THU-70250403, Convex Optimization (Fall 2021) **Homework: 10**

Convex Relaxation and Geometric Problems (continue)

Lecturer: Li Li `li-li@tsinghua.edu.cn`

Student:

(Please finish Problem 1 + either Problem 2 or Problem 3.)

Problem 1

Please derive a mathematical programming model to find the smallest square (each of whose surfaces is parallel to $(n-1)$ axis) center at the original point to cover the ellipse $\boldsymbol{x}^T Q \boldsymbol{x} \leq 1$, $\boldsymbol{x} \in \mathbb{R}^n$, $Q \in \mathbb{S}^n_{++}$. Please numerically determine such a square for $Q = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

Problem 2

Please solve a special case of the GilbertPollak conjecture [1]-[2].

1) Determine one point, the sum of whose distances to the four vertexes of the square reaches the minimum.

2) Suppose we have two points. Let us link them with one edge, link the four vertexes of the square to the closest one of these two points with edges. Determine a special pair of such two points, the sum of whose lengths of the edges reaches the minimum.

3) Compare these two sums of distances.

Problem 3

Please roughly explain how to use convex relaxation to solve trajectory planning problem for soft landing optimal control problem [3]-[4].

References

[1] E. N. Gilbert, H. O. Pollak, "Steiner minimal trees," *SIAM Journal on Applied Mathematics*, vol. 16, no. 1, pp. 1-29, 1968.

[2] D.-Z. Du, F. K. Hwang, "A proof of the GilbertPollak conjecture," *Algorithmica*, vol. 7, no. 2-3, pp. 121-135, 1992.

[3] B. Açikmeşe, L. Blackmore, "Lossless convexification of a class of optimal control problems with non-convex control constraints," *Automatica*, vol. 47, no. 2, pp. 341-347, 2011.

[4] B. Açikmeşe, J. M. Carson III, L. Blackmore, "Lossless convexification of nonconvex control bound and pointing constraints of the soft landing optimal control problem," *IEEE Transactions on Control System Technology*, vol. 21, no. 6, pp. 2104-2113, 2013.