Quantum Cryptocurrency Arbitrage Detection

Zhiyang Deng

Ph.D student of Financial Engineering
School of Business, Stevens Institute of Technology
https://www.overleaf.com/project/638693725e6203d8e7dcf9ce zdeng10@stevens.edu

June 29, 2023

Cryptocurrency Arbitrage

Linear Programming Formulation

Suppose that we have m cryptocurrencies and let r_{ij} denote the given currency exchange rate for converting currency i to currency j, for $i, j = 1, 2, \cdots, m, j \neq i$, which can be understood as a directed graph $(E, V) = (m(m-1), m)$.

Besides, we let binary variables $x_{ij} \in \{0, 1\}$ be the decision variables ($x_{ij} = 1$ means "exchanging from currency i to currency j" is part of our arbitrage chain, and vice versa). Then, we have following optimization problem

$$\max \sum_{(i,j)\in E} c_{ij} x_{ij}, \textit{where } c_{ij} = \log r_{ij}. \tag{1}$$

subject to

(1) the flow conservation constraint: $\sum_{j,(i,j)\in E} x_{ij} = \sum_{j,(j,i)\in E} x_{ji}$ for all $i \in V$

(2) Forbidden to pass through a node twice: $\sum_{j,(i,j)\in E} x_{ij} \leq 1$ for all $i \in V$

Cryptocurrency Arbitrage

Previous QUBO Formulation (Incorrect Version)

The the most profitable arbitrage detection can reformulated as a quadratic unconstrained binary optimization (QUBO) problem:

$$\max \sum_{(i,j)\in E} c_{ij}x_{ij} + \lambda \sum_{i} \Big(\sum_{j,(i,j)\in E} x_{ij} - \sum_{j,(j,i)\in E} x_{ji} \Big)^2 + \mu \sum_{i} \Big(\sum_{j,(i,j)\in E} x_{ij} - 1 \Big) \quad (2)$$

where two constraints are rewritten as the penalty terms, and $\lambda \& \mu$ are Lagrangian multipliers.

QUBO Formulation (Correct Version)

The the most profitable arbitrage detection can reformulated as a quadratic unconstrained binary optimization (QUBO) problem:

$$\max \sum_{(i,j)\in E} c_{ij}x_{ij} + \lambda \sum_{i} \Big(\sum_{j,(i,j)\in E} x_{ij} - \sum_{j,(j,i)\in E} x_{ji} \Big)^2 + \mu \sum_{i} \sum_{j,(i,j)\in E} x_{ij} \Big(\sum_{j,(i,j)\in E} x_{ij} - 1 \Big) \quad (3)$$

where $\lambda \& \mu$ are Lagrangian multipliers for penalty term.

Cryptocurrency Arbitrage

Data Source for Cryptos (Kaiko)

1. Four different cryptos (Bitcoin, ETH, BCH, LTC)

2. Order Book Aggregation data (daily price) and time interval is one month interval (from 2023-03-08 to 2023-04-07).

3. Chose the best bid and ask price (highest bid price and lowest ask price) for each currency and calculate their exchange rates.

4. Implemented QAOA (IBM, incorrect implementation), Quantum Annealing (D-WAVE)

5. Compared the numerical results with CPLEX and Traversal Algorithm (classical algorithms).

Cryptocurrency Arbitrage

Data Source for Classical Currencies (Yahoo Finance)

1. Four different classical currencies (USD, EUR, GBP, CHF)

2. Chose daily "close price" and time interval is one month interval (from 2023-03-08 to 2023-04-07).

3. Implemented Quantum Annealing (D-WAVE)

4. Compared the numerical results with CPLEX and Traversal Algorithm (classical algorithms).

Numerical Results for Crypto Currencies

Table 1 Optimal Arbitrage Detection of Traversal Algorithm, CPLEX and Quantum Annealing (D-WAVE) for past 30 days (4 Cryptos: ETH, BTC, BCH, ILT)

Date	Traversal	CPU Time	CPLEX	CPU Time	D-WAVE	CPU Time	Return Ratio
2023-03-08	(0, 2, 1, 0)	0.0001287s	(0, 0, 0, 0)	0.0697956s	No	0.3108678s	0.898529
2023-03-09	(0, 2, 1, 0)	0.0001014s	(0, 0, 0, 0)	0.0643461s	No	0.2624893s	0.899191
2023-03-10	(0, 2, 1, 0)	0.0001159s	(0, 0, 0, 0)	0.0408667s	No	0.2261098s	0.899078
2023-03-11	(1, 0, 2, 1)	0.0001027s	(0, 0, 0, 0)	0.0452669s	No	0.2266676s	0.900312
2023-03-12	(0, 2, 1, 0)	0.0001615s	(0, 0, 0, 0)	0.0517139s	No	0.2340727s	0.898315
2023-03-13	(0, 2, 1, 0)	0.0001063s	(0, 0, 0, 0)	0.0421877s	No	0.2760107s	0.898999
2023-03-14	(0, 2, 1, 0)	0.0001034s	(0, 0, 0, 0)	0.0378162s	No	0.2462074s	0.898134
2023-03-15	(0, 3, 1, 0)	0.0001026s	(0, 0, 0, 0)	0.1210791s	No	0.2485968s	0.898296
2023-03-16	(0, 2, 1, 0)	0.0001125s	(0, 0, 0, 0)	0.0835017s	No	0.2300412s	0.898366
2023-03-17	(0, 2, 1, 0)	0.0001626s	(0, 0, 0, 0)	0.0761472s	No	0.2240705s	0.898858
2023-03-18	(0, 3, 1, 0)	9.9621e-5s	(0, 0, 0, 0)	0.0736194s	No	0.2583420s	0.898168
2023-03-19	(0, 3, 1, 0)	9.9813e-5s	(0, 0, 0, 0)	0.0499179s	No	0.2435133s	0.898320
2023-03-20	(0, 2, 1, 0)	9.9302e-5s	(0, 0, 0, 0)	0.0805395s	No	0.2945682s	0.898615
2023-03-21	(0, 2, 1, 0)	0.0001002s	(0, 0, 0, 0)	0.0712949s	No	0.2326663s	0.898717
2023-03-22	(0, 3, 1, 0)	0.0002852s	(0, 0, 0, 0)	0.0977219s	No	0.2570551s	0.905242
2023-03-23	(0, 2, 1, 0)	0.0001135s	(0, 0, 0, 0)	0.0547286s	No	0.2187551s	0.898658
2023-03-24	(0, 3, 1, 0)	0.0001004s	(0, 0, 0, 0)	0.0400997s	No	0.2278457s	0.899137
2023-03-25	(2, 1, 0, 2)	9.7608e-5s	(0, 0, 0, 0)	0.0564897s	No	0.2648345s	0.899430
2023-03-26	(2, 1, 0, 2)	9.4794e-5s	(0, 0, 0, 0)	0.0704001s	No	0.2463384s	0.898888
2023-03-27	(0, 2, 1, 0)	9.3460e-5s	(0, 0, 0, 0)	0.0884236s	No	0.2384909s	0.899055
2023-03-28	(0, 2, 1, 0)	9.5154e-5s	(0, 0, 0, 0)	0.0676328s	No	0.2531631s	0.898476
2023-03-29	(0, 3, 1, 0)	9.4774e-5s	(0, 0, 0, 0)	0.1428454s	No	0.2271842s	0.898314
2023-03-30	(0, 2, 1, 0)	9.9051e-5s	(0, 0, 0, 0)	0.1181347s	No	0.2279079s	0.898314
2023-03-31	(0, 2, 1, 0)	9.2597e-5s	(0, 0, 0, 0)	0.0653288s	No	0.2699792s	0.899179
2023-04-01	(2, 1, 0, 2)	9.2115e-5s	(0, 0, 0, 0)	0.0730868s	No	0.2296044s	0.898835
2023-04-02	(0, 2, 1, 0)	0.0001022s	(0, 0, 0, 0)	0.0749110s	No	0.2363743s	0.899031
2023-04-03	(1, 0, 2, 1)	9.1404e-5s	(0, 0, 0, 0)	0.0665275s	No	0.2685084s	0.899494
2023-04-04	(0, 2, 1, 0)	9.2033e-5s	(0, 0, 0, 0)	0.0377184s	No	0.3467756s	0.898513
2023-04-05	(1, 0, 2, 1)	9.1435e-5s	(0, 0, 0, 0)	0.0391821s	No	0.2211354s	0.898421
2023-04-06	(0, 2, 1, 0)	9.6175e-5s	(0, 0, 0, 0)	0.1717798s	No	0.2577657s	0.898555
2023-04-07	(0, 2, 1, 0)	9.2200e-5s	(0, 0, 0, 0)	0.0362277s	No	0.2556961s	0.898342

Numerical Results for Classical Currencies

Table 2 Optimal Arbitrage Detection of Traversal Algorithm, CPLEX and Quantum Annealing (D-WAVE) for past 30 days (4 Traditional Currencies: CHF, EUR, GBP, USD)

Date	Traversal	CPU Time	CPLEX	CPU Time	D-WAVE	CPU Time	Return Ratio
2023-03-08	(0, 1, 3, 0)	0.0001252s	(0, 1, 3, 0)	0.0860032s	Yes	0.1719861s	1.0000532
2023-03-09	(1, 2, 3, 1)	9.6498e-5s	(1, 3, 1)	0.0541646s	No	0.1922679s	0.9999295
2023-03-10	(0, 3, 1, 0)	9.4752e-5s	(0, 3, 1, 0)	0.0400733s	Yes	0.1591266s	1.0000064
2023-03-13	(0, 3, 1, 0)	9.3219e-5s	(0, 0, 0, 0)	0.0410824s	No	0.1900348s	0.9999530
2023-03-14	(0, 2, 3, 0)	9.3184e-5s	(0, 3, 0)	0.0986906s	No	0.1968059s	0.9999862
2023-03-15	(0, 3, 1, 0)	0.0001002	(2, 3, 2)	0.1164716s	No	0.2107165s	0.9999752
2023-03-16	(0, 3, 1, 0)	9.4137e-5s	(0, 3, 1, 0)	0.1088298s	Yes	0.1596797s	1.0001705
2023-03-17	(0, 3, 1, 0)	9.6505e-5s	(0, 3, 0)	0.0886378s	No	0.2038638s	0.9999762
2023-03-20	(1, 2, 3, 1)	9.2511e-5s	(2, 3, 2)	0.0858606s	No	0.1906189s	0.9999200
2023-03-21	(1, 2, 3, 1)	9.1975e-5s	(1, 2, 3, 1)	0.0798848s	Yes	0.2079759s	1.0000683
2023-03-22	(0, 3, 1, 0)	9.3123e-5s	(1, 3, 1)	0.0522734s	No	0.2453234s	0.9999762
2023-03-23	(0, 3, 1, 0)	9.2806e-5s	(2, 3, 2)	0.0872442s	No	0.2497159s	0.9999867
2023-03-24	(0, 3, 1, 0)	9.2988e-5s	(0, 3, 1, 0)	0.0418490s	Yes	0.2138448s	1.0001565
2023-03-27	(0, 2, 3, 0)	9.3071e-5s	(0, 2, 3, 0)	0.0411740s	Yes	0.1779079s	1.0008026
2023-03-28	(0, 3, 1, 0)	9.2709e-5s	(2, 3, 2)	0.0711443s	No	0.2235346s	0.9999608
2023-03-29	(0, 3, 2, 0)	9.7499e-5s	(2, 3, 2)	0.0764389s	No	0.1794511s	0.9999248
2023-03-30	(0, 3, 1, 0)	9.6654e-5s	(0, 3, 1, 0)	0.1734708s	Yes	0.1551328s	1.0001164
2023-03-31	(0, 3, 1, 0)	9.2066e-5s	(0, 3, 0)	0.1472245s	No	0.1904517s	0.9999910
2023-04-03	(0, 3, 1, 0)	9.5160e-5s	(0, 3, 0)	0.0963806s	No	0.1709726s	0.9999022
2023-04-04	(1, 3, 2, 1)	9.4697e-5s	(1, 3, 2, 1)	0.0876040s	Yes	0.1713712s	1.0002145
2023-04-05	(0, 3, 2, 0)	9.3415e-5s	(0, 3, 2, 0)	0.0673915s	Yes	0.1621442s	1.0003018
2023-04-06	(0, 3, 1, 0)	9.1579e-5s	(2, 3, 2)	0.0412772s	No	0.2080176s	0.9999946

Cryptocurrency Arbitrage

Next Step

1. Implementation of corrected version of QAOA algorithm on IBM machine.

2. Increase the number of cryptos (from 4 currencies to 20 currencies, then the binary variables of QUBO will be increased from 12 variables to 380 variables) and add more trading exchange center (Our dataset is limited in two trading exchanges. Cross-Exchange Arbitrage is more reasonable).

3. Intraday Optimal Arbitrage Detection for Cyptos.

Thank You!