THU-70250403, Convex Optimization (Fall 2021) **Homework: 6**

Dual Problems and Practical Problems

Lecturer: Li Li li-li@tsinghua.edu.cn

Student:

Problem 1

Please drive the corresponding dual problem of the following problem

$$\max_{\boldsymbol{x}} \quad \frac{1}{|\boldsymbol{c}^T\boldsymbol{x} - d|_2} \tag{1}$$
$$s.t. \quad |A\boldsymbol{x} - \boldsymbol{b}|_2 \leq \lambda \tag{2}$$

where $\boldsymbol{x}, \boldsymbol{c} \in \mathbb{R}^n$, $\boldsymbol{b} \in \mathbb{R}^m$, $d, \lambda \in \mathbb{R}$, $A \in \mathbb{R}^{m \times n}$, and $\text{rank}(A) = n$.

Problem 2

Please find the optimal solution of the following problem using KKT condition.

$$\min_{x_1, x_2} \quad x_1^2 - x_2 \tag{3}$$
$$\text{s.t.} \quad x_1 \geq 1,\ x_1^2 + x_2^2 \leq 26,\ x_1 + x_2 = 6 \tag{4}$$

Problem 3

(This problem is Optional)

Twin Support Vector Machine (TWSVM) aims to classify two classes of data by finding two non-parallel hyper-planes in such a manner that the first hyper-plane locates near to the data samples of the first class while distant from the second class of data samples. Vice verse, the second hyper-plane locates near to the data samples of the second class while distant from the first class of data samples [1]-[2].[1]

Let the samples of positive and negative classes are denoted by m and n respectively and positive and negative class data samples are represented by data matrices $X_1 \in R^{k \times m}$ and $X_2 \in R^{k \times n}$. $X_1 = (\boldsymbol{x}_{11}, \ldots, \boldsymbol{x}_{1m})$ and $X_2 = (\boldsymbol{x}_{21}, \ldots, \boldsymbol{x}_{2n})$.

Suppose two non-parallel hyper-planes in k-dimensional real space R_k are parameterized as

$$\boldsymbol{w}_1^T\boldsymbol{x} + b_1 = 0 \tag{5}$$
$$\boldsymbol{w}_2^T\boldsymbol{x} + b_2 = 0 \tag{6}$$

[1] Some researchers argued that the Twin SVM cannot be viewed as a SVM, since all the data points are active to generate the solution of the optimization problems and no sparse supporting vectors exist.

where, symbols $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ indicate normal vectors to the hyperplane,b_1 and b_2 are bias terms. The formulation of TWSVM for linear case is obtained.

The primal problem of soft margin Linear TWSVM can be formulated as

$$\min_{\boldsymbol{w}_1, b_1, \boldsymbol{\xi}} \quad \frac{1}{2}\sum_{i=1}^{m}\left(\boldsymbol{w}_1^T\boldsymbol{x}_{1i} + b_1\right)^2 + C_1\sum_{j=1}^{n}\xi_j \tag{7}$$

$$\text{s.t.} \quad -\left(\boldsymbol{w}_1^T\boldsymbol{x}_{2j} + b_1\right) + \xi_j \geq 1, \ \ j = 1, \ldots, n \tag{8}$$

$$\xi_j \geq 0, \ \ j = 1, \ldots, n \tag{9}$$

$$\min_{\boldsymbol{w}_2, b_2, \boldsymbol{\eta}} \quad \frac{1}{2}\sum_{j=1}^{n}\left(\boldsymbol{w}_2^T\boldsymbol{x}_{2j} + b_2\right)^2 + C_2\sum_{i=1}^{m}\eta_i \tag{10}$$

$$\text{s.t.} \quad -\left(\boldsymbol{w}_2^T\boldsymbol{x}_{1i} + b_2\right) + \eta_i \geq 1, \ \ i = 1, \ldots, m \tag{11}$$

$$\eta_i \geq 0, \ \ i = 1, \ldots, m \tag{12}$$

where slack variables and penalty parameters are represented by $\boldsymbol{\xi}$,$\boldsymbol{\eta}$ and C_1,C_2 respectively.

Please derive their dual problems.

Please write a program code snippet (a CVX based matalb program code is preferred) to implement both primary and dual solution of the Twin SVM. Test your code with some data and show the outcomes. Compare Twin-SVM with SVM on some benchmark problems.

References

[1] Jayadeva, R. Khemchandani, S. Chandra, "Twin support vector machine for pattern classification," *IEEE Tractions on Pattern Analysis and Machine Intelligence*, vol. 29, no. 5, pp. 905-910, 2007.

[2] D. Tomar, S. Agarwal, "Twin Support Vector Machine: A review from 2007 to 2014," *Egyptian Informatics Journal*, vol. 16, pp. 55-69, 2015.