和抽象，为大家在广泛的行业中取得成功做好准备。

课程大致分为三个部分：凸分析、凸优化问题求解算法、和凸优化建模应用，课时分配大致为 1:1:1。

0.1. Course Information

1. 现实生活中，很多问题都属于凸优化问题。例如，线性规划就是凸优化问题的特例。因此，凸优化问题的分析和求解具有重要的研究意义和实用价值。

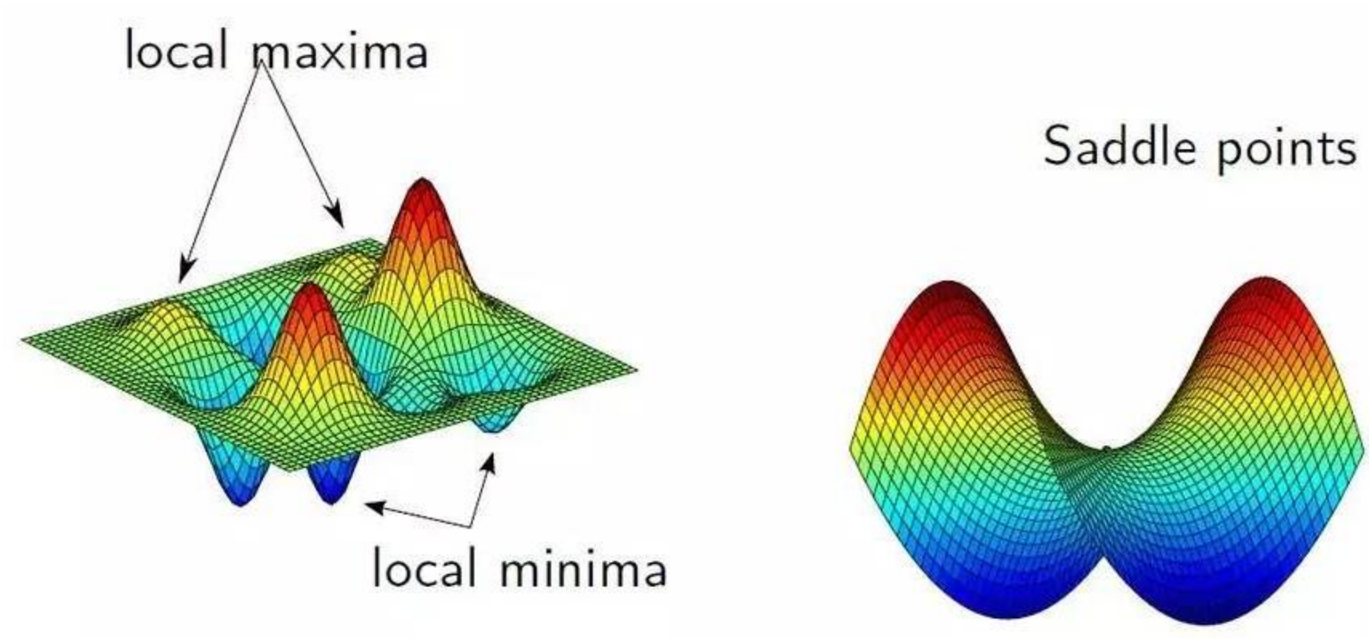
2. 凸优化问题性质好，可以进行深入的理论分析。很大一部分凸优化问题是多项式时间可解问题（P问题），我们已经建立了十分有效的求解算法，可以快速求得全局最优解。

3. 凸优化是研究连续变量优化的起点和基础。目前很多非凸优化问题中非凸性的刻画都脱胎于凸优化，相关问题的求解也和凸优化联系在一起，常常有赖于找到这些非凸优化问题中“凸”的结构。

很多非凸优化或NP-Hard的问题可以近似转化为多项式时间可解的凸优化问题，并由此给出原问题的界。

0.1. Course Information

凸优化不是万能的，也不能迷信



很多非连续变量优化问题，凸优化方法不太适用

0.1. Course Information

数学系的优化偏理论分析，有较多较难的数学证明和算法理论

工业工程系的优化学偏应用，注意力多在建模及优化软件的使用上

我们希望同学们具备 **建模** → **分析** → **求解** 的完整能力

1. master basic convex analysis
2. remember some important conclusions and know the proof
3. recognize/formulate problems as convex optimization
4. develop code for problems of moderate size
5. characterize optimal solution, analyze performance, etc.

斯坦福或麻省理工的计算机系比清华的强在哪？

关注问题 写回答 邀请回答 好问题 58 1 条评论 分享 ···

50 个回答 默认排序

齐鹏
机器学习话题下的优秀答主

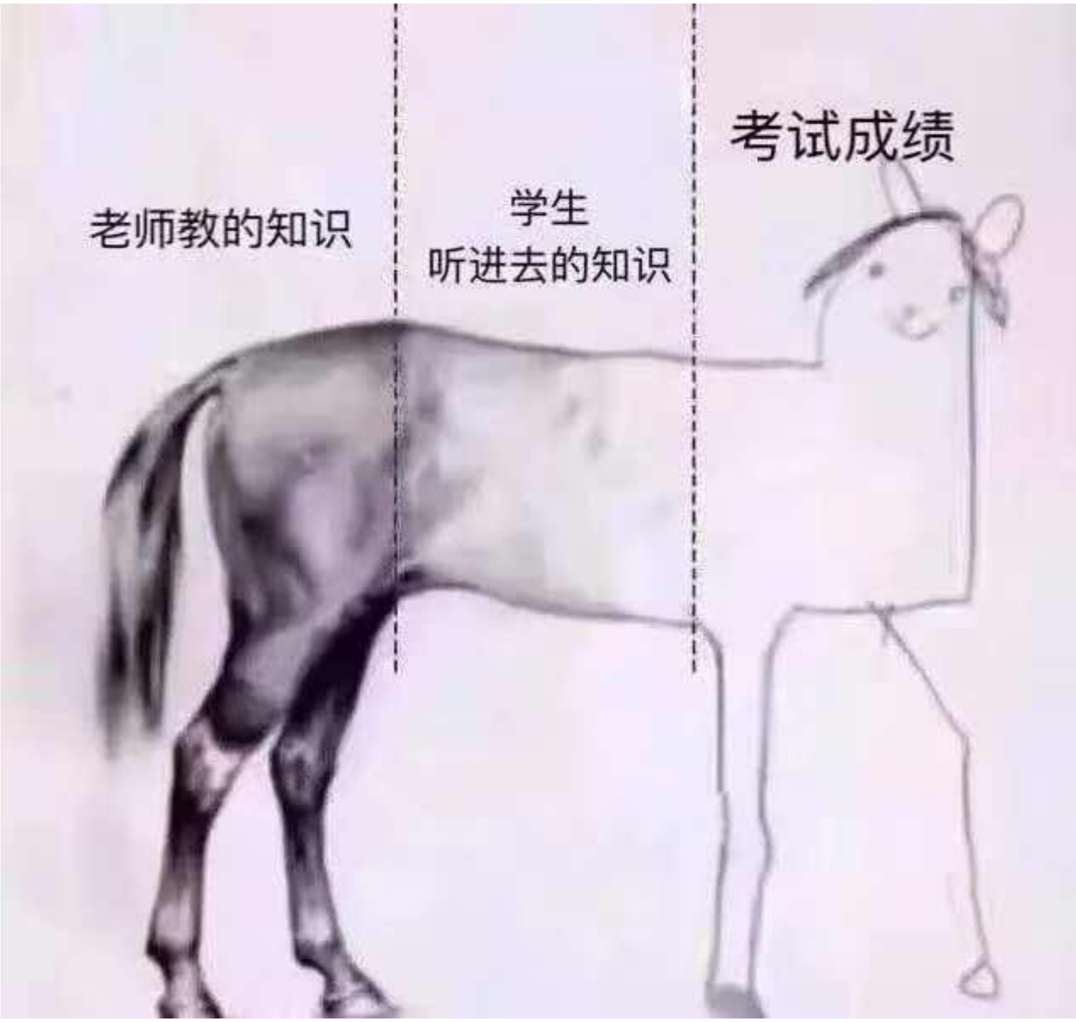
先说对学生。业界联系、地利人和（硅谷+校友）这种大家都看得到的自不必说——比如，毕竟并不是所有学校都能造出（至少是经手）Larry Page这样的人才。给我感触最深的一点是，到斯坦福才算真正理解了一句“国内老师是讲书的，国外老师是写书的”的差距。如果说一门传统意义上的数学课——至少是数学知识要求比较高的课，上课的时候几乎所有人都全神贯注，偶尔还会跟着老师的玩笑全班大笑；老师经常用各个专业的问题类比正在讨论的问题，或者举浅显易懂的例子帮你理解，让课堂上的同学都觉得学这门课是一种乐趣——你一定会惊讶。我们对数学课（尤其是大学数学课）的印象多半是照本宣科、催人睡下，更别说让人心情愉悦、积极课堂互动、开拓思路了；或者应该说，除了班上少有的几个数学方面比较有天赋的同学之外，大多数人不能从这些课里体会到这些数学知识的魅力。对了，我是不是忘了说，我说的这门“有趣”的数学课是课号300系列的，1xx是本科基础课，2xx是本科进阶/研究生基础，3xx是研究生进阶课（通常master会懒得去上）？这（两）门课就是Stephen Boyd开的凸优化I和II (EE364A/B)，也是Convex Optimization课本的作者（之一）。援引Boyd原话（大意）：“... In many parts of the world they are teaching this course wrong. They spend the whole time talking about the first couple of chapters in the book, but the really useful things are in the second half.” 凸优化I让学生学会将一个给定问题建模/近似到凸优化问题并用现有的优化软件解决，而II则深入浅出地穿插了四十年来凸优化领域各种算法的发展——以一种正常人听得懂、并能体验得到其中原理奥妙的形式。如果你想说这只是个个例，我可以说我听过的许多在清华会显得枯燥乏味的理论课或者导论课，在斯坦福都有很好的体验。当然也有一些课在清华的讲授专业性远强于斯坦福（本科课程），但这和国内外本科教育思路不同有关，在此就不深入探讨了。只能说，个人感觉是，如果你知道你最感兴趣的专业是什么、知道今后想从事什么方向的工作，国内本科无疑是很好的选择，因为你会受到更系统、集中、深入的专业知识教育；但如果还不知道自己想做什么（大多数人的情况），那国外灵活的本科教育能给你更宽阔的视野。

0.1. Course Information

我们尽量覆盖各个知识点，突出主要知识点，部分证明不做深入要求

众口难调，大家万请见谅！有问题烦请联系助教和教师

0.1. Course Information



课堂讲授只是最基本的教学环节，自己思考和练习是关键

0.2. Course Outline

Theory

Lecture 1	Sep. 14	Introduction
Lecture 2	Sep. 21	Convex Sets 凸集
Lecture 3	Sep. 28	Convex Functions 凸函数
~~ ~~	~~Oct. 5~~	~~National Day Break~~
Lecture 4	Oct. 5	Optimization Problems 优化问题
Lecture 5	Oct. 12	Duality 对偶理论

Applications

Lecture 6	Oct. 19	Classification 分类问题
Lecture 7	Oct. 26	Parameter Estimation 参数估计问题
Lecture 8	Nov. 2	Norm Approximation, Regularization
Lecture 9	Nov. 9	Convex Relaxation

0.2. Course Outline

Lecture 10	Nov. 16	Geometric Problems, Other Problems

Algorithms

Lecture 11	Nov. 23	Intro. Solving Algorithms 算法概论
Lecture 12	Nov. 30	无约束问题求解
Lecture 13	Dec. 7	等式约束问题求解
Lecture 14	Dec. 14	The State-of-the-Art Algorithm I
Lecture 15	Dec. 21	The State-of-the-Art Algorithm II

The course schedule MAY be alternated. Any change will be announced in advance.

0.3. Textbooks and References

凸分析

---

[1] R. V. Gamkrelidze, ed., *Analysis II: Convex Analysis and Approximation Theory*, Springer, 1990.

[2] R. T. Rockafellar, *Convex Analysis*, Princeton University Press, 1996.

[3] G. G. Magaril-ll'yaev, V. M. Tikhomirow, *Convex Analysis: Theory and Applications*, American Mathematical Society, Providence, RI, 2003.

[4] A. Barvinok, *A Course in Convexity*, Graduate Studies in Mathematics, vol. 54, American Mathematical Society, 2002.

[5] J. B. Hiriart-Urruty, C. Lemarechal, *Fundamentals of Convex Analysis*, abridged, Springer, 2004.

[6] A. D. Alexandrov, *Convex Polyhedra*, Springer, 2005.

[7] A. J. Kurdila, M. Zabarankin, *Convex Function Analysis*, Birkhauser, 2005.

[8] Y. Nesterov, A. Nemirovsky, *Interior Point Polynomial Algorithms in Convex Programming*, SIAM, Philadelphia, 1994.

[9] J. Renegar, *A Mathematical View of Interior-Point Methods in Convex Optimization*, SIAM, Philadelphia, 2001.

[10] E. de Klerk, *Aspects of Semidefinite Programming: Interior Point Algorithms and Selected Applications*, Kluwer Academic Publishers, 2004.

[11] D. P. Bertsekas, *Convex Optimization Algorithms*, Athena Scientific, 2015.

[12] S. Bubeck, "Convex optimization: Algorithms and complexity," *Foundations and Trends® in Machine Learning*, vol. 8, no. 3-4, 231-357, 2015.

综合教材和课件等

[13] J. B. Hiriart-Urruty, C. Lemarechal, *Convex Analysis and Minimization Algorithms, Part 1, Fundamentals*, corrected edition, Springer, 1996.

[14] J. B. Hiriart-Urruty, C. Lemarechal, *Convex Analysis and Minimization Algorithms, Part 2, Advanced Theory and Bundle Methods*, corrected edition, Springer, 2001.

[15] J. M. Borwein, A. S. Lewis, *Convex Analysis and Nonlinear Optimization Theory and Examples*, Springer, 2000
http://convexoptimization.com/TOOLS/Borwein.pdf

[16] A. Ben-Tal, A. Nemirovski, *Lectures on Modern Convex Optimization*, SIAM, Philadelphia, 2001.
http://www2.isye.gatech.edu/~nemirovs/Lect_ModConvOpt.pdf

[17] L. D. Berkovitz, *Comvexity and Optimization in* R^n, John Wiley & Sons, 2002.

[18] D. P. Bertsekas, A. Nedic, A. E. Ozdaglar, *Convexity, Duality, and Lagrange Multipliers*, Lecture Notes for MIT 6.291, 2001.

[19] D. P. Bertsekas, A. Nedic, A. E. Ozdaglar, *Convex Analysis and Optimization*, Athena Scientific Press, 2003.
[20] Y. Nesterov, *Introductory Lectures on Convex Optimization: A Basic Course*, Springer, 2003.
[21] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004. http://www.stanford.edu/~boyd/cvxbook/ http://www.ee.ucla.edu/~vandenbe/cvxbook
[22] J. Dattorro, *Convex Optimization & Euclidean Distance Geometry*, Meboo Publishing, 2005. http://www.stanford.edu/~dattorro/mybook.html
[23] S. K. Mishra, S.-Y. Wang, K. K. Lai, *Generalized Convexity and Vector Optimization*, Springer, 2009.
[24] O. Güler, *Foundations of Optimization*, GTM 258, Springer, 2010.
[25] V. Barbu, T. Precupanu, *Convexity and Optimization in Banach Spaces*, 4th edition, Springer, 2012.
[26] J. Peypouquet, *Convex Optimization in Normed Spaces: Theory, Methods and Examples*, Springer, 2015.

凸优化建模应用

[27] D. P. Palomar, Y. C. Eldar, eds., *Convex Optimization in Signal Processing and Communications*, Cambridge University Press, 2009.

[28] L. Li, *Selected Applications of Convex Optimization*, Springer, 2015.

网络课程

[29] Prof. Stephen Boyd, *EE364a: Convex Optimization I*, Stanford University, http://www.stanford.edu/class/ee364a/
http://academicearth.org/courses/convex-optimization-i

[30] Prof. Stephen Boyd, *EE364b: Convex Optimization II*, Stanford University, http://www.stanford.edu/class/ee364b/
http://academicearth.org/courses/convex-optimization-ii

[31] D. P. Bertsekas, *6.253 Convex Analysis and Optimization*, MIT, http://web.mit.edu/dimitrib/www/teaching.htm

[32] 文再文，凸优化, https://bicmr.pku.edu.cn/~wenzw/opt-2020-fall.html

应用软件

[33] CVX, http://cvxr.com/cvx/

[34] SeDuMi, http://sedumi.ie.lehigh.edu/

[35] SDPT3, http://www.math.nus.edu.sg/~mattohkc/sdpt3.html

[36] YALMIP, https://yalmip.github.io/

0.3. Textbooks and References

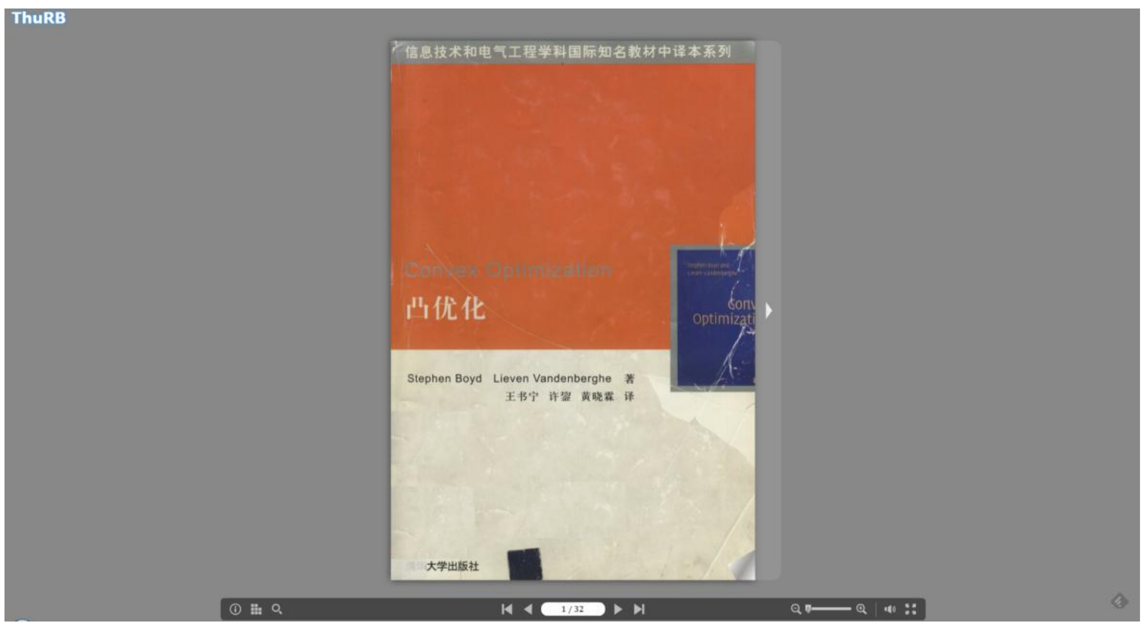
在线教材：Convex optimization 中文翻译版 凸优化

原作者：Stephen Boyd, Lieven Vandenberghe

译者：王书宁，许鋆，黄晓霖

北京 ：清华大学出版社 2013

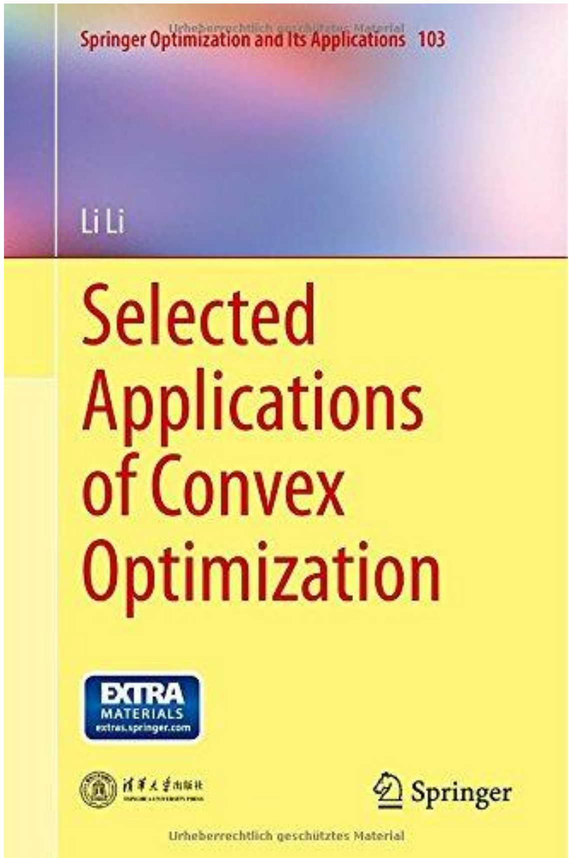
http://reserves.lib.tsinghua.edu.cn/Search/BookDetail?bookId=c86cc465-ac46-490f-8787-395725ea3965



0.3. Textbooks and References

在线教材：Li Li, Selected Applications of Convex Optimization, Springer, 2015.

https://link.springer.com/book/10.1007%2F978-3-662-46356-7



0.3. Textbooks and References

学生点评¤

1.课程或授课教师的特色 ¤

¤

2.希望和建议¤

希望有更加易于理解的讲解|感觉讲课的逻辑关联性还可以再加强一下，感觉有时对于某些定理背后的逻辑讲得不是很深入，但这也有可能是学时所限的原因吧。|希望课程作业与课本的知识点结合更紧密一些，即多一些实践的作业。¶
比如第四次作业 LMI 中全都是矩阵的证明题，若改为建立与求解 LMI 问题是否会更加合适？|李老师很认真，在课堂上也很有激情，并且所学也都跟学科前沿联系起来，使我收获颇多|老师课上对一些内容的分析讲解不够充分，有时总是听的还不明白然后就没有了|讲课过程舒缓，能学到更多实用又不缺乏理论支持的东西，只是教材比较简陋，希望有更多充实的内容。¤

0.4. History and People

Prehistory: Early 1900s - 1949
- Caratheodory, Minkowski, Steinitz, Farkas
- Basics of convex analysis: convex sets and convex functions

Fenchel - Rockafellar era: 1949 - mid 1980s
- Duality theory
- Minimax/game theory (von Neumann)
- (Sub)differentiability, optimality conditions, sensitivity

Modern era - Paradigm shift: Mid 1980s - present
- Change of the fundamental viewpoint underlying the field
- Nonsmooth analysis (a theoretical/esoteric direction)
- Practical algorithms and complexity analysis
- Diversified Applications

0.4. History and People

1947: simplex algorithm for linear programming (Dantzig)

1960s: early interior-point methods (Fiacco & McCormick, Dikin, ...)

1970s: ellipsoid method and other subgradient methods

1980s: polynomial-time interior-point methods for linear programming (Karmarkar 1984)

late 1980s-now: polynomial-time interior-point methods for nonlinear convex optimization (Nesterov & Nemirovski 1994)

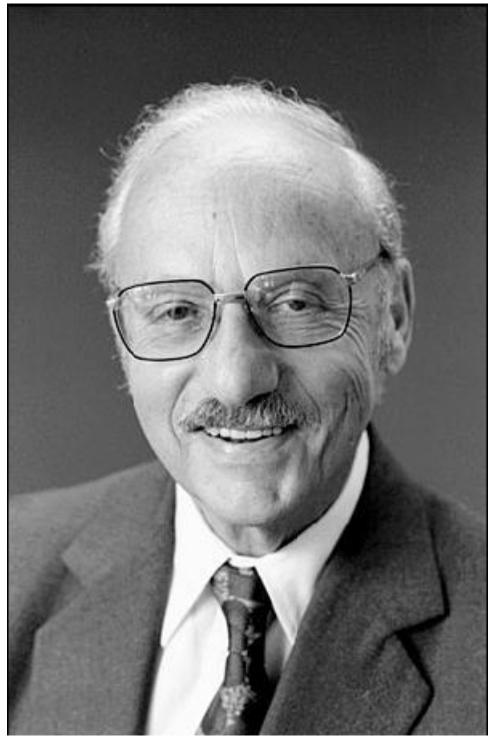
before 1990: mainly in math or operations research; few in engineering

since 1990: many new applications in engineering (control, signal processing, communications, ...)

0.4. History and People

Werner Fenchel

May 3, 1905-Jan 24, 1988

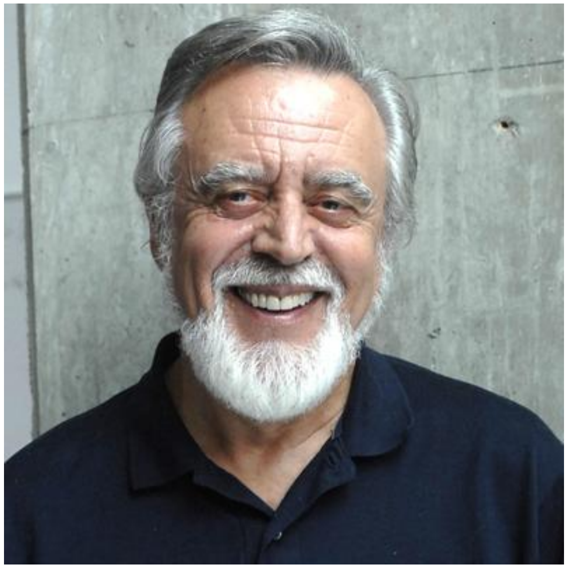


George Dantzig

Nov 8, 1914-May 13, 2005

Terry Rockafellar

0.4. History and People



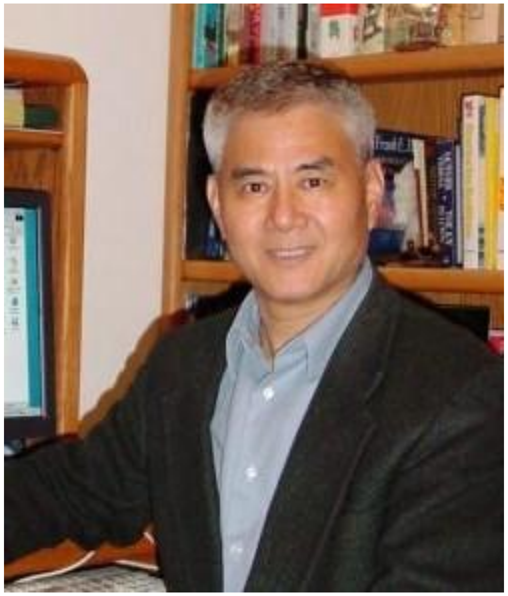
Dimitri P. Bertsekas Stephen P. Boyd Lieven Vandenberghe

0.4. History and People

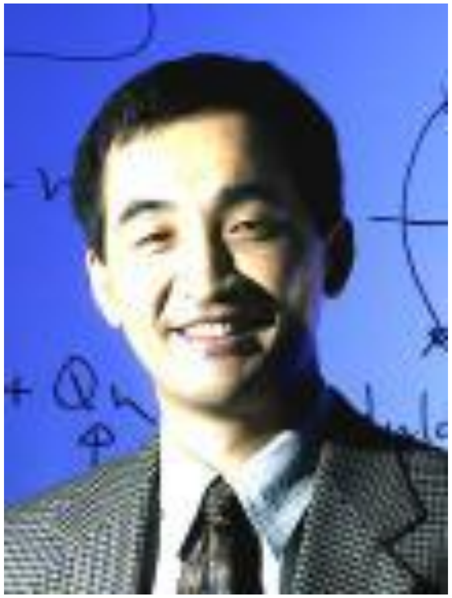
Arkadi Nemirovski

Yurii Nesterov

0.4. History and People



Yinyu Ye



Zhiquan Luo

Shu-Cherng Fang

0.5. Acknowledgement

Thanks to Prof. Stephen Boyd for his kind permission to reuse material of his Course EE364, Stanford University!

Thanks to Ms. Yin Wang (DA Graduate 2008), Mr. Ying Zhu (DA Graduate 2008), Mr. Jun Ding (DA Graduate 2009), Mr. Jiajie Zhang (DA Graduate 2011) for helping typing a part of these slides!

Thanks to Dr. Xiaoling Huang, Dr. Zhengpeng Wu, Mr. Kaidi Yang, Mr. Yingde Chen, Dr. Wei Guo for typing some parts of this book, and Mr. Jiajie Zhang for developing most Matlab code snippets as well as all figures for this book. Thanks to our students for debugging the typos of the book.