| Page | Points | Score |
|--------|--------|-------|
| 2↓ | 6 | |
| 3↑ | 8 | |
| 4↓ | 4 | |
| 5↓ | 4 | |
| 6↓ | 4 | |
| 7↓ | 4 | |
| Total: | 30 | |

- 1. Determine whether each statement below is true or false, then explain how you know. Note: that if the statement is false, then it might be easiest to provide a counterexample as justification.
- (a) [/2] If a linear system has n variables and m equations, then the augmented matrix has n columns.
- (b) [/ 2] An inconsistent system can be made consistent by performing a sequence of elementary row operations.
- (c) [/2] A consistent system whose augmented matrix has 3 rows and 5 columns will have infinitely many solutions.

- 2. Create an augmented matrix for the scenarios below or explain why it is impossible to do so. The associated system:
 - (a) [/ 2] has infinite solutions, but the augmented matrix has no row of zeros.
 - (b) [/2] has exactly one solution, but the augmented matrix has two rows of zeros.
 - (c) [/ 2] is consistent, but has more equations than unknowns.
 - (d) [/ 2] is inconsistent, but the augmented matrix has a row of zeros.

3. Consider the linear system whose augmented matrix is

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

- (a) Determine a value for h so that the system is **consistent**.
- (b) Determine a value for h so that the system is **inconsistent**.

4. **[** /2] Show that x = 1, y = 2, and z = 3 is not a solution to the following system.

$$\begin{cases} x + y + 2z = 9 \\ 2x + 4y - 2z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

5. [/2] Find a solution to the following system of linear equations:

$$-450x_1 + -22x_2 + 1x_3 + 1x_4 + 0x_5 + 333x_6 = 0$$

$$3x_1 + 2x_2 + 1x_3 + 0x_4 + 900x_5 + 0x_6 = 0$$

$$-\pi x_1 + 0x_2 + 88x_3 + 45x_4 + 1x_5 + 0x_6 = 0$$

$$7x_1 + 12x_2 + 300x_3 + 0x_4 + 9x_5 + 0x_6 = 0$$

$$1x_1 + 3x_2 + 9x_3 + 27x_4 + 81x_5 + 243x_6 = 0$$

6. **[/2]** Suppose the matrix below is the augmented matrix of a system of linear equations. Write the general solution in parametric vector form (as a linear combination of vectors some scaled by parameters).

$$\begin{bmatrix} 1 & 0 & -4 & 0 & 2 & -1 \\ 0 & 1 & 8 & 0 & -7 & 9 \\ 0 & 0 & 0 & 1 & 0 & 3 \end{bmatrix}$$

7. [/2] Let
$$\mathbf{A} = \begin{bmatrix} -0 & -4 \\ 3 & 9 \end{bmatrix}$$
, $\mathbf{B} = \begin{bmatrix} -9 & 1 \\ -9 & 8 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 7 & 5 \\ 3 & -3 \end{bmatrix}$.

Determine:
$$-8c + 2(5A - 3B) + 4C - 10A + 4(C + 2B)$$

8. [/ 2] Fact: the vector equation below is consistent

$$2\begin{bmatrix}1\\4\end{bmatrix} + 3\begin{bmatrix}2\\-12\end{bmatrix} - 5\begin{bmatrix}7\\0\end{bmatrix} + 6\begin{bmatrix}-3\\5\end{bmatrix} = \begin{bmatrix}-45\\2\end{bmatrix}$$

Use that fact to find a solution to the matrix equation $\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b}$ where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 7 & -3 \\ 4 & -12 & 0 & 5 \end{bmatrix}$$
 and $\mathbf{b} = \begin{bmatrix} -45 \\ 2 \end{bmatrix}$

9. [/ 2] Define a transformation

$$T: \mathbb{R}^3 \to \mathbb{R}^4$$
$$\boldsymbol{x} \to \boldsymbol{A}\boldsymbol{x}$$

where
$$\mathbf{A} = \begin{bmatrix} -5 & -4 & 1 \\ 3 & 2 & -1 \\ -4 & 0 & 8 \\ 7 & 0 & 9 \end{bmatrix}$$
. It is a fact that $T \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = \begin{bmatrix} 3 \\ -3 \\ 28 \\ 43 \end{bmatrix}$

Use this fact to produce a solution to the system of linear equations below

$$-5x_1 + -4x_2 + 1x_3 = 3$$

 $3x_1 + 2x_2 + 1x_3 = -3$
 $-4x_1 + 0x_2 + 38x_3 = 28$
 $7x_1 + 0x_2 + 9x_3 = 43$