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1   Vectors

Interpretations of Vectors

▷ **Vector**: an ordered list of numbers.

▷ Possible notations: $\vec{v} = \boldsymbol{v}$ are most common.

▷ **Dimensionality**: the number of the elements in a vector.

▷ **Geometric vector**: an object with a magnitude and direction.

▷ Standard position: when the vector beings at the origin.

▷ Vectors must have same dimensionality for addition and subtraction.

▷ **Unit vector**: a vector with a norm (length) of 1. Notation: $\hat{u} = \dfrac{\boldsymbol{u}}{|\boldsymbol{u}|}$

Vector Multiplication

▷ **Scalar**: scales each element in a vector, does not change direction. Generally represented with greek letters.

▷ **Dot product**: a single number that provides information about the relationship between two vectors. Must have same dimensionality.

▷ Notation for dot product: $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}^T \boldsymbol{b} = \langle \boldsymbol{ab} \rangle = \sum a_i b_i$

▷ *Algebraic* dot product properties:

  ○ *Associative*: $\boldsymbol{a}^T(\boldsymbol{b}^T \boldsymbol{c}) \neq (\boldsymbol{a}^T \boldsymbol{b})^T \boldsymbol{c}$

  ○ *Distributive*: $\boldsymbol{a}^T(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a}^T \boldsymbol{b} + \boldsymbol{a}^T \boldsymbol{c}$

  ○ *Commutative*: $\boldsymbol{a}^T \boldsymbol{b} = \boldsymbol{b}^T \boldsymbol{a}$

  ○ Vector magnitude/length: $\|\boldsymbol{v}\| = \sqrt{\boldsymbol{v}^T \boldsymbol{v}}$

▷ *Geometric* dot product properties:

  ○ Magnitudes of vectors scaled by angle between them. i.e. $\vec{a} = |a||b| \cos(\theta_{ab})$

  ○ Geometric and algebraic are really the same. The above equation can be rewritten as the algebraic vector length, i.e. $\boldsymbol{a}^T \boldsymbol{b} = \cos(\theta_{ab})|a||b|$

▷ Dot product features based on θ:

  ○ If $\cos(\theta) > 0$ then $\alpha > 0$

  ○ If $\cos(\theta) < 0$ then $\alpha < 0$

  ○ If $\cos(\theta) = 0$ then $\alpha = 0$; termed **Orthogonal**

  ○ If $\cos(\theta) = 1$ then $\alpha = |a||b|$

▷ *Hadamard vector multiplication*: elementwise multiplication of two vectors of equal dimensionality.

▷ **Outer product**: $vw^T = n \times m$; a matrix resulting from the product of vectors with dimensions *n* and *m*.

▷ **Cross product**: defined only between two 3D vectors; produces another 3D vector that is perpendicular to both original vectors, or normal, to the plane containing them.

▷ *Complex conjugate*: the inverse sign of imaginary componenet of a number.

▷ **Hermitian transpose**: or conjugate transpose, is transpose of a vector or matrix containing imaginary numbers using the complex conjugate.

▷ Notation for Hermitian transpose on a matrix: M^H or M^*

Vector Space

▷ **Dimension**: in linear algebra dimension represnets a new element of infomation about a vector. Geometrically, each dimension represent a new direction.

▷ **Fields**: a set of numbers on which a set of arithmetic are valid.

▷ Field notations:

    ○ $\mathbb{R}$: real numbers.

    ○ $\mathbb{C}$: complex numbers.

    ○ $\mathbb{K}$: real complex numbers.

    ○ $\mathbb{Z}$: integers.

    ○ $\mathbb{N}$: poisite integers.

    ○ $\mathbb{Q}$: rational numbers.

    ○ Dimensionality in fields is written with superscripts, e.g. $\mathbb{R}^N$

▷ **Subspace**: The set of all vectors that can be created by a linear combination of some vectors or a set of vectors, i.e. $\lambda v, \ \lambda \in \mathbb{R}$

    ○ More formal definition: a vector subspace (V) must be closed under addition and scalar multiplication, and must contain the zero vector.

    ○ $\forall v, w \in V; \ \forall \lambda, \alpha \in \mathbb{R}; \ \lambda v + \alpha w \in V$

    ○ Geometric: all possible scaled versions of vector produces a line for a single vector, and a 2d plane between two linear independent vectors.

▷ **Subset**: a set of points that satisfies some conditions; doesn't need to include toe origin, doesn't need to be closed, and can have boundaries.

▷ **Span**: all possible linear combinations of vectors within a subspace.

▷ **Linear Independence**: a property of an entire set of vectors that is true only if no vector in the set can be written as a linear combination of the others.

  ○ Geometric perspective: a set of M vectors is independent if each vector points in a geometric dimension not reachable using other vectors in the set.

  ○ Any set of $M > N$ vectors in $\mathbb{R}^N$ is dependent.

  ○ Any set of $M \leq N$ vectors in $\mathbb{R}^N$ *could be* independent.

▷ **Basis**: a combination of span and independence. More formally: if every element of a vector space V over a field is a linearly independent subset of V that spans V.