Spatial interpolation and kriging

EDS 222

Tamma Carleton

Fall 2021

Announcements/check-in

- Final projects guidelines

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- **Change in office hours** today 1:30pm-2:30pm (by appointment) in Bren Hall 4327

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- Final project presentations: 12/2 9:30-10:45am (Bren Hall 1414); 12/7 8-10:30am (Bren Hall 14**2**4)
    - You will *randomly* be assigned a slot (slots announced 11/24)

Today

Refresher: types of spatial data

Points, vector, raster/field, dynamic raster/field

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A common challenge: spatial interpolation

Points to fields, interpolation

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A common challenge: spatial interpolation

Points to fields, interpolation

Kriging: a powerful form of interpolation

Variogram, kriging

Types of spatial data

Spatial data

Spatial Data can generally split into:

- **Vector** Data

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Examples : Elevation. Temperature. Wind direction.

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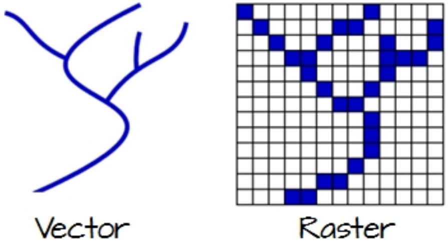

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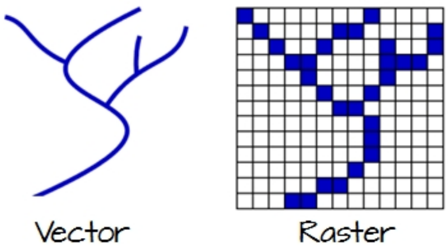

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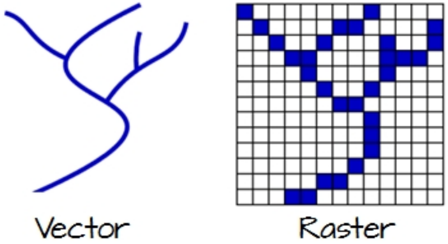
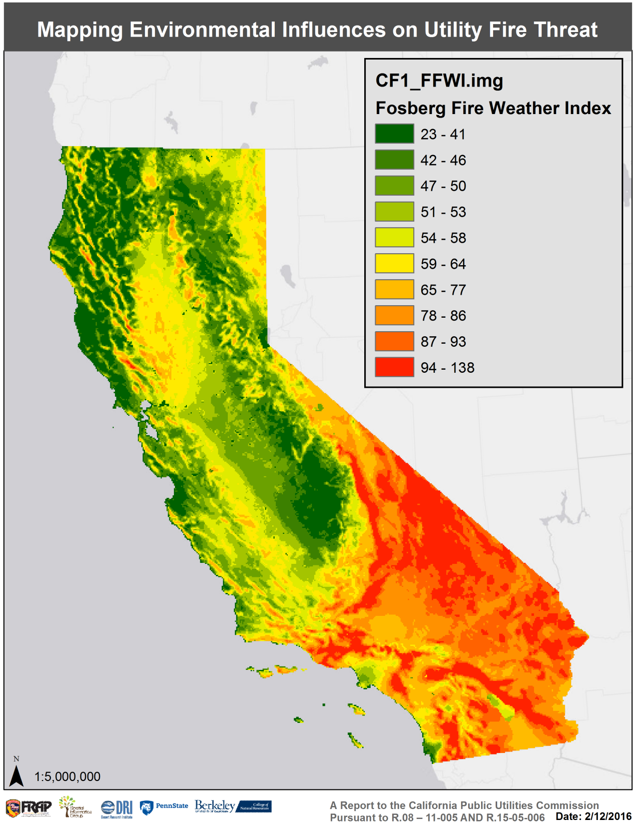


- Usually it will be easier to represent **objects** with **vector data** and **fields** with **raster** data, but ultimately this depends on what analysis you want to run
- Luckily, `R` makes it easy to switch back and forth (but we need to be careful and intentional when transforming!)

Spatial interpolation

Spatial interpolation

In environmental data science, we are **often interested in modeling fields**



Spatial interpolation

But we are doing **statistics!**

Spatial interpolation

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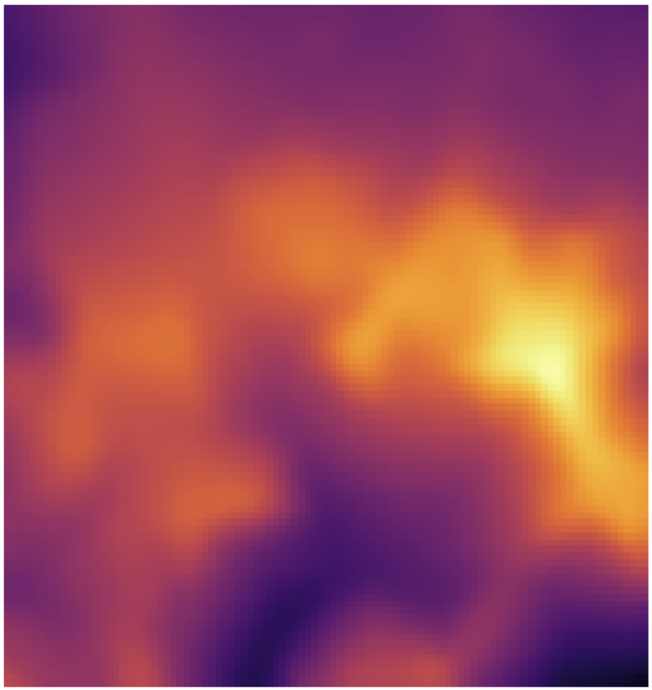
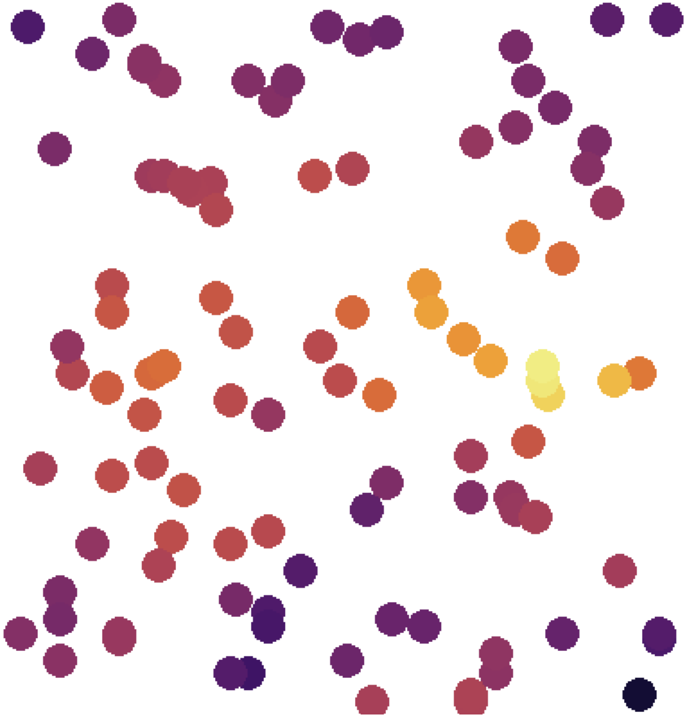
That means we only have data from a *sample*, not a census of the *population*





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Spatial interpolation is the process of using a **sample** of observed points to estimate values for **all locations** in a study region

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For example:

- Predicting "gold grades" across South Africa using a few borehole samples (the problem of Daniel *Krige*!)
- Predicting depth to groundwater across California using monitoring wells
- Predicting air pollution across China using monitoring stations

Spatial interpolation in math

- Let $Z(x_0)$ indicate the value (e.g., elevation) at a location x_0 that was *not* sampled

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Spatial interpolation aims to predict $Z(x_0)$ using a linear combination of the values in the sampled locations:

$$\hat{Z}(x_0) = \sum_{i=1}^{m} \lambda_i Z(x_i)$$

where λ_i are weights applied to each sampled location.

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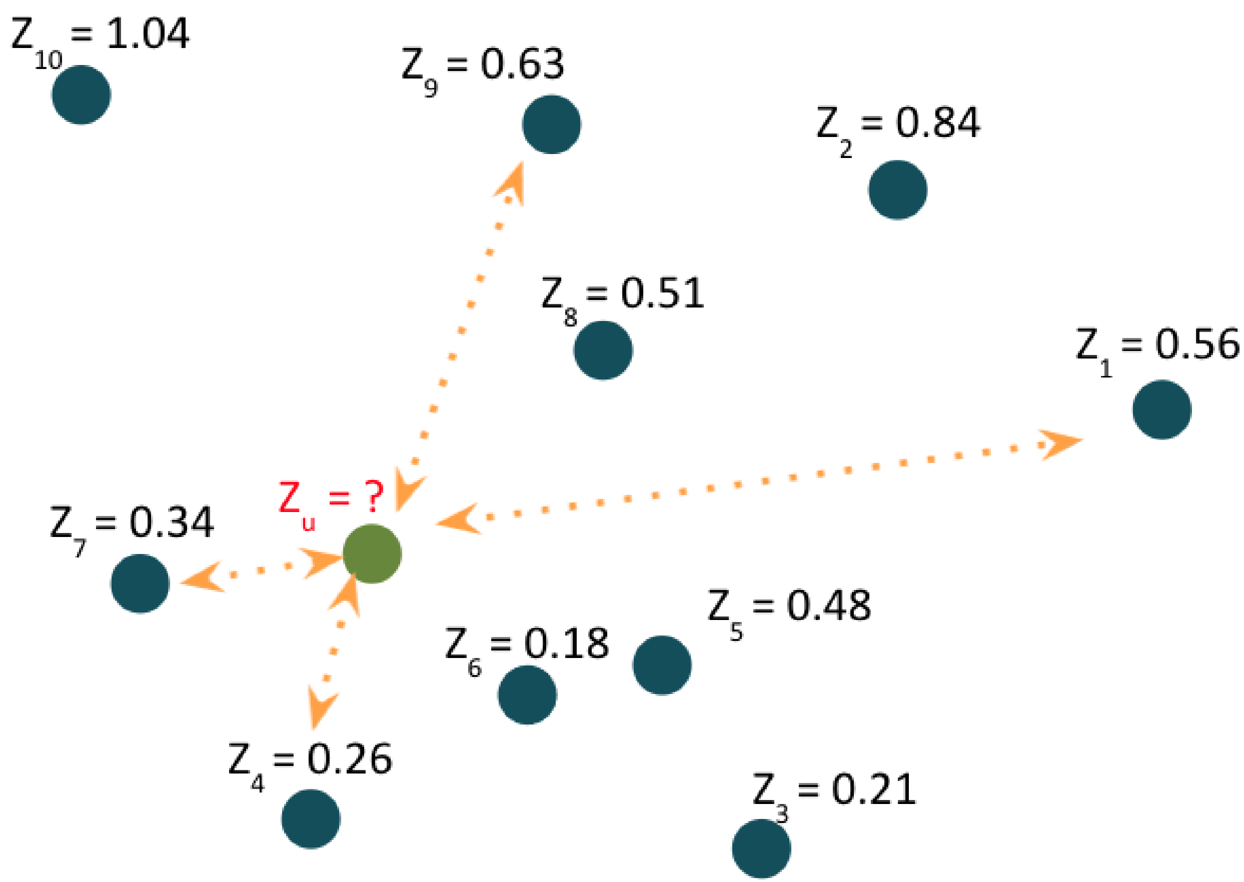
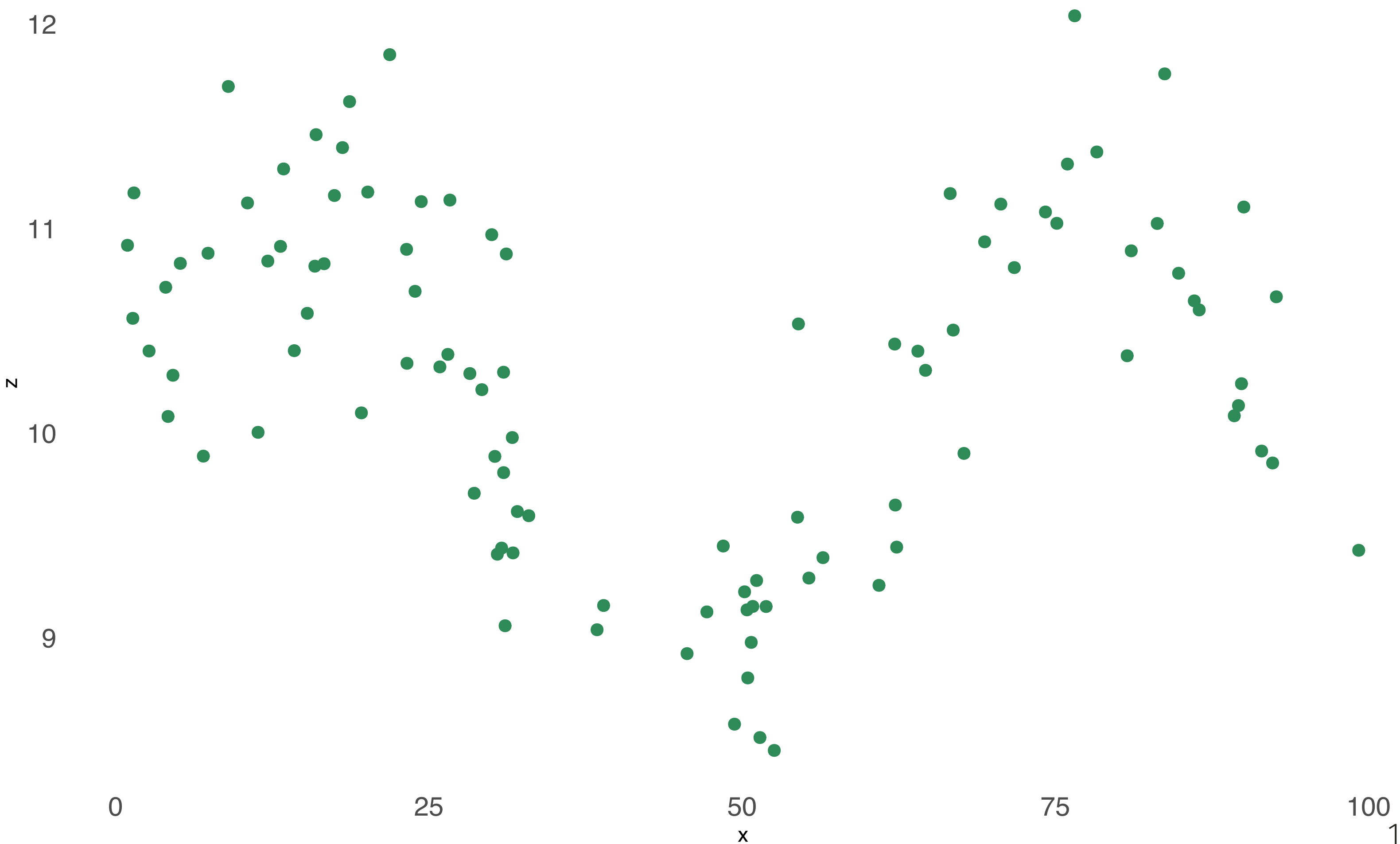
- All spatial interpolation methods assume or derive a set of λ's to compute $\hat{Z}$'s

Interpolation in pictures



Interpolation in one dimension

Consider one-dimensional space where values y depend on location x



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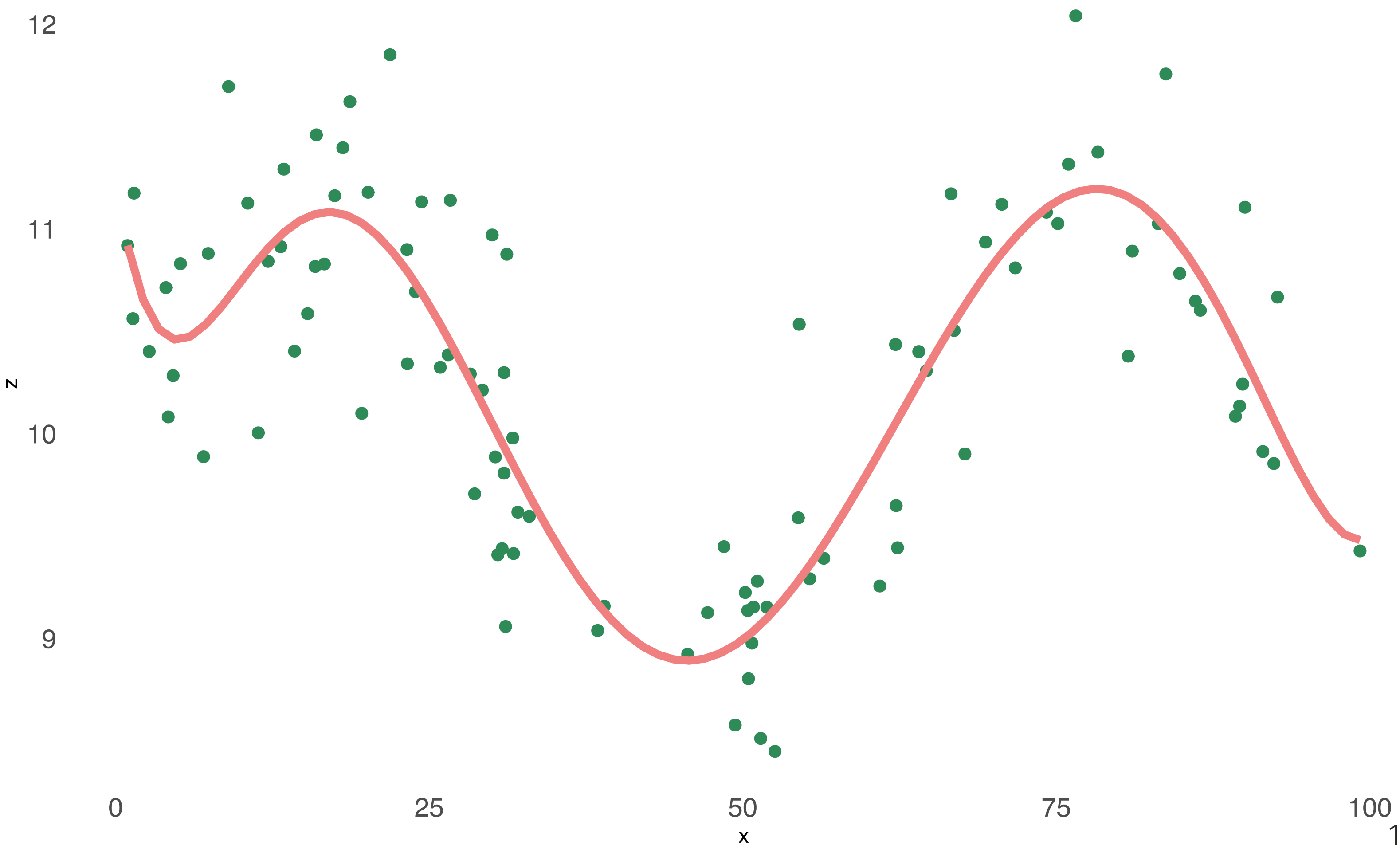

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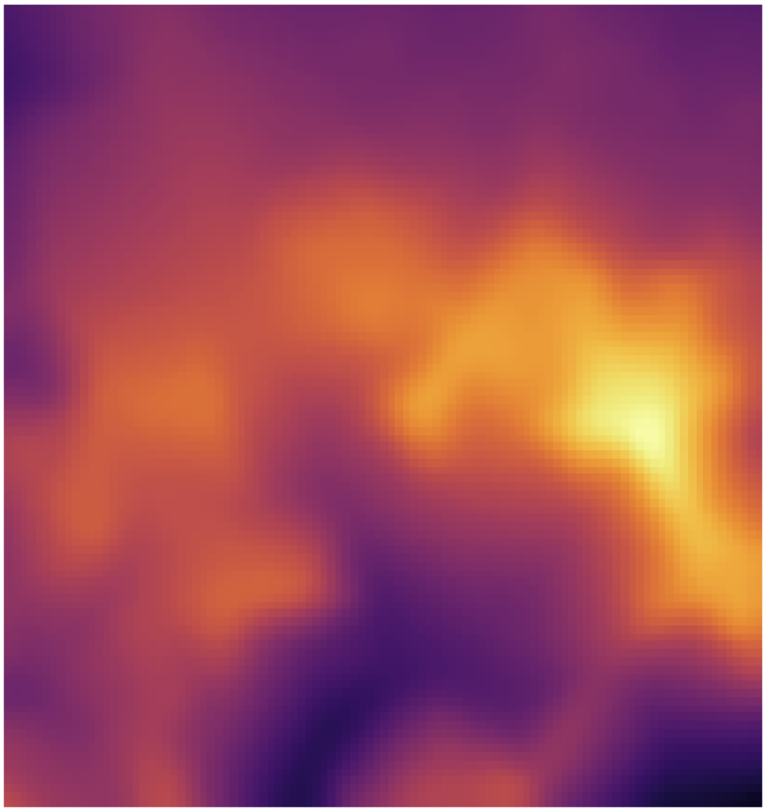

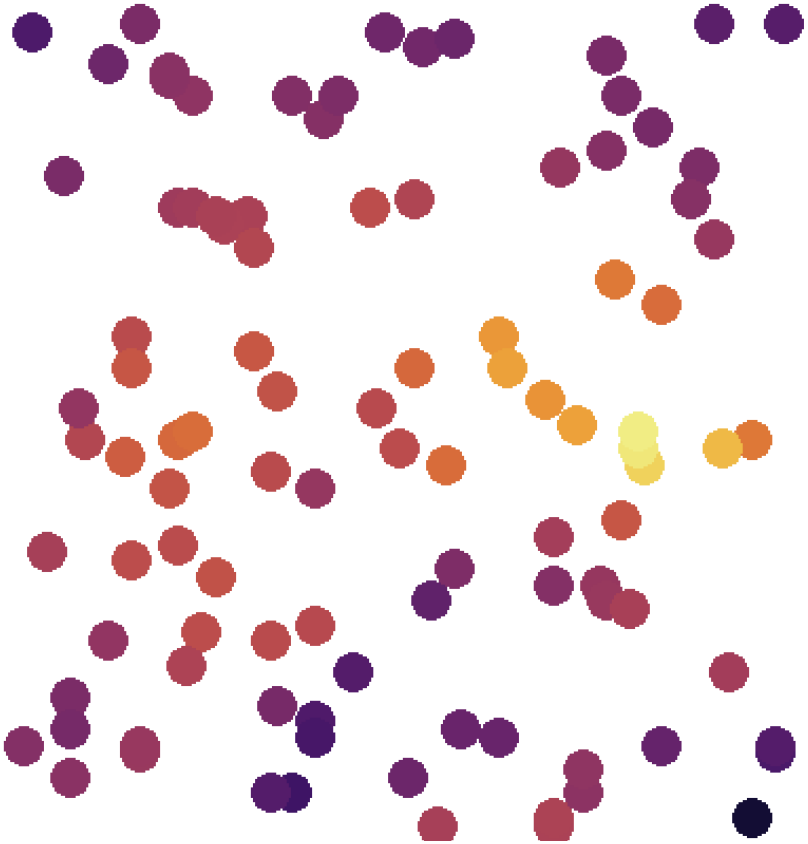

Interpolation methods

Polynomial regression

- In one-dimensional space:

$$\hat{Z}(x_0) = \hat{\beta}_0 + \hat{\beta}_1 x_0 + \hat{\beta}_2 x_0^2 + \ldots + \hat{\beta}^p x_0^p$$

- In two-dimensional space with (x_0, y_0) the unknown value:

$$\hat{Z}(x_0, y_0) = \hat{\beta}_0 + \hat{\beta}_1 x_0 + \hat{\beta}_2 y_0 + \hat{\beta}_3 x_0 y_0 + \hat{\beta}_4 x_0^2 + \hat{\beta}_5 y_0^2 + \ldots$$

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Exact: Predicts a value identical to the measured value.

Inexact: Does *not* predict a value identical to the measured value.

Polynomial regression interpolation

This is just **multiple linear regression** using spatial information as the independent variables

```
mod = lm(z~poly(x,8))
predictions = augment(mod)$.fitted
```

Interpolation methods

Nearest Neighbors (NN)

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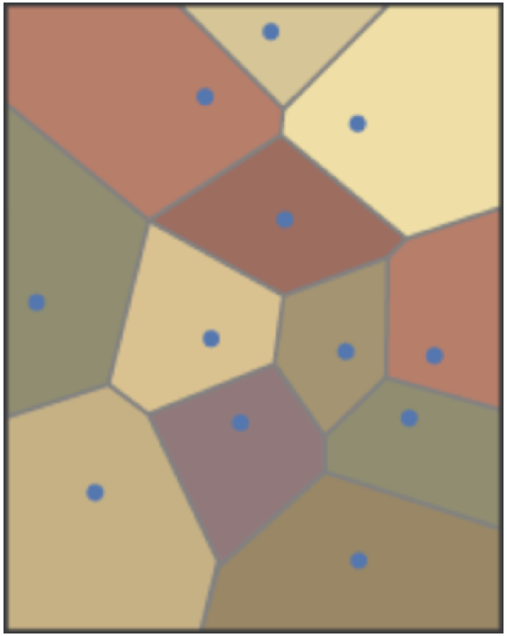

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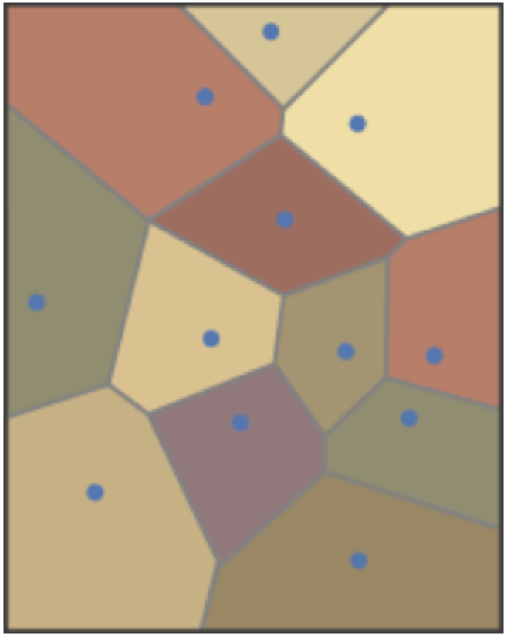

- Creates what are called "Theissen Polygons", which allocate space to the nearest sampled point

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Implementation in `R`

- Easy with the `voronoi()` function from the `dismo` package:

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library(dismo)
v ← voronoi(dta)
plot(v)
```

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- Helpful tutorial here

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Inverse distance weighting

Basic idea: weights are a decreasing function of distance from $\boldsymbol{x}_0$ to $\boldsymbol{x}_i$

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Basic idea: weights are a decreasing function of distance from x_0 to x_i

$$\hat{Z}(x_0) = \sum_{i=1}^{m} \frac{Z(x_i) Dist(x_i, x_0)^{-p}}{\sum_{i=1}^{m} Dist(x_i, x_0)^{-p}}$$

Equivalently:

$$\lambda_i^{IDW} = \frac{1/Dist(x_i, x_0)^p}{\sum_{i=1}^{m} 1/Dist(x_i, x_0)^p}$$

where p is the "power parameter" determining how fast the weight declines as the distance between the points grows larger

Interpolation methods

Inverse distance weighting

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- **Cons:** Difficult/computationally intensive (you need to compute distances for *all* pairs of points in the region!), all sampled observations influence $\hat{Z}(x_0)$, have to choose p somehow, result can be "clumpy"

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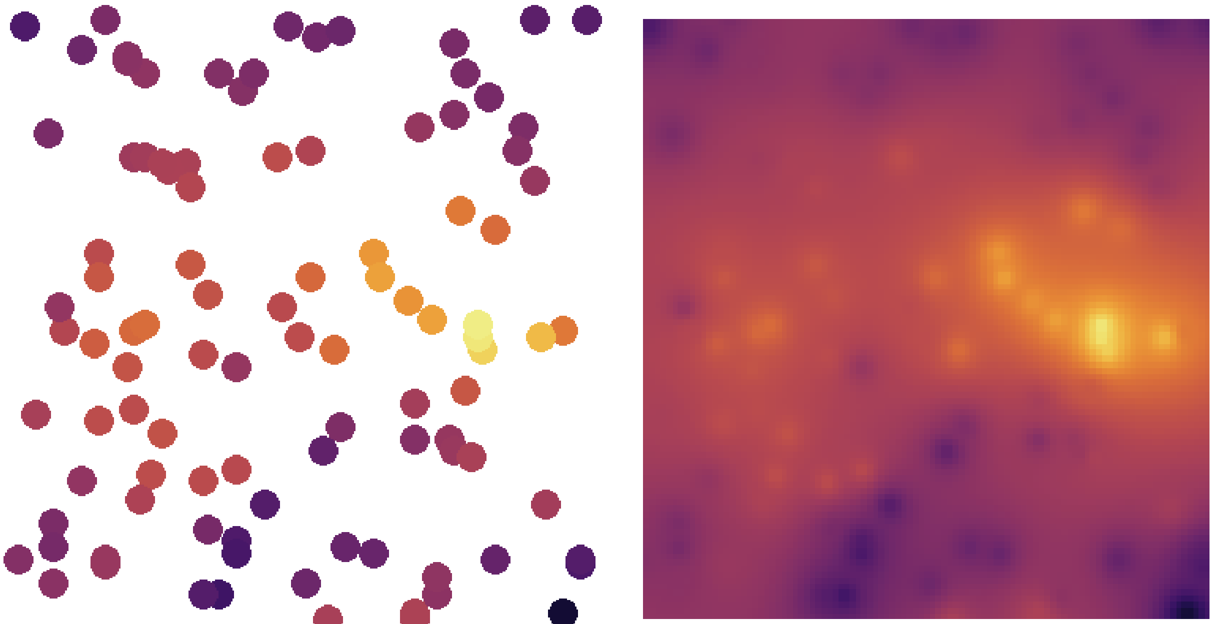

Interpolation methods

Inverse distance weighting

Implementation in `R`

```r
library(phylin)
idw(values, coords, grid, method = "Shepard", p = 2, R = 2, N = 15,
    distFUN = geo.dist, ...)
```

- Note the `method` argument: "Shepard" follows the math on the previous slide
- Note the `p` argument: Need to specify power parameter

Interpolation methods

There are many more!

- Piecewise linear interpolation / Delany triangulation
- Local polynomial regression
- Radial basis function (RBF)
- Kriging (of many forms)
- Many new machine-learning based methods
- Learn more in Li and Heap (2014)

Enter: Kriging

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Why?

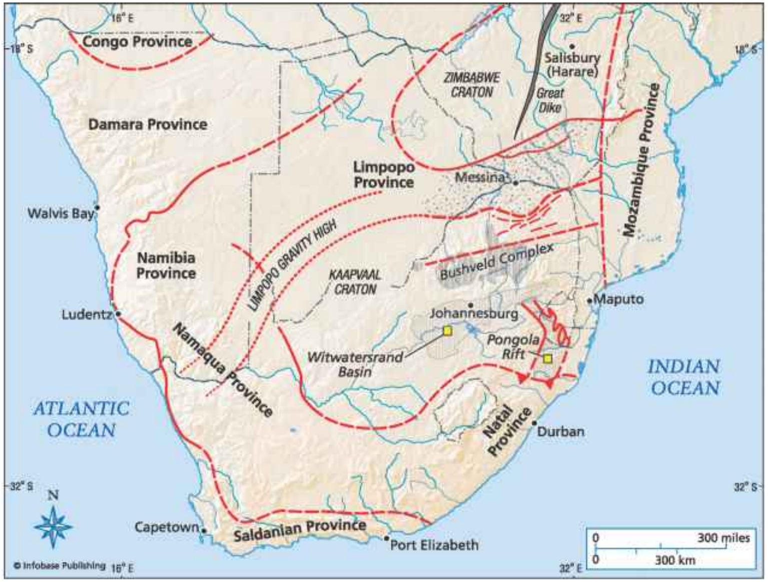
- It is *flexible* (i.e., less researcher decisions, more data-driven)
- Under certain assumptions it is the "best linear unbiased estimate" (sound like OLS yet??)
- You can recover an estimate *and* a standard error (i.e., it is *stochastic*)

Next up: Kriging details!

Kriging

Kriging: an origin story

The Witwatersrand ("Rand") in South Africa is known for its gold content. Mining engineers wanted to know where in the Rand was most likely to have a high gold content per block of ore.



Kriging: an origin story

- Many individual ore samples have been taken (**vector** data -- points)
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Spatial interpolation is highly valuable!

- **Danie Krige's solution:** [in his master's thesis!]
  - Use an estimator that minimizes the **mean squared prediction error** (very similar to OLS)
  - Show that it has a bunch of nice properties relative to other forms of spatial interpolation

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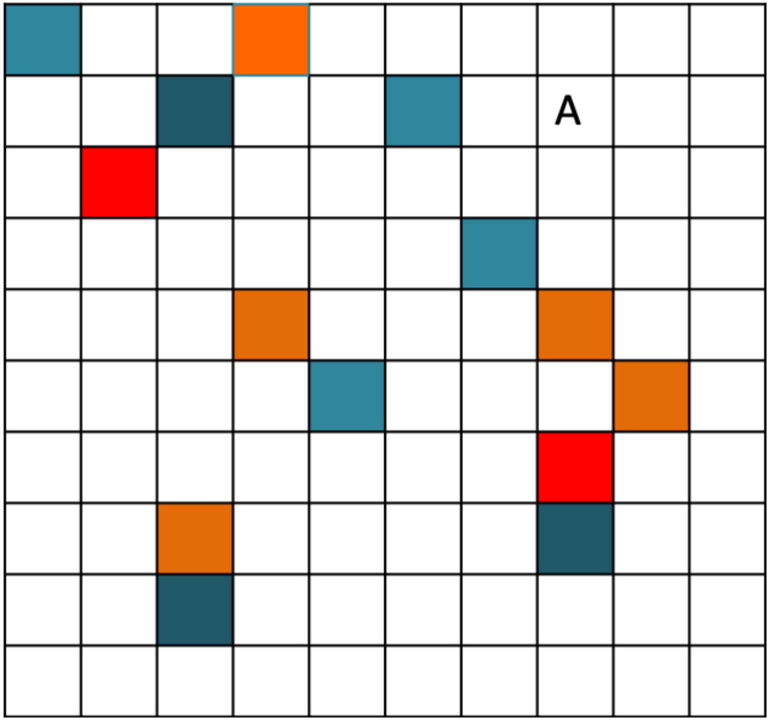
A STATISTICAL APPROACH TO SOME BASIC MINE VALUATION PROBLEMS ON THE WITWATERSRAND

By D. G. KRIGE, M.Sc. (Eng.) (Rand)

Correlations in space

Q: If there is **no correlation** between values in nearby locations, can we predict new values based on our sample?



- Blue = low value; Red = high value
- **Zero** correlation between values in nearby locations
- Can you predict the value in location A based on this sample?

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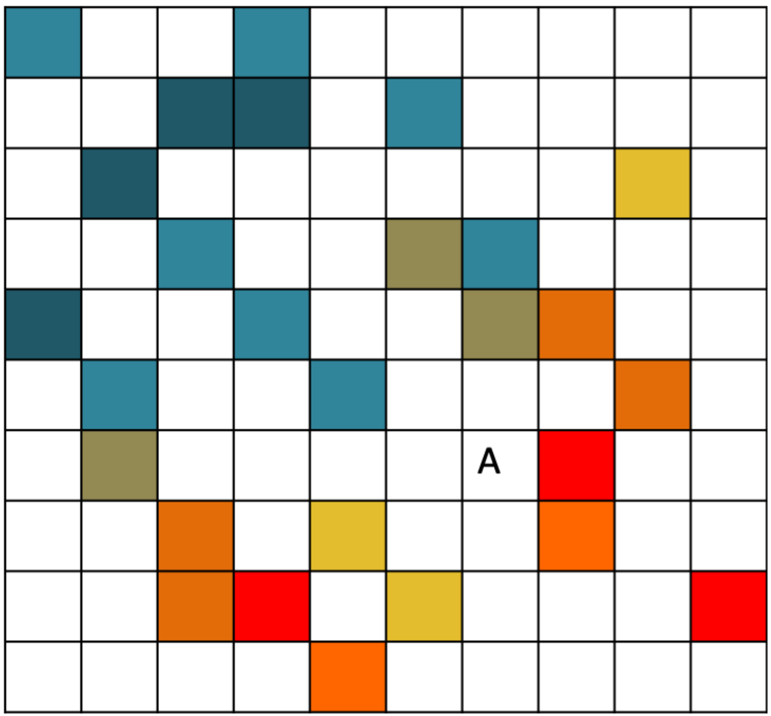

- Blue = low value; Red = high value
- **Positive** correlation between values in nearby locations
- Can you predict the value in location A based on *this* sample?

Variogram

Key takeaway: quantifying spatial dependence is key to spatial interpolation

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Key concept: Variograms give us a way of understanding how correlated spatial observations are to those around them, and how that correlation "decays" as points get further apart

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Key concept: Variograms give us a way of understanding how correlated spatial observations are to those around them, and how that correlation "decays" as points get further apart

Mining example: Variogram gives a measure of how much two samples taken from the mining area will vary in gold percentage depending on the distance between the samples. Samples farther apart will vary more than those taken close together.

Variogram

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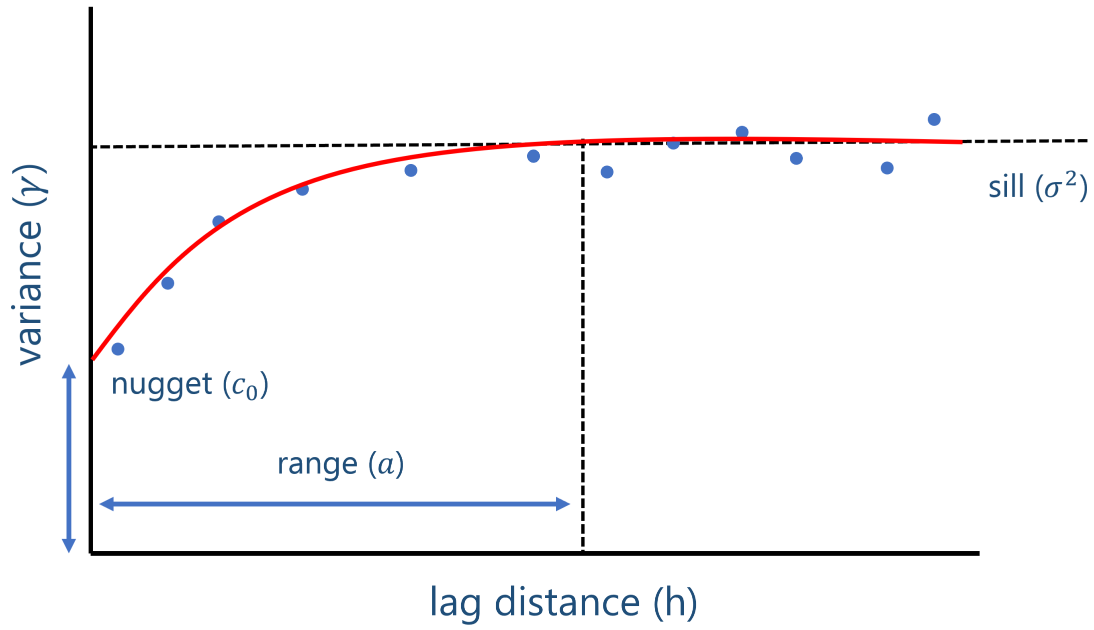
Why? Recall:

$$var(a-b) = var(a) + var(b) - 2cov(a,b)$$

So, for a "stationary" variogram, we have $\gamma(x+h,x) = var(Z(x)) - cov(Z(x), Z(x+h))$

Variogram: in pictures



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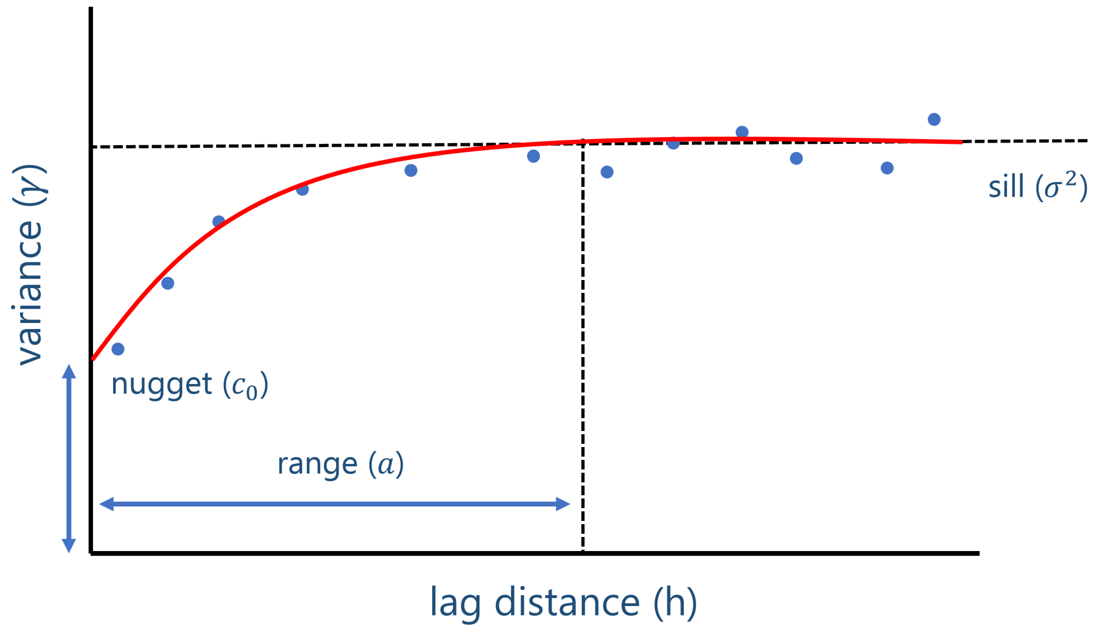

- **Nugget:** At $h = 0$, residual variance is from microscale effects or measurement error
- **Sill:** The stationary maximum variance -- no more covariance
- **Range:** Separation distance beyond which there is no covariance

Estimating a (semi)variogram

Empirical semivariogram

$$\hat{\gamma}(h \pm \delta) = \frac{1}{2N(h \pm \delta)} \sum_{(i,j) \in N(h \pm \delta)} |z_i - z_j|^2$$

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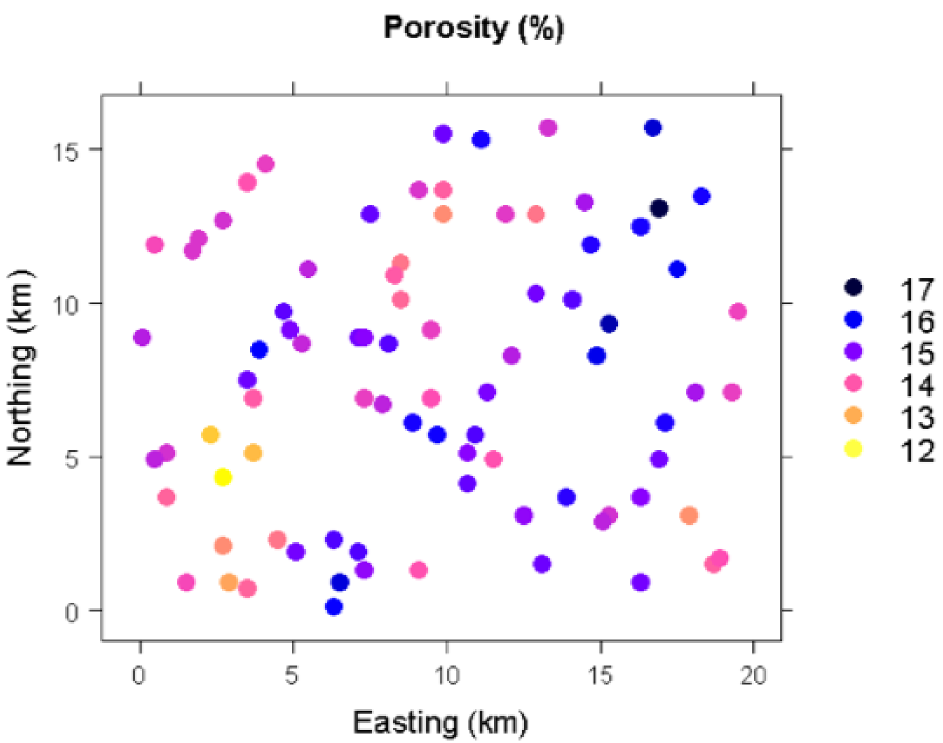
How?

- Draw "donuts" of width δ and average distance h around each point
- Compute differences in values for each pair of points, square them
- Take an average!

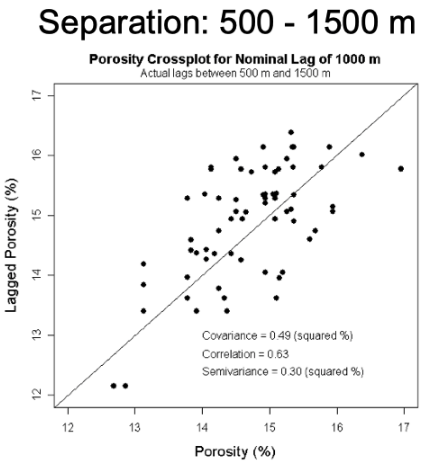
Empirical variogram example

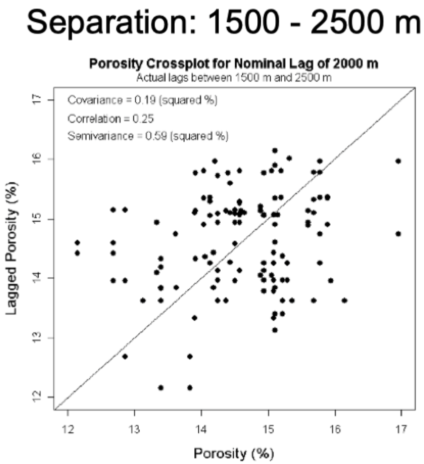
- Bohling's *Introduction to Geostatistics and Variogram Analysis*
- Porosity values in a bean field
- 85 wells sampled

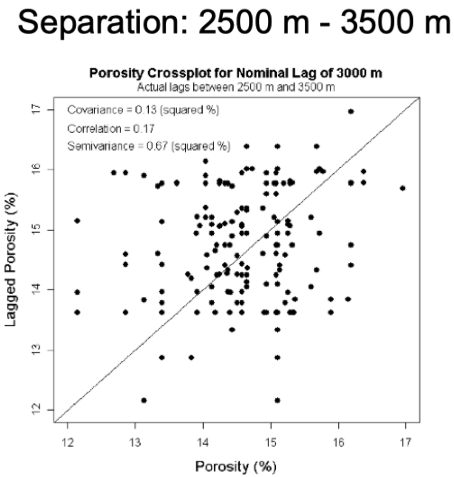


Empirical variogram example

For various values of h and a fixed δ, compute semivariance:

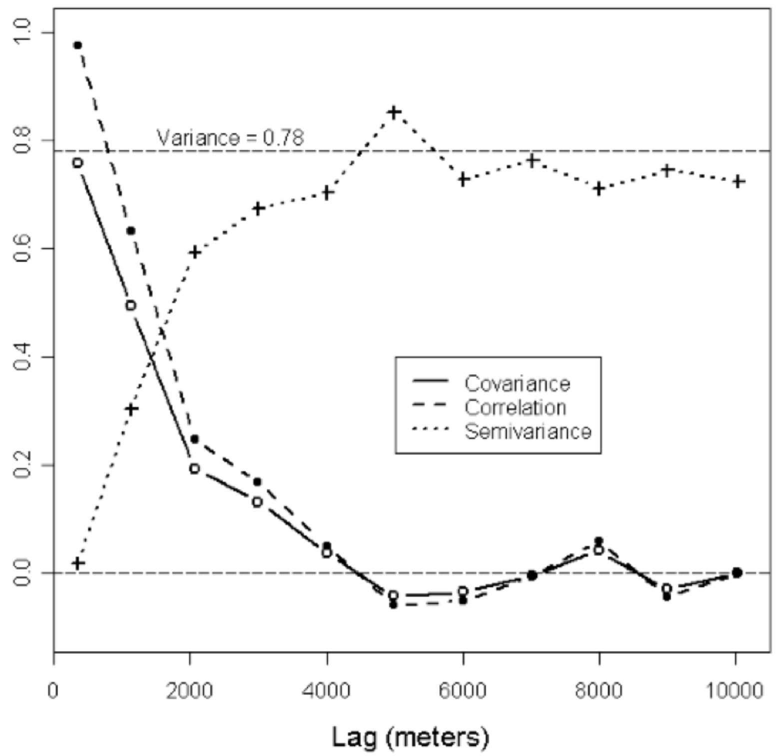






Empirical variogram example

Plot your semivariances:



Empirical variogram example

Then choose (or optimize) a **variogram model** to fit through the semivariance points:

- Exponential
- Spherical
- Gaussian
- ...

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Many more details on variograms here or in any geostatistics textbook (e.g., Cressie and Wikle, 2011)

Back to kriging

Recall that our goal is a prediction of a value $\hat{Z}(x_0)$ based on observations in all sampled locations:

$$\hat{Z}(x_0) = \sum_{i}^{m} \lambda_i Z(x_i)$$

Back to kriging

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In **kriging** (and many spatial interpolation methods), the λ_i weights **decay** as distance between x_0 and x_i grows larger

--

> How do we find the weights in kriging?

Kriging weights

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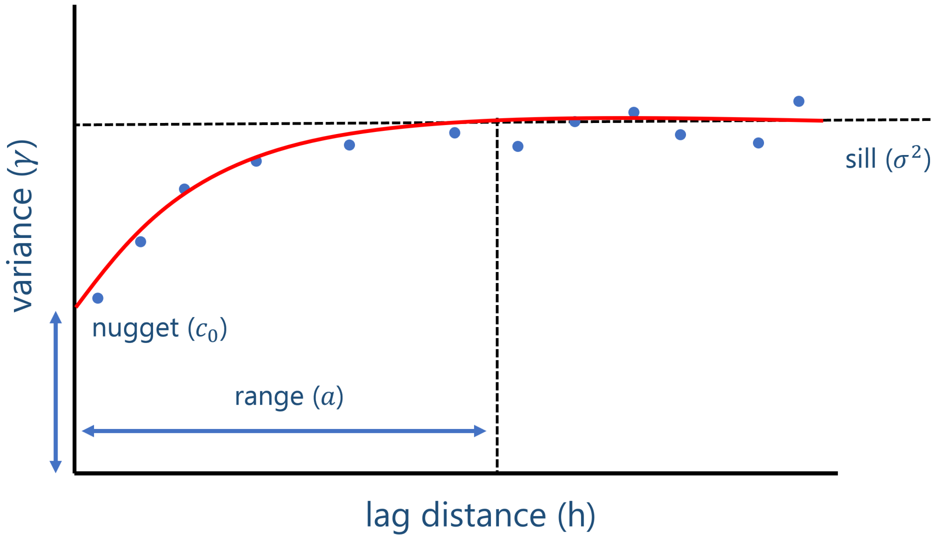
Hint:



The **variogram** tells us how correlated values are with other values near them, and how this correlation falls as distance grows. It is a **key input** into the kriging solution.

Deriving the kriging solution

Note: full derivation in Cressie and Wikle (2011) [this is a very shorthand version]

Goal: minimize mean squared prediction error

$$min_\lambda \; E[(Z(x_0) - \sum_i^m \lambda_i Z(x_i))^2] \text{ subject to } \sum_i^m \lambda_i = 1$$

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To solve:

1. Take derivatives with respect to each λ_i
2. Set each first order condition = 0
3. Solve system of equations for λ_i^* values that minimize mean squared error

Deriving the kriging solution

Result:

$$\hat{Z}(x_0) = \underbrace{\{\tilde{\gamma}(x_0) + \mathbf{1}(1 - \mathbf{1}'\mathbf{\Gamma}_Z^{-1}\tilde{\gamma}(x_0))/(\mathbf{1}'\mathbf{\Gamma}_Z^{-1}\mathbf{1})\}'\mathbf{\Gamma}_Z^{-1}}_{\hat{\lambda}} Z$$

- where $\tilde{\gamma}(x_0)$ is the vector containing the semivariogram evaluated between x_0 and every other point, and
- $\mathbf{\Gamma}_Z$ is the $m \times m$ matrix containing all semivariogram evaluations for all sampled point pairs.

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Other helpful resources here

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- We will work on implementation in `R` in the next lab.

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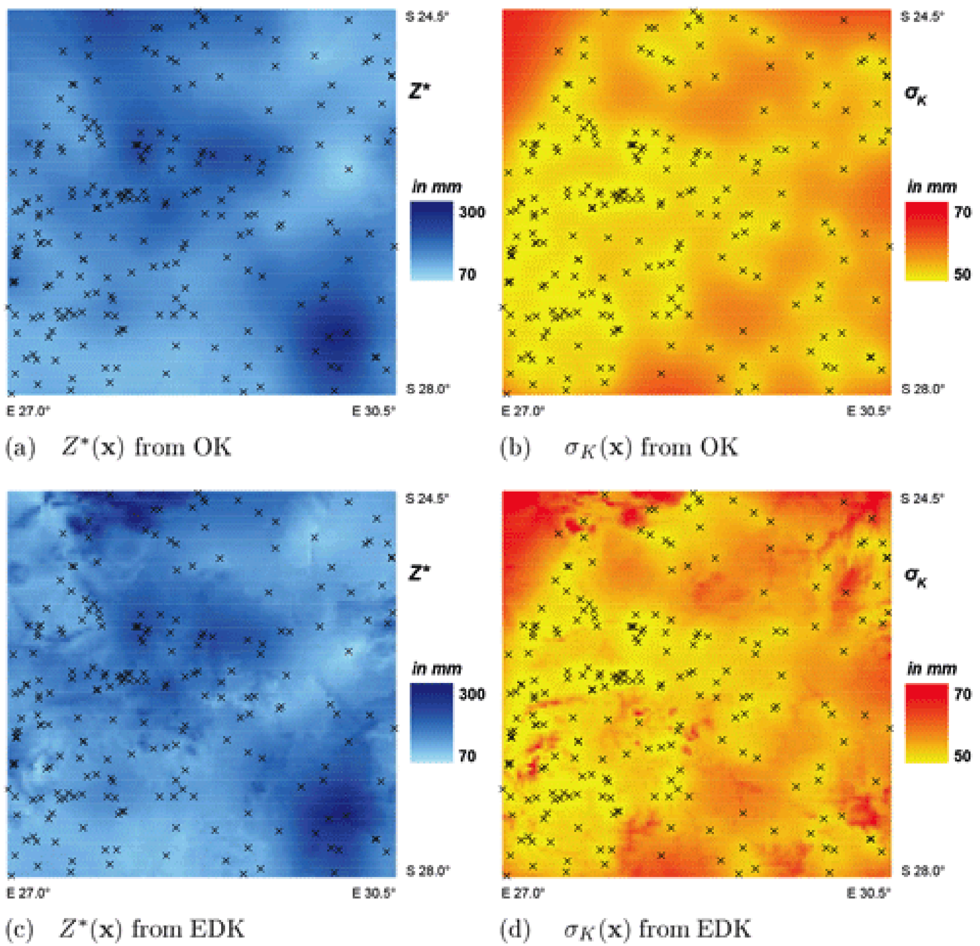

(a) $Z^*(\mathbf{x})$ from OK (b) $\sigma_K(\mathbf{x})$ from OK
(c) $Z^*(\mathbf{x})$ from EDK (d) $\sigma_K(\mathbf{x})$ from EDK

Source: Lebrenz and Bardossy (2019)

Kriging summary

Pros:

- Under each set of assumptions specific to the kriging form, kriging is the best linear unbiased predictor ("BLUP")
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> Unlike other interpolation methods, kriging allows the data to directly inform the weights (e.g., compare to inverse distance

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All spatial interpolation approaches should be used cautiously, especially if:

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- You don't have very many observations

Slides created via the R package **xaringan**.