第 5 次作业

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1. Expanded Michaelis-Menten kinetics

解：根据反应过程，可知

$$\begin{cases} [E_T] = [E] + [EA] + [EAB] \\ \dfrac{dA}{dt} = -k_1[E][A] + k_{-1}[EA] \\ \dfrac{dB}{dt} = -k_2[EA][B] + k_{-2}[EAB] \\ \dfrac{dE}{dt} = -k_1[E][A] + k_{-1}[EA] + k_3[EAB] \\ \dfrac{dP}{dt} = k_3[EAB] \\ \dfrac{dQ}{dt} = k_3[EAB] \end{cases}$$

对EA和EAB使用 quasi-steady-state approximation，则有：

$$\begin{cases} \dfrac{d}{dt}(EA) = k_1[E][A] - k_{-1}[EA] - k_2[EA][B] + k_{-2}[EAB] \approx 0 \\ \dfrac{d}{dt}(EAB) = k_2[EA][B] - k_{-2}[EAB] - k_3[EAB] \approx 0 \end{cases}$$

故

$$\begin{cases} [EAB] = \dfrac{k_2[EA][B]}{k_{-2} + k_3} \\ [EA] = \dfrac{k_1[E][A] + k_{-2}[EAB]}{k_{-1} - k_{-2}[B]} \end{cases}$$

综上，则有

$$\begin{cases} \dfrac{dP}{dt} = \dfrac{dQ}{dt} = k_3[EAB] = \dfrac{V_{max}[AB]}{K + [A][B]} \\ V_{max} = k_3[E_T] \\ K = \dfrac{k_3 + k_{-2}}{k_2}[A] + \dfrac{k_3}{k_1}[B] + \dfrac{k_{-1}(k_2 + k_3)}{k_1 k_2} \end{cases}$$

2. Gene production rate

解：对该过程，我们有微分方程

$$\frac{dY}{dt} = \beta - \alpha Y$$

进行拉普拉斯变换，则有

$$sY(s) - Y(0) = \frac{\beta}{s} - \alpha Y(s)$$

因此

$$Y(s)$$

$$= \frac{Y(0) + \frac{\beta}{s}}{s + \alpha}$$
$$= \frac{sY(0) + \beta}{s(s + \alpha)}$$
$$= \frac{Y(0)}{s + \alpha} + \frac{\beta}{\alpha}\left(\frac{1}{s} - \frac{1}{(s + \alpha)}\right)$$

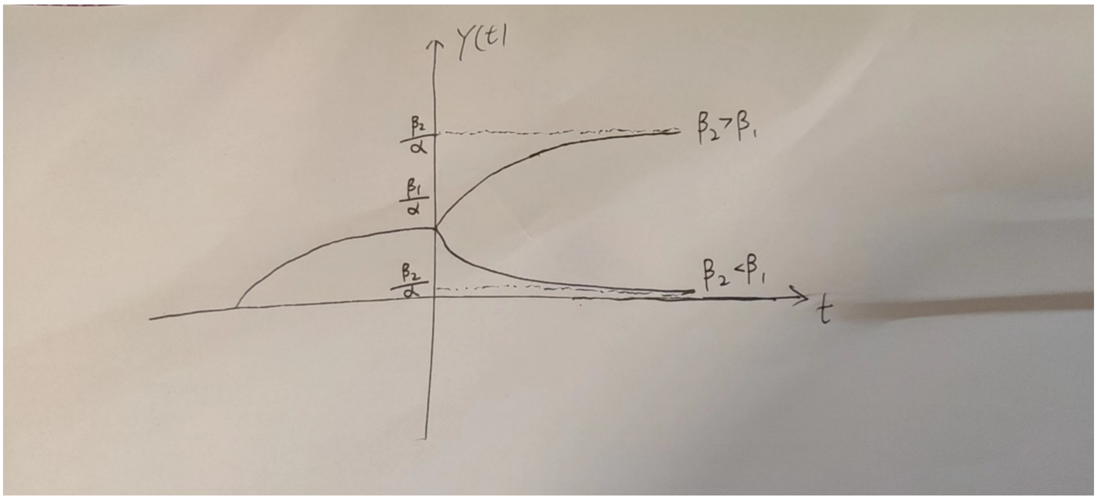
得到

$$Y(t) = (Y(0) - \beta)e^{-\alpha t} + \frac{\beta}{\alpha}, t \geq 0$$

故$Y(\infty) = \frac{\beta}{\alpha}$，且有$\frac{\beta_1 - \beta_2}{\alpha} e^{-\alpha t} = \frac{\beta_1 - \beta_2}{2\alpha}$

故响应时间为$\frac{\ln 2}{\alpha}$

$Y(t)$的图像如下：



3. Competitive regulation

解：对该过程，有

$$\begin{cases} [D_T] = [D] + [DX] + [DY] \\ \frac{d[DX]}{dt} = k_{onX}[X][D] - k_{offX}[DX] \\ \frac{d[DY]}{dt} = k_{onY}[Y][D] - k_{offY}[DY] \end{cases}$$

对EA和EAB使用 quasi-steady-state approximation，则有：

$$\begin{cases} \frac{d}{dt}[DX] \approx 0 \\ \frac{d}{dt}[DY] \approx 0 \end{cases}$$

故输入函数为

$$\beta\frac{[DX]}{[DT]} = \frac{\beta\frac{k_{onX}}{k_{offX}}[X]}{1 + \frac{k_{onX}}{k_{offX}}[X]\frac{k_{onY}}{k_{offY}}[Y]}$$

将$K_i = \frac{k_{offi}}{k_{oni}}$代入进上式，得到输入函数为

$$\frac{\beta \frac{[X]}{K_X}}{1 + \frac{[X]}{K_X} + \frac{[Y]}{K_Y}}$$

若这是相互独立的组合，则有

$$\beta \frac{[DX]}{[DT]} = \frac{\beta \frac{[X]}{K_X}}{1 + \frac{[X]}{K_X} + \frac{[Y]}{K_Y}}$$