Calculus

Limits and Continuity

Limits

Limit (Wikipedia) | Thomas' Calculus (2.2–2.6)

- **Limit** $\lim_{x \to c}$: the value of a function (or sequence) approaches as the input (or index) approaches some value (informal definition)
  - Limits are used to define continuity, derivatives, and integrals

Limits of a Functions and Sequences

Limit of a function (Wikipedia) | Limit of a sequence (Wikipedia)

  - **Limit of a function**: a fundament concept in calculus and analysis concerning the behavior L of a function near a particular input p, i.e.,

$$\lim_{x \to p} f(x) = L$$

    - Reads as "f of x tends to L as x tends to p"
  - ε, δ **Limit of function**: a formalized definition, wherein $f(x)$ is defined on an open interval, except possibly at c itself, leading to above definition, if and only if:
    - For every real $\varepsilon > 0$, there exists a corresponding real $\delta > 0$ such that for all real x, $0 < |x - p| < \delta$ implies that $|f(x) - L| < \varepsilon$, i.e.,

$$f : \mathbb{R} \to \mathbb{R},\ p, L \in \mathbb{R} \Rightarrow \lim_{x \to p} f(x) = L$$
$$\Updownarrow$$
$$\forall \varepsilon > 0 \, (\exists \delta > 0 : \forall x, 0 < |x - p| < \delta \Rightarrow |f(x) - L| < \varepsilon)$$

  - Functions do not have a limit when the function:
    - has a unit step, i.e., it "jumps" at a point;
    - is not bounded, i.e., it tends towards infinity;
    - or does not stay close to any single number, i.e., it oscillates too much.
  - **Limit of a sequence**: the value that the terms of a sequence (x_n) "tends to" (and not to any other) as n approaches infinity (or some point), i.e.,

$$\lim_{n \to \infty} x_n = x$$

  - ε **Limit of sequence**: for every measure of closeness ε, the sequence's term eventually converge to the limit, i.e.,

$$\forall \varepsilon > 0 \, (\exists N \in \mathbb{N} \, (\forall n \in \mathbb{N} \, (n \geq N \Rightarrow |x_n - x| < \varepsilon)))$$

    - **Convergent**: when a limit of a sequence exists.
    - **Divergent**: a sequence that does not converge.

Limit Laws and Theorems

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Continuity

- Sources:

Continuity at a Point

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Continuous Functions

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Intermediate Value Theorem

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Limits Involving Infinity

- Sources:

Limits at Infinity

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Infinite Limits

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Derivatives

Integrals

Multiple Integrals