

# Calculus



## Limits and Continuity

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### Applications of Integrals

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### Vectors and Vector-Valued Functions

### Partial Derivatives

### Multiple Integrals

### Vector Calculus

### Second-Order Differential Equations

# Limits and Continuity



## Limits

📖 [Limit \(Wikipedia\)](#) 📖 | [Thomas' Calculus \(2.2–2.6\)](#) 📖

- **Limit**  $\lim_{x \rightarrow c}$ : the value of a function (or sequence) approaches as the input (or index) approaches some value (informal definition)
  - Limits are used to define [continuity](#) ↓, [derivatives](#) ↓, and [integrals](#) ↓

## Limits of a Functions and Sequences

📖 [Limit of a function \(Wikipedia\)](#) 📖 | [Limit of a sequence \(Wikipedia\)](#) 📖

- **Limit of a function**: a fundamental concept in calculus and analysis concerning the behavior  $L$  of a function near a particular input  $p$ , i.e.,

$$\lim_{x \rightarrow p} f(x) = L$$

- Reads as “ $f$  of  $x$  tends to  $L$  as  $x$  tends to  $p$ ”
- $\epsilon, \delta$  **Limit of function**: a formalized definition, wherein  $f(x)$  is defined on an open interval, except possibly at  $c$  itself, leading to above definition, if and only if:
  - For every real  $\epsilon > 0$ , there exists a corresponding real  $\delta > 0$  such that for all real  $x$ ,  $0 < |x - p| < \delta$  implies that  $|f(x) - L| < \epsilon$ , i.e.,

$$f : \mathbb{R} \rightarrow \mathbb{R}, p, L \in \mathbb{R} \Rightarrow \lim_{x \rightarrow p} f(x) = L$$
$$\Updownarrow$$

$$\forall \epsilon > 0 (\exists \delta > 0 : \forall x, 0 < |x - p| < \delta \Rightarrow |f(x) - L| < \epsilon)$$

- Functions do not have a limit when the function:
  - has a unit step, i.e., it “jumps” at a point;
  - is not bounded, i.e., it tends towards infinity;
  - or does not stay close to any single number, i.e., it oscillates too much.
- **Limit of a sequence**: the value that the terms of a sequence  $(x_n)$  “tends to” (and not to any other) as  $n$  approaches infinity (or some point), i.e.,

$$\lim_{n \rightarrow \infty} x_n = x$$

- $\epsilon$  **Limit of sequence**: for every measure of closeness  $\epsilon$ , the sequence’s term eventually converge to the limit, i.e.,

$$\forall \epsilon > 0 (\exists N \in \mathbb{N} (\forall n \in \mathbb{N} (n \geq N \Rightarrow |x_n - x| < \epsilon)))$$

- **Convergent**: when a limit of a sequence [exists](#).
- **Divergent**: a sequence that [does not](#) converge.

## Limit Laws and Theorems

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## Continuity

- Sources:

### Continuity at a Point

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### Continuous Functions

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### Intermediate Value Theorem

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## Limits Involving Infinity

- Sources:

### Limits at Infinity

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### Infinite Limits

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# Derivatives



# Applications of Derivatives



# Integrals





# Applications of Integrals



# Transcendental Functions



# Techniques of Integration



# Infinite Sequences and Series



# First-Order Differential Equations



# Parametric Equations and Polar Coordinates



# Vectors and Vector-Valued Functions



# Partial Derivatives





# Multiple Integrals



# Vector Calculus



# Second-Order Differential Equations

