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Multivariate Data Analysis

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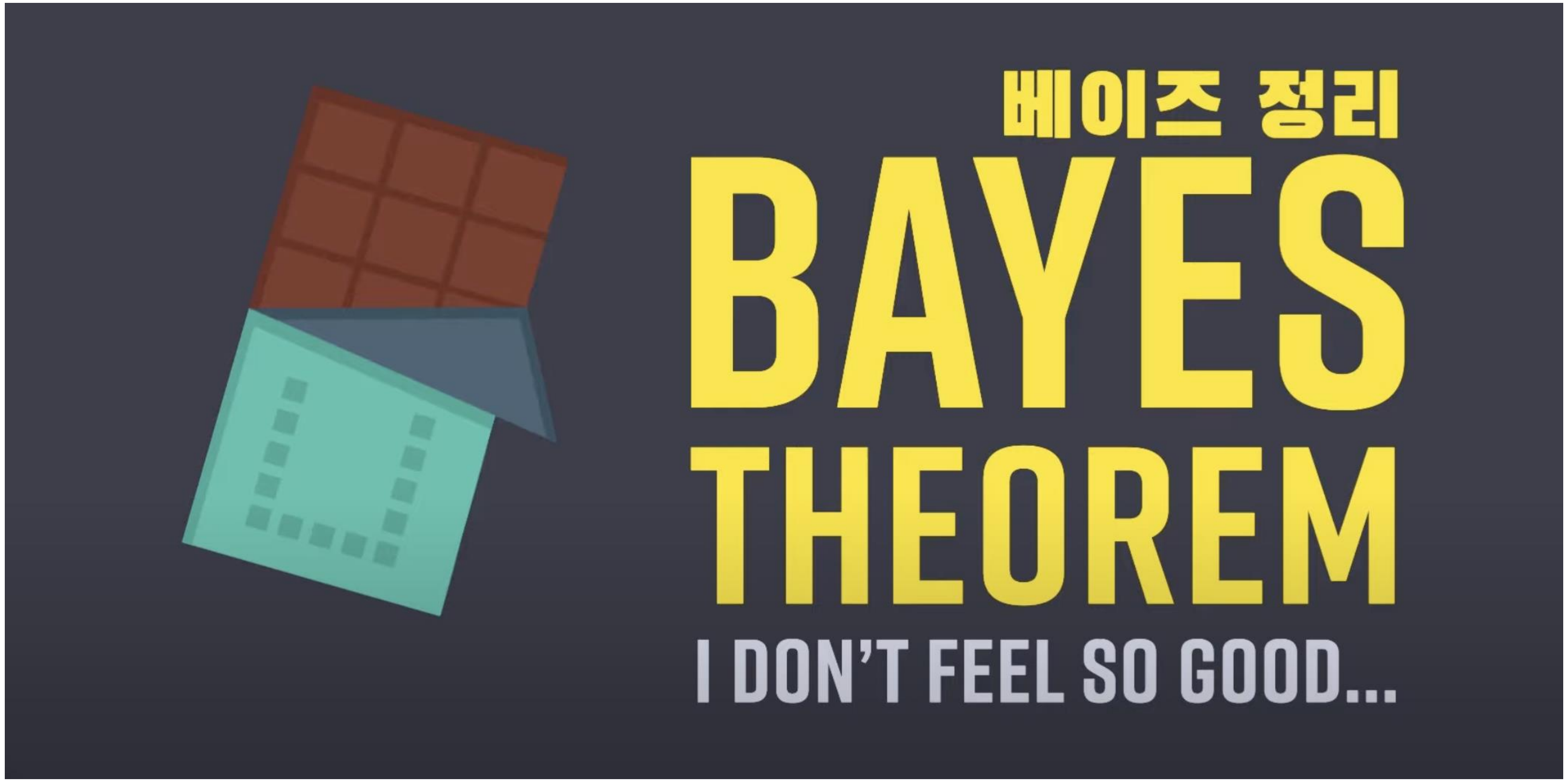
Bayes Theorem

◆ **The easiest way to understand Bayes Theorem**

➢ https://youtu.be/Y4ecU7NkiEI?si=X3oaMmM3A_3YQ6yG



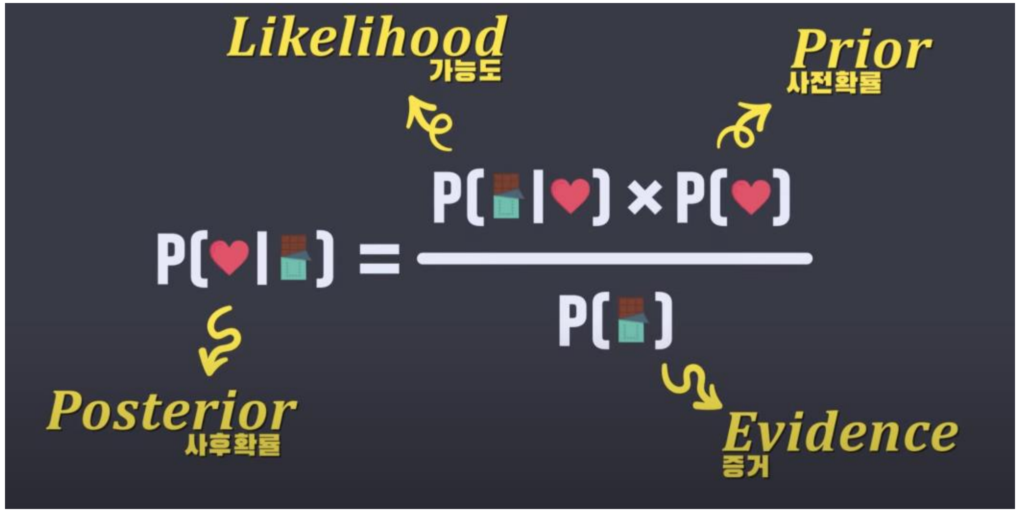
Bayes Theorem

Likelihood Prior

Posterior

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

Evidence



- **H** : **Hypothesis**. It means a hypothesis or 'a claim that an event occurred.'
- **E** : **Evidence**. It means an evidence or 'new information'.
- **P(H)** : This is the **prior probability**. It reflects the initial belief about the probability of the hypothesis H before considering the evidence E. This might be based on previous knowledge or statistical data prior to the new evidence.
- **P(E)** : Known as the **marginal probability** of E, this term acts as a normalizing constant ensuring that the probabilities sum up to 1. It is calculated by summing the probabilities of observing E across all possible outcomes of H, weighted by their respective probabilities:
- **P(E|H)** : Known as the **likelihood**, this term describes the probability of observing the evidence *E* assuming that the hypothesis *H* is true. For example, this could be the probability of observing certain symptoms given a particular disease.
- **P(H|E)** : This is the **posterior probability**. It represents the probability of the hypothesis H being true given that evidence E has occurred. For instance, in a medical diagnosis scenario, this could be the probability of having a disease H given the presence of symptoms E.

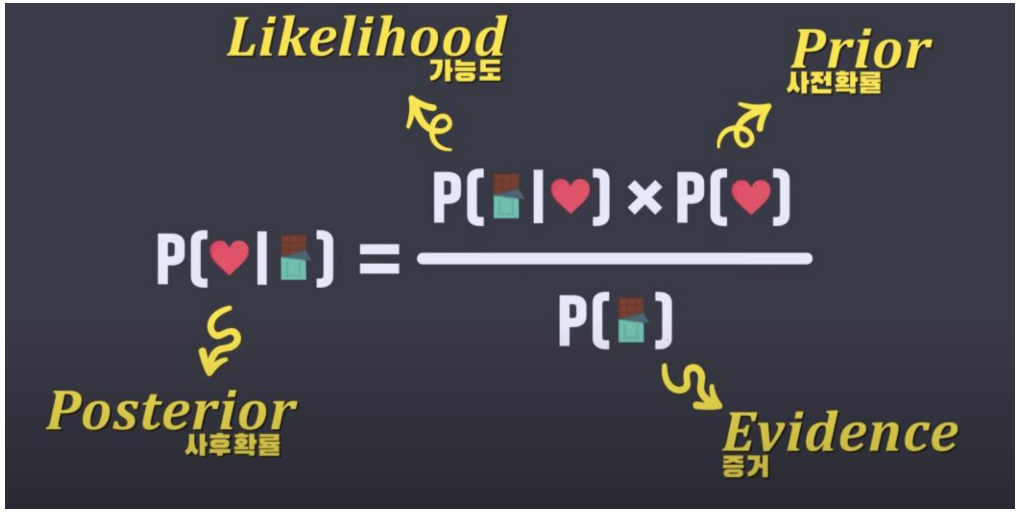
Bayes Theorem

Likelihood Prior

Posterior

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

Evidence



- Example:

35% Obervation (Evidence) 55%

P(H) + P(E) P(H | E)

Bayes Theorem

Likelihood

$P(E|H)$

Deductive Reasoning

vs.

Posterior

$P(H|E)$

Inductive Reasoning

- Deductive reasoning and inductive reasoning are two fundamental approaches in logic and reasoning, each serving a unique purpose in problem-solving and decision-making processes. Here's a brief explanation of each:
- **Deductive Reasoning**
  - ➢ Deductive reasoning starts with a general statement or hypothesis and examines the possibilities to reach a specific, logical conclusion. It uses the top-down approach. If the premises are true and the reasoning is valid, the conclusion must also be true. For example, if we know all humans are mortal (general statement) and John is a human (specific instance), then we can deduce that John is mortal (conclusion). Deductive reasoning is often used in mathematical proofs and logical problems where the goal is to derive a conclusion strictly bound by the premises.
- **Inductive Reasoning**
  - ➢ Inductive reasoning works in the opposite direction, moving from specific observations to broader generalizations and theories. This is known as a bottom-up approach. Inductive reasoning involves making numerous observations, discerning a pattern, making a generalization, and inferring an explanation or a theory. It is inherently probabilistic, meaning that the conclusions reached are likely but not necessarily certain. For example, observing that the sun has risen every morning in our lifetime, we may induce that the sun will rise every morning (general conclusion). Inductive reasoning is commonly used in scientific inquiry and everyday decision-making when deriving general rules from specific examples.
- In summary, deductive reasoning provides conclusions that are as certain as the premises themselves, whereas inductive reasoning offers probabilistic conclusions that generalize beyond the original observations.

Bayes Theorem

- Let's solve the problem using Bayes' theorem.
  - (Example1) The incidence of cancer A is known to be 0.1%.
    Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - If a person is diagnosed with cancer, what is the probability that this person really has the cancer?

Bayes Theorem

- Let's solve the problem using Bayes' theorem.
  - (Example1) The incidence of cancer A is known to be 0.1%.
    Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - If a person is diagnosed with cancer, what is the probability that this person really has the cancer? **-> P(H|E)?**

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

(Quiz) Calculate it!

Bayes Theorem

- Let’s solve the problem using Bayes’ theorem.

  - (Example1) The incidence of cancer A is known to be 0.1%.
    Let’s suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - If a person is diagnosed with cancer, what is the probability that this person really has the cancer? **-> P(H|E)?**

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

- Hypothesis: “this person really has the cancer.“
- Evidence: this person was diagnosed as having a cancer.

- The probability that a random person has this cancer : **P(H)** = 0.001
- The probability of being diagnosed as having this cancer when it actually exists (sensitivity) : **P(E|H)** = 0.99
- The probability of being diagnosed as not having the cancer when it does not actually exist (specificity) : **P(~E|~H)** = 0.98
- How to get the value for **P(E)** ?

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + \underline{P(E|H^c)P(H^c)}}$$

We don’t know this as well

Confusion Matrix

Predicted Category	Actual Category: Positive	Actual Category: Negative
Positive	TP	FP
Negative	FN	TN

- Accuracy(정확도) = $\dfrac{TP + TN}{TP + TN + FP + FN}$
- Recall(재현도) = $\dfrac{TP}{TP + FN}$
- Precision(정밀도) = $\dfrac{TP}{TP + FP}$
- F-Score = $2 \times \dfrac{(precision) \times (recall)}{(precision + recall)}$

Confusion Matrix

Actual Category

Predicted Category

	Positive	Negative
Positive	TP	FP
Negative	FN	TN

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative
	Positive	TP	FP
	Negative	FN	TN

P(H)
=0.001

- P(H) = 0.001
  : Actual(H)=1

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative
	Positive	TP	FP
	Negative	FN	TN

P(~H)
=0.999

- P(H) = 0.001
  : Actual(H)=1

- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

Confusion Matrix

Actual Category

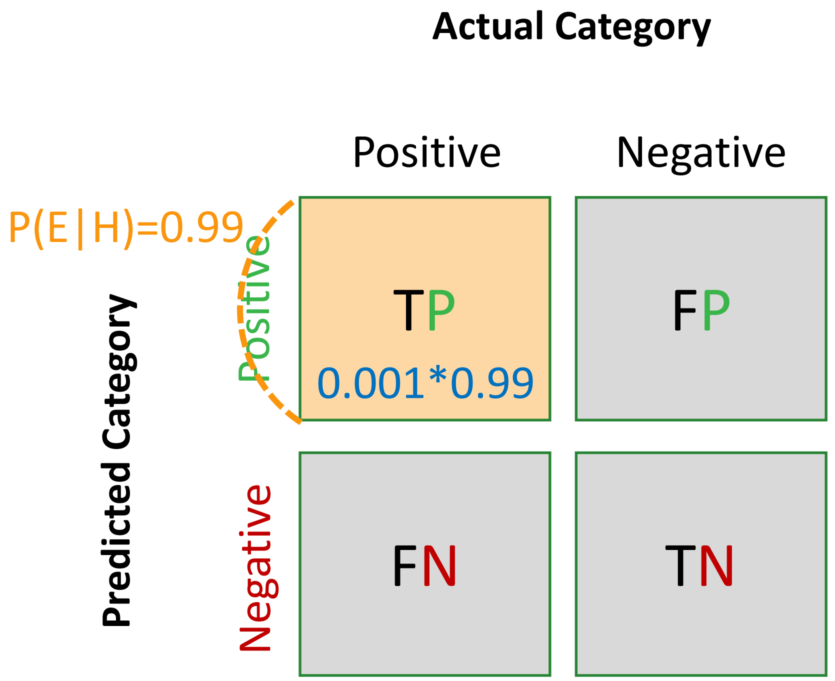
Predicted Category		Positive	Negative
	Positive	TP	FP
	Negative	FN	TN

P(E|H)=0.99

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP

Confusion Matrix

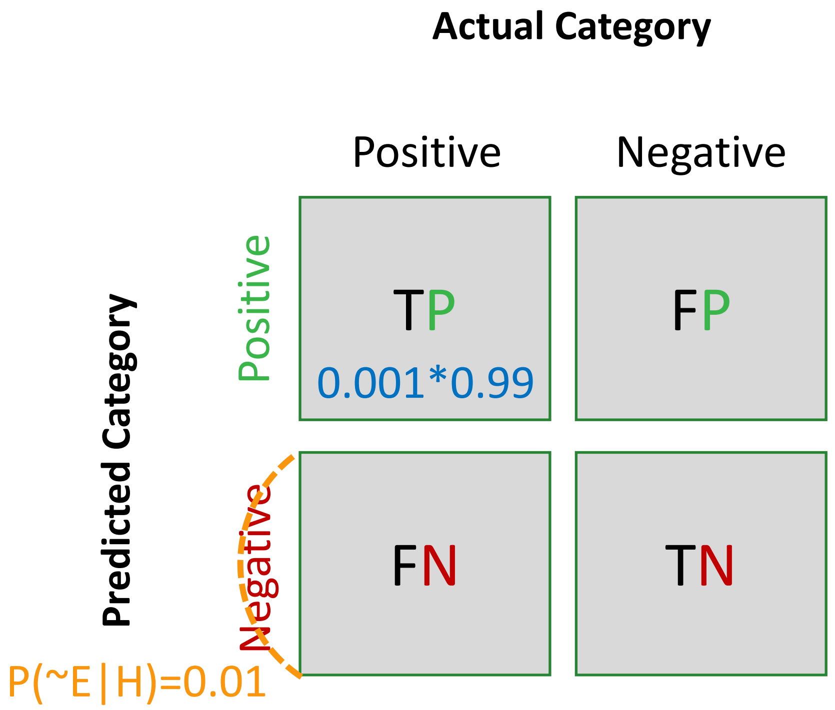


- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99

Confusion Matrix



- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99

Confusion Matrix

		Actual Category	
		Positive	Negative
Predicted Category	Positive	TP 0.001*0.99	FP
	Negative	FN 0.001*0.01	TN

P(~E|H)=0.01

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative
	Positive	TP 0.001*0.99	FP
	Negative	FN 0.001*0.01	TN

P(~E|~H)
=0.98

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01

Confusion Matrix

Predicted Category		Actual Category: Positive	Actual Category: Negative
	Positive	TP 0.001*0.99	FP
	Negative	FN 0.001*0.01	TN 0.999*0.98

P(~E|~H) =0.98

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01
- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Confusion Matrix

		Actual Category	
		Positive	Negative
Predicted Category	Positive	TP 0.001*0.99	FP
	Negative	FN 0.001*0.01	TN 0.999*0.98

P(E|~H)
=0.02

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01
- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative
	Positive	TP 0.001*0.99	FP 0.999*0.02
	Negative	FN 0.001*0.01	TN 0.999*0.98

P(E|~H)
=0.02

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01
- P(~H) * P(E|~H) = 0.01998
  = 0.999 * 0.02
- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Confusion Matrix

Actual Category

Predicted Category	Positive	Negative
Positive	TP 0.001*0.99	FP 0.999*0.02
Negative	FN 0.001*0.01	TN 0.999*0.98

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01
- P(~H) * P(E|~H) = 0.01998
  = 0.999 * 0.02
- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Confusion Matrix

		Actual Category	
		Positive	Negative
Predicted Category	Positive	TP 0.001*0.99	FP 0.999*0.02
	Negative	FN 0.001*0.01	TN 0.999*0.98

P(E)
=0.02079

- P(E) = 0.02097
: Predicted(E)=1
= 0.001*0.99 + 0.999*0.02 = 0.02079

- P(H) = 0.001
: Actual(H)=1

- P(~H) = 0.999 (1-0.001)
: Actual(H)=0

- P(E|H) = 0.99
: Actual(H)=1, Predicted(E)=1
= TP

- P(~E|H) = 0.01 (1-0.99)
: Actual(H) = 1, Predicted(E)=0
= FN

- P(E|~H) = 0.02 (1-0.98)
: Actual(H)=0, Predicted(E)=1
= FP

- P(~E|~H) = 0.98
: Actual(H) = 0, Predicted(E)=0
= TN

- P(H) * P(E|H) = 0.00099
= 0.001 * 0.99

- P(H) * P(~E|H) = 0.00001
= 0.001 * 0.01

- P(~H) * P(E|~H) = 0.01998
= 0.999 * 0.02

- P(~H) * P(~E|~H) = 0.97902
= 0.999 * 0.98

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative	
	Positive	TP 0.001*0.99	FP 0.999*0.02	
	Negative	FN 0.001*0.01	TN 0.999*0.98	P(~E) =0.97903

- P(E) = 0.02097
  : Predicted(E)=1
  = 0.001*0.99 + 0.999*0.02 = 0.02079

- P(~E) = 0.97903
  : Predicted(E)=0
  =0.001*0.01 + 0.999*0.98 = 0.97903

- P(H) = 0.001
  : Actual(H)=1

- P(~H) = 0.999 (1-0.001)
  : Actual(H)=0

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP

- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN

- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP

- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99

- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01

- P(~H) * P(E|~H) = 0.01998
  = 0.999 * 0.02

- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Bayes Theorem

- Let’s solve the problem using Bayes’ theorem.

> - (Example1) The incidence of cancer A is known to be 0.1%.
>   Let’s suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
> - If a person is diagnosed with cancer, what is the probability that this person really has the cancer? **-> P(H|E)?**

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

- Hypothesis: “this person really has the cancer.“
- Evidence: this person was diagnosed as having a cancer.

(Quiz) Calculate it!

- The probability that a random person has this cancer : **P(H)** = 0.001
- The probability of being diagnosed as having this cancer when it actually exists (sensitivity) : **P(E|H)** = 0.99
- The probability of being diagnosed as not having the cancer when it does not actually exist (specificity) : **P(~E|~H)** = 0.98
- How to get the value for **P(E)** ?

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)}$$

Bayes Theorem

- Let's solve the problem using Bayes' theorem.

> - (Example1) The incidence of cancer A is known to be 0.1%.
>   Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
> - If a person is diagnosed with cancer, what is the probability that this person really has the cancer? **-> P(H|E)?**

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.02 \times 0.999}$$

$$= 0.047$$

- Hypothesis: "this person really has the cancer."
- Evidence: this person was diagnosed as having a cancer.
- The probability that a random person has this cancer : **P(H)** = 0.001
- The probability of being diagnosed as having this cancer when it actually exists (sensitivity) : **P(E|H)** = 0.99
- The probability of being diagnosed as not having the cancer when it does not actually exist (specificity) : **P(~E|~H)** = 0.98
- How to get the value for **P(E)** ?

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)}$$

Bayes Theorem

- Let's solve the problem using Bayes' theorem.
  - (Example 2) The incidence of cancer A is known to be 0.1%.
    Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - A patient was already diagnosed with cancer in the past.
  - If this person is diagnosed with cancer a second time, what is the probability that this person really has the cancer? **-> P(H|E)?**

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)}$$

(Quiz) Calculate it!

Bayes Theorem

- Let's solve the problem using Bayes' theorem.
  - (Example 2) The incidence of cancer A is known to be 0.1%.
    Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%, and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - A patient was already diagnosed with cancer in the past.
  - If this person is diagnosed with cancer a second time, what is the probability that this person really has the cancer? **-> P(H|E)?**

- Hypothesis: "A patient was already diagnosed with cancer in the past, and this person really has the cancer."
- Evidence: this person was diagnosed as having a cancer.

- The probability that a random person has this cancer : **P(H)** = Example1's **P(H|E)** = 0.047
- The probability of being diagnosed as having this cancer when it actually exists (sensitivity) : **P(E|H)** = 0.99
- The probability of being diagnosed as not having the cancer when it does not actually exist (specificity) : **P(~E|~H)** = 0.98
- How to get the value for **P(E)** ?

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)}$$

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative
	Positive	TP 0.047*0.99	FP 0.953*0.02
	Negative	FN 0.047*0.01	TN 0.953*0.98

- P(H) = 0.047
  : Actual(H)=1
- P(~H) = 0.953 (1- 0.047)
  : Actual(H)=1

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.047 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.047 * 0.01
- P(~H) * P(E|~H) = 0.01998
  = 0.953 * 0.02
- P(~H) * P(~E|~H) = 0.97902
  = 0.953 * 0.98

Confusion Matrix

		Actual Category	
		Positive	Negative
Predicted Category	Positive	TP 0.047*0.99	FP 0.953*0.02
	Negative	FN	TN

P(E)
=0.06559

- P(E) = 0.02097
  : Predicted(E)=1
  = 0.047*0.99 + 0.953*0.02 = 0.06559

- P(H) = 0.001
  : Actual(H)=1
- P(~H) = 0.999 (1-0.001)
  : Actual(H)=0

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP
- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN
- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP
- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99
- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01
- P(~H) * P(E|~H) = 0.01998
  = 0.999 * 0.02
- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Confusion Matrix

Actual Category

Predicted Category		Positive	Negative	
	Positive	TP 0.047*0.99	FP 0.953*0.02	
	Negative	FN 0.047*0.01	TN 0.953*0.98	P(~E) =0.93441

- P(E) = 0.02097
  : Predicted(E)=1
  = 0.047*0.99 + 0.953*0.02 = 0.06559

- P(~E) = 0.97903
  : Predicted(E)=0
  =0.047*0.01 + 0.953*0.98 = 0.93441

- P(H) = 0.001
  : Actual(H)=1

- P(~H) = 0.999 (1-0.001)
  : Actual(H)=0

- P(E|H) = 0.99
  : Actual(H)=1, Predicted(E)=1
  = TP

- P(~E|H) = 0.01 (1-0.99)
  : Actual(H) = 1, Predicted(E)=0
  = FN

- P(E|~H) = 0.02 (1-0.98)
  : Actual(H)=0, Predicted(E)=1
  = FP

- P(~E|~H) = 0.98
  : Actual(H) = 0, Predicted(E)=0
  = TN

- P(H) * P(E|H) = 0.00099
  = 0.001 * 0.99

- P(H) * P(~E|H) = 0.00001
  = 0.001 * 0.01

- P(~H) * P(E|~H) = 0.01998
  = 0.999 * 0.02

- P(~H) * P(~E|~H) = 0.97902
  = 0.999 * 0.98

Bayes Theorem

- Let's solve the problem using Bayes' theorem.
  - (Example 2) The incidence of cancer A is known to be 0.1%.
    Let's suppose that the probability of being diagnosed as having this cancer when it actually exists (sensitivity) is 99%,
    and the probability of being diagnosed as not having the cancer when it does not actually exist (specificity) is 98%.
  - A patient was already diagnosed with cancer in the past.
  - If this person is diagnosed with cancer a second time, what is the probability that this person really has the cancer? **-> P(H|E)?**

- Hypothesis: "A patient was already diagnosed with cancer in the past, and this person really has the cancer."
- Evidence: this person was diagnosed as having a cancer.

- The probability that a random person has this cancer : **P(H)** = Example1's **P(H|E)** = 0.047
- The probability of being diagnosed as having this cancer when it actually exists (sensitivity) : **P(E|H)** = 0.99
- The probability of being diagnosed as not having the cancer when it does not actually exist (specificity) : **P(~E|~H)** = 0.98
- How to get the value for **P(E)** ?

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)} = \frac{0.047 \times 0.99}{0.047 \times 0.99 + 0.02 \times 0.953} = 0.709$$

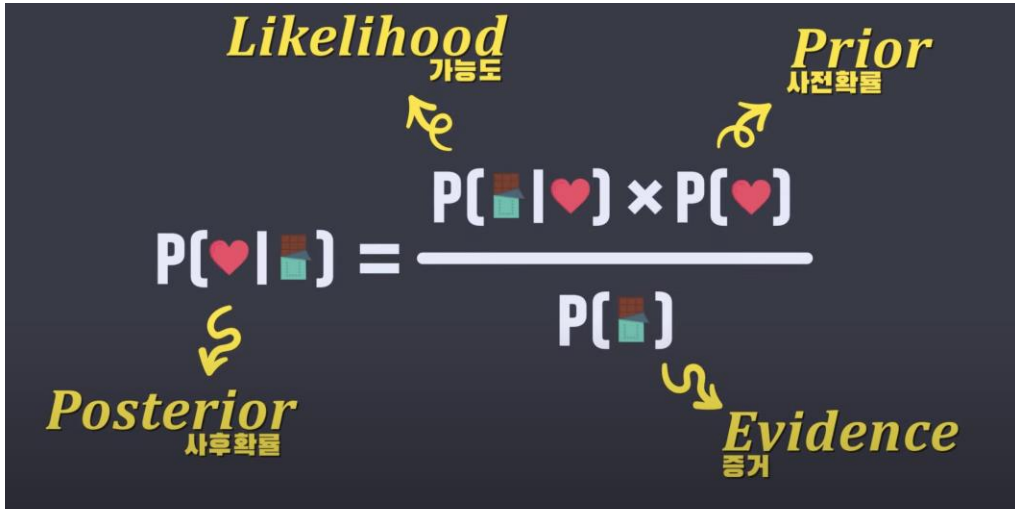
Bayes Theorem

Likelihood Prior

Posterior

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

Evidence



- Example:

4.7%

P(H)

Obervation
(Evidence)

+ P(E)

70.9%

P(H | E)

Bye Bye