

1. Consider the matrix below to be the augmented matrix of a system of linear equation

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \\ 3 & 4 & 5 & 6 \end{array} \right]$$

(a) Write the associated system of linear equations.

$$\begin{cases} 1x_1 + 2x_2 + 3x_3 = 4 \\ 5x_1 + 6x_2 + 7x_3 = 8 \\ 9x_1 + 0x_2 + 1x_3 = 2 \\ 3x_1 + 4x_2 + 5x_3 = 6 \end{cases}$$

(b) Define matrix  $\mathbf{A}$  and vectors  $\mathbf{x}$  and  $\mathbf{b}$  so that the system of equations above can be represented as  $\mathbf{Ax} = \mathbf{b}$

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 0 & 1 \\ 3 & 4 & 5 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 4 \\ 8 \\ 2 \\ 6 \end{bmatrix}$$

(c) Define vectors  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ , and  $\mathbf{a}_3$  and then write the above system as a single vector equation.

$$\mathbf{a}_1 = \begin{bmatrix} 1x_1 \\ 5x_1 \\ 9x_1 \\ 3x_1 \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} 2x_2 \\ 6x_2 \\ 0x_2 \\ 4x_2 \end{bmatrix} \quad \mathbf{a}_3 = \begin{bmatrix} 3x_3 \\ 7x_3 \\ 1x_3 \\ 5x_3 \end{bmatrix}$$

$$\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 = \mathbf{b}$$

2. Parametrize the solution set (write the general solution) to the equation  $x + 2y + 3z = 5$  as a linear combination of vectors.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$