Contents

1 Vectors

Interpretations of Vectors

- **Vector**: an ordered list of numbers.
- Possible notations: $\vec{v} = \boldsymbol{v}$ are most common.
- **Dimensionality**: the number of the elements in a vector.
- **Geometric vector**: an object with a magnitude and direction.
- *Standard position*: when the vector beings at the origin.
- Vectors must have same dimensionality for addition and subtraction.
- **Unit vector**: a vector with a *norm* (length) of 1. Notation: $\hat{u} = \frac{\boldsymbol{u}}{|\boldsymbol{u}|}$

Vector Multiplication

- **Scalar**: scales each element in a vector, does not change direction. Generally represented with greek letters.
- **Dot product**: a single number that provides information about the relationship between two vectors. Must have same dimensionality.
- Notation for dot product: $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}^T\boldsymbol{b} = \langle \boldsymbol{ab} \rangle = \sum a_i b_i$
- *Algebraic* dot product properties:
  - **Associative: False**; $\boldsymbol{a}^T(\boldsymbol{b}^T\boldsymbol{c}) \neq (\boldsymbol{a}^T\boldsymbol{b})^T\boldsymbol{c}$
  - **Distributive: True**; $\boldsymbol{a}^T(\boldsymbol{b}+\boldsymbol{c}) = \boldsymbol{a}^T\boldsymbol{b} + \boldsymbol{a}^T\boldsymbol{c}$
  - **Commutative: True**; $\boldsymbol{a}^T\boldsymbol{b} = \boldsymbol{b}^T\boldsymbol{a}$
  - Vector magnitude/length: $\|\boldsymbol{v}\| = \sqrt{\boldsymbol{v}^T\boldsymbol{v}}$
- *Geometric* dot product properties:
  - Magnitudes of vectors scaled by angle between them.
  - $\vec{a} = |a||b|\cos(\theta_{ab})$
  - Geometric and algebraic are really the same. The above equation can be rewritten as the algebraic vector length, i.e. $\boldsymbol{a}^T\boldsymbol{b} = \cos(\theta_{ab})|a||b|$
- Dot product features based on θ:
  - If $\cos(\theta) > 0$ then $\alpha > 0$
  - If $\cos(\theta) < 0$ then $\alpha < 0$
  - If $\cos(\theta) = 0$ then $\alpha = 0$; termed **Orthogonal**

- If $\cos(\theta) = 1$ then $\alpha = |a||b|$

- ▷ *Hadamard vector multiplication*: elementwise multiplication of two vectors of equal dimensionality.
- ▷ **Outer product**: $\boldsymbol{v}\boldsymbol{w}^T = n \times m$ matrix resulting from vectors with dimensions n and m.
- ▷ **Cross product**: defined only for two 3D vectors; produces another 3D vector that is perpendicular to both original vectors, or *normal*, to the plane containing them.
- ▷ *Complex conjugate*: inverse sign of imaginary componenet of a number.
- ▷ **Hermitian transpose**: or conjugate transpose, is transpose of a vector or matrix containing imaginary numbers using the complex conjugate.
- ▷ Notation for Hermitian transpose on a matrix: $\boldsymbol{M}^H$ or $\boldsymbol{M}^*$

Vector Space

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