Alexandre Kirillov and Luiza Churaeva (wife) Highlights

Moscow, September 13, 1989

A. Before the University

E. D. Today is Wednesday, September 13, 1989. We are in the hotel of the Academy of Sciences of the USSR, speaking with Alexandre Kirillov and his wife Luiza Churaeva. Let's start with you, Sasha. You must have developed a keen interest in mathematics already in high school —as far as I remember, you excelled in mathematical Olympiads and were later involved in organizing them. So tell us about the beginning of your mathematical career.

A. K. In the first grade I wanted to become a pilot, but in the seventh my sights were already firmly set on mathematics. There must have been some kind of transition in between, but I don't remember how it came about. In the fifth and the sixth grade my teacher sent me to regional mathematical Olympiads. I liked them a lot. I solved all the problems and even received some prizes. However, at the time I still didn't pay any particular attention to mathematics.

My first mentor was Igor Arshon, a university student who ran a mathematical circle and was a big enthusiast of math education. We met by pure chance. He was a classmate of the older brother of one of my classmates. They told him that I was capable of solving any math problem. Arshon met with me in my classmate's apartment. He gave me some problems to work on, and I solved most of them right away. He said that he would come up with more difficult ones, but I solved them as well. We met a few more times and that was it.

About the same time I already started to read popular books on mathematics, especially the series *Library of Mathematical Circle*. My first books in the series I received as a prize and later started buying them in bookstores. They were extremely cheap at the time.

While in high school, every year I participated in the Moscow Mathematical Olympiad. I was the winner for three consecutive years. However, I was shy and rarely attended math circles. I preferred the Olympiads; there I just answered questions in

writing, whereas in a circle I had to talk. It was not until my last year of high school that I joined the circle of Sasha Krylov, then a senior undergrad at MSU.

My third mentor was Anatoli Vitushkin,[1] to whom I and some other freshmen had been recommended by Sasha Krylov. Vitushkin was a graduate student at the time and continued the famous circle founded by Alexander Kronrod. Its ideology can be traced back to the Lusin School of real analysis: we were not encouraged to read books, but had to prove all the basic theorems (including e.g., Radon-Nicodym theorem for measures, differentiability almost everywhere of a monotone function, etc) by ourselves.

The fourth mentor and my first real teacher were you, Evgeni Borisovich. The fifth was Gelfand. This is the full list of my teachers.

B. First research experience

E. D. Do you remember my algebra course which you took as a freshman?

A. K. Yes, I liked it a lot, especially the part about the Mikhailov curve. Dima Arnold and I wrote a paper on it together.

E. D. The topic seems a little bit outdated. My source was a conversation with Misha Postnikov who taught a freshman algebra course at the same time.

A. K. It was completely new to us then. We have seen nothing like that in our textbooks. Arnold and I were very excited about this result and decided to consider not only half-plane but also other domains. Already then we approached the problem differently: Arnold thought it was more important to describe the general case, whereas I was convinced that it would be interesting to study the singular points as well. Perhaps these were the first signs of our future rivalry in various fields of mathematics.

As a freshman I never read mathematical papers before. So when we set about writing our paper I didn't have the slightest idea how to begin. Arnold said, "Let's begin with the following: *Let φ be* ..." I was shocked. I couldn't imagine one could start a paper in such an abrupt way. Later, after many years, Arnold himself used to poke fun at this style of writing.

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[1] http://en.wikipedia.org/wiki/Anatoli_Georgievich_Vitushkin

The paper was not published because Arnold discovered our result in a 1913 algebra textbook of D. A. Grave, a mathematician from Kiev.

E. D. Your first published paper was about spherical vector fields, right?

A. K. Yes, I wrote it in your seminar, but it turned out to have been not entirely original either: Naum Yakovlevich Vilenkin[2] felt a bit offended that I didn't mention his paper on this subject.

E. D. It was my fault. I wasn't aware of his work. But even though your result could be deduced from his, it was not stated there explicitly.

A. K. True, Vilenkin's result was more general. Nowadays I could assign to derive my formula from his general theorem as an exercise to my students. However, thirty years ago this wasn't so obvious.

E. D. Your paper was published in the *Doklady*, wasn't it?

A. K. Yes, and I was very proud of this publication.

C. "Mind reading" in Lie groups seminar

E. D. I remember you told me a funny story about an impression made on you by Piatetski-Shapiro[3] at our seminar on Lie groups. It deserves to be recorded.

A. K. Yes. One time Berezin[4] was giving a talk at your seminar. As he was about to conclude, Piatetski-Shapiro stood up in the back row of the auditorium, waited until all eyes were on him, and said, "Here you can play up the following …" He then fell silent. There was a long pause, as nobody knew how to react. Berezin took some time to think and replied, "No you can't play it up." Piatetski-Shapiro took this remark very seriously. He came forward to the blackboard, propped himself against the rostrum and said, "Yes, you can. I did it". He then went back to his seat. The auditorium burst into laughter. Nobody had the slightest idea what he was talking about.

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[2] http://en.wikipedia.org/wiki/Naum_Vilenkin
[3] His interview is a part of the present Collection.
[4] http://en.wikipedia.org/wiki/Felix_Berezin

D. The Golden Years of Mekhmat

A. K. The general atmosphere of those years (1955-65) was splendid, and it has never been the same again. What happens in Mekhmat today does not stand comparison. I was lucky to belong to a group of 20-25 students who were united in their admiration for talent and were genuinely interested in each other's work. I always considered it my duty to get to know the best students in my year and to learn about their accomplishments. Even on our winter ski expeditions we would find time to discuss math problems.

E. D. Were there many strong students in your year?

A. K. Yes, we had Arnold, Vinberg, Freidlin, Arkhangelski, Palamodov, Chernavski, Shur ... I may forget some of them, but we had ten or fifteen students who were very strong. There were also many good students a couple of years before and after: Sinai, Tikhomirov, Anosov, Gorin, Lin, Novikov, Fuchs, Tyurina... Then there was a ten year gap before the new wave arrived: Kazhdan, Margulis, Bernstein etc. About fifteen students from my year became Doctors of Physical and Mathematical Sciences. I would say that the most famous person from my year is Arnold. Many of these people have become distinguished mathematicians, well-known in Russia and abroad.

This positive atmosphere persisted for a few years and then gradually dissipated. There was not a trace of it in ten years. You witnessed at least part of the period when things started to get worse. I remember how Gorin, in my presence, was explaining to his students the difference between students from Mekhmat and students from other departments or from other universities. He said, "Take any university student and tell him that he can have the diploma without finishing his studies. They would take it and leave without hesitation. Students from Mekhmat, by contrast, would say that they are interested primarily in mathematics and that diploma comes second". I suspect this distinction no longer holds true.

E. D. What is the current situation in Mekhmat?

A. K. I would say we are experiencing a kind of revival. The number of applications went up. For some reason there is a renewed interest in mathematics.

E. D. When has this trend started?

A. K. I have reached this conclusion on the basis of the growing number of

participants in the Moscow Mathematical Olympiads. Eight years ago when my son[5] (now a graduate student) participated in the Olympiad, being in the 7th grade, we had one thousand five hundred participants instead of the expected three hundred. Also there were a number of bright students in his entering class. Unfortunately, many of them were drafted into the army, and when they returned they were no longer focused on mathematics.

E. D. Were university students drafted for several consecutive years?

A. K. All of them were subject to draft for three consecutive years, and then for another two years some of them were and others weren't. My son was lucky. He wasn't drafted. He had a concussion in high school and was disqualified.

E. D. Do you call this "lucky"?

A. K. Well, obviously it's awful to suffer this kind of injury, but at least it served him well later.

E. The Gelfand seminar

E. D. Coming back to your student years, what do you remember about the seminar of Gelfand during those years? In 1944 I was one of its first participants. It was very small then. In this seminar I talked about the works of Hermann Weyl and Van der Warden, and this was the origin of my simple roots diagrams.

A. K. When I joined it, it was already quite big. I was brought there by Berezin, who told Gelfand that I could solve any math problem. Gelfand pshawed in disbelief and asked me about the structure of ideals in matrix rings. He put me on the spot. I couldn't give him an instant answer. He pshawed again and let me go. In a week I found the answer and told him.

As for his pedagogical approach, it surprised me from the very first day. He was concise and unceremonious at the same time.

E. D. He always focused on the subject at hand.

A. K. Yes, and he could be rude with people of any rank.

E. D. And yet he treated different people differently. For example, he was always

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[5] Alexander Kirillov, Jr.(see http://www.math.sunysb.edu/~kirillov/)

lenient toward you.

A. K. Yes, especially when I was a student, but this is no longer the case.

E. D. So what's going on with his seminar now and how is he doing?

A. K. Gelfand is still doing a great job because the last year's seminar was again a huge success. There were a group of bright students, mostly freshmen and sophomores who were supervised by several fifth year undergraduates and first year graduates. Together they formed the nucleus of the seminar. As Gelfand himself used to joke, "the seminar was intended primarily for schoolboys and talented PhD students. Only a few of the professors can get in."

E. D. What's the main focus of the seminar these days?

A. K. It is difficult to isolate one particular topic. More recently, Gelfand's research interests veered toward general hypergeometric functions and their connection to combinatorics, algebraic geometry, and representation theory.

E. D. At Cornell there is a good specialist in combinatorics, Billera.[6] I have just learned that he is going to teach a seminar focused on Gelfand's work on hypergeometric functions. I am not sure though if Gelfand's most recent works on the subject are available to him.

A. K. Many of them have been published, but many are still in progress or in review. I know for a fact that in his seminar Gelfand went beyond the scope of his published works.

E. D. … as is usually the case. Can you get preprints here?

A. K. There is no such thing at MSU. You can get them in the Institute of Applied Mathematics, where I work part time, but not at the university. As far as I understand, the administration couldn't care less, and nobody is interested in what others think about it.

F. Mekhmat in 1989

E. D. Who has the most impact on students in Mekhmat today?

A. K. It's a difficult question. There are several good seminars. Aside from that of Gelfand, there is a big seminar of Novikov, a seminar of Arnold which fluctuates in terms of enrollment numbers. In the last three years the most popular has been the seminar of

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[6] http://www.math.cornell.edu/People/Faculty/billera.html

Manin, as well as the whole field of algebraic geometry. For some reason discrete mathematics is also very popular, although I wouldn't say we have strong faculty in this field. Maybe the name itself carries some attraction, just as in the past people went into cybernetics without knowing what it actually is.

E. D. How much time does Arnold devote to students? I know that he works part time now.

A. K. Yes, but he still offers his seminar. He always had many good students but a little less so now, probably because he recently taught a course on differential equations which was quite intense.

E. D. Do you run any seminars?

A. K. Yes, I have two seminars: one on representation theory [for my former students?] and a seminar for younger students called "Algebra and Functional Analysis". In it I usually have ten students from each year.

E. D. have you had any talented students lately?

A. K. I would name Neretin[7] and Olshanski.[8]

E. D. Are they PhD students?

A. K. Neretin recently defended his candidate dissertation and will soon defend the doctorate too. As for Olshanski, he defended his candidate dissertation a while ago but, given his shyness and timid character, is still not a doctor. But I am sure he will defend soon.

E. D. What about younger students?

A. K. I have one first year PhD student, Denis Yuriev, who is very talented but also unusual. He is interested in all kinds of things: history, literature, languages. Mathematics is only one of them, and so I don't know yet what he will end up doing.

E. D. Is Sinai still here?

A. K. Yes, he works part time. He teaches a very well attended seminar with me, Anosov, and Stepin.

E. D. I don't know what the situation is today, but Sinai always used to have many good students.

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[7] http://www.mat.univie.ac.at/~neretin/
[8] http://www.iitp.ru/en/userpages/88/

A. K. Yes, and I think he kept his position in the university precisely in order to recruit young students. Traditionally, we used to play volleyball, my seminar against his, but it has not often happened of late.

E. D. You are not involved in the admission process, aren't you?

A. K. For seventeen or eighteen years I participated in mathematics exams only in other departments. Last year I wrote a letter to the chair of the Mekhmat admissions committee saying that it was about time that I administer entrance exams in Mekhmat as well.

E. D. Rumor has it there were a lot of problems with entrance exams last year.

A. K. There were no problems in my auditorium, except for the fact that apart from me there were twelve people I didn't know.

E. D. Examiners?

A. K. Yes, they were examiners. It is absolutely incredible that, after working in Mekhmat for thirty years, I couldn't recognize any of these people. There are quite a few people in Mekhmat these days whom nobody knows.

E. D. Are they instructors or what?

A. K. Well, on paper they are vaguely defined as employees.

E. D. So they could be lab staff or something like that?

A. K. I have no idea, but many of them erred grading students papers so that it was necessary to go over them a second time.

E. D. I've heard there has been a change for the better this year.

A. K. I don't know all the details, but my son told me that there fewer problems with entrance exams compared to the previous year. However, last year there were six requests to appeal the exam grade, five of which from students who took their exam in the auditorium next to mine. So there must have been some problems after all.

G. Stumbling blocks for young talent

E. D. Are there any new good professors in Mekhmat?

A. K. It's a difficult question. The first one after us was Sergei Novikov, and then nobody for quite a long time.

E. D. I heard that some people were appointed professors for achievements in something other than mathematics. Is that true?

A. K. Yes, for a long time only secretaries of the Komsomol organization were appointed in Mekhmat. As I said earlier, there are many people in the department whom I don't know. I assume not all of them used to be Komsomol secretaries, but how they managed to get appointed I have no idea.

E. D. They are not known for their achievements in mathematics, are they?

A. K. Not at all! Moreover, a number of good people were hired for ancillary positions in the Lab of Computational Methods, not in the department itself. Today all good young prospects work outside of the university.

E. D. Well, this is a big problem of Soviet science as a whole. In the West in general, and in the U.S. in particular, young talented mathematicians are in high demand, and their careers skyrocket overnight, while here they are relegated to minor positions. Consider for example Margulis.

A. K. Well, Margulis has now attained his share of celebrity. So he is doing just fine.

E. D. Yes, but still he is not where he is supposed to be. He has to be either in the Mathematical Institute or at MSU and not in the Institute for Information Transmission Problems, where his research is not directly relevant.

A. K. Also, it is a general tendency that less and less young people turn to mathematics.

E. D. What can you say about Beilinson?[9]

A. K. Beilinson is a very nice person in every respect, and very knowledgeable as well.

E. D. Is he a student of Gelfand?

A. K. Strictly speaking he is a student of Manin but is also in many ways a self-made man.

E. D. I've heard from Stroock that there are plans to have him spend one semester each year at MIT. Has it happened yet?

A. K. I've heard about it too, and I think he is in the States right now.

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[9] http://en.wikipedia.org/wiki/Alexander_Beilinson

E. D. What is his position in Moscow?

A. K. I have no idea. All I know is that he held a number of random jobs rather tenuously related to his research. I heard he worked at the Institute of Cardiology or something like that. I also think he used to work in Gelfand's council on cybernetics, but I'm not sure about that.

E. D. So I can see that "*Perestroika*" and *"Glasnost"* so far have had very little impact on science, in the sense of promoting young talent and getting rid of mediocre people capable only of political posturing.

A. K. It is hard to get rid of anyone in the Soviet system, and promotion opportunities are few and far between. Whenever there is a new vacancy, it has to be filled right away, although talented people may come along in a year or two.

H. Mathematical economics

E. D. What about you, Luiza?

L. Ch. I work in the Department of Economics, at the chair of Mathematical Methods founded by Nemchinov. From the very beginning we've been battling the traditional Soviet political economists of the old generation.

E. D. And who is the winner?

L.Ch. Not we, I suppose. Every year about 250 students graduate from our department. Most of them are economists of the old type who can teach only political economy. Only 50 or less are mathematical economists who can apply mathematical methods.

A. K. Apropos mathematics in economic sciences, I have an interesting story for you. In 1983 Elena Korkina, one of my former students in Mekhmat, defended her thesis at the Central Economic Mathematical Institute (CEMI).

L. Ch. She worked in the All-Union Institute for Systems Research under Djerman Gvishiani[10] but defended her thesis in CEMI.

A. K. At her defense I was struck by the following exchange. You can also think of it as a riddle. After she wrote her formulas on the blackboard and gave her presentation, the

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[10] http://keywiki.org/index.php/Djerman_Gvishiani

floor was open for questions. Question from the audience, "I can see on the blackboard that you have used the partial derivative. Why? What do you mean?" I didn't understand this question at all. But I was even more surprised by the answer: "Here we have a function of two variables, but it is homogeneous. So we can compute a partial derivative as an ordinary one".

As I learned later from other participants, the key to understanding the question and the answer is that at the time the use of partial derivatives in economics was forbidden — or at least strongly "discouraged." Rumor has it that one of the Party bosses discovered somewhere in a math encyclopedia the definition of partial derivative as a derivative with respect to one variable while all other variables are fixed. This was at odds with one of the main principles of dialectical materialism,[11] according to which everything is in constant flux, everything changes, and everything is correlated, and therefore one cannot fix any part of the variables.

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[11] Official ideology of the Communist Party.