THU-70250403, Convex Optimization (Fall 2021) **Homework: 14**

Constrained Minimization (continue)

Lecturer: Li Li li-li@tsinghua.edu.cn

Student:

Problem 1

Consider the following optimization problem:

$$\begin{aligned} \min_{x,y \in \mathbb{R}} \quad & -\log(x+1) - \log(y+2) && (1) \\ \text{s.t.} \quad & x \geqslant 0,\ y \geqslant 0,\ x+y=3 && (2) \end{aligned}$$

1) Please prove that the above problem is a convex optimization problem.

2) Please find the optimal solution and optimal value for the above problem.

Problem 2

Consider the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{R}^n, z \in \mathbb{R}} \quad & z && (3) \\ \text{s.t.} \quad & |\boldsymbol{A}_i \boldsymbol{x}| \leqslant z,\ \forall i = 1, \ldots, m && (4) \\ & \boldsymbol{c}^\top \boldsymbol{x} = 1 && (5) \end{aligned}$$

where $A \in \mathbb{R}^{m \times n}$, $\operatorname{rank}(A) = n$, $\boldsymbol{A}_i$ is the i-th row of A, and $\boldsymbol{c} \in \mathbb{R}^n$, $\boldsymbol{c} \neq \boldsymbol{0}$.

1) Please derive the Lagrangian of the above problem and show the KKT conditions.

2) Let $z^\star$ be the optimal value. Please prove that the optimal dual variable of equality constraint $v^\star \neq 0$ and $v^\star = z^\star$.

Problem 3

Consider the following quadratic programming problem:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \frac{1}{2} \boldsymbol{x}^\top P \boldsymbol{x} + \boldsymbol{q}^\top \boldsymbol{x} \\ \text{s.t.} \quad & A\boldsymbol{x} = \boldsymbol{b}, \\ & \boldsymbol{x} \geqslant \boldsymbol{0}. \end{aligned} \qquad (6)$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $P \in \mathbb{S}_+^n$, $\boldsymbol{q} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $\boldsymbol{b} \in \mathbb{R}^m$.

1) Solve the above problem using barrier method with tolerance $\epsilon = 10^{-10}$. Please plot the logarithmic duality gap $\log(n/t)$ versus Newton step k and give the optimal primal-dual solution $\boldsymbol{x}^\star$, $\boldsymbol{\lambda}^\star$, $\boldsymbol{\nu}^\star$ and the optimal value $p^\star$. Note that $\mu = 10$ is suggested for barrier method.

2) Solve the above problem using primal-dual interior-point method with norm of primal residual $\|\boldsymbol{r}_{\text{pri}}\|_2 \leqslant 10^{-10}$, norm of dual residual $\|\boldsymbol{r}_{\text{dual}}\|_2 \leqslant 10^{-10}$ and surrogate duality gap $\hat{\eta} \leqslant 10^{-10}$. Please plot the logarithmic surrogate duality gap $\log(\hat{\eta})$ and logarithmic norm of the primal and dual residuals $\log\left(\sqrt{\|\boldsymbol{r}_{\text{pri}}\|_2^2 + \|\boldsymbol{r}_{\text{dual}}\|_2^2}\right)$ versus iteration number k, respectively, and give the optimal primal-dual solution $\boldsymbol{x}^\star$, $\boldsymbol{\lambda}^\star$, $\boldsymbol{\nu}^\star$ and the optimal value $p^\star$.

You need to submit the program, the calculation results and the analysis documents electronically, and submit them through the "Course Assignment" section of the Online Learning Center, using the data provided in the folder, where we give P, $\boldsymbol{q}$, A, $\boldsymbol{b}$ and the initial point $\boldsymbol{x}_0$, $\boldsymbol{\lambda}$, $\boldsymbol{\nu}$ with $m = 100$ and $n = 200$. log means logarithm with natural logarithm as the base. Please make sure that the program can be run directly in the folder when submitting the assignment.

References